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PICKING CLUMPY ORDERS ON A CAROUSEL

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Carousels are rotatable closed-loop storage systems for small items, where items are stored in bins along the loop. An order at a carousel consists of (say) n different items stored there. We analyze two problems: (1) minimizing the total time to fill an order (travel time) and (2) order delays as they arrive, are filled, and depart. We define *clumpy* orders and the nearest-end-point heuristic (NEPH) for picking them. We determine conditions for NEPH to be optimal for problem (1), and under a weak stochastic assumption, we derive the distribution of travel time. We compare NEPH with the nearest-item heuristic. Under Poisson arrivals and assumptions much weaker than in the literature, we show that problem (2) may be modeled as an $M/G/1$ queue.

1. INTRODUCTION

Carousel systems are inventory systems in the form of a rotatable closed loop. They are used in automated warehouses to facilitate the storage and retrieval of small parts. Along the loop are a large number of storage bins, shelves, or drawers, where each bin stores a particular kind of part.

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We will analyze the filling of orders at a warehouse, where an *order* consists of a set of (say) *n items* (kinds of part) stored on the carousel. An order may call for more than one part of the same kind from the same bin. This possibility causes no difficulty in our analysis.

To fill an order, the carousel rotates to a point where a *picker* (human or otherwise), in a fixed location, selects an item, then rotates again for the picker to select another item, and so on until all items are selected.

We will consider two problems: (1) minimizing the total time to fill an order (the *fill time*) and (2) the analysis of the delays experienced by orders as they arrive, are filled, and depart (a *queuing analysis*).

The fill time consists of the sum of times for the carousel to move from one location to the next (the total *travel time*) and for the picker to pick each item (a picking time). We assume that picking times are independent of the *picking sequence*—the sequence that items in an order are picked. Thus, the time to fill an order with a fixed set of items depends only on this sequence. We assume that the carousel moves at constant velocity v, and for convenience we choose $v = 1$, so travel time is equal to travel distance. Under these assumptions, minimizing travel distance also minimizes travel time and fill time.

Because the carousel moves and the picker is in a fixed location, we will adopt a convention found to be convenient by other authors: We pretend that the picker moves and the carousel is fixed.

As in Bartholdi and Platzman [1], Litvak and Adan [5], Litvak, Adan, Wessels, and Zijm [6], and Rouwenhorst, van den Berg, van Houtum, and Zijm [7], where the above assumptions are made, we model the carousel as a circle with circumference of length 1; facing the circle, we use the shorthand *right* for clockwise and *left* for counterclockwise movements.

For an order of n items in known locations and the picker in some initial location, there are $n!$ possible picking sequences for the picker to select the items. An optimal sequence (not necessarily unique) minimizes travel time. For many of these sequences, the picker passes by some items without picking them, only to have to come back to them later. Such sequences clearly are not optimal, in the weak sense that for any such sequence there is another that is at least as good (pick items at the first opportunity). Thus, we consider sequences where the picker starts off in one direction, say to the right, picking up items along the way, until either to completion (all items are picked) or, after picking some item, switches (changes direction, this time to the left) and picks items in that direction either to completion or until the picker switches again, and so on.

The swings back and forth as the picker switches become wider. Consider any sequence F with more than one switch, and suppose that after the last switch, the picker is moving left. Replace this sequence with the single-switch sequence H, where the picker initially moves right to the last item picked on the widest swing of F, and then moves left. The travel distance (and time) of H is smaller than that of F. This proves the following result, previously shown in Bartholdi and Platzman [1] and Stern [8]:

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THEOREM 1.1: *The optimal picking sequence switches direction at most once.*

Thus for an order of n items, the optimal sequence is one of the following $2n$ sequences: The picker moves initially either to the right or the left and, for each direction, either continues to completion or switches after picking the jth item, $j = 1, \ldots, n-1$, and then continues to completion.

From this theorem, the computational effort to determine the optimal solution is modest, unless n is large. Nevertheless, various heuristics have been considered because they are easier to implement, and under certain assumptions they have been shown to be "near" optimal. In one, which we call *sweep*, the picker moves in one direction until all items are picked. For example, Bartholdi and Platzman [1] consider sweep in an arbitrary direction, sweep in the direction of the nearest item to the picker, and what is called the *nearest-item heuristic* (NIH), where the picker initially picks the nearest item, and after each selection, he picks the nearest unpicked item, until all items are picked.

Whereas Bartholdi and Platzman [1] analyzed policies in a deterministic setting, Litvak and Adan [5], Litvak et al. [6], and Rouwenhorst et al. [7] did most of their work under an assumption that we call $\mathcal{R}$ (for random): The locations of the n items in an order are independent and identically distributed (i.i.d.), with the uniform distribution over the circumference of the carousel. Implicit in $\mathcal{R}$ is a continuous approximation for a large number of individual bins.

In Litvak et al. [6], NIH was analyzed under $\mathcal{R}$; among other results, they obtained closed-form expressions for the mean and the variance of the (total) travel time to pick an order. In Litvak and Adan [5], these results were extended to the distribution and arbitrary moments of the travel time. Corresponding results for the optimal travel time under $\mathcal{R}$ are unknown.

In Rouwenhorst et al. [7], a carousel was modeled as a single-server queue, where arriving orders are Poisson, orders are served first-in-first-out (FIFO), order sizes are i.i.d., and independent of all else, an order of size n satisfies $\mathcal{R}$. For any policy such as NIH, sweep, or optimal, it is readily seen that order service times are i.i.d. under these assumptions, and the queuing model becomes a standard $M/G/1$ queue. When the time to pick the items is negligible, the average delay in queue is a function of the first two moments of the travel time. As these moments are unknown for optimal travel times, the article spends considerable effort approximating these moments for this case.

Sometimes, assumptions such as $\mathcal{R}$ develop a life of their own, and many articles are written that analyze consequences of these assumptions, without questioning whether they are reasonable. In a particular application, of course, data will either support or challenge modeling assumptions. More generally, modeling choices should be based on our understanding of the class of intended applications, which, in this case, is carousels.

A designer of a carousel assigns items to bins. When there are a large number of (say) k bins and the designer assigns the set of items to the set of bins such that all $k!$ possible assignments are equally likely, then $\mathcal{R}$ will be a good approximation. In this

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connection, we mention Ha and Hwang [2]. Although the model is quite different (e.g., items are stored and retrieved individually), they show that performance is improved when some assignments are more likely than others.

Here, we are dealing with orders, which are subsets of the set of items. For items that often occur together in the same order, would it not make sense to assign them to locations that are near each other? Although it may be true that the location of a randomly selected item is uniformly distributed over the circumference, some assignment *subsets* should be more likely than others. One defense of $\mathcal{R}$ is simply lack of information. However, good design may undermine this assumption, and when it does, we believe that an analysis based on $\mathcal{R}$ will overestimate real travel times.

How to make assignments in a way that effectively reduces travel times is an interesting research topic that is not investigated in this article. Instead, we investigate the stochastic performance of a carousel under a formulation that is different from $\mathcal{R}$ and captures the idea that items in the same order should be stored near each other.

In Section 2, we define *clumpy* orders and the simple nearest-end-point heuristic (NEPH) for picking them. We determine conditions for NEPH to be optimal, and under an assumption denoted by $\mathcal{P}$, which is much weaker than $\mathcal{R}$, we derive the distribution of total picking time under NEPH. In Section 3, we analyze NIH under an assumption similar to $\mathcal{R}$, where, now, item locations are randomly distributed over the clump. Unlike what occurs under $\mathcal{R}$, the performance of NIH gets worse as n increases. When arriving orders are Poisson and under assumptions much weaker than $\mathcal{R}$, we show in Section 4 that order delays may be modeled as those arriving from an $M/G/1$ queue and well-known elementary formulas apply. This is true under NEPH, NIH, or any other rule for picking orders. In Section 5, we analyze a modification of NEPH.

2. CLUMPY ORDERS AND THE NEAREST-END-POINT HEURISTIC

For an order of $n \geq 2$ items, let l be the length of arc A, the longest arc between successive items on the circumference of the carousel (may not be unique), and C be the complementary arc, with length $r = 1 - l$. The entire order lies within C, including one item at each end. For $n = 1$, we define C as the point where this item is located and $r = 0$. As the spacing between the items in C is smaller than l, we call C a *clump* and say that orders are *clumpy*.

By our definition, *all* orders are clumpy; this is just a way of conceptualizing the problem. In a stochastic setting, we believe that this approach is useful when the items in an order tend to be closer together than $\mathcal{R}$ would predict.

If the items are located as points on a linear segment (straight line) rather than on the circumference of a circle, the optimal policy (for minimizing travel time) is obvious: Call the points that are farthest apart the two end points and suppose that the picker starts between them. The picker should first go to the nearest end point, then switch direction and go to the farthest end point. This suggests the following heuristic, where now an end point is an end point of C.

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NEAREST-END-POINT HEURISTIC (NEPH): *When starting in A, the picker sweeps in the direction of the nearest end point. When starting in C, the picker moves to the nearest end point, then switches direction and moves to the farthest end point.*

Notice that when the picker starts in A, NEPH and NIH are identical. They may be quite different when the picker starts in C. We now present conditions where NEPH is optimal.

THEOREM 2.1: *When the picker starts in A, NEPH is optimal. When the picker starts in C and* $r \leq \frac{1}{2}$, *NEPH is optimal.*

PROOF: When the picker starts in A, it is easily shown that switching direction (once) cannot be optimal and, hence, NEPH is optimal. Suppose the picker starts in C. For $n = 2$, NEPH is optimal. Adding points (items) to an order cannot decrease the optimal travel time. For fixed r and picker location, adding points within C does not change the travel time under NEPH. Hence, NEPH is optimal for $n > 2$ as well. ■

When the picker starts in C, it is easy to show that NEPH may not be optimal when r is large because traversing A may reduce the travel time. Here is an example that shows this can happen for any $r > \frac{1}{2}$.

Example 2.2: Locate three items at 0, $\frac{1}{2} - 2\epsilon$, and $\frac{1}{2} + \epsilon$, where $\epsilon > 0$. C is arc$[0, \frac{1}{2} + \epsilon]$ and $r = \frac{1}{2} + \epsilon$. Suppose that the picker starts at $\frac{1}{4}$. NEPH gives travel time $\frac{3}{4} + \epsilon$, which is longer than $\frac{3}{4}$, the travel time by moving toward $\frac{1}{2} + \epsilon$ and sweeping.

This is just an example. For $r > \frac{1}{2}$, NEPH may be optimal, depending on the initial location of the picker and the location of points within C. In particular, suppose that the picker is within s of its nearest end point. Then, NEPH is optimal if $r + 2s \leq 1$. When the points are closely spaced and $\frac{1}{2} < r < \frac{2}{3}$, NEPH will be optimal or nearly so.

2.1. Stochastic Performance of the Nearest-End-Point Heuristic

Let T^E be the (total) travel time (or distance) by the picker to pick an order under NEPH. Whether the picker starts in C or A, we write T^E as the sum

$$T^E = Z^E + r, \tag{1}$$

where Z^E is the time to travel to the nearest end point of C, and once there, r is the time to travel to the other end point. Notice that for fixed r, T^E is independent of the number and the location of points within C.

We now make assumption $\mathcal{P}$, for the random *picker*: The location of the picker is uniformly distributed over the circumference and is independent of the location of the clump. The rationale for $\mathcal{P}$ is that when orders are diverse, the location of the picker when finished with one order is unrelated to the location of the clump for the next. This assumption is implied by the formulation of the queuing model in Section 4 and is much weaker than $\mathcal{R}$.

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Let $U(a,b)$ denote a uniform distribution on interval (a,b) and the symbol $\sim$ mean "distributed as." Under $\mathcal{P}$, the picker starts in C with probability r, and, given that this occurs, it is readily shown that $Z^E \sim U(0,r/2)$. (Split the arc C into two arcs of equal length and condition on the picker starting in either one.) Similarly, the picker starts in A with probability $1-r$, and, given that this occurs, $Z^E \sim U(0,(1-r)/2)$. Thus, under $\mathcal{P}$, the distribution of Z^E is a mixture of uniforms:

$$Z^E \sim rU(0,r/2) + (1-r)U(0,(1-r)/2), \tag{2}$$

and from Eq. (1), so is the distribution of T^E. The mean and (after some algebra) the variance of T^E readily follow from Eqs. (1) and (2):

$$E(T^E) = \frac{1+2r+2r^2}{4} \quad \text{and} \quad V(T^E) = V(Z^E) = \frac{1}{48} - \frac{r^2(1-r)^2}{4}. \tag{3}$$

3. NEAREST-ITEM HEURISTIC FOR CLUMPY ORDERS

Example 2.2 shows that NEPH may not be optimal for clumpy orders when $r > \frac{1}{2}$. NIH is shown to perform well under $\mathcal{R}$ (Litvak and Adan [5], Litvak et al. [6]). A natural question is whether NIH performs well when orders are clumpy. A stochastic analysis of NIH for clumpy orders requires a model of how items within a clump are dispersed. We will make an assumption much like $\mathcal{R}$, but with locations restricted to the clump.

When $r \leq \frac{1}{2}$, the picker will not traverse A under either NEPH (which is optimal) or NIH; the treatment of this case is similar to a linear segment of length r.

3.1. Stochastic Performance of the Nearest-Item Heuristic on a Unit Segment

Consider an order of $n \geq 2$ items on a unit interval such that there is one item at each end point. We assume that the locations of the remaining $n-2$ items and the initial location of the picker are i.i.d. uniform on this interval. Thus, there are $n-1$ "inside" points, and our analysis will be based on the corresponding order statistics.

Label all the points from left to right as $0,1,\ldots,n$, where points 0 and n are the two end points. Let V_i be the distance between points i and $i-1$, $i=1,\ldots,n$. It is well known that $(V_1,\ldots,V_n)$ has the same distribution as $(X_1/S_n,\ldots,X_n/S_n)$, where $X_1,\ldots,X_n$ are i.i.d. exponential random variables and $S_n = \sum_{i=1}^n X_i$ (Karlin and Taylor [3, p. 105]). For $S_k = \sum_{i=1}^k X_i$, $k=1,\ldots,n$, Litvak et al. [6] show that

$$E\left(\frac{\min(S_i, X_{i+1})}{S_n}\right) = \frac{1}{n}\left(1 - \frac{1}{2^i}\right), \qquad i = 1,\ldots,n-1. \tag{4}$$

We will use Eq. (4) to find properties of T_n^I, the total time to pick an order of n items under NIH. We write T_n^I as

$$T_n^I = Z_n^I + 1, \tag{5}$$

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where Z_n^I is the time for the picker to pick the first of the end points and 1 is the time to travel from the first picked to the other end point. In the limit when $n \to \infty$, we use T_∞^I and Z_∞^I for T_n^I and Z_n^I, respectively.

Let Y_n be the index of the initial location of the picker before any move, $Y_n \in \{1, \ldots, n-1\}$. As these locations are equally likely, we write

$$E(Z_n^I) = E\left[E(Z_n^I|Y_n)\right] = \frac{\sum_{y=1}^{n-1} E(Z_n^I|Y_n = y)}{n-1}.$$

To find $E(Z_n^I|Y=1)$, let $Z_{j,n}^I$ be the additional time to travel to one of the end points given that the picker is at point j and any items at points $1, \ldots, j$ *have been picked*; $Z_{n,n}^I = 0$. The indices of $X_{(\cdot)}$ are labeled according to their order of being considered by the heuristic. For $j = 1, \ldots, n-1$,

$$Z_{j,n}^I = \frac{\min(S_j, X_{j+1})}{S_n} + \begin{cases} 0 & \text{if } S_j < X_{j+1} \\ Z_{j+1,n}^I & \text{otherwise.} \end{cases} \tag{6}$$

Applying Eq. (6) recursively and making use of Eq. (4) along the way, we have

$$E(Z_{j,n}^I) = \frac{1}{n}\left[\sum_{k=j}^{n-1}\left(\prod_{l=j}^{k}\left(1 - \frac{1}{2^l}\right)\right)\right],$$

$$E(Z_n^I|Y_n = 1) = E(Z_{1,n}^I) = \frac{1}{n}\sum_{k=1}^{n-1}\left(\prod_{l=1}^{k}\left(1 - \frac{1}{2^l}\right)\right). \tag{7}$$

$E(Z_n^I|Y_n = y)$ for $y > 1$ may be found by conditioning, but the set of equations does not give a neat analytical expression. From these quantities, one can compute $E(T_n^I)$. For example, $E(T_2^I) = 1.25$, $E(T_3^I) = 1.2917$, $E(T_4^I) = 1.3229$, and $E(T_5^I) = 1.3466$.

For large n, let m_n be the positive integer such that the probability of switching direction on the m_nth move, $P(S_{m_n} < X_{m_n}+1) = 1/2^{m_n}$, is small enough to make the total effect of any switch after m_n moves negligible. As $n \to \infty$, the total time to visit m_n points is insignificantly small, and after these m_n moves, the picker is equally likely to be moving left or right from any initial location. Thus, as $n \to \infty$, $Z_\infty^I \to \tau \sim U(0,1)$; $E(T_\infty^I) = E(\tau) + 1 = 1.5$ and $V(T_\infty^I) = V(\tau) = \frac{1}{12}$.

To assess the performance of NIH, we compare $E(T_n^I)$ with (optimal) $E(T^E)$ on this line segment, where it is easily shown that $E(T^E) = 1.25$. Let the *overshoot* be the percentage by which $E(T_n^I)$ exceeds the optimal value. For $n = 2$, NIH is the same as NEPH, and the overshoot is zero. The overshoot is as follows: For $n = 3$, it is 3.33%; for $n = 4$, it is 5.83%; for $n = 5$, it is 7.73%; and in the limit as $n \to \infty$, it is 20%.

3.2. The Limiting Performance of NIH on Clumpy Orders

Consider a clumpy order with $r \leq \frac{1}{2}$. We now assume what we call $\mathcal{RC}$: the locations of the $n-2$ items within the clump are i.i.d. $\sim$ uniform on C, an arc of length r. We

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also assume $\mathcal{P}$. Redefine T_∞^I and Z_∞^I from a unit interval to arc C. Then, $T_\infty^I = Z_\infty^I + r$. Given that the picker starts in A, an event with probability $1 - r$, $Z_\infty^I \sim U(0,(1-r)/2)$, the same as under NEPH. Given that picker starts in C, an event with probability r, we have a rescaling of the analysis of NIH on a unit segment and, hence, $Z_\infty^I \sim U(0,r)$. Combining, we have

$$T_\infty^I = Z_\infty^I + r,$$

where

$$Z_\infty^I \sim rU(0,r) + (1-r)U(0,(1-r)/2). \tag{8}$$

After some algebra, we get

$$E(T_\infty^I) = \frac{1+2r+3r^2}{4} \quad \text{and} \quad V(T_\infty^I) = \frac{1}{48} - \frac{r^2(6-16r+9r^2)}{16}. \tag{9}$$

Unlike the analysis of NIH under $\mathcal{R}$ (Litvak and Adan [5], Litvak et al. [6]), performance does not approach optimality as n increases, but gets worse, as the picker has a higher probability, approaching $\frac{1}{2}$, of hitting the farthest end point first. The direction of the nearest point provides no information about the direction of the nearest *end* point. Under the assumption that the points within C are equally spaced (not random), rather than i.i.d. uniform, the distribution in Eq. (8) is exact.

4. QUEUING ANALYSIS FOR CLUMPY ORDERS

We now model the carousel under a set of assumptions we call $\mathcal{Q}$: The carousel operates as a single-server (picker) queue, where arriving orders are Poisson at rate λ. Orders are served FIFO and properties of orders, from order to order, are i.i.d. For order j, these properties include the following: Clump location (measured from either end point) is uniformly distributed on the circumference, clump arc length, R_j, and the number of items in the order, N_j, have some joint distribution. To analyze the queuing model under policies such as NIH, the distribution of points within the clump would also have to be specified.

Under any of the policies we have considered, let T_j be the travel time to pick order j. The service time S_j of this order is $S_j = T_j$ when picking times are negligible. (More generally, we can write $S_j = T_j + aN_j$, where a is the time to pick an individual order; our approach still applies, but we omit this complication in the following.) Under $\mathcal{Q}$, the picker satisfies assumption $\mathcal{P}$ and the service times S_j are i.i.d. Thus, the queuing model is an $M/G/1$ queue. It is not necessary to assume the much stronger $\mathcal{R}$ to have this simple queue. We now present some results for this model. For them and additional properties of the $M/G/1$ queue, see Wolff [9, Chap. 8].

Let *delay* D_j be the time from the arrival of order j to when service (travel time) on that order begins, let $S = T$ be a generic service time, and let R be a generic arc length. When $\rho \equiv \lambda E(S) < 1$, all customers are served, and the long-run average delay d is given by

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$$d = \frac{\lambda E(S^2)}{1-\rho}. \tag{10}$$

Under NEPH, the conditional distribution of $(T|R=r)$ is given by Eqs. (1) and (2), and the conditional mean and variance is given by Eq. (3). Thus, moments of S may be found from moments of R. This is clearly appropriate when $P(R \leq \frac{1}{2}) = 1$ and is also appropriate when the points within C are closely spaced and R_j rarely exceed $\frac{2}{3}$.

Analytic expressions for the moments of S are not required to apply Eq. (10). They can be estimated easily by working out optimal travel times and service times (whether or not picking times are negligible) for a sample of orders.

The simplicity of this model is a consequence not only of $\mathcal{P}$ but also of the FIFO assumption. Particularly when most of the R_j are small, there are advantages to abandoning FIFO. Is it not foolish to pass over an order that could be picked, on the way to picking an order that arrived earlier? The *nearest-order* heuristic (NOH) (rather than nearest item within an order) deserves consideration and was analyzed in Krouse and Schmidt [4] under the special case where an order consists of a single item and picking time is not negligible. We are currently investigating NOH and other alternatives to FIFO.

5. LARGE r AND THE MODIFIED NEAREST-END-POINT HEURISTIC

For $r \leq \frac{1}{2}$, NEPH is optimal and there is no reason to consider any other way of picking orders. For the rest of this section, we assume $\mathcal{RC}$ and $\mathcal{P}$.

Now, consider the range $\frac{1}{2} < r \leq \frac{2}{3}$. Suppose that the picker starts in C. NEPH will be asymptotically optimal as the number of items $n \to \infty$ because, on reaching the nearest end point, the picker will find that switching direction is better than traversing A. Under NIH, the picker may (foolishly) reach the farthest end point first and may find that traversing A is better than switching direction. Thus, Eqs. (8) and (9) are not valid in this range. Nevertheless, we expect NEPH to be better than NIH in this range when n is large.

Now, consider the range $r > \frac{2}{3}$. Suppose the picker starts in C. Asymptotically, NEPH will not be optimal when the picker starts close to the middle of C because on reaching the nearest end point, traversing A will be better than switching direction.

We now propose a simple modification of NEPH that performs well for large r and n.

Modified Nearest-End-Point Heuristic (MNEPH): *MNEPH is the same as NEPH, except in one situation: Under MNEPH, and when $r > \frac{2}{3}$ and the picker starts in C, the picker sweeps in the direction of the nearest item.*

For the rest of this section, assume $r > \frac{2}{3}$. Let T_n^M be the travel time to pick an order of n items under MNEPH.

For ease of reference, we use the same symbols A and C to denote the events that the picker starts in arcs A and C, respectively. Given A, the time to pick the first of the end points is $\sim U(0,(1-r)/2)$, and the remaining time is r:

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$$(T_n^M|A) \sim U\left(r, \frac{1+r}{2}\right) \quad \text{with moments}$$

$$E(T_n^M|A) = \frac{1+3r}{4}, \qquad V(T_n^M|A) = \frac{(1-r)^2}{48}.$$

Given C, the picker starts in some interval between two items. Then, $T_n^M = 1 - L$, where L is the longer of the distances between the picker and the two neighboring items. The analysis is related to the shortest-distance heuristic (SDH) (Litvak et al. [6], Rouwenhorst et al. [7]); for SDH, they find the mean and variance of the time to pick n items to be $1 - (3/2(n+1))$ and $(5n-4)/4(n+1)^2(n+2)$, respectively.

For our case, arc C is of length r and a clumpy order of $n \geq 2$ items corresponds to an order of $n-1$ items for SDH. Hence,

$$E(T_n^M|C) = 1 - \frac{3r}{2n}, \qquad V(T_n^M|C) = \frac{(5n-9)r^2}{4n^2(n+1)}.$$

Unconditioning, we have

$$E(T_n^M) = \frac{1+6r-3r^2}{4} - \frac{3r^2}{2n},$$

$$V(T_n^M) = \frac{(1-r)^3}{48} + \frac{(5n-9)r^3}{4n^2(n+1)} + 9r(1-r)\left(\frac{1-r}{4} - \frac{r}{2n}\right)^2.$$

These results may be used in the queuing model in Section 4.

A further modification when $r > \frac{2}{3}$ and the picker starts in C is as follows: Use the NEPH when the picker starts within $1-r$ of an end point and have the picker sweep in the direction of the nearest item when it starts closer to the middle of C.

6. CONCLUDING REMARKS

We have reformulated the order-picking problem in terms of the clumpiness of orders, introduced the elementary NEPH to pick them, and obtained the travel time distribution under this heuristic and much weaker stochastic assumptions than in the literature. We have obtained conditions for NEPH to be optimal and discussed other circumstances where we expect it to perform well. Under an alternative to $\mathcal{R}$, we have shown that NIH performs poorly, and believe that one should be wary of using it when $\mathcal{R}$ does not hold.

Our queuing analysis of order delays holds under much weaker stochastic assumptions than in the literature and under NEPH, NIH, or any other rule for picking orders. As done in Rouwenhorst et al. [7], we assumed FIFO, which greatly simplifies the analysis. As explained at the end of Section 4, there are alternatives to FIFO that we expect to perform much better. They are under investigation.

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