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RESEARCH ARTICLE

A categorical flop in dimension one

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Abstract

In this note we observe that the categorical structure of a flop occurs for some well-known noncommutative resolutions of a nodal curve. We describe the flop-flop spherical twists, and give a geometric interpretation in terms of Landau–Ginzburg models. The resolutions are all weakly crepant but not strongly crepant, and we formulate an intermediate condition that distinguishes the smaller ones.

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1. Introduction

Nonuniqueness of crepant resolutions is conjectured to be addressed by an equivalence at the level of derived categories [BO], [Kaw]. It is then natural to ask what algebraic structure on derived categories arises from a resolution of singularities. One approach is that of [Kuz1], where categorical resolutions were defined in the following manner. A resolution $\pi : \widetilde{X} \to X$ gives rise to adjoint functors

$$\begin{array}{ccc} \mathfrak{Perf}(X) & \xrightarrow{\pi^*} & D^b(\widetilde{X}) \\ \downarrow & \swarrow_{\pi_*} & \\ D^b(X), & & \end{array}$$

and when X has rational singularities $\pi_* \circ \pi^*$ is naturally isomorphic to the identity on $\mathfrak{Perf}(X)$. Kuznetsov then defined a *categorical resolution of singularities* to be a smooth triangulated category $\mathscr{C}$ replacing $D^b(\widetilde{X})$ in the above diagram, with functors π_* and π^* having these same properties.

It was argued in [KL] that this definition is reasonable even when X has irrational singularities, where $D^b(\widetilde{X})$ actually fails to give a categorical resolution. The authors constructed an enlargement of $D^b(\widetilde{X})$ which *does* satisfy the definition, even for nonreduced irrational singularities (in characteristic 0).

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As an example, consider the case of a nodal curve $X = \{xy = 0\} \subset \mathbb{A}^2$, with normalization $\widetilde{X} = \mathbb{A}^1_x \amalg \mathbb{A}^1_y$. The categorical resolution from [KL] is equivalent to a module category $D^b(\mathcal{A})$ over the Auslander order $\mathcal{A} = H^0 \operatorname{End}_X(\mathcal{O}_X \oplus \pi_* \mathcal{O}_{\widetilde{X}})$, introduced as a resolution for nodal and cuspidal curves in [BD]. It is well-known that this resolution is not minimal, containing two exceptional objects which can be contracted to give (equivalent) smaller resolutions; cf. [Kuz2, §3.5], [LP1, §4.5].

Our main observation is that these two contractions match the behaviour of the functors involved in a 3-fold flop, forming a categorical structure which has already been studied for that reason (e.g., [Bar, §4]). Identical structures were described for higher-dimensional nodes in [Kuz3, Proposition 3.15], using spinor bundles. We find the same phenomena occurring for curves, even though the general methods break down; categorical resolutions extend the behaviour of 'nice' geometric resolutions to these more degenerate irrational singularities, supporting the philosophy of [KL].

1.1. Overview

The smaller resolutions come from partial versions of $\mathcal{A}$:

$$\mathcal{A}_x = H^0 \operatorname{End}_{\mathcal{O}_X}(\mathcal{O}_X \oplus \pi_* \mathcal{O}_{\mathbb{A}^1_x}) \quad \text{and} \quad \mathcal{A}_y = H^0 \operatorname{End}_{\mathcal{O}_X}(\mathcal{O}_X \oplus \pi_* \mathcal{O}_{\mathbb{A}^1_y}),$$

over which $\mathcal{A}$ is a roof via certain bimodules:

$$\begin{array}{ccccc} & & D^b(\mathcal{A}) & & \\ & \swarrow & & \searrow & \\ D^b(\mathcal{A}_x) & & & & D^b(\mathcal{A}_y) \\ & \searrow & & \swarrow & \\ & & D^b(X). & & \end{array} \tag{1.1}$$

We can then consider the push-pull composition

$$\begin{array}{ccccc} & & D^b(\mathcal{A}) & & \\ & \nearrow & & \searrow & \\ D^b(\mathcal{A}_x) & & \xrightarrow{\Phi} & & D^b(\mathcal{A}_y), \end{array} \tag{1.2}$$

and this turns out to be a derived equivalence. Composing with the reversed version gives an autoequivalence of $D^b(\mathcal{A}_x)$, which is the twist around a spherical object.

For comparison, recall the Atiyah flop over a 3-fold node $Y = \{uv = st\} \subset \mathbb{A}^4$. This has two crepant resolutions $V_u = \operatorname{Bl}_{\{u=s=0\}} Y$ and $V_v = \operatorname{Bl}_{\{v=s=0\}} Y$, with the roof $V = \operatorname{Bl}_0 Y$ giving a derived equivalence:

$$\begin{array}{ccccc} & & D^b(V) & & \\ & \overset{(\pi_u)^*}{\nearrow} & & \overset{(\pi_v)_*}{\searrow} & \\ D^b(V_u) & & \xrightarrow{\simeq} & & D^b(V_v). \end{array}$$

The flop-flop autoequivalence on $D^b(V_u)$ is the twist around a spherical object, given by a line bundle on the exceptional divisor.

Remark 1.1. One interpretation of $\mathcal{A}_x$, $\mathcal{A}_y$ and $\mathcal{A}$ is as 'noncommutative blowups'. For a ring R and ideal I one can define such a thing by the algebra $\operatorname{End}_R(R \oplus I)$, as in [Leu, §R]. We apply this to the two branches of the node and also their intersection, in analogy with V_u, V_v and V.

Remark 1.2. As described in [BKS] for the geometric situation, these diagrams of categories can be seen as giving examples of perverse schobers on $\mathbb{C}$ with one marked point, which categorify perverse sheaves on the hyperplane arrangement $(\mathbb{C}, 0)$.

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1.2. Landau–Ginzburg models

The above resolutions are also equivalent to categories of B-branes on partial compactifications of a Landau–Ginzburg model, from the 'exoflop' construction of [Asp]. These LG-models were already described in the context of mirror symmetry in [LP2], and we briefly sketch an A-side picture of the flop in §4.4. The geometry of these LG-models suggests a version of the flop for *smooth* curves using root stacks, which we outline in Remark 4.3.

It also explains the crepancy properties of the resolutions. Kuznetsov defined weak and strong notions of crepancy for categorical resolutions, both equivalent to usual crepancy when $\mathscr{C} = D^b(\widetilde{X})$. They are characterized by the behaviour of a relative Serre functor on the resolution, which for the LG-models is given by the canonical bundle. Strong crepancy asks for the canonical bundle to be globally trivial, while weak crepancy only requires it to be trivial on a certain open set, so categorical resolutions from compactifications of LG-models are typically weakly crepant; see [FK].

In particular, all the above resolutions (including the roof) are weakly crepant. On the other hand, they are not strongly crepant; the partial compactifications are not Calabi–Yau. This confirms that weak crepancy is too weak to ensure minimality of the resolution, since $D^b(\mathcal{A})$ is clearly not minimal. Kuznetsov conjectured that strongly crepant resolutions are minimal, although here we see that what is presumably the minimal resolution, $D^b(\mathcal{A}_x)$, fails to be strongly crepant.

Motivated by this, we introduce an intermediate condition (Definition 3.7): that the kernel subcategory $\ker(\pi_*)$ is Calabi–Yau with respect to the relative Serre functor, that is, the relative Serre functor restricts to a homological shift on $\ker(\pi_*)$. This is strictly in-between weak and strong crepancy, and seems effective at identifying minimal resolutions. Optimistically, one could conjecture that it identifies minimal categorical resolutions in general; see §3.2 for further discussion.

1.3. Generalizations

The algebra $\mathcal{A}$ was shown to give a categorical resolution for any nodal or cuspidal curve singularity in [BD], and we show that a diagram with the same properties as Eq. (1.1) exists for any nodal curve in §3.3. As mentioned above, the roof also has a geometric analogue using root stacks (Remark 4.3).

It would be nice to unify our results with [Kuz3, Proposition 3.15], including for example the Atiyah flop. A preliminary remark in this direction: $\mathcal{A}$ and $\mathcal{A}_x$ seem to fit into the higher Auslander–Reiten theory of [Iya, §3] as 1- and 2-Auslander algebras, so a good understanding of the results in [BIKR] may suffice to conclude that similar pictures exist for simple curve singularities of type A_n with n odd and D_m with m even – perhaps also in higher dimensions.

Conventions

We work over $\mathbb{C}$. Functors will be derived without changing notation. Modules are *right* modules, and quiver representations are *contravariant*.

2. Background

We recall some of [Kuz1]. Experienced readers may prefer to skip straight to §3.

Definition 2.1. A *weak categorical resolution of singularities* for a variety X is a (dg-enhanced) triangulated category $\mathscr{C}$ with (quasi-)functors

$$\begin{array}{ccc} \mathfrak{Perf}(X) & \xrightarrow{\pi^*} & \mathscr{C} \\ {\scriptstyle\iota}\downarrow & \swarrow{\scriptstyle\pi_*} & \\ D^b(X), & & \end{array}$$

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where ι is the inclusion $\mathfrak{Perf}(X) \subseteq D^b(X)$, satisfying three conditions:

1. π^* and π_* are adjoint, in the sense that

$$\mathrm{Hom}_{\mathscr{C}}(\pi^*(-), -) \simeq \mathrm{Hom}_{D^b(X)}(\iota(-), \pi_*(-))$$

   as (quasi-)functors on $\mathfrak{Perf}(X)^{\mathrm{op}} \otimes \mathscr{C}$,
2. $\pi_* \circ \pi^* \simeq \iota$, so that π^* is fully faithful, and
3. $\mathscr{C}$ is smooth.

It is *weakly crepant* if π^* is also a *right* adjoint for π_* in the sense of (1).

Remark 2.2. We say $\mathscr{C}$ is *smooth* if $\mathscr{C} \simeq \mathfrak{Perf}(A)$ where A is a smooth dg-algebra, that is, the diagonal bimodule A is a perfect complex over $A^e = A^{\mathrm{op}} \otimes_{\mathbb{C}} A$. This is independent of choices, and equivalent to $\mathrm{gl.dim} < \infty$ if $A = H^0(A)$ [Lun, §3].

Example 2.3. If $\pi : \widetilde{X} \to X$ is a geometric resolution of a variety X with *rational* singularities, meaning that $\mathcal{O}_X \simeq \mathbf{R}\pi_*\mathcal{O}_{\widetilde{X}}$, then $D^b(\widetilde{X})$ with the usual functors π^* and π_* gives a categorical resolution of X by the projection formula. If π is also crepant, then the categorical resolution is weakly crepant; the adjoints to π_* differ iff the relative canonical bundle is nontrivial: $\pi^! = \omega_{\widetilde{X}/X} \otimes \pi^*$.

In [Lun] it is proved in broad generality that the category $D^b(X)$ is smooth. It follows that $\mathscr{C} = D^b(X)$, $\pi^* = \iota$, $\pi_* = \mathrm{id}_{D^b(X)}$ is a universal weakly crepant categorical resolution unless we make further restrictions on the behaviour of $\mathscr{C}$.

Example 2.4. For $X = \mathrm{Spec}\,\mathbb{C}[w]/w^2$ we have a strong generator $D^b(X) = \langle \mathcal{O}_X/w \rangle$, so $D^b(X) \simeq \mathfrak{Perf}(A)$ for $A = \mathrm{Ext}^*_X(\mathcal{O}_X/w, \mathcal{O}_X/w) = \mathbb{C}[\theta]$ with $|\theta| = 1$. This is classical Koszul duality. Here A is smooth, but with unbounded cohomology; it fails to give a relatively proper resolution, being far from finite over $\mathbb{C}[w]/w^2$.

Various strengthenings of the definition exist. A key idea is that the category $\mathscr{C}$ should be local on X, formalized by requiring $\mathscr{C}$ to be a module for the monoidal category $\mathfrak{Perf}(X)$ – in the affine case just an $\mathcal{O}_X$-linear dg-category with (π^*, π_*) being $\mathcal{O}_X$-linear – as introduced in [Kuz1]. This is one step towards having the structure of a noncommutative resolution à la Van den Bergh [VdB].

With this structure one obtains relative versions of properties of $\mathbb{C}$-linear derived categories, by using the $\mathcal{O}_X$-module structure on $\mathrm{Hom}_{\mathscr{C}}(x, y)$. For example $\mathscr{C}$ is proper relative to X if the $\mathcal{O}_X$-linear chain complex $\mathrm{Hom}_{\mathscr{C}}(x, y)$ gives an element of $D^b(X)$ for all $x, y \in \mathscr{C}$.

Definition 2.5. Assume X is affine and Gorenstein. We will call categorical resolutions which are $\mathcal{O}_X$-linear and relatively proper in the above sense *strong categorical resolutions*. In this setting $S : \mathscr{C} \to \mathscr{C}$ is a *relative Serre functor* for $\mathscr{C}$ if we have a natural isomorphism

$$\mathrm{Hom}_{\mathscr{C}}(x, Sy) \simeq \mathrm{Hom}_{\mathscr{C}}(y, x)^\vee := \mathrm{Hom}_{D^b(X)}(\mathrm{Hom}_{\mathscr{C}}(y, x), \mathcal{O}_X).$$

Kuznetsov defined the resolution to be *strongly crepant* if the identity is a relative Serre functor.

Remark 2.6. By [Kuz1, Lemma 3.6] we then get $S \circ \pi^*$ as a right adjoint for π_*, so weak crepancy of a strong categorical resolution is equivalent to S restricting to the identity on $\pi^*(\mathfrak{Perf}(X))$.

Example 2.7. If $\pi : \widetilde{X} \to X$ is a resolution of rational singularities, then $D^b(\widetilde{X})$ is a strong categorical resolution with relative Serre functor $\omega_{\widetilde{X}/X} \otimes -$, and hence is strongly crepant iff π is a crepant resolution. In this case strong and weak crepancy are equivalent.

Example 2.8. Revisiting $X = \mathrm{Spec}\,\mathbb{C}[w]/w^2$, there is a strong categorical resolution given by the graded Kronecker quiver

$$Q = \left(\bullet_0 \underset{w}{\overset{\theta}{\rightrightarrows}} \bullet_1 \right),$$

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where $|\theta| = 1$.[1] We can embed $\mathfrak{Perf}(X)$ as the subcategory generated by

$$G = \operatorname{cone}(P_0[-1] \xrightarrow{\theta} P_1) = \left(\mathbb{C} \underset{1}{\overset{0}{\leftleftarrows}} \mathbb{C} \right),$$

since $\mathbf{R}\operatorname{Hom}(G, G) = \mathbb{C}[w]/w^2$. This quiver is derived equivalent to the Auslander algebra $H^0 \operatorname{End}_X(\mathcal{O}_X \oplus \mathcal{O}_X/w)$, coming from a full exceptional collection described in [KL, Example 5.16] and [KS1, §3]. It hence has an $\mathcal{O}_X$-linear structure, giving a strong categorical resolution with a relative Serre functor S as in Eq. (3.2). One can compute that $S(G) = G$, so the resolution is weakly crepant, but it is not strongly crepant since $\operatorname{Hom}(P_1, P_0) = 0 \neq \operatorname{Hom}(P_0, P_1)$.

3. Flops and crepancy

We return to the notation from the introduction, where $X = \{xy = 0\} \subset \mathbb{A}^2$. To aid computations, write $\mathcal{A}$ as the path algebra of a quiver with relations:

$$\mathcal{A} = \mathbb{C}\left(\begin{array}{ccc} & q_x \curvearrowright & \\ & \bullet_{\mathbb{A}^1_x} & \\ \bullet_X & \curvearrowleft i_x & \\ & i_y \curvearrowright & \\ & \bullet_{\mathbb{A}^1_y} & \\ & q_y \curvearrowright & \end{array} \right) \Big/ (q_x i_y = 0 = q_y i_x).$$

Then $\mathcal{A}_x$ and $\mathcal{A}_y$ are obtained by omitting vertex projectives:

$$\mathcal{A}_x = \operatorname{End}_{\mathcal{A}}(P_X \oplus P_x), \qquad \mathcal{A}_y = \operatorname{End}_{\mathcal{A}}(P_X \oplus P_y).$$

The functors of Eq. (1.1) come from the bimodules $P_X \oplus P_x$ and $P_X \oplus P_y$, together with P_X mapping down to $D^b(X)$.

It follows that $\mathfrak{Perf}(\mathcal{A}_x)$ and $\mathfrak{Perf}(\mathcal{A}_y)$ are identified with the subcategories

$${}^{\perp}S_y := \{M \in \mathfrak{Perf}(\mathcal{A}) : \mathbf{R}\operatorname{Hom}(M, S_y) = 0\} \quad \text{and} \quad {}^{\perp}S_x (:= \text{ similar})$$

orthogonal in $\mathfrak{Perf}(\mathcal{A})$ to simple modules at the omitted vertices. These simple modules have the following projective resolutions:

$$P_y \xrightarrow{i_y} P_X \xrightarrow{q_x} P_x \to S_x, \qquad P_x \xrightarrow{i_x} P_X \xrightarrow{q_y} P_y \to S_y, \tag{3.1}$$

from which one computes that they are exceptional objects, so we will refer to them as $E_x := S_x$ and $E_y := S_y$. It follows that ${}^{\perp}E_x$ and ${}^{\perp}E_y$ are admissible subcategories of $D^b(\mathcal{A})$, defined by the semiorthogonal decompositions

$$D^b(\mathcal{A}) = \langle E_x, {}^{\perp}E_x \rangle = \langle E_y, {}^{\perp}E_y \rangle.$$

Proposition 3.1. *The categories $D^b(\mathcal{A})$, $D^b(\mathcal{A}_x)$ and $D^b(\mathcal{A}_y)$ give strong categorical resolutions of X with relative Serre functors.*

Proof. We have $\operatorname{gl.dim}(\mathcal{A}) = 2$ by [BD, Theorem 2.6], so $\mathfrak{Perf}(\mathcal{A}) = D^b(\mathcal{A})$ is smooth by [Lun, Proposition 3.8]. The admissible subcategory $\mathfrak{Perf}(\mathcal{A}_x)$ is smooth by [LS, Theorem 3.24], and hence also $\mathfrak{Perf}(\mathcal{A}_x) = D^b(\mathcal{A}_x)$. The same holds for $\mathcal{A}_y$. These give $\mathcal{O}_X$-linear categories which are relatively proper as categorical resolutions, since the algebras are coherent over $\mathcal{O}_X$.

[1] This is smooth, being a regrading of the Beilinson quiver for $\mathbb{P}^1$ (which is smooth). It compactifies the $\mathbb{A}^1_\theta$ of Example 2.4. Failure of strong crepancy is no surprise; $\mathbb{P}^1$ is not Calabi–Yau.

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Serre functors for such algebras are given by a formal construction. We define

$$S : D^b(\mathcal{A}) \xrightarrow{\mathrm{Hom}_{\mathcal{A}}(-,\mathcal{A})} D^b(\mathcal{A}^{\mathrm{op}}) \xrightarrow{\mathrm{Hom}_X(-,\mathcal{O}_X)} D^b(\mathcal{A}), \tag{3.2}$$

which is a Serre functor by tensor-hom adjunction:

$$\begin{aligned}\mathrm{Hom}_X(\mathrm{Hom}_{\mathcal{A}}(M,N),\mathcal{O}_X) &\simeq \mathrm{Hom}_X(N \otimes_{\mathcal{A}} \mathrm{Hom}_{\mathcal{A}}(M,\mathcal{A}),\mathcal{O}_X)\\ &\simeq \mathrm{Hom}_{\mathcal{A}}(N,\mathrm{Hom}_X(\mathrm{Hom}_{\mathcal{A}}(M,\mathcal{A}),\mathcal{O}_X)).\end{aligned}$$

The same construction works over $\mathcal{A}_x$ and $\mathcal{A}_y$. □

Remark 3.2. From Eq. (3.1) we find $S(E_x) = E_y[1]$, $S(E_y) = E_x[1]$. Moreover

$$\mathrm{Hom}_{\mathcal{A}}(-,E_*)^{\vee_X} = \mathrm{Hom}_{\mathcal{A}}(-,E_*)^{\vee_{\mathbb{C}}}[-1]$$

for $* \in \{x,y\}$, since $\mathrm{Hom}_X(\mathcal{O}_0,\mathcal{O}_X) = \mathcal{O}_0[-1]$. So $S[1]$ is close to being a Serre functor over $\mathbb{C}$.

3.1. Spherical twists

The functor Φ of Eq. (1.2) is the restriction to ${}^{\perp}E_y$ of the projection to the admissible subcategory ${}^{\perp}E_x$ in $D^b(\mathcal{A})$. It is an equivalence:

Lemma 3.3. *Let $F : D^b(\mathbb{C}) \oplus D^b(\mathbb{C}) \to D^b(\mathcal{A})$ be the functor mapping $\mathbb{C} \oplus 0$ and $0 \oplus \mathbb{C}$ to E_x and E_y respectively (Fig. 1). Then F is spherical, and Φ is obtained by restricting the associated dual twist $T^{-1} : D^b(\mathcal{A}) \to D^b(\mathcal{A})$ to ${}^{\perp}E_y$.*

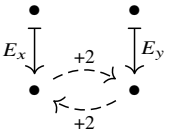


Figure 1. *The spherical functor F.*

Proof. The right adjoint of F is $R = \mathrm{Hom}(E_x,-) \oplus \mathrm{Hom}(E_y,-)$, and as a consequence of Remark 3.2 there is also a left adjoint $L = RS[1]$. The cotwist $C = \mathrm{cone}(1 \to RF)[-1]$ is the following bimodule over $\mathbb{C} \oplus \mathbb{C}$:

$$\begin{bmatrix} 0 & \mathrm{Hom}(E_x,E_y) \\ \mathrm{Hom}(E_y,E_x) & 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathbb{C}[-2] \\ \mathbb{C}[-2] & 0 \end{bmatrix}.$$

This is a shift of the symmetry swapping the two points – an autoequivalence – and satisfies $R = CL[1]$ because

$$\begin{aligned}\mathrm{Hom}(E_x,-) &= \mathrm{Hom}(-,S(E_x))^{\vee_X}\\ &= \mathrm{Hom}(-,E_y[1])^{\vee_X} = \mathrm{Hom}(E_y,S(-))[-1].\end{aligned}$$

Hence F is spherical by the two-out-of-four theorem [AL, Theorem 1.1].

It follows that the twist $T = \mathrm{cone}(FR \to 1)$ is an autoequivalence, with inverse the dual twist $T^{-1} = \mathrm{cone}(1 \to FL)[-1]$. As $L = RS[1]$, this is given by

$$T^{-1}(G) = \mathrm{cone}\Big(G \to \big(\mathrm{Hom}(G,E_x)^{\vee_{\mathbb{C}}} \otimes E_x\big) \oplus \big(\mathrm{Hom}(G,E_y)^{\vee_{\mathbb{C}}} \otimes E_y\big)\Big)[-1],$$

which agrees with the projection to ${}^{\perp}E_x$ when $G \in {}^{\perp}E_y$. □

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Hence Φ is an equivalence, as T^{-1} is an autoequivalence. By symmetry the same functor T^{-1} also gives the other direction of the flop. We write $\Phi = T^{-1}$ for this extension of both functors as a slight abuse of notation.

By [Bar, Theorem 4.1.3] there is a simple way of expressing the flop-flop autoequivalence $\Phi^2|_{D^b(\mathcal{A}_x)}$ as a spherical twist. Since $D^b(\mathcal{A}_x) \subset D^b(\mathcal{A})$ is the complement of an exceptional object, it is the dual twist around a spherical object: the image of E_x in ${}^{\perp}E_y = D^b(\mathcal{A}_x)$. Similar holds for $D^b(\mathcal{A}_y)$. Explicitly, these spherical objects are $F_x := \operatorname{cone}(E_x[-1] \to E_y[1])$ and $F_y := \operatorname{cone}(E_y[-1] \to E_x[1])$.

Remark 3.4. The kernel of $D^b(\mathcal{A}) \to D^b(X)$ is generated by E_x and E_y, and its image in $D^b(\mathcal{A}_x)$ is the kernel of $D^b(\mathcal{A}_x) \to D^b(X)$, which is therefore just $\langle F_x \rangle$. This is in line with results for other nodal singularities [Sun], [CGL+], [KS1].

Here F_x is 3-spherical, even though X is 1-dimensional, and the disparity means that $D^b(\mathcal{A}_x)$ cannot be strongly crepant. On the other hand $D^b(\mathcal{A})$ and $D^b(\mathcal{A}_x)$ are weakly crepant, since the kernels are Serre invariant [KS2, Lemma 5.8].

Remark 3.5. Despite being spherical in their respective subcategories, F_x and F_y are not spherical in $D^b(\mathcal{A})$. Indeed, we find $S(F_x) = F_y[1]$; this is not a shift of F_x. They satisfy the sphere-cohomology part of the definition of a spherical object, but fail the Serre-invariance condition.

Remark 3.6. It turns out that $\Phi = S$. This is almost certainly a formal consequence of $S(E_x) = E_y[1]$ and $S(E_y) = E_x[1]$, together with the description of Φ in terms of mutations, but we have not been able to find a proof in that vein. It can be deduced indirectly from the equivalence with Landau–Ginzburg models outlined in §4, by matching Remark 4.5 with Lemma 3.3. A rather involved proof of the analogous claim for the nodal cubic was given in [Sun, §3].

One consequence is that the Serre functor on $D^b(\mathcal{A}_x)$ is the restriction of S^2, which explains how F_x becomes spherical in the subcategory: $S^2(F_x) = F_x[2]$.

3.2. Crepancy

In view of Remarks 3.2 and 3.4, we make the following definition.

Definition 3.7. Suppose $(\mathscr{C}, \pi_*, \pi^*)$ is a strong categorical resolution of X, with a relative Serre functor S. We say it is *fairly crepant* if $S(\ker(\pi_*)) \subset \ker \pi_*$ and there is some n such that $S|_{\ker(\pi_*)} = [n]$. We call n the *index of crepancy*.

Recall that strong crepancy was the condition $S = \mathrm{id}_{\mathscr{C}}$, while weak crepancy is the condition that $S(\ker(\pi_*)) \subset \ker(\pi_*)$ by [KS2, Lemma 5.8], provided we revise our definition of 'strong categorical resolution' to also require π_* to be a categorical contraction [KS2, Definition 5.1]. It is then clear that strong crepancy implies fair crepancy with index 0, and fair crepancy of any index implies weak crepancy. The reverse implications do not hold in general.

Example 3.8. Consider the categorical resolution of $X = \operatorname{Spec} \mathbb{C}[w]/w^2$ given by the graded Kronecker quiver from Example 2.8. The kernel is generated by $\operatorname{cone}(P_0[-d] \xrightarrow{w} P_1)$, which is a 2-spherical object, so this resolution is fairly crepant of index 2.

For a nontrivial grading $|w| = d$ of $\mathbb{C}[w]/w^2$ we can still consider categorical resolutions, as the definitions given above for affine schemes generalize immediately to graded algebras. (One can even define categorical resolutions for more general dg-categories [KS2, §5]). After setting $|\theta| = 1 - d$ the graded Kronecker quiver still gives a categorical resolution which is fairly crepant of index $2 - d$.

When $d = 2$ this resolution is fairly crepant of index 0, but it is not strongly crepant by the same argument as in Example 2.8. Hence fair crepancy of index 0 is weaker than strong crepancy, at least in this more general context. (We are not aware of a geometric example of the distinction; cf. Remarks 3.12 and 3.13).

Example 3.9. From Remark 3.2 we have that $D^b(\mathcal{A})$ is weakly crepant but not fairly crepant, while $D^b(\mathcal{A}_x)$ is fairly crepant of index 2 from Remark 3.4 (cf. Remark 3.6). The index is the discrepancy

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between the dimension of the spherical object in the kernel (in this case 3) and that of the base variety X (in this case 1).

If we assume that $\mathscr{C}$ is suitably birational, so that $\mathrm{Hom}_{\mathscr{C}}(x, y)$ for $x, y \in \ker(\pi_*)$ is a module supported at the singularity in X, then for an isolated Gorenstein singularity the relative Serre functor on $\ker(\pi_*)$ is a shift of the absolute Serre functor (as in Remark 3.2). In this situation fair crepancy of index n is equivalent to requiring that $\ker(\pi_*)$ is $(n + \dim X)$-Calabi–Yau, implying that $\ker(\pi_*)$ has no nontrivial semiorthogonal decompositions. The resolution then cannot be made smaller via semiorthogonal decomposition, assuming $\mathscr{C}$ is connected, so we find it reasonable to expect that fairly crepant categorical resolutions do not factor nontrivially through further categorical resolutions. Note that 'minimal' categorical resolutions are conjectured to have a universal factorization property [BO], and in particular should be unique, although the correct definitions to ensure this are unclear.

Questions.

1. Does the index of crepancy reduce to a known numerical invariant?
2. In what generality do fairly crepant categorical resolutions exist?
3. Are they universal minimal resolutions? In particular, are they unique?

Assuming uniqueness, nonvanishing of the index of crepancy would be a sufficient nonexistence criterion for (geometric) crepant resolutions of rational singularities, and also other strongly crepant categorical resolutions, such as NCCRs. We find results consistent with this prediction in Remarks 3.12 and 3.13.

3.3. General nodal curves

It was shown in [BD] that $\mathcal{A}$ gives a categorical resolution for any nodal or cuspidal curve. We briefly note that exceptional objects with the same properties as E_x and E_y exist in $D^b(\mathcal{A})$ for all nodal curves.

Indeed, suppose X is a curve with a node at p, and take the normalization $\pi : \widetilde{X} \to X$. Then $\pi^{-1}(p)$ consists of two reduced points q_1 and q_2, and these give simple modules $E_1 = \pi_*\mathcal{O}_{q_1}$ and $E_2 = \pi_*\mathcal{O}_{q_2}$ over $\mathcal{A} = \mathrm{End}_X(\mathcal{O}_X \oplus \pi_*\mathcal{O}_{\widetilde{X}})$.

Proposition 3.10. *We have the following:*

1. *E_1 and E_2 are exceptional objects in $D^b(\mathcal{A})$.*
2. *$S(E_1) = E_2[1]$ and $S(E_2) = E_1[1]$, where S is as in Eq. (3.2).*

In particular, the results of §3.1 apply to ${}^{\perp}E_1, {}^{\perp}E_2 \subset D^b(\mathcal{A})$.

Proof. We assume without loss of generality that X is affine. The algebra $\mathcal{A}$ has projective modules P_X and $P_{\widetilde{X}}$, with

$$\begin{aligned} \mathrm{End}(P_X) &= \mathcal{O}_X, \qquad & \mathrm{Hom}(P_X, P_{\widetilde{X}}) &= \mathcal{O}_{\widetilde{X}}, \\ \mathrm{End}(P_{\widetilde{X}}) &= \mathcal{O}_{\widetilde{X}}, \qquad & \mathrm{Hom}(P_{\widetilde{X}}, P_X) &= \mathcal{I}. \end{aligned} \tag{3.3}$$

Here $\mathcal{I}$ is the ideal cutting out $\mathrm{Sing}(X)$, equal to the conductor $\mathrm{Ann}_X(\mathcal{O}_{\widetilde{X}}/\mathcal{O}_X)$. This is all from [BD, Proposition 2.2]. The points $q_i \in \widetilde{X}$ are divisors $\{s_i = 0\}$, and $g = s_1 s_2$ satisfies $\mathcal{I} = \mathcal{O}_{\widetilde{X}} \cdot g$. We then have the following resolutions:

$$\begin{aligned} P_{\widetilde{X}} &\xrightarrow{(-g, s_2)^T} P_X \oplus P_{\widetilde{X}} \xrightarrow{(1, s_1)} P_{\widetilde{X}} \to E_1, \\ P_{\widetilde{X}} &\xrightarrow{(-g, s_1)^T} P_X \oplus P_{\widetilde{X}} \xrightarrow{(1, s_2)} P_{\widetilde{X}} \to E_2, \end{aligned}$$

with maps understood via Eq. (3.3). Using these (1) is a direct computation, and we also find that $(E_i)^{\vee_{\mathcal{A}}} = E_j[-2]$, where $\{i, j\} = \{1, 2\}$ and E_j is viewed as a left-module. Then (2) follows from $\mathrm{Hom}_X(\mathcal{O}_p, \mathcal{O}_X) = \mathcal{O}_p[-1]$. □

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Table 1. *Indices of crepancy for categorical resolutions of nodes.*

dim	0	1	2	3	4	5	6	7	⋯
index	2	2	0	0	−2	−2	−4	−4	⋯

Corollary 3.11. *For any isolated nodal curve singularity X, there is a categorical resolution with kernel generated by a single 3-spherical object.*

Such a resolution is fairly crepant of index 2, but not strongly crepant since $2 \neq 0$. This extends the description of [CGL$^+$] for $\dim(X) \geq 2$, continuing the pattern of 2- or 3-spherical objects by parity of dimension.[2] The zero-dimensional node also follows this pattern; see Example 3.8.

Remark 3.12. In particular, fairly crepant resolutions exist for isolated nodal singularities of all dimensions. The indices of these resolutions are displayed in Table 1. Note that the index is 0 in exactly the two dimensions where geometric crepant resolutions exist; the constructed categorical resolutions are equivalent to the geometric resolutions in these cases, and are therefore strongly crepant.

Remark 3.13. In [Fie], a description of the kernels of certain categorical resolutions of simple hypersurface singularities of type A_2 is given. They appear to be fairly crepant of index $2 - d$, where d is the dimension.[3] Again, this index vanishes iff a geometric crepant resolution exists.

4. Landau–Ginzburg models

Recall that a *Landau–Ginzburg model* (following [Seg1]), at least on the B-side, is a smooth variety U with a *superpotential* $W \in H^0(U, \mathcal{O}_U)$, together with an action of a rank one torus $\mathbb{C}^*_R$ on U called the *R-charge*, satisfying

1. $\{\pm 1\} \subset \mathbb{C}^*_R$ acts trivially on U, and
2. W has weight 2 for the $\mathbb{C}^*_R$ action.

There is then an associated dg-enhanced triangulated category $D^b(U, W)$ of *matrix factorizations* of W, or *B-branes*, whose objects are $\mathbb{C}^*_R$-equivariant sheaves with an endomorphism d of weight 1 satisfying $d^2 = W$, having a suitable notion of quasi-isomorphism by [Orl].

4.1. The exoflop

We apply the idea of [Asp] to our situation as follows. The derived category of $X = \{xy = 0\}$ is equivalent to a category of matrix factorizations $D^b(\mathbb{A}^3, xyz)$, where the R-charge weights are $|x| = |y| = 0$ and $|z| = 2$. This is an instance of Knörrer periodicity ([Knö], cf. [Shi]), which relates the derived category of a complete intersection – or more generally factorizations on a complete intersection – to factorizations on the associated vector bundle. Now $\widetilde{X}$ lies inside the blowup $\mathrm{Tot}_{\mathbb{P}^1_{x:y}} \mathcal{O}(-1)$ of the plane, cut out by the section xy of $\mathcal{O}(2)$, and by Knörrer periodicity it is also derived equivalent to an LG-model. We get a square

$$\begin{array}{ccc} D^b(X) & \xrightarrow{\simeq} & D^b(\mathbb{A}^3, xyz) \\ \uparrow & & \uparrow \\ D^b(\widetilde{X}) & \xrightarrow{\simeq} & D^b(\mathrm{Tot}_{\mathbb{P}^1}(\mathcal{O}(-1) \oplus \mathcal{O}(-2)), xyz), \end{array}$$

[2]The fact that our resolutions are categorical contractions in the sense of [KS2] follows from [BD, Theorem 4.8], composing with the admissible inclusion $D^b(\mathcal{A}_x) \hookrightarrow D^b(\mathcal{A})$.

[3]In odd dimensions a minor extension of Fietz's results is required for this to be proven.

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where z is the coordinate on $\mathcal{O}(-2)$. This LG-model has a weighted flip:

$$\mathrm{Tot}_{\mathbb{P}^{1:1}_{x:y}}(\mathcal{O}(-1)_w \oplus \mathcal{O}(-2)_z) \dashrightarrow \overline{U} \coloneqq \mathrm{Tot}_{\mathbb{P}^{1:2}_{w:z}} \mathcal{O}(-1)^2_{x,y},$$

with $\mathbb{P}^{1:2}$ viewed as an orbifold. By [BFK] there is a decomposition

$$D^b(\overline{U}, xyz) = \langle D^b(\mathrm{pt}), D^b(\widetilde{X})\rangle, \tag{4.1}$$

where the exceptional object is the sheafy matrix factorization $\mathcal{O}_{\mathbb{P}^{1:2}}(-1)$ on the zero section.[4] Here $D^b(\widetilde{X})$ is embedded via Knörrer periodicity followed by the flip.

The key observation is that $\overline{U}$ is a partial compactification of the original LG-model $(\mathbb{A}^3, xyz)$, which is the open set $\{w \neq 0\}$. Restriction to this open set gives a functor $D^b(\overline{U}, xyz) \to D^b(X)$, and the adjoint pushforward is defined for factorizations supported at $\{z = 0\}$, which corresponds to $\mathfrak{Perf}(X)$ under Knörrer periodicity. Restriction and pushforward compose to the identity, and $D^b(\overline{U}, xyz)$ is smooth (from compactification of the critical locus, or just Eq. (4.1)), so this is a categorical resolution. It can be checked to be equivalent to $D^b(\mathcal{A})$.[5]

Remark 4.1. Notice how three completely different constructions – that of [KL], [BD], and [Asp] – all produce the same categorical resolution $D^b(\mathcal{A})$. This kind of coincidence can be seen as evidence for the conjectural existence of a universal minimal categorical resolution, even though we expect there to be no strongly crepant resolution in this case. One slightly odd point is that the nonminimal resolution $D^b(\mathcal{A})$ seems to crop up more naturally than the minimal one $D^b(\mathcal{A}_x)$.

4.2. Removing orbifold points

To better understand these LG-models, we should consider the critical locus of the superpotential (Fig. 2). For $(\mathbb{A}^3, xyz)$ this is the node itself in the plane $\{z = 0\}$, together with a vertical $\mathbb{A}^1_z$ branch through the singularity, with positive R-charge. This indicates the unbounded Ext groups supported at the singularity of X; indeed, the open set $\{z \neq 0\}$ of the LG-model is equivalent by Knörrer periodicity to $D^b(\mathbb{A}^1_z - \{0\})$, where $|z| = 2$ is positively graded. The nontrivial part of the compactification lies in this open set, and if we apply Knörrer periodicity to $\{z \neq 0\} \subset \overline{U}$ we instead get $D^b([\mathbb{A}^1_w/\mathbb{Z}_2])$, because the branch $\mathbb{A}^1_z$ has been compactified to an orbifold $\mathbb{P}^{1:2}$. This compactification of the positively graded part of the critical locus is why we get a relatively proper categorical resolution.

A more obvious compactification just uses $\mathbb{P}^1$. We can achieve it algebraically by removing an exceptional object, motivated by the decomposition $D^b([\mathbb{A}^1_w/\mathbb{Z}_2]) = \langle \mathcal{O}_0, \mathcal{O}\rangle = \langle D^b(\mathrm{pt}), D^b(\mathbb{A}^1_{w^2})\rangle$ for the chart $\{z \neq 0\}$. There are two choices, since there are two twists of the skyscraper sheaf in $[\mathbb{A}^1/\mathbb{Z}_2]$, giving two exceptional objects $\mathcal{O}_{\{x=w=0\}}$ and $\mathcal{O}_{\{y=w=0\}}$ in $D^b(\overline{U}, xyz)$. Their orthogonal complements give two smaller categorical resolutions, matching §3.

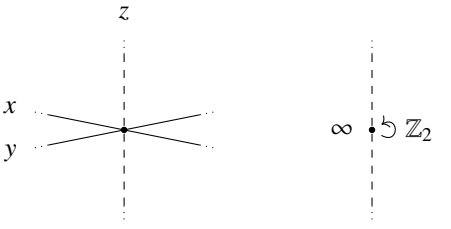


Figure 2. *The critical locus of $(\mathbb{A}^3, xyz)$, and a chart near $z = \infty$ for the partial compactification with an orbifold point.*

[4] By 'sheafy matrix factorization' we mean that the differential d is zero.
[5] A suitable generator is $\mathcal{O}_{\{x=0\}} \oplus \mathcal{O}_{\{y=0\}} \oplus \mathcal{O}_{\{z=0\}}$.

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Remark 4.2. The smaller resolutions are also equivalent to factorizations on another LG-model: $(\mathrm{Tot}_{\mathbb{P}^1_{w:z}}(\mathcal{O}_x \oplus \mathcal{O}(-1)_y), xyz)$. Here the critical locus is the obvious compactification. This LG-model description appears in [LP2].

Remark 4.3. These descriptions suggest the following distillation of the essence of the flop: for a smooth point p on a curve C, the derived category of the root stack $\sqrt[2]{p/C}$ has an exceptional object, the skyscraper sheaf at p, with orthogonal complement the pullup of $D^b(C)$. It can be twisted by the $\mathbb{Z}_2$-isotropy, giving a different subcategory equivalent to $D^b(C)$ and a diagram like Eq. (1.2). The flop-flop autoequivalence on $D^b(C)$ is then $- \otimes \mathcal{O}(p)$. We are looking at an analogue of this construction for the noncommutative curve $\mathcal{A}_x$.

This is an example of the general principle that root stacks behave a lot like blowups, especially when viewed through the lens of derived categories; see [BLS, §4].

Example 4.4. When $C = \mathbb{A}^1$ and $p = 0$ the root stack is $[\mathbb{A}^1/\mathbb{Z}_2]$, which was the affine patch at infinity for the critical locus of the LG-model considered above.

4.3. Canonical bundles

The Serre functor on the LG-model is just given by the canonical bundle $\mathcal{O}(-1)$, by a local version of [FK, Theorem 1.2]. Weak crepancy holds since the line bundle is trivial near $\{z = 0\}$, using Remark 2.6 together with the fact that $\mathfrak{Perf}(X)$ corresponds to factorizations supported at $\{z = 0\}$. Of course, this local condition does not imply global triviality (i.e., strong crepancy). The orbifold point at $\{z = \infty\}$ locally obstructs strong crepancy, contributing a local nontriviality to the canonical bundle, but there is also the global obstruction that the toric compactifications of $(\mathbb{A}^3, xyz)$ are not Calabi–Yau, so the smaller resolution is still not strongly crepant (cf. Remark 4.2).

The kernel subcategory consists of factorizations which restrict to zero on the open set $\{z \neq 0\}$, that is, those with support at $\{z = \infty\}$, and so fair crepancy is equivalent to local triviality of the canonical bundle near $\{z = \infty\}$. This fails only for the orbifold compactification, where the isotropy group gives an obstruction.

Remark 4.5. As a line bundle, the Serre functor is the inverse twist along the spherical functor given by the divisor $\{w = 0\}$ with ideal sheaf $\mathcal{O}(-1)$. The source category is the restricted LG-model $D^b([\mathbb{A}^2/\mathbb{Z}_2], xy)$, which is equivalent by Knörrer periodicity to $D^b([\mathrm{pt}/\mathbb{Z}_2]) = D^b(\mathrm{pt}) \oplus D^b(\mathrm{pt})$, recovering the spherical functor F.[6]

4.4. The A-side

Let us briefly sketch an equivalent picture under homological mirror symmetry. For more details, the reader should consult [LP1] and [LP2].

Our starting point is a standard result, that the LG-model $(\mathbb{A}^3, xyz)$ is equivalent to a wrapped Fukaya category $\mathcal{W}(\Sigma)$, where Σ is the pair-of-pants surface.[7] It was further shown in [LP1] that the categorical resolution $D^b(\mathcal{A})$ is equivalent to a *partially* wrapped Fukaya category $\mathcal{W}(\Sigma; \Lambda)$, where two stops $\Lambda = \{s_x, s_y\}$ have been added to one of the boundary components (the one with positive grading). Generators realizing the equivalence with $D^b(\mathcal{A})$ are horizontal lines in Fig. 3. The forgetful functor $\mathcal{W}(\Sigma; \Lambda) \to \mathcal{W}(\Sigma)$ is a localization, which contracts two exceptional objects (E_x and E_y) given by small arcs bounding the two stops (also depicted).

The two smaller resolutions then correspond to removing just one of these stops, giving the equivalent categories $\mathcal{W}(\Sigma; s_x)$ and $\mathcal{W}(\Sigma; s_y)$, with $\mathcal{W}(\Sigma; \Lambda)$ providing a roof via the forgetful functors. Adjoint to these forgetful functors are the inclusions as admissible subcategories, which can be described on objects

[6]Some care is needed for R-charge here. To invert z we re-gauge so that $|z| = 0$, giving $|x| = |y| = 1$. These odd weights are possible because of the $\mathbb{Z}_2$-action.

[7]In order to grade $\mathcal{W}(\Sigma)$ a line field must be chosen, whose winding numbers around the three punctures of Σ correspond to the R-charge weights of x, y and z; see [Seg2].

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(purely algebraically) by choosing a lift to the roof, and then projecting to the orthogonal complement of the relevant exceptional object via a mapping cone. This reproduces the spherical twists of §3; the arcs near the stop in the single-stopped case are spherical objects, and in the double-stopped case they are the exceptional objects of Lemma 3.3.

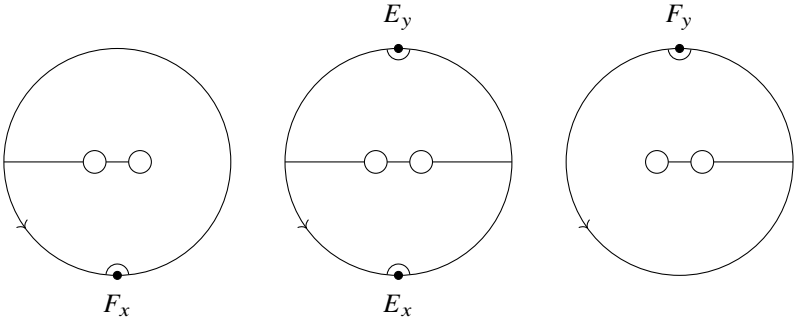


Figure 3. *Objects in* $\mathcal{W}(\Sigma; s_x)$, $\mathcal{W}(\Sigma; \Lambda)$ *and* $\mathcal{W}(\Sigma; s_y)$.

These spherical twists are examples of the 'wrap-once' autoequivalence for a swappable stop from [Syl], which is always the twist for a natural associated spherical functor – the Orlov functor – and is defined in terms of a 'swapping' self-isotopy of the stopping divisor. This is a full circuit around the boundary for our single-stopped surface, but can be a half twist in the double-stopped case (swapping the two stops), which explains why one autoequivalence is the square of the other.

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