J. Plasma Physics (1998), *vol.* 60, *part* 1, *pp.* 65–68. Printed in the United Kingdom

© 1998 Cambridge University Press

The large-Larmor-radius Rayleigh–Taylor instability

M. FAGHIHI and F. EBRAHIMI†

Department of Physics, Amirkabir University of Technology (Tehran Polytechnic), Hafez Avenue, Tehran 15914, Iran

(Received 20 March 1997)

The effect of a large ion Larmor radius on the Rayleigh–Taylor instability is investigated using the Vlasov fluid model. The results are compared with an ideal magnetohydrodynamic model. It is found that this effect reduces the growth rate of the Rayleigh–Taylor instability with respect to the ideal magnetohydrodynamic growth rate.

1. Introduction

The Rayleigh–Taylor instability in the limit of a large ion Larmor radius (i.e. $kr_{Li} \gg 1$, $\Omega_i \ll \omega$, where k and ω are the wavenumber and frequency scales of interest and r_{Li} and Ω_i are the ion gyroradius and gyrofrequency) is of much interest in the context of the stability of collisionless plasmas. Finite- or kinetic-Larmor-radius effects under certain conditions are particularly important for Z pinches and theta pinches. Early studies of Z-pinch stability (Faghihi and Scheffel 1988; Scheffel and Faghihi 1989) in the collisionless regime were based on the fluid equations, taking the ion Larmor radius to be zero or small. Since in many high-β experiments the ions are collisionless and have finite gyro-orbits, the fluid models used to investigate plasma instabilities do not predict the proper growth rates for the instability modes observed experimentally. Kinetic study of Z-pinch (Arber *et al.* 1994), rigidly rotating theta-pinch (Seyler 1979) and screw-pinch (Turner 1977*a*, *b*) stability have shown a reduction in the growth rate of instability modes in these configurations.

The Rayleigh–Taylor instability in a plasma appears at the plasma – vacuum interface with a magnetic field parallel to the boundary surface. In a hot collisionless magnetized plasma, the ion orbits may be large, i.e. of comparable size to a typical wavelength of the perturbed boundary surface. A detailed study of the instability mechanism shows that the large ion orbits change the growth rate. Various models have been used to study this effect. Fluid models have been shown to be inadequate, since they do not include kinetic effects. Thus stability is in general overestimated in fluid models. It is therefore necessary to use kinetic models, which treat the plasma as an ensemble of interacting particles.

Gary and Thomsen (1982) have made a kinetic study of the localized linear Rayleigh–Taylor instability with the further assumption of low β, and have found new unstable modes not encountered in fluid analysis. Also, the study of

† Present address: Alfvén Laboratory, Royal Institute of Technology, SE-100 44 Stockholm, Sweden.

https://doi.org/10.1017/S0022377898006515 Published online by Cambridge University Press

Rayleigh–Taylor instability with the aid of an ideal one-fluid Hall model (Velikovich 1991) showed that the global Rayleigh–Taylor instability mode, whose growth rate is $\gamma = (gk)^{1/2}$, still exists in this case – but is not necessarily the dominant mode. However, the one-fluid Hall magnetohydrodynamic (MHD) model is not a universally accepted model of large-Larmor-radius plasmas (Lehnert and Scheffel 1988). In Sect. 2 below, we derive the dispersion relation using the Vlasov fluid model, and our conclusions are presented in Sect. 3.

2. The dispersion relation

The Rayleigh–Taylor instability in the limit of a large ion Larmor radius using the Vlasov fluid model (Friedberg 1972) is studied here. In this model the ions are treated kinetically using the collisionless Boltzmann (Vlasov) equation. The electrons are assumed to form a massless, pressureless ($T_e = 0$) fluid tied to the magnetic field, satisfying $\mathbf{E} + \mathbf{U}_e \times \mathbf{B} = 0$, where $\mathbf{U}_e$ is the electron velocity. The basic equations of the Vlasov fluid model using Maxwell's equations are given by

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{1}$$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f + \frac{e}{m_i} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \boldsymbol{\nabla}_{\mathbf{v}} f = 0, \tag{2}$$

$$(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} = \mu_0 e \int (\mathbf{E} + \mathbf{v} \times \mathbf{B}) f \, d\mathbf{v}. \tag{3}$$

The plasma is considered to be acted upon by a uniform magnetic field $\mathbf{B} = B_0 \mathbf{e}_z$. The gravitational field $\mathbf{g} = -g\mathbf{e}_x$ and density gradient $\boldsymbol{\nabla} n$ are oppositely directed. The ions circulating with speed $v_\perp$ transverse to the unperturbed magnetic field have orbits

$$x(t') = x(t) + \frac{v_\perp}{\Omega_i} [\sin(\Omega_i \tau - \phi) + \sin \phi], \tag{4}$$

$$y(t') = y(t) - \frac{v_\perp}{\Omega_i} [\cos(\Omega_i \tau - \phi) - \cos \phi] + g \frac{\tau}{\Omega_i}, \tag{5}$$

$$z(t') = z(t) + v_z \tau, \tag{6}$$

$$\mathbf{v}(\tau) = v_\perp \cos(\Omega_i \tau - \phi) \, \mathbf{e}_x + v_\perp \sin(\Omega_i \tau - \phi) \, \mathbf{e}_y + v_z \mathbf{e}_z, \tag{7}$$

where $\tau = t' - t$ and $\Omega_i = eB_0/m_i$ is the ion gyrofrequency. Using the constants of motion for this system,

$$\varepsilon = \tfrac{1}{2} m_i v^2 + m_i g x, \tag{8}$$

$$P_z = m_i v_z, \quad P_y = m_i v_y + \Omega_i m_i x, \tag{9}$$

the zeroth-order time-independent distribution function is constructed:

$$f_0(x, v) = n_i \left(\frac{m_i}{2\pi k_B T_i} \right)^{3/2} \left[1 + \varepsilon_n \left(x + \frac{v_y}{\Omega_i} \right) \right] \exp\left[-\left(\frac{m_i v^2}{2 k_B T_i} + \frac{m_i g x}{k_B T_i} \right) \right], \tag{10}$$

where

$$\varepsilon_n = \frac{1}{n_i} \frac{d n_i^{(0)}(x)}{dx} + \frac{m_i g}{k_B T_i}, \tag{11}$$

https://doi.org/10.1017/S0022377898006515 Published online by Cambridge University Press

k_B and T_i are Boltzmann's constant and the ion temperature, and $n_i^{(0)}(x)$ is the first velocity moment of this distribution. Linearizing (3) and following Freidberg (1972) by defining the potentials $\mathbf{A}_\perp = \boldsymbol{\xi}_\perp \times \mathbf{B}_0$, $\mathbf{A}_{LL} = 0$ and $\Phi_1 = \boldsymbol{\xi}_\perp \cdot \mathbf{E}_0$, a relation between ω^2 (Vlasov fluid model) and ω_m^2(MHD model) can be obtained:

$$\omega_m^2 \kappa_m = -\tfrac{1}{2} i\omega e \int \boldsymbol{\xi}_\perp^* \cdot \left[n_0 \boldsymbol{\xi}_\perp \times \mathbf{B}_0 + \int \left(\mathbf{E}_0 + \mathbf{v} \times \mathbf{B}_0 \right) \frac{\partial f_0}{\partial \varepsilon} S_1 \, d\mathbf{v} \right] d\mathbf{r}, \tag{12}$$

where

$$S_1 = e \int_{-\infty}^{t} \boldsymbol{\xi}_\perp \cdot (\mathbf{E}_0 + \mathbf{v} \times \mathbf{B}_0) \, dt', \tag{13}$$

$$k_m = \tfrac{1}{2} \int \rho_0 |\boldsymbol{\xi}_\perp|^2 \, d\mathbf{r}. \tag{14}$$

S_1 is the trajectory integral along the ion orbits and $\boldsymbol{\xi}_\perp$ is the perpendicular MHD displacement vector.

In order to analyse the Rayleigh–Taylor instability in a slab configuration, the displacement vector $\boldsymbol{\xi}_\perp$ is represented as a Fourier transform perpendicular to the x direction, the direction of the inhomogeneity:

$$\boldsymbol{\xi}_\perp (\mathbf{x}, t) = \boldsymbol{\xi}(x) \exp [i(kz - \omega t)]. \tag{15}$$

To obtain $\boldsymbol{\xi}_\perp(\mathbf{x}, t)$, it is not necessary to solve the eigenvalue problem for the slab configuration. Instead, we consider the MHD displacement vector (15) with a linear dependence on x. With this assumption, the trajectory integral S_1 becomes

$$S_1 = -m_i \int_{-\infty}^{t} (v_x' + i v_y')^2 \exp[i(kz' - \omega t') \, dt'. \tag{16}$$

The S_1 integral can be evaluated by the change of variable $\tau = t - t'$ and the periodicity properties of the ion orbits, $v'(\tau + T) = v'(\tau)$, where T is period of the orbit:

$$\begin{aligned} S_1 = \frac{-m_i \exp(-i\omega t)}{1 - \exp[i(kv_z T + \omega T)]} \Bigg\{ & \frac{-ig^2 + ig^2 \exp[iT(kv_z + \omega)]}{\Omega_i^2 (kv_z + \omega)} \\ & + \frac{-2 \exp(-i\phi) g v_\perp + 2 \exp[-i\phi + iT\Omega_i + iT(kv_z + \omega)] g v_\perp}{\Omega_i (kv_z + \Omega_i + \omega)} \\ & + \frac{i \exp(-2i\phi) v_\perp^2 + i \exp[-2i\phi + 2iT\Omega_i + iT(kv_z + \omega)] v_\perp^2}{kv_z + 2\Omega_i + \omega} \Bigg\}. \end{aligned} \tag{17}$$

For large ion Larmor radius ($kr_{Li} \gg 1, \Omega_i \gg \omega$), the last two terms of (17) are small and may be neglected. Substituting this result into (12) and performing the phase integral, we obtain the required dispersion relation:

$$\omega^2 = w_m^2 - I, \tag{18}$$

where

$$\begin{aligned} I = {} & \frac{\omega e g^2 m_i}{2 k_m \Omega_i^2} \left[\frac{n_i}{k_B T_i} \left(\frac{m_i}{2\pi \kappa_B T_i} \right)^{3/2} \right] \left[\frac{1 + \frac{1}{n_i} \frac{dn_i^{(0)}(x)}{dx} + \frac{m_i g}{k_B T_i} \left(\frac{k_B T_i}{m_i g} \right)^2}{k^2} \right] \\ & \times \left\{ \frac{k_B T_i}{m_i} + \frac{2\pi}{\Omega_i} \left[\frac{1}{n_i} \frac{dn_i^{(0)}(x)}{dx} + \frac{m_i g}{k_B T_i} \right] \left(\frac{M_i}{2 k_B T_i} \right)^{-3/2} \right\} \left[\frac{\pi^{1/2}}{2k} Z_p \left(\frac{1}{k} \left(\frac{m_i}{2 k_B T_i} \right)^{1/2} \omega \right) \right]. \end{aligned} \tag{19}$$

https://doi.org/10.1017/S0022377898006515 Published online by Cambridge University Press

In this case, the dispersion function reduces to $Z_p(\zeta) \approx i\pi^{1/2}\exp(-\zeta^2) - \zeta^{-1}$, where $\zeta = \omega/kv_{Ti}$ and $v_{Ti} = (m_i/2k_B T_i)^{-1/2}$ is the thermal speed transverse to the magnetic field. Setting $\omega = \omega_r + i\gamma$, and writing $\omega_m^2 = -\gamma_m^2$, the ratio of the Vlasov fluid growth rate to the ideal MHD growth rate is given by

$$\frac{\gamma}{\gamma_m} = \left[1 - \frac{en_0(1+\varepsilon_n)}{16\pi\gamma_m^2 (kr_{Li})^2}\left(1 + \frac{2\pi^{1/2}\varepsilon_n}{v_{Ti}^{3/2}}\right)\right]^{1/2}, \tag{20}$$

where we have used $r_{Li} = (2k_B T_i/m_i\Omega_i^2)^{1/2}$. In the limit of a large ion Larmor radius ($kr_{Li} \gg 1$), that is, with increasing ion temperature, (20) indicates a substantial reduction in growth rate. Therefore, with a more realistic model where the ion Larmor radius is considered large, a reduction in the growth rate of the Raleigh–Taylor instability with respect to the ideal MHD growth rate occurs.

When the ion Larmor radius is sufficiently small, the growth rate of Vlasov fluid model becomes approximately the same as the ideal MHD growth rate. In the case of a small ion Larmor radius (the MHD limit), $Z_p(\zeta) \approx i\pi^{1/2}$ and using this function in evaluating the phase integral yields $\gamma \approx \gamma_m$.

3. Conclusions

The dispersion relation for the Rayleigh–Taylor instability in a slab configuration in the presence of a gravitational field, as described by the Vlasov fluid model, has been obtained. The main aim of the computations has been to compare the results with the MHD fluid model. The discrepancy between the present results and those of MHD theory appear to be due to the kinetic contribution of a large ion Larmor radius.

Acknowledgement

The authors wish to express their deep appreciation to Dr Jan Scheffel at the Alfvén Laboratory for his suggestions and encouragement throughout the course of this research.

References

Arber, T. D. and Coppins, M. 1989 *Phys. Fluids* **B1**, 2289.
Arber, T. D., Coppins, M. and Scheffel, J. 1994 *Phys. Rev. Lett.* **72**, 2399.
Faghihi, M. and Scheffel, J. 1988 *J. Plasma Phys.* **38**, 495.
Freidberg, J. P. 1972 *Phys. Fluids* **15**, 1102.
Freidberg, J. P. and Wesson, J. A. 1970 *Phys. Fluids* **13**, 1117.
Gary, P. and Thomsen, M. 1982 *J. Plasma Phys.* **28**, 551.
Lehnert, B. and Scheffel, J. 1988 *Phys. Rev. Lett.* **61**, 897.
Rosenbluth, M. N., Krail, N. A. and Rostoker, N. 1962 *Nucl. Fusion Suppl.* **1**, 143.
Scheffel, J. and Faghihi, M. 1989 *J. Plasma Phys.* **41**, 427.
Seyler, C. E. 1979 *Phys. Fluids* **22**, 2324.
Turner, L. 1977*a* *Phys. Fluids* **20**, 654.
Turner, L. 1977*b* *Phys. Fluids* **20**, 662.
Velikovich, A. L. 1991 *Phys. Fluids* **B3**, 492.

https://doi.org/10.1017/S0022377898006515 Published online by Cambridge University Press