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Charging of a spherical dust particle on a plasma-facing wall

YUKIHIRO TOMITA[1], R. SMIRNOV[2], T. TAKIZUKA[3] and D. TSKHAKAYA[4]

[1]National Institute for Fusion Science, 322-6 Oroshi-cho, Toki 509-5292, Japan

[2]The Graduate University for Advanced Studies (Soken-dai), 322-6 Oroshi-cho, Toki 509-5292, Japan

[3]Naka Fusion Research Establishment, Japan Atomic Energy Research Institute, Naka, Ibaraki 311-0193, Japan

[4]Association Euratom-ÖAW, Institute of Theoretical Physics, University of Innsbruck, Technikerstrasse 25/II, Innsbruck A-6020, Austria

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Abstract. The induced charge of a spherical conducting dust particle on a plasma-facing wall is studied analytically. An external non-uniform electric field is approximated by the polynomial of the axial coordinate and the bipolar coordinate is introduced in order to employ the boundary condition. The result is applied to the dust in the non-uniform electric field in the Debye sheath in front of the conducting wall. The results are useful for analyzing the shielding effects of the plasma surrounding a dust particle.

1. Introduction

Dust in plasmas is an interesting topic with a developing science community connected to astrophysical, space, laboratory, and processing plasma applications as well as nuclear fusion. The dust in fusion plasmas is important because it may influence the plasma as a source of impurities or other confinement limitations, and presents safety and operational challenges for a nuclear fusion facility, such as ITER, due to absorption of radioactive tritium [1]. In this study, we investigate analytically the charge formation of a spherical conducting dust particle on a plasma-facing wall. The two-dimensional structure of the sheath around the spherical dust is formed. The non-uniform electric field induces the charge on the surface of the dust particle. The plasma shields the induced charge. We study analytically the induced charge on the spherical dust immersed in the non-uniform electric field, which is formed in the Debye sheath in front of the plasma-facing wall.

2. Electrostatic potential

A spherical conducting dust particle with a radius R_d is attached on an infinitely extended conducting plane wall. The potential ϕ_in due to the induced charge on the dust satisfies the Laplace equation

$$\left[\frac{\partial}{\partial\alpha}\left(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial}{\partial\beta}\right)\right]\phi_\mathrm{in}=0 \tag{2.1}$$

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in the bipolar coordinate (α, β) [2], where $\alpha = R_\mathrm{d} r/(r^2+z^2)$, $\beta = R_\mathrm{d} z/(r^2+z^2)$, and the origin of the conventional cylindrical coordinates r and z is the contact point of the dust on the wall. The plane $z=0$, i.e. $\beta=0$, corresponds to the wall surface and $\beta = 1/2\,(\equiv \beta_0)$ indicates the surface of the spherical dust particle. The general solution of the Laplace equation (1) is given by

$$\phi_\mathrm{in}(\alpha,\beta) = \sqrt{\alpha^2+\beta^2}\sum_{n=1}^{\infty} c_\mathrm{n}\int_0^\infty d\lambda\,\lambda^n e^{-\beta_0\lambda}\frac{\sinh(\beta\lambda)}{\sinh(\beta_0\lambda)}J_0(\alpha\lambda)(\alpha,\beta) \tag{2.2}$$

where the coefficients c_n are determined by a boundary condition at the surface of the spherical dust. In order to analyze analytically the induced charge on the spherical dust, the non-uniform electrostatic potential is approximated by the polynomial of the axial coordinate z:

$$\phi_\mathrm{ex}(\alpha,\beta) = \sum_{k=0}^{k_\mathrm{max}} h_k z^k = \sum_{k=0}^{k_\mathrm{max}} h_k \left(\frac{R_\mathrm{d}\beta}{\alpha^2+\beta^2}\right)^k \tag{2.3}$$

where the coefficient h_0 corresponds to the biased wall potential ϕ_w. This external potential can be expressed by the functions

$$(-1)^n\frac{d^n}{d\beta^n}\frac{1}{\sqrt{\alpha^2+\beta^2}} = \int_0^\infty d\lambda\,\lambda^n e^{-\beta\lambda}J_0(\alpha\lambda) \tag{2.4}$$

From the boundary condition at the surface of the spherical dust, $\beta=\beta_0=\frac{1}{2}$: $\phi_\mathrm{ex}+\phi_\mathrm{in}=\phi_\mathrm{w}$, the unknown coefficients c_n in (2.2) can be expressed by the known coefficients h_k. In the case of $k_\mathrm{max}=3$, the following relations are obtained:

$$c_1 = -\left(h_1 R_\mathrm{d} + \frac{h_2 R_\mathrm{d}^2}{3\beta_0} + \frac{h_3 R_\mathrm{d}^3}{5\beta_0^2} + \frac{h_4 R_\mathrm{d}^4}{7\beta_0^3}\right), \quad c_2 = -\left(\frac{h_2 R_\mathrm{d}^2}{3} + \frac{h_3 R_\mathrm{d}^3}{5\beta_0} + \frac{h_4 R_\mathrm{d}^4}{7\beta_0^2}\right)$$

$$c_3 = -\left(\frac{h_3 R_\mathrm{d}^3}{15} + \frac{2h_4 R_\mathrm{d}^4}{35\beta_0}\right), \quad \text{and} \quad c_\mathrm{n} = 0 : n > 3 \tag{2.5}$$

Finally we obtain the total local potential consisting of the external potential ϕ_ex, (2.3) and the induced potential by substituting (2.5) into (2.2).

3. Induced charge

The induced charge Q_din is calculated from the electric field to the normal direction E_n at the dust surface $(\beta=\frac{1}{2})$:

$$Q_\mathrm{din} = \varepsilon_0\int_{S_\mathrm{d}} E_\mathrm{n}|_{\beta=1/2}\,dS = -2\pi\varepsilon_0 R_\mathrm{d}^2(d_1 h_1 + d_2 h_2 R_\mathrm{d} + d_3 h_3 R_\mathrm{d}^2 + \cdots), \tag{3.1}$$

where S_d is the dust surface area. In the case of $k_\mathrm{max}=3$, the numerical coefficients d_k are

$$d_1 = I_{qc2}, \quad d_2 = \frac{1}{3}\left(\frac{I_{q1}}{\beta_0^2} + \frac{I_{qc2}}{\beta_0} + I_{qc3}\right), \quad \text{and}$$

$$d_3 = \frac{1}{15}\left[\frac{6I_{q1}}{\beta_0^3} + \frac{6(I_{q2}+I_{qc2})}{\beta_0^2} + \frac{3I_{qc3}}{\beta_0} + I_{qc4}\right]. \tag{3.2}$$

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where

$$I_{qn} = \frac{\Gamma(n)}{(2\beta_0)^2} \quad \text{and} \quad I_{qcn} = 2\frac{\Gamma(n)\zeta(n)}{(2\beta_0)^2}. \tag{3.3}$$

Here $\Gamma(n)$ and $\zeta(n)$ are the Gamma and Riemann's Zeta functions, respectively.

4. Non-uniform electrostatic potential in the Debye sheath

The above result is applied to the dust in the non-uniform electrostatic potential of the Debye sheath. Our model of the Debye sheath formation is the following: (1) ions are monoenergetic; (2) electrons have a truncated Maxwellian velocity distribution due to the absorption of the high-energy component by the wall; (3) the electric field is vanishing at the sheath entrance; and (4) the external electrostatic potential $\phi_{\rm ex}$ is expressed by the polynomial of degree three, where the coefficients h_k are determined by the potential drop across the Debye sheath $\phi_{\rm w}$ and the sheath width $z_{\rm se}$. The density of ions in a collisionless sheath is derived as a function of the local potential ϕ from the conservation of both the particles and the energy:

$$n_{\rm i}(\phi) = n_{\rm ise}/\sqrt{1 - 2Z_{\rm i}e\phi/m_{\rm i}V_{\rm ise}^2}. \tag{4.1}$$

Here $n_{\rm ise}$ and $V_{\rm ise}$ are the ion density and the ion flow velocity at the sheath edge, where the potential $\phi = 0$. The electron density is also expressed by the local potential:

$$n_{\rm e}(\phi) = n_{\rm ese}\exp(e\phi/T_{\rm e})\frac{1 + {\rm erf}(\sqrt{e(\phi - \phi_{\rm w})/T_{\rm e}})}{1 + {\rm erf}(\sqrt{-e\phi_{\rm w}/T_{\rm e}})}, \tag{4.2}$$

where $n_{\rm ese}$ is the electron density at the sheath edge and $T_{\rm e}$ is the uniform electron temperature. With the aid of Poisson's equation, the electric field at the wall $E_{\rm w}$ is obtained as a function of the wall potential drop:

$$E_{\rm w}^2 = \frac{2n_{\rm se}T_{\rm e}}{\varepsilon_0}\left\{\frac{1}{1 + {\rm erf}(\sqrt{-e\phi_{\rm w}/T_{\rm e}})}[e^{e\phi_{\rm w}/T_{\rm e}} - 1 - {\rm erf}(\sqrt{-e\phi_{\rm w}/T_{\rm e}}) + \frac{2}{\sqrt{\pi}}e^{e\phi_{\rm w}/T_{\rm e}}\sqrt{-e\phi_{\rm w}/T_{\rm e}}] + \frac{1}{Z_{\rm i}}(\sqrt{1 - 2Z_{\rm i}e\phi_{\rm w}/T_{\rm e}} - 1)\right\}, \tag{4.3}$$

where the charge neutrality condition at the sheath edge $n_{\rm se} \equiv n_{\rm ese} = Z_{\rm i}n_{\rm ise}$ and the Bohm condition $V_{\rm ise} = c_{\rm s} = \sqrt{T_{\rm e}/m_{\rm i}}$ are employed.

The approximation coefficients h'_k ($k_{\max} = 3$) in (2.3) are expressed by the wall potential drop $\phi_{\rm w}$ and the sheath width $z_{\rm se}$:

$$h_0 = \phi_{\rm w}, \quad h_1 = -E_{\rm w}(\phi_{\rm w}), \quad h_2 = \frac{2}{z_{\rm se}}E_{\rm w}(\phi_{\rm w}) - \frac{3}{z_{\rm se}^2}\phi_{\rm w},$$

$$h_3 = -\frac{E_{\rm w}(\phi_{\rm w})}{z_{\rm se}^2} + \frac{2}{z_{\rm se}^3}\phi_{\rm w}, \quad \text{and} \quad h_k = 0 : k > 3. \tag{4.4}$$

These coefficients give the induced charge $Q_{\rm din}$ in (3.1), which is shown as a function of the dust radius $R_{\rm d}$ in Fig. 1 for the relatively shallower potential drop $-e\phi_{\rm w}/T_{\rm e} = 3.5$ and the deeper potential drop of 10.0. In this case the Debye sheath width is $z_{\rm se}/\lambda_{\rm Dse} = 8.0$, where $\lambda_{\rm Dse}$ is the electron Debye length at the sheath entrance. The stronger electric field at the wall, which is made by the deeper wall

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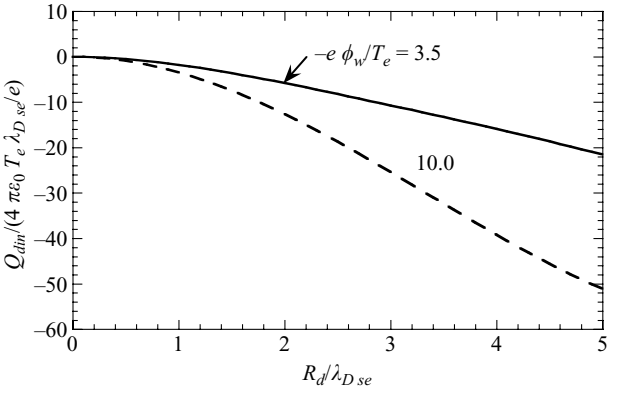


Figure 1. Induced dust charge as a function of the dust radius R_{d} for the cases of the potential drop $-e\phi_{\mathrm{w}}/T_{\mathrm{e}} = 3.5$ and 10.0, where the Debye sheath width is $8.0\lambda_{\mathrm{Dse}}$.

potential, the higher induced charge. In case of dust smaller than the Debye length, the effects of the potential drop and the electric field at the wall are small.

5. Conclusion

An induced charge of a spherical conducting dust particle on the plasma-facing wall was studied analytically. An external non-uniform electric field was approximated by the polynomial of the axial coordinate and the bipolar coordinate was introduced in order to employ the boundary condition on the dust surface. This analysis was applicable to the non-uniform electric field in the Debye sheath in front of the conducting wall. This theoretical result can be compared to the result of the particle simulation [3], where both induced charge and plasma shielding are included. These results are useful to understand the dust behavior in the boundary plasma [4].

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