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FOUR BASIC LOGICAL ISSUES

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Abstract. The paper addresses what we see as the four major issues in logic. The overriding issue is that of the choice of logic. We start with some discussion of the preliminary issue of whether there is such a 'one true logic,' but we reserve the main discussion for the first issue of 'classical logic versus non-classical logic.' Here, we discuss the role of meaning and truth, the relation between classical logic and classical negation, and whether and, if so, how classical logic should reside at the base world. Given the argument in favor of an overall use of non-classical logic, the second issue is that of the choice of non-classical logic. Brady's logic MC of meaning containment is argued for, with some comparison made with other relevant logics. For the remaining two issues, we make a case for relevant deduction, in comparison with classical deduction, and we explore possibilities for the appropriate meta-logic, comparing classical and non-classical approaches.

Among the diversity of logical issues, the most profound and fundamental concern the choice of the (inferential) logic itself, as almost all else flows from such a logic via its properties, its extensions, and its applications. However, there is also the *preliminary issue of whether there is such a 'one true logic' to be chosen*, or whether there is just a plethora of logics to be used for different types of applications, as argued recently by Beall & Restall (2000, 2001), under the heading of logical pluralism. Before setting out the issues dealing with the choice of logic, let us very briefly say something about this preliminary issue. The idea of a single logic is that through its conceptualization one should be able to see how it would apply in different contexts, thus avoiding the need for a plethora of logics. If such a conceptualization can be found, and we will be arguing for this below, then this would avoid what would be a complicated pooling together of a range of logics, each to be used in its appropriate circumstances.[1]

Note, though, it might be argued that as soon as you admit that logic can or should be revised, particularly on the basis of nonlogical (e.g., empirical) considerations, you are also embracing logical nonapriorism (see Bueno & Colyvan, 2004). That is, as soon as we talk of *choosing* a (or 'the') logic at all, we talk of defending or, in this case, replacing the logic currently in favor. The most obvious examples of such nonlogical considerations are empirical counter examples.

It is also possible to run an argument to the effect that logical nonapriorism leads to logical pluralism. The argument runs along the following lines: if we can claim of no one set of logical axioms or rules that they are certain, or a priori (i.e., if any logic is, in general,

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[1] Note that one can still tinker with the axiomatization of a logic to try to ensure that it precisely represents the intended conceptualization of the logic.

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open to scrutiny or vulnerable to empirical counter example), then this situation—nonapriorism—can be interpreted as logical pluralism: that what counts as a logical law should be changed according to the context or debate we're in, or according to the domain we're considering.[2]

Logical pluralism versus logical monism is, of course, a debate deserving more discussion than can be included here (more discussion is included in Brady & Rush, in preparation, specifically regarding monism for both pure and applied logics, as well as the problem of selecting specific criterion for a universal logic). The aim here is simply to suggest that the search for one true logic—or a universal logic—is indeed well founded.

Two brief points should suffice to support this. First: once we have established a new conceptualization, such as that mentioned above, there need be no further appeal to non-logical issues. That is, the use of empirical counter examples and arguments against a given logic, need not by themselves, constitute a commitment to the general applicability of such considerations to all logics. This position can be articulated and defended in a few different ways. For a thorough discussion of these, see Brady & Rush (in preparation). Notable among them is that inspired by Wright's (2004a), in which it is argued that we are, in some cases, *entitled* to our faith in some logical laws. The strategy Wright employs is able to be adapted to our ends, and offers a neat framework in which logical apriorism can sit alongside the use of empirical counter examples. If we can make a good case that a given set of axioms (e.g., those presented here) ought to, or in fact do, underlie deductive reasoning in general, or explicate deduction itself, then we can make a good case for considering these axioms to be true a priori (i.e., the burden can be shifted from the proponents—that is, from showing that they are axioms hold; to the opponents—that is, to showing how the assumption that they hold could be inadmissible or flawed). In short, if there is one true logic, then, once we've reason to believe that we've found it, we can consider ourselves entitled to our trust, even belief, in its axioms.[3]

Second: even if we considered all logics (including the one true logic) to be nonapriori, the potential step linking nonapriorism to pluralism seems avoidable. That is, a nonapriorist monist could argue that the particular goals inherent in the monist program are important and productive enough to establish this perspective as one worth 'fixing' so that, within this framework, logics properly 'compete' or can be judged according to the criterion established by the fixed aim that there can *only* be one logic; even while, within other (pluralist) frameworks, logics can properly coexist. Of course, this has the effect of shifting the logical monist versus pluralist question to something more like framework (or perspective) monism versus framework pluralism, but the question, 'is there one true perspective?' need not undermine the monist program, especially given that the goals fixed by the monist

[2] Bueno & Colyvan (2004, p. 157) briefly outline this line of reasoning as follows: "It has been argued that the variety of alternatives to classical logic – all of which seem adequate for at least some domains – presents problems for logical monism, so issues about theory choice in logic are not so much about which is the one true logic, but, rather, which logic is adequate for a given domain."

[3] The terminology here is yet to be fully worked out—Wright (2004b) extends his treatment of the issues, in which he also outlines some of the problems attendant to the basic concept of entitlement. Nonetheless, the notion is useful here to capture our belief in the axioms, which goes someway beyond a conviction that they are true, and shares something of our attitude toward the sorts of indispensable assumptions Wright outlines: namely, an attitude something like acceptance, or 'taking for granted', coupled, in the case of logical axioms, with an active belief in their truth.

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framework lead to fruitful enquiry. The search for a set of axioms applicable to the widest variety of contexts tends to yield intuitively acceptable principles whose application to other areas follows naturally. Quite different results occur when axioms are tailored to a particular application. In this case, the intuitive acceptability of the axioms is emphasized less than an often quite specific set of results.

Another way of putting the point is to say that a logic's overarching aim may be monist—that is, its aim may be to correctly express as many arguments as possible, across as many domains as possible, within its (single) expressive framework.

So, we think it is fair to proceed with the issues concerning the choice of logic, returning to the question of logical pluralism later.

This paper also serves the purpose of explaining in further philosophical detail, via the four issues below, why we have pursued a particular logical road, splintering somewhat between Brady's 'past' work in Brady (2006)[4] and Brady (1996), and his more recent work, especially in Brady (2007)[5] and to some extent in Brady's forthcoming, Brady & Rush, *Entailment Logic*. Also, in discussing the first issue, we will introduce three important sub-issues which will help explain the difference between his 'earlier' Brady (2006) and (Brady, 1996) and his later (Brady, 2007; forthcoming, Brady & Rush, *Entailment Logic*).

§1. The four issues. The first issue to deal with in the choice of logic is that between classical logic and nonclassical logic. Despite the arguments of people like Routley in chapter 1 of Routley *et al.* (1982) and many others over the years, classical logic is pervasive in the logical community and so this issue must still be addressed, and arguments additional to these fairly familiar arguments, will need to be found to convince the large bulk of logicians that there are problems with classical logic, in the first place, and that there are viable nonclassical renderings of its problem-free machinery, as well as that classical logic is no longer satisfactory, as a universal logic, given the considerations above.

A further related outstanding issue is that of the presence of a certain inconsistency here between what many logicians say in criticism of classical logic and their use of classical logic in their own work (e.g., Ellis, 1979, where an opportunity to use nonclassical logic was not taken up in the formalization of his rational belief systems). Classical logic has been and still is, a sort of 'default logic' in this sense. Noting this, we could render the search for a universal logic in just these terms, that is, a universal logic is a logic designed to be used across the board, including where classical logic has traditionally been the 'default option'. In this sense, then, in seeking to create a universal logic, we are seeking to create a new 'default logic', whose generality can be presupposed in just the same way that the generality of classical logic has traditionally (or by default) been presupposed.

The next issue is the determination of a particular nonclassical logic, in the light of such arguments. Here, there is quite an array of possibilities, but again the conceptualization of the logic is of paramount importance in choosing a particular logic over others. In accordance with this, we will indeed choose the logic MC based on meaning containment.

The third issue concerns the use of classical deduction of conclusions from premises, which is usually applied to logics even if they are nonclassical. There also seems to be a lack of consistency between the nature of the theorems of the logic on the one hand and the

[4] Brady, *Universal Logic*, was more or less completed in 1996, and is thus referred to in the past tense.

[5] Brady, "Entailment Logic—A Blueprint" is, as the title suggests, an overview of a position on a particular logic of entailment, which does not contain a lot of detail.

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deductions from assumptions on the other hand. We need to explore the use of a deduction system which behaves in accordance with the theorems of the chosen logic.

An important point to note here is that, in the logic MC presented here, the truth preserving rule $\Rightarrow$ is 'driven by' the meaning preserving connective $\rightarrow$. That is, $\Rightarrow$ is, in effect, applied meaning containment ($\rightarrow$). Another way of characterizing this would be to note that "one can correctly infer from a formula no more than its meaning allows" (Read, 1988, p. 173). Putting it another way, in MC, we cannot have truth preservation without a meaning connection. Or, more accurately, the formalized concept of truth preservation (if not always the rule) depends on the formalized concept of a meaning connection.

The last issue concerns the use of classical meta-logic, which is again used by virtually all logicians for logics whether they be classically based or not. So, once again, there seems to be a lack of consistency between the logic on the one hand and its meta-logic, or some aspect of it, on the other hand. There needs to be an investigation of nonclassical meta-logics which will serve the purpose.

These are surely the four fundamental issues concerning the choice of logic.

§2. Classical logic versus nonclassical logic.

Meaning and truth.

Dummett (1978, p. xxiii) claims that "no theory of meaning that is not based on... two valued semantics, and, in particular, no theory of meaning that involves the rejection of bivalence, can take the notion of truth as its fundamental notion."

Similarly, we could note that whenever the fundamental notion of a logic, or the primary notion upon which it is conceptualized, is something other than truth, then truth by necessity, gets conceptualized only relative to, or as a derivative of, the primary notion.

And again, that when truth is the primary notion of a logic, as is the case for classical logic, then that logic is critically impoverished, in that some crucial aspect (or aspects) of meaning is lost.

To make sense of these observations, we note first that one of the main conceptualizations behind deductive logic is the preservation of truth. But, we would argue that it is also something more than this. One of the principle conceptualizations behind deductive logic in general is the preservation of something like our *entitlement* outlined above. Deductive logic explores the process whereby we deduce from a set of supposed entitlements whatever conclusions preserve those entitlements. When it comes to formally expressing this aspect of the conceptualization, though, classical logic flounders. That is, for classical logic, the preservation of this sort of entitlement can amount to no more than the preservation of truth. The notion of classical validity—"if the premises are true then the conclusion is true also" (Brady, 2007, p. 110)—does not leave enough room for a full logical analysis of the role of the meanings of the terms and formulas involved, and so does not capture the sense in which a given set of entitled beliefs can contain further entitled beliefs: namely those we can deduce from the first.

The problem for classical logic is that this aspect of deductive reasoning seems to involve more than simple truth preservation. For one thing, entitlement or belief, as singular concepts, involve more than 'truth' as a singular concept can hope to convey. That is, even were we to claim that what we are entitled to take for granted is true, or what is true we can take for granted, it seems clear that the intuitive concept behind the notion of entitlement and that behind truth each convey quite different things. The natural question then becomes, apart from truth, what else is needed to express this conceptualization of

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deduction? The concept of meaning and of meaning containment is the obvious candidate, given that meanings are needed anyway to start assessing logical arguments.[6]

There is a close connection, perhaps to the extent of inextricability, between meaning and truth, such that it may even be the case that both (minimally) are needed to express not only the nature of deduction, but also truth itself. And even if we were to suppose that knowing the meaning of *p* necessarily involves knowing when *p* is true, this does not entail that knowing the truth conditions for *p* gives us its meaning. In fact, though, we do not grant the former, since, for example, truth-value gaps are meaningful. Noting this gives further weight to the notion that meaning is a primary, irreducible logical particle. And so truth conditions alone should not determine validity. Indeed, the more closely we examine the separation of truth from meaning (in this context), the more artificial and ad hoc such a separation appears. If the two concepts are (minimally) intertwined, our logic should reflect this.

All of this may still appear relatively unproblematic for classical logic. Note, particularly, that an emphasis on truth and a shunning of meaning can be seen as one of the driving forces behind the development, not only of classical logic, but of formalization in general. That meanings can be put to one side or expunged altogether has been perceived as a virtue of formal systems at least since the inception of modern classical logic.[7]

In fact, as Coffa (1991, p. 66) notes, "[Frege's] idea was to produce a language in which, even though inference was based on meaning, one need no longer think about meanings … since one could now restrict oneself to the signs 'present to the senses' and their symbolic correlations." The extreme end of this idea is, of course, Hilbert's formalism: "draining the expressions occurring within the system of all meaning [the idea being that] when a system has been completely formalized, the derivation of theorems from postulates is nothing more than the transformation (pursuant to rule) of one set of such 'strings' [the resultant meaningless expressions] into another set of 'strings"' (Nagel & Newman, 2001, pp. 25–26).

Formalism historically rejected the classical and contentual conceptions of proof, wherein "we know a thing best when we know it through its 'cause' … [which was] taken to reside in basic definitions and principles of construction" (Defletsen, 2005, p. 237]). In a sense, then, relevant logic, and in particular the logic MC presented here, can be seen as a move back to these earlier conceptions of proof. Nonetheless, there is something right about the formalist's conception. In accordance with its fixed 'universal' aim, the logic presented here takes what is right from both conceptions and incorporates each into a single logical framework (that MC successfully runs this middle line between the two conceptions is argued for in more depth in Brady & Rush, in preparation).

The first point to note in response to the formalist outlook outlined above is that, as mentioned, it seems that the very idea that one can refer to rules of use or to truth conditions, but not to meaning, is itself misguided. The one may not so easily be separated from the other. Putting the same point more metaphysically, note that this sort of observation

[6] Other strategies have been deployed. These include modalizing on meanings and using quantification over interpretations. These and other strategies will be compared with meaning containment in Brady & Rush (in preparation).

[7] And long before, but note too, that so has the desirability of an emphasis on meaning. It could be argued that Frege's classical logic, at least at its inception, did not deny the importance of meaning—as is the intended reading of the Coffa quote on Frege.

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provides one possible way in which it can be argued that truth is *dependent* on meaning.[8] Minimally, the basis of such a claim would be that there is more to the meaning of p than the truth conditions for p, and that meaning is 'logical' in Read's (1988, p. 172) sense, in that it "plays a substantive role in matters of consequence."

Of course, that truth may depend on meaning (among, perhaps, other items: e.g., the real world) is a fairly contentious claim. We note, though, that Read's observation above not only provides a basis for such a claim, but leads quite naturally to this and similar conclusions about the nature of truth. A less contentious version may be the familiar claim that 'rules of use' are an aspect of meaning (or even that they are all there is to meaning). Indeed, there is wide acceptance of (or of versions of) the slogan 'meaning is use'. But even this less contentious claim seems, in an important sense, to contain the former more contentious claim about truth, insofar as it is hard to imagine rules of use that do not involve truth conditions in some way.

Perhaps the best way to put it is that the way truth formally behaves is dependent on the way meaning formally behaves; that is (formal), truth is intensional. This in itself is not a particularly radical philosophical position—the importance of meaning, and its intimate connection with truth has been appreciated for some time (in fact, also since Frege). Indeed, the increasing awareness of the centrality of meaning to a host of philosophical problems is a key ingredient of the entire 'semantic tradition' (Coffa, 1991).

Thus, the admittance of meaning into our logical system, as a logical particle, can itself be viewed simply as the formalization of what has long been (informally) accepted. In this way, the logic MC can be seen as the logical 'fleshing out' of the formal role of meaning. So long as we grant the informal philosophical importance of meaning, the universality, expressive power, and applicability of MC should come as no great surprise.

The second point to note is that the neglect of the importance of meaning has led to all sorts of undesirable results. There has been much written on the demerits of classical logic. In particular, Routley, in §1.2 of Routley *et al.* (1982), gives a raft of examples (mostly due to Hunter, 1972) of arguments, all of which are classically valid but are all informally invalid.

For example, 'If you throw both switch A and switch B then the motor will start. Therefore, either if you throw switch A the motor will start, or if you throw switch B the motor will start.'

This argument has the shape of the classically valid argument,

$$(A\&B) \supset C \Rightarrow (A \supset C) \vee (B \supset C).$$

The informal inferences between the switches being thrown and the starting of the motor clearly involve more than just truth functionality and this suffices to yield the informal invalidity. Indeed, the inferences depend on the way the switches are connected together and to the motor. Further, if the switches are in series with the motor, the premises would be true, while the conclusion is false. Such empirical concerns should not be able to influence logical laws which apply as a matter of necessity. So, the above classically valid argument cannot be considered a logical law which would be expected to hold for all contexts.

It is surprising that most professional logicians have not taken heed of all this work. Indeed, many of Brady's students over the years have thought, independent of any solicitations from him, that there are problems with classical logic, particularly with the placement

[8] We are talking here about 'formal truth', that is, truth as it is captured by truth conditions, and need not, at this stage, commit to any further (e.g., Metaphysical) thesis.

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of the three *T*'s in the truth table for '⊃' and the lack of relevance between antecedent and consequent of some material implications and between premise(s) and conclusion of some classical arguments. The fallaciousness of the latter irrelevance is clearly so for these students as it accords with a group of informal fallacies of relevance, which they would have previously studied in their text, in chapter 3 of Hurley (2000).

Taking this relevance point further, the Relevance Condition (RC) was introduced by Anderson & Belnap (1975) in [ENT1] to distinguish relevant logics that satisfy (RC) from those that could be called irrelevant. For sentential logics L with at least the inference connective '→', the Relevance Condition states:

If $A \rightarrow B$ is a theorem of the logic L then A and B share a sentential variable.

Thus, irrelevant logics L include an inference $A \rightarrow B$, where A and B have no sentential variable in common, and hence a consequent can be inferred from an antecedent with no logical content whatsoever in common. The Relevance Condition acts as a necessary condition to filter out these logics.

Further, the foregoing considerations all suggest that the RC is a necessary condition on a good *universal* logic.

They also serve as an argument that the RC is a natural feature, not only of language, but of argumentation and of logical consequence itself. That is, the RC is a natural desideratum for any logic. This is highlighted both by students' awareness of the problem of irrelevance and by the significant amount of work already accomplished in the field of relevant logic. This in turn suggests that there is a natural relationship between meaning and consequence that ought not be abstracted, or expunged from a logic (i.e., that meaning must be kept as a logical particle; in particular, meaning cannot be considered *alongside* logic, it must be considered *as part of* logic itself).

Classical logic and classical negation.

And it seems that logicians in general do agree that there is some problem with classical entailment. In Brady's experience, the main sticking point for logicians is in fact not with the revision of classical entailment, but with the revision of classical negation. That is, it seems that many would prefer implication or entailment to be treated nonclassically (and this will be further dealt with in §3 when discussing the choice of nonclassical logic) but the nonclassical treatment of negation seems not, at least not at first glance, to preserve its natural or intuitive sense.

It turns out, though, that it is not possible, in general, for logicians to stick with classical negation and have nonclassical inferences; that is, it may be that classical negation and classical inference are inseparable. This is borne out by the following relation between classical negation and classical inference. By modifying an argument of Priest (2001, p. 14) given classical negation and its two key properties of the Law of Excluded Middle (LEM: $A \vee \sim A$) and Disjunctive Syllogism (DS: $\sim A, A \vee B \Rightarrow B$),[9] material implication can be used to represent inference, provided every rule in the inference is also available in its disjunctive form: If $A \Rightarrow B$ then $C \vee A \Rightarrow C \vee B$.[10] Specifically, if the inference '$A \Rightarrow B$' is assumed then $\sim A \vee B$ holds, and conversely. This is proved as follows:

[9] The DS is deductively equivalent to the disjunctive form of Explosion (Exp: $A, \sim A \Rightarrow B$), with help of the LEM. [The disjunctive form of Exp is: $C \vee A, C \vee \sim A \Rightarrow C \vee B$.]

[10] Note that the disjunctive form of Classical Modus Ponens can be readily proved using LEM and DS. Also, in relevant logic, the rules of logics with reduced Routley–Meyer semantics or with Priest–Restall simplified semantics all have disjunctive forms.

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$\underline{L \rightarrow R}$. Let '$A \Rightarrow B$' and let $\sim A \vee A$. Then, we place '$\sim A \vee$' in front of every step in the inference until $\sim A \vee B$ is established.

$\underline{R \rightarrow L}$. Let $\sim A \vee B$ and let A. Then, by Disjunctive Syllogism, B follows.

(A general result of this sort also appears in Brady (1993, pp. 581–584) as a Classical Deduction Theorem.)

So, in the contrapositive, if one wishes to reject material inference we will need to reject classical negation as well. Though the former rejection seems reasonable enough, for many logicians the latter rejection would be more difficult because it is hard to see what negation would mean other than the change of truth-value, as in the classical account.

The link between classical negation and classical inference seems to suggest that, if we grant that there are some fundamental or deep problems with classical inference, we in fact also grant (or ought to grant) that there are similar problems with the classical conception of negation. Perhaps the problems with classical entailment are one manifestation of the problems with classical negation, which, due to the perceived intuitiveness of the latter, we might not otherwise have uncovered. That is, we *could* see the classical entailment problems as problems not just for classical entailment but for the other concepts necessarily involved in its conception. Of course, we *need* not see it this way, but if we do, then the problems for classical entailment, or more positively, the various solutions put forward (such as the RC and the logic of Meaning Containment itself) might be understood as new, perhaps necessary, ways of conceptualizing these other concepts that may prove more powerful, reasonable, and indeed more natural than the original, classical rendition—especially so if the RC is at least as natural and intuitive a feature of consequence as classical negation initially appears in general.

But before exploring such a possible alternative conception of negation, note that the connection between classical negation and material inference is further solidified by the fact that there is also an easy semantic argument that shows that one can build classical logic on top of classical negation. We do this in a four-valued background where, initially at least, formulae can be both true (T) and false (F) and neither true nor false, as well as taking a single truth-value. The usual classical truth conditions are used for both truth and falsity, as occurs in Dunn's (1976) semantics for E_{fde}:

(i) $I(\sim A) = T$ iff $I(A) = F$.
$I(\sim A) = F$ iff $I(A) = T$.

(ii) $I(A\&B) = T$ iff $I(A) = T$ and $I(B) = T$.
$I(A\&B) = F$ iff $I(A) = F$ or $I(B) = F$.

(iii) $I(A \vee B) = T$ iff $I(A) = T$ or $I(B) = T$.
$I(A \vee B) = F$ iff $I(A) = F$ and $I(B) = F$.

We let A and B be classically evaluated, that is, $I(A)$ and $I(B)$ each take only one value, that is, T or F. Then, it clearly follows that $\sim A$, $A\&B$, and $A \vee B$ are also classically evaluated.

Before proceeding, let us reiterate the point and look at another semantic argument that lends further support to our above proof theoretic argument relating classical negation with classical inference. We consider the Routley–Meyer semantics, which not only applies to relevant logics but to many irrelevant logics as well. Indeed, most nonclassical logics can be captured within its ambit (e.g., see Brady, 1982, for some finite matrix logics). However, the Routley–Meyer semantics employs a nonclassical negation in that it allows both $I(A, a) = T$ and $I(\sim A, a) = T$, as well as $I(A, a) = F$ and $I(\sim A, a) = F$, for worlds a.

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Taking this argument further in the contrapositive direction, if classical negation is allowed at every world a, then this validates the formulae, $p \to q \vee \sim q$ and $p \& \sim p \to q$, both of which fail the condition (RC) (they both hold in the CL and KL systems of Routley *et al.* (1982), in their respective forms). Indeed, to invalidate these two formulae, one must allow q and $\sim q$ to be both false and p and $\sim p$ to be both true, that is, both nonclassical valuations of '$\sim$' must be included in the semantics, as happens with the Routley–Meyer semantics.

Classicality at the base world.

The discussion of classical negation leads naturally to the problem of preserving classicality in general. There seems to be little controversy regarding the desirability of preserving at least some aspects of classical logic. In the above semantics, for instance, nonclassical valuations may not be needed at all worlds, and one might still make a reasonable case for classicality at the base world, where validity is determined. This would let in the LEM and the DS as these, by not containing '$\to$', only impact on the base world (and its *-world). Indeed, a relevant logic with both the LEM and the DS is Ackermann's (1956) system Π'.

So, with all these arguments in mind, it is worth discussing the issue of classical negation and with it the issue of classicality in general, at greater depth, focusing on the base world. To do this, we first discuss the following basic *subissue concerning what deductive logic is all about*. Is it about the physical world or based on the physical world, as Quine[11] and Passmore[12] would have us believe, or does it apply to arguments wherever one finds them, as would appear to be the case in most textbooks, including Hurley (2000) and Priest (2001)?

Let us examine the *first alternative*. This would involve restricting the application of logic to the physical world, reasonably with inclusion of abstractions to deal with concepts based on the physical world, and with possible inclusion of idealizations to deal with arithmetic, set theory, meta-theory, and the like. We take it that the general idea of the proponents would be that classical logic itself is abstracted from the physical world due to the fact that statements concerning the physical world are all negation consistent and negation complete.

In an important sense, then, restricting logic to the physical world involves restricting it to an application. This is opposed to the general motivation behind the creation of a 'universal' logic, whose scope of application is to be kept as wide as possible. As noted earlier, formalism can be seen as one approach to this desideratum for universality (or for the universality of entitled deduction) in that 'emptying' the formalisms of meaning, allows an argument to be valid in virtue of its form alone, rather than by reference to any possible subject matter. The logic of meaning containment similarly abstracts away from all possible subject matter, but without sacrificing the logical importance of meaning. That is, without needing to specify particular (applied) contents (meanings), it captures the way contents (meanings) behave *in general*.

In general, in a universal logic, there should be no empirical constraints on meaning nor on truth, but there should be a 'meaning constraint' on truth and on valid deduction

[11] Recall Quine's maxim, 'To be is to be the value of a bound variable', which rules out logic applying to fictional contexts and to idle theorizing.

[12] Passmore, in conversation with Brady in 1978, said that he thought logic was a physical science, as opposed to Brady's then view of logic as a social science.

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sufficient to express the conceptualization of deduction as preservation, not only of truth, but also of entitlement.

But besides all this, there is, as we shall see, no guarantee that even the physical world satisfies negation consistency and negation completeness. It turns out that the belief that it does may be a significant philosophical commitment.

Proponents of the notion that the physical world is the proper domain of logic may engage one of two strategies at this point: (1) stop at abstractions, or (2) include idealizations, especially infinite ones, in the proposed domain as well. A concern for the *first strategy* is that just how one could exclude idealizations is unclear: idealizations are certainly part of the physical world (or of descriptions of the physical world) and of our reasoning about extensions of the physical world, and there seems to be no way to translate them—that is, to reexpress what idealizations express some other way, for example, finitely. The problem here is analogous to the mathematical constructivist's problem when deciding how or whether to represent the higher reaches of set theory, specifically the infinities of 'Cantor's paradise'. It seems that on this general strategy, 'groundedness'—the insistence upon a tie or link between our experience, or intuition, or understanding, and so forth, and whatever it is that logic and mathematics can apply to—inevitably disastrously restricts our logic's or mathematics' expressive power. That is: this is a problem not only for mathematics but also for the meta-theory of formal logical systems.

Moreover, if the logic is classical, then there is a further difficulty with Heisenberg's Uncertainty Principle, which states that, for some particles, either their position or their momentum cannot be determined. This goes to show that the physical world, at least at its edges, is indeterminate, upsetting negation completeness as far as physical determination goes (see Bueno & Colyvan, 2004, for a discussion of the impact on classical logic). Thus, any abstraction of classical logic from the physical world may need to be restricted to its core aspects.

And, of course, even this may not suffice. There is, it seems, no simple delineation between classicality and nonclassicality. While classicality may apply to the physical world, it is not adequate to describe the outer reaches of the universe. Because of the time taken for light to reach the earth, we cannot even know what is happening there *now.* Thus, we cannot describe the outer reaches of the universe and this gives rise to negation incompleteness in a similar way to that for Heisenberg's Uncertainty Principle, that is, there is a physical impediment involved. Also, as Mares points out, classical logic is not able to capture even relatively mundane aspects of the physical world (or, at least, of our description of the physical world). Specifically, the classical theory of negation cannot accommodate Mares' partially defined and overdefined predicates. Mares gives examples of such predicates along with an argument that they are irreducibly part of our language and of our understanding of the physical world. Which may mean that the way that we understand the world is irreducibly paraconsistent (or at least nonclassical), even in fairly ordinary cases (an example from Mares, 2004, is the predicate 'is a shadow'). So, it is unclear just how much could be gained by such finite reductions anyway, since even the (finite) physical world seems not so well behaved as (classical) logic would require.

Sharpening what's at issue here, note that there is a subtext argument present to the effect that 'if logic is all about the physical world, then logic ought to be classical'. Responding to this, we note first that a universal logic is, clearly, not about the physical world. It is about argument and deduction in general, wherever they occur. Second, when we come to apply logic to the physical world, we discover that there is no definitive guide enabling the separation of subjects into, for example, those to which the law of excluded middle applies, and those to which it does not. While some general assumptions regarding its use

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seem reasonable; such as the assumption that the LEM holds for some 'core' aspects of the physical world, we cannot regard such assumptions as anything stronger than just that: assumptions. The question of whether a given subject behaves classically or not, then, is a contingent issue, requiring a justification separate from that enjoyed by the (pure) logical axioms and rules.

For the *second strategy*, there is less general difficulty as most of what one might want to reason about is included. However, even this strategy falls short of a full accommodation of all we might want to express. That is, for classical logic, Gödel's Incompleteness Theorem similarly impacts on the second strand, as it deals with the infinity of arithmetic, again upsetting negation completeness, given negation consistency, as far as deduction is concerned.

All of the above considerations seem to point to the same conclusion, namely that classical logic is definitely limited in its application and hence is ruled out as a possible universal logic. Surely, in the context of the problem set at the beginning of this paper—that of finding one logic, to be applied everywhere—a logic that is more generally applicable is to be preferred over one whose reach has definite limits.

In contrast, we will maintain the alignment of provability and truth, following this into the four-valued logic of the second alternative. Before looking at the four-valued alternative, though, note that the preservation of an alignment of truth with proof is another expression of the general relationship of truth with meaning outlined earlier. That is, the idea that logical or deductive entitlement is best ensured when meaning, as a logical component, 'carries' or enables the derivation itself, leads naturally to the notion that truth preservation is not all there is to logical entitlement. And this latter idea is supported by the alignment of truth and provability: entitlement, deduction, and truth are inescapably tied to meaning, and no one of these concepts can overreach the others.

Let us examine the *second alternative*. Recall that this alternative takes logic to be applicable to arguments in the most general sense—'wherever one finds them'. This generally removes the restriction to the physical world, and allows logic to apply to any context at all, real or imagined, or even to nonsense expressions. Moreover, reasoning can still take place in imagined scenarios. In a fictional context, inferences could still be appropriately drawn concerning a character who might, say, use disjunctive syllogism to choose which road to take at a fork. Such an inference could be justified on the grounds that the setting of the story is in a physical world largely similar to ours, where such an inference would be justified by mapping into the core physical world.

In textbooks such as Hurley (2000), where classical logic is studied, it is emphasized that the logical form of an argument is the sole determinant for the validity of that argument. On p. 57, nonsense terms such as adlers, bobkins, and crockers are used as examples of what can appear in such valid arguments (Hurley drops this in his eighth edition in 2003). Making some sense of this, we would have to rely on the meanings of the logical connectives and quantifiers to achieve validity, regardless of what the nonlogical terms might mean. That is, the terms 'adlers', 'bobkins', and 'crockers' can be given arbitrary referents, as in van Fraassen's (1966) supervaluations. This is sensible as vagueness is something we should be able to deal with and the supervaluation approach does not involve changing the underlying logic.

Note too that this approach does not take 'nonsense' as 'nonmeaning', and that this stance is of a piece with the notion of a universal logic itself. A comparison with the formalist approach may be useful here. Meaninglessness, according to the theory presented here, is a much more slippery phenomenon than the formalist's approach seems to indicate. That is, a noncontentual term may not be as easily come by as it may initially appear.

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Moreover, there are a great many terms with undeniable significance—for example, 'quark', 'infinity', even 'set', whose exact reference we'd be hard pressed to pin down. Nonetheless, they clearly participate in meaningful sentences, and a universal logic should be able to deal with such terms, that is, to represent the meaningful sentences in which they occur, *whatever the exact meanings of these sentences are*. The inclusion of nonsense terms is simply the extreme end of this reasonable requirement.

The requirement also serves to highlight the fact that meaning in our logic is an abstracted or formal item itself. Accordingly, an uninterpretable term is not a meaningless term, and (even formalist) symbolic reasoning needs to take this into account. Meaninglessness occurs only in the case of a term occurring outside the scope of any predicate, and this is not a phenomena easily generalized upon.

Taking on at least one important aspect of the formalist's program, though, note that we do not intend to fully represent meaning as it is known preformally, or discussed in theories of meaning, or even as it is understood in everyday use. Rather, we formalize those aspects of meaning that "play a key role in the determination of validity and in the relation of terms, relations, and sentences" (Brady, 2006, p. 12). That is, it is only the logical or formal role of meaning in deductions that we need capture here.[13]

Further, Priest (2001, pp. 125–131) argues in favor of four-valued logic. Priest argues that the truth could well be negation inconsistent or negation incomplete. He gives examples of negation inconsistency (or gluts) in the framing of laws and in the paradoxes of self-reference, and examples of negation incompleteness (or gaps) in denotation failure and future contingents (however, his handling of the paradoxes of self-reference differs from our account). It appears to Brady too, that the Gödel sentence G of formal arithmetic is in a stronger position to be a truth-value gap than a future contingent event, which might be decided as true or false at some stage. G, on the other hand, would require a nonrecursive set of truths to make it true on the standard account of truth.

There are two strategies here as well: (1) the two-sorted approach as in Brady's 'past' work in Brady (2006), where he had classical sentences applying to the physical world or whatever can be argued to be classical, or (2) the one-sorted approach as in his more recent work in Brady's (2007; forthcoming, Brady & Rush, *Entailment Logic*), dubbed the Metacomplete Way (see also Brady (forthcoming)).

Let us examine the *first strategy*. In Brady (2006), Brady distinguished classical formulae in a two-sorted system by adding primes to the symbols for formulae. Thus, A', B', C', ... represent classical formulae, while A, B, C, ... represent formulae in general. Brady axiomatized classical formulae using the LEM and the DS, thus:

CA1. $A' \vee \sim A'$.

CR1. $\sim A'$, $A' \vee B \Rightarrow B$.

As in the first alternative, there is the issue of whether classicality applies to abstractions and idealizations on the physical world (see Brady, 1996), and Brady (2006), also, Brady (2007). Certainly, the two-sorted approach requires some domain of classicality for these classical formulae to apply to.

Here, there is a spin-off *subissue of a guaranteed domain of classicality*. Despite the above points about Heisenberg's Uncertainty Principle and Mares' shadows, it still appears

[13] Of course, our formalized 'meaning' ought to bear some relation to our everyday or informal notion of meaning, not least to our informal intuition that a meaning connection between premises and conclusion is an essential element of a valid deduction. We have already given some justification that our formalized meaning captures this element of everyday meaning, and that it captures key features of modern theories of meaning has likewise been argued for above.

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that descriptions concerning the large part of the physical world are classical. Apart from such descriptions, it would appear that classicality is guaranteed when there is a mapping of a described situation or theory into the large part of the physical world. Indeed, examples of Disjunctive Syllogism given by Burgess (1983) are of this type (see p. 46 for two examples). However, this leaves the issue of classicality as not that of finding a logical kind nor of finding a natural kind, as would seem to be required by the use of the classical sort in this context. Classicality would naturally be determined deductively, just by proving the LEM and the DS, as axiomatized above. This would be done in a case-by-case manner and leads us into the *second strategy*.

Before examining the second strategy, the classicality issue needs further comment. It seems clear that there are situations, claims, and general sentences that do fall neatly into one or other of the two categories suggested so far—classical and nonclassical. There are, for instance, the above two examples: the former clearly presenting a nonclassical scenario and so suggesting a nonclassical treatment, the latter clearly presenting a classical scenario and so suggesting the inclusion of the LEM and the DS, that is, a classical treatment. But we cannot assume every formula presents so obviously as one or the other. Nor, both for the reasons given above and because it is unlikely that the set of classical formulas would be consistent (see Brady, 2006, p. 166), should we *want* to specify two such sharply delineated domains. Rather, then, than attempting to provide a 'hands down' criterion for the division of formulas as classical and nonclassical, we suggest that 'classical' and 'nonclassical' be treated as two possible perspectives or interpretations on any formula. That is, for any formula A, we can treat A as if it was classical or we can treat A as if it was nonclassical. Which treatment is the most appropriate may not be easily resolved. But where, say, a nonclassical treatment gives a more coherent technical result than a classical treatment, we would, for practical purposes prefer the former interpretation over the latter. In sum, any valuation (i.e., classical or nonclassical) can be taken for a formula; but where appropriate, classical valuations can be ensured (via the use of the LEM or DS), and, (less commonly) wherever we have $A \leftrightarrow \sim A$, a nonclassical valuation is ensured.

So, let us examine the second strategy. With just the one sort, we still need to be able to derive the axiom CA1 and/or the rule CR1 of classical formulae when appropriate. The LEM, $A \vee \sim A$, is easily provable by Addition through one of its disjuncts.

In metacomplete logics, such as the logic MC below (see Slaney, 1984, 1987, on metacompleteness), the LEM is only provable this way, due to the Priming Property: 'if $\vdash A \vee B$ then either $\vdash A$ or $\vdash B$', and it is this which solves a number of paradoxes, such as the Liar and Russell's. In the latter, neither the self-membership of the Russell class $R \in R$ nor its negation $R \notin R$ is establishable within Naive Class Theory, as demonstrated by consistent models. Thus, due to the Priming Property, $R \in R \vee R \notin R$ is not derivable and, given $R \in R \leftrightarrow R \notin R$, nor is $R \in R \& R \notin R$ derivable.

Further, the Priming Property does not hold under entailment hypothesis or rule hypothesis, since, given $A \rightarrow B \vee C$, $A \rightarrow B$ or $A \rightarrow C$ does not generally follow, and similarly, given $A \Rightarrow B \vee C$, $A \Rightarrow B$, or $A \Rightarrow C$ does not generally follow (these can be seen by replacing A by $A\&(B \vee C)$, B by $A\&B$, and C by $A\&C$). It would seem that analytic disjunctions satisfy the Priming Property, where the meaning of the disjunction is foremost, thus breaking the disjunction up into the two disjuncts. Many empirical disjunctions will depend on background knowledge which will require a hypothesis, as above.

To complete this approach, we can use rejection to derive the other half of classicality, that is, the DS. Given the Priming Property: $\vdash A \vee B \Rightarrow \vdash A$ or $\vdash B$, and its rejection-form:

$\dashv A, \dashv B \Rightarrow \dashv A \vee B$ (where the comma means conjunction), then the uncontroversial form of the DS is: $\dashv A, \vdash A \vee B \Rightarrow \vdash B$. To obtain the classical DS, we would need the

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consistency of A in the form: $\vdash\!\sim A \Rightarrow \dashv A$. However, in such circumstances, there may well be a direct derivation of B in place of $A \vee B$, as one might expect using a metacomplete logic (there is more on metacomplete logics in §3).

Further, we can use rejection to express the classicality of a formula A in a single-sorted logic, as follows:

$\vdash A, \dashv\!\sim A$ would represent the classical truth of A.

$\vdash\!\sim A, \dashv A$ would represent the classical falsity of A.

This is especially useful for atomic sentences, which would replace use of the classical sort in the two-sorted approach.

Given this accommodation of classicality, and the nonpluralist perspective previously argued for, perhaps the initial characterization of this issue as a choice between classical and nonclassical logic is misleading, in that such a characterization suggests that the choice at hand is between domains or argument types. A universal logic aims to capture most of what we are able to express in classical logic while removing its problematic inferences, and to capture as many more arguments as possible besides. That is, given our aim of capturing arguments wherever we may find them, and noting that the universal domain of arguments includes most of those captured by classical logic, and more besides, the choice at hand is perhaps better described as between expressive frameworks rather than logical systems and what these individual systems can capture.

Again, if a nonclassical logic can be found which retains (most of) the expressive power of classical logic, while eliminating its problematic inferences and theorems, then the choice is better described as between expressive frameworks rather than between logical systems and 'their' domains. That is, where arguments in general are the domain with which we are concerned, those particular arguments captured by classical logic are no longer viewed as a particular (classical) domain: rather, they are simply a part of the universal domain of arguments.

A universal logic captures arguments according to an initial conceptualization (i.e., here a general, reasonable, restriction on what constitutes an argument: the content containment interpretation of the RC presented below) and according to the universal aim: to express as much of these arguments as possible. In this sense, a UL can be seen as a logic that 'collects' the real arguments representable in all and any logics, and captures as many of these as it can.

As has been observed elsewhere (e.g., Garfield, 2004) it is apparent that people do reason, and that arguments can be formed, in more ways than is captured by classical logic; for example, arguments can have inconsistent premises and conclusions. It is also clear that the classical paradoxes can be resolved (see Brady, 2006). Hence, from a universal perspective, MC is a more inclusive logic than classical logic. To recap, MC does not exclude most of the arguments that classical logic can capture and is able to accommodate inconsistency, incompleteness, and classicality, and so MC is the more obvious choice, a more detailed case being made for it in the next section.

§3. The choice of nonclassical logic. Given the argument in §2, it is fair to say that a nonclassical logic is needed, and hence a nonclassical negation is needed for worlds in general and, to capture arguments as we find them, for the base world in particular. Classicality should then be introduced using the one-sorted rather than the two-sorted approach. Also, as in §2, the Relevance Condition helps to pin down a suitable logic. However, there is still a big range of relevant logics, going from Routley and Meyer's basic system B through to the system R of relevant implication. Systems E and weaker are considered entailments, while RW and stronger are considered implications.

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However, with stronger relevant logics such as R, there are some problems, a number of which are set out in Brady (1996). It appears that the main problem is, in fact, the lack of a purely positive implicational concept. That is, R is generally understood in a positive, plus a negative, way, that is, as classical logic but without irrelevant implications. So, R is generally formalized as an implicational substitute for classical logic, but stronger than an entailment. This is difficult conceptually as it is somewhere in between an entailment concept and that of classical logic. Intuitively, a universal deductive logic should be the expression of a clear central conceptualization. Compare R with intuitionist logic, which, in having a single, clear concept, positively posited, is the simpler logic (at least in this respect), and so is more accessible and indeed (in this respect) more universal, than R. Another problem with stronger logics such as R is the lack of decidability at the sentential level, which is unusual, as logics go, and demonstrates a deep technical complexity, despite the simple natural deduction systems these logics have.

In contrast, in Brady (1996) and Brady (2006), entailment is conceptualized, in a singular positive way, as meaning containment. Logical contents and ranges of atomic formulae are introduced, to which reasonable properties are added, which in turn enable these contents and ranges to extend to compound formulae. Entailment is introduced as content containment and the set theoretic properties of containment are reflected in those of entailment. The basic properties of the three connectives, negation, conjunction, and disjunction, are such as to interact with entailment, thus dropping the LEM and the DS, where no entailment is present. The resultant logic MC, standing for meaning containment, can accommodate the above approach to classicality and is axiomatized as follows:

MC.
Primitives: $\sim$, $\&$, $\vee$, $\rightarrow$.
Axioms.

1. $A \rightarrow A$.
2. $A\&B \rightarrow A$.
3. $A\&B \rightarrow B$.
4. $(A \rightarrow B)\&(A \rightarrow C) \rightarrow .A \rightarrow B\&C$.
5. $A \rightarrow A \vee B$.
6. $B \rightarrow A \vee B$.
7. $(A \rightarrow C)\&(B \rightarrow C) \rightarrow .A \vee B \rightarrow C$.
8. $A\&(B \vee C) \rightarrow (A\&B) \vee (A\&C)$
9. $\sim\sim A \rightarrow A$.
10. $A \rightarrow \sim B \rightarrow .B \rightarrow \sim A$.
11. $(A \rightarrow B)\&(B \rightarrow C) \rightarrow .A \rightarrow C$.

Rules.

1. $A, A \rightarrow B \Rightarrow B$.
2. $A, B \Rightarrow A\&B$.
3. $A \rightarrow B, C \rightarrow D \Rightarrow B \rightarrow C \rightarrow .\ A \rightarrow D$.

Meta-rule.

1. If $A \Rightarrow B$ then $C \vee A \Rightarrow C \vee B$.

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Then, soundness and completeness theorems are proved for the logic MC with respect to the above content semantics. As can be seen in Brady (1996) and Brady (2006), the above axiomatization is very largely pinned down by the content semantics. Each of the axioms, rules, and meta-rule are shown to satisfy the meaning considerations, expressed as logical contents. Further, a number of key nontheorems, such as $A \rightarrow B \rightarrow .B \rightarrow C \rightarrow .A \rightarrow C$; $A \rightarrow .A \rightarrow B \rightarrow B$; $A \rightarrow \sim A \rightarrow \sim A$; $A\&(A \rightarrow B) \rightarrow B$, are shown to fail the content containment requirements.

Let us examine some features of the logic MC. Firstly, MC is a metacomplete logic, satisfying the Priming Property. As can be seen from Slaney (1984, 1987), such logics emphasize entailments as the primary logical theorems, as is the above focus (one should note that the example at the beginning of §2 would yield theorems that would fail the Priming Property rather than the Relevance Condition). Secondly, the logic MC provides a basis for the solution of the set theoretic and semantic paradoxes, as set out in Brady (2000). This is due to the simple consistency of naive set theory, consisting of the Axioms of Comprehension and Extensionality, and proved in Brady (2006), which is based on a previous proof for the logic TW in Brady (1983).

Noting the comprehensive, indeed universal applications and uses of the logic MC of meaning containment, its accommodation of classicality at the base world, and given the concerns about the use of the logic R of relevant implication, it makes sense to raise the question: 'What else is there?'. The two basic semantic concepts of meaning and truth are accounted for in MC and classicality. Anything else would seem to be derivative upon these logics. Indeed, an enthymematic version of MC is obtainable by defining its implication $A \mapsto B$ as $A\&t \rightarrow B$, where t is understood as the conjunction of all truths and axiomatized by the two-way rule: $A \Leftrightarrow t \rightarrow A$. Further, there is no need for a separate necessitated entailment as $\Box(A \rightarrow B) \leftrightarrow .A \rightarrow B$ would be a theorem of any modal extension of MC. Given all this, there seems to be no room for any further primitive inferences and thus no need for a plethora of logics, in accordance with Beall and Restall's logical pluralism.

Further, the Relevance Condition and the logic MC capture a fundamental part of the preformal notion of deduction as preservation of entitlement—as mentioned earlier, this much is evidenced both by the history of relevant logic: notably by its persistence and the increasing recognition of its value and application; and by such examples of intuitive content as the student's common objections to 'explosion'. Thus, all of the informal or natural concepts constituting consequence itself are likewise fully acknowledged and represented in MC.

Against these monist claims, though, Bueno (2002, p. 548) argues that "in consistent contexts [paraconsistent] and classical logic yield the same results … [and that this means that] … [both] such logics are equally adequate to deal with consistent domains [so pluralism results]."

The key to addressing such an argument is to note that we need not wholeheartedly, or unrestrictedly agree with the first half of the argument. That is, while it is true that some paraconsistent logics yield the same results as classical logic (i.e., that classical logic is contained in some paraconsistent logics), not all paraconsistent or nonclassical logics can be described this way. A more accurate description, in the case of MC, would be to say that, in consistent contexts, MC and classical logic yield *much* the same results, insofar as they capture the same entities, where these entities are correctly definable (according to MC's conceptualization) as arguments. Now the second half of Bueno's argument need not follow. Also, while it may be the case that two logics both deal adequately with the one

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domain, in the technical sense of yielding the same results, it may still be the case that only one of these logics is the *best* expression of the entities of that one domain.

There is an alternative picture here: rather than two logics, each dealing adequately with one domain, a better understanding of the situation may be to see one logic, in two stages of proof theoretic refinement. Thus, the net is cast once, then again with more finesse or control, but it is nonetheless, the *same net* (i.e., according to this understanding, classical logic is better described as *subsumed* rather than *contained*).

This offers an alternative monist story on the undeniable presence of 'other' logics: rather than presenting each of these as properly competing, this (above) scenario presents the competition as apparent only—as a series of refinements on the whole. Note that either one of these pictures suffices to motivate a Universal Logic: we look for one true logic either because there *is* only one logic, or we look for one true logic because only one logic (of the many) is the *true* logic. Whichever way we look at it, the notion of a universal logic remains well motivated.

This line of thought also addresses Bueno's (2002, p. 548) more generalized argument, that: "To the extent that in consistent domains paraconsistent logics agree with classical logic, a substantial form of pluralism will immediately emerge." Again, the agreement claimed here may only extend to the sharing of results. Recall the scenario wherein the same result or the same argument is expressed in two entirely different ways (or captured according to two quite different initial conceptualizations). The characterization of such a scenario as of two logics 'in agreement' is somewhat unnatural, if not entirely inappropriate.

§4. Classical deduction versus relevant deduction. Given the arguments in §1 and §2, one would expect the relevance of the logic to carry through to deductions and the definition of a valid deductive argument. To see that it does, we need to distinguish between classical deduction and relevant deduction. The textbook definition of a classically valid deductive argument is generally that of truth preservation in the base world of a semantics, for all interpretations of the variables involved. If there is only one world, as in the case of classical sentential logic, truth preservation applies to it. If there is no base world and an arbitrary number of worlds, then truth preservation applies to each world.

All these definitions allow validity of the argument to hold in the case where the conclusion, by itself, is valid, where validity here is defined as above for arguments but with no premises. In such a case, the premises are immaterial in the determination of validity. Similarly, any argument from inconsistent premises is valid, independently of the conclusion. Both of these are incongruous with a relevant logic. Indeed, the same sort of relevance argument that applies to implications and entailments should extend to arguments from premises to conclusion. One would not expect meaning containment of a conclusion in the premises, but there should be some relevant connection between them. Indeed, one would expect such a relevant connection to be due to the inferences between the steps used in getting from the premises to the conclusion, so that the inference is determined through the use of meaning containment, as earlier discussed in §2. This brings us to a proof theoretic approach.

For classical deduction, the conclusion can follow from premises that are unrelated to it, but for relevant deduction we would expect, as a minimal requirement, that at least one premise is used, in an essential way, in the derivation of the conclusion. We take this as a definition of *relevant deduction*. This concept has been discussed in chapter 2 of Routley *et al.* (1982) and set up in a Fitch-style natural deduction system by Batens &

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van Bendegem (1985). A similar version also appears in chapter 12 of Brady (2003), the author being unaware of the earlier work. A special subscript 0 is used to trace the effect of the premises in the derivation of the conclusion.

We simply end up with relevant deduction, where at least one premise is used, as distinct from theorem derivation, where no premises are used. However, there are three different types of relevant deduction, developed in part II of Brady (forthcoming, Brady & Rush, *Entailment Logic*), the first and most general type being only restricted in the application of the $T\&I$ and $F\vee I$ rules. In application, relevant deduction is a suitable substitute for many uses of material implication. Other deductive concepts have also been defined in Brady (1993), relating to the usage of specific rules in the derivation and yielding a variety of Deduction Theorems.

§5. Classical meta-logic versus nonclassical meta-logic. Given the argument of §4, it would also seem sensible for the meta-logic to reflect the logic of the object language. This is in spite of the fact that meta-logical reasoning can be regimented in classical logic, even when the object logic is nonclassical. There are two major points on this issue that have been made:

(1) Intensional logic requires an intensional semantics.

This has been strongly argued by Meyer & Routley (1977) and in the appendix of Brady (2003), rejecting extensional reduction in favor of a semantics that more directly captures the meaning of the object logic. But, what nonclassical meta-logic is appropriate? Does this mean that it is the same logic as that of the object language? This leads us to the second point.

(2) Trivial semantics (or toil-less semantics) is just that.

Meyer and Sylvan (né Routley) have also made this point, in conversation. Here, such a semantics interprets each connective according to the logic of the object language, as follows:

$I(p)$ is assigned T or not assigned T, for each p.
$I(\sim A) = T$ iff not $-(I(A) = T)$.
$I(A\&B) = T$ iff $I(A) = T$ and $I(B) = T$.
$I(A\vee B) = T$ iff $I(A) = T$ or $I(B) = T$.
$I(A \rightarrow B) = T$ iff, if $I(A) = T$ then $I(B) = T$.

A formula A is valid iff $I(A) = T$, for all assignments to its variables.

The Soundness and Completeness Theorems are then trivial. For completeness, for a nontheorem N, $I(N) = T$ would fail as a result of the failure of the corresponding meta-logical law, applied to totally unconstrained assignments to its variables.

There seems to be a paradox here, which needs resolution. In order to do this, we need to find a meta-logic which is something of a compromise between mimicking the logic of the object language and using the standard classical meta-logic.

Perhaps we can answer this question best by using the distinction between *interpretational and underlying meta-logic*, made by Brady in chapter 13 of Brady (2003), where the interpretational meta-logic interprets the connectives and quantifiers in an appropriately nonclassical way, while the underlying meta-logic interprets the logic of the meta-linguistic infrastructure supporting the interpretation of the connectives and quantifiers. This includes the provability of formulae and the truth and falsity of assignments to formulae under interpretation. Provability statements are classical in a decidable logic such as MC, but this would not extend to some statements in an undecidable logic, neither the provability

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nor unprovability of which is establishable in a finite manner.[14] Assignment statements, however, are all classical as they are inductively stipulated.

We examine the content semantics of Brady (1996) and Brady (2006) and the free semantics of Brady (accepted subject to revision), which provide examples of differing approaches to meta-logic. In the content semantics, as in algebraic semantics, each connective is interpreted as a two-place or one-place function on the domain of algebraic elements. In the Lindenbaum canonical model, the elements are logically equivalent sets of formulae, which in the content semantics are taken to be logical contents. So, there is no interpretational meta-logic as it is subsumed into the relations between the elements or sets. This leaves the meta-logic as wholly underlying and classical. And such classicality can be added to the logic MC by the addition of the LEM and the DS, both taken as assumptions for this purpose.

However, in the free semantics, the interpretational meta-logic is embodied in the choice of natural deduction system which forms the basis of the semantics. This has the effect of sameness for the interpretational meta-logic, while the underlying meta-logic is classical. However, unlike the trivial semantics, quite a deal of work is required to prove soundness and completeness for free semantics, which importantly has the advantage of providing a usable method of invalidating nontheorems as well as validating theorems (see Brady accepted subject to revision). Thus, this proof theoretic semantics will provide much of the technical results one needs for the logic MC, while the content semantics provides the real semantics, pinning down the logic in relation to other logics, as can be seen in chapter 1 of Brady (2006).

The foregoing all points to the further development and application of the logic MC(Q), including relevant deduction and meta-theory. Some of this work has been undertaken in work in preparation, including discussions on what's wrong with Cantor's Diagonal Argument in set theory, which also impacts on Peano Arithmetic. Finally, MC(Q) will be applied on a much broader front in the book *Entailment Logic: Theory and Application*, Brady & Rush (in preparation).

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[14] Brady owes thanks to Graham Priest for pressing him on the possibility of nonclassicality in the underlying meta-logic.

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