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Order and chaos in the magnetic electron drift vortex mode

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Ordered structures as well as chaotic motion of the magnetic electron drift vortex (MEDV) mode in a unmagnetized nonuniform plasma are discussed. In the collisionless limit, the nonlinear MEDV waves can self-organize into dipolar vortices or vortex chains. The latter exist in the regime where dipolar vortices are forbidden. When dissipation is included, the governing equations for the MEDV mode can be cast into the form of the Lorenz equations, so that chaotic solutions can exist.

It is well known (Jones 1983, 1988; Yu and Stenflo 1985; Stenflo and Yu 1986; and Yu and Xiao 1987) that a non-uniform electron plasma with stationary ion background supports magnetic electron drift vortex (MEDV) modes. The latter are driven by density and temperature gradients but are not associated with density perturbations. The nonlinear equations (Nycander *et al.* 1987) that govern weakly interacting MEDV waves admit stationary solutions describing dipolar (Shukla 1991; Petviashvili and Pokhotelov 1992) and monopolar (Shukla 1989; Nycander and Pavlenko 1991) vortices as well as vortex chains (Stenflo 1994). In this paper, we present a new stationary MEDV structure and show that deterministic chaos can occur in MEDV wave dynamics.

The nonlinear propagation of the MEDV mode in a dissipative medium is governed by (Yu and Xiao 1987; Nycander et al. 1987; Pavlenko and Uby 1993)

$$(1-\lambda^2\nabla^2)\partial_t B = \lambda^4 J(B,\nabla^2 B) - \frac{1}{mn_0}J(n_0,T_0+T_1) - \frac{\eta_0}{mn_0}\lambda^2\nabla^4 B + 0.51\nu\lambda^2\nabla^2 B\,, \tag{1}$$

$$(\partial_t - D\nabla^2)T_1 + \lambda^2 J(B,T_0+T_1) + \frac{2T_0}{3n_0}\lambda^2 J(n_0,B) = 0\,, \tag{2}$$

where $\mathbf{B} \equiv eB(x,y)\hat{\mathbf{z}}/mc$ (with m and e the electron mass and charge magnitude, and c the speed of light), $\lambda \equiv c/\omega_{pe}$ is the collisionless electron skin depth, ω_{pe} is the electron plasma frequency, $n_0(x)$ and $T_0(x)$ are the unperturbed electron number density and temperature respectively, ν is the electron–ion collision frequency, $\eta_0 = 0.73n_0T_0/\nu$ is the viscosity, $D = 3.16T_0/m\nu$ is the diffusion coefficient, and T_1

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($\ll T_0$) is the temperature perturbation. The Jacobian $J(f,g)$ is defined by $J(f,g) = (\boldsymbol{\nabla} f \times \boldsymbol{\nabla} g)\cdot \hat{\mathbf{z}}$.

We first consider the dissipation-free case where the characteristic time scale of the MEDV mode is much shorter than the electron collision and diffusion rates. Ignoring ν and D, we rewrite (1) and (2) in the forms (Pavlenko and Uby 1993)

$$\partial_t q + J(B,q) = -\alpha \partial_y T_1 \,, \tag{3}$$

$$\partial_t T_1 + J(B,T_1) = -\beta \partial_y B \,, \tag{4}$$

where $q = B - \nabla^2 B$ is the vorticity, $\alpha = d_x \ln n_0$ is the density inhomogeneity and $\beta = \frac{2}{3}\alpha - d_x \ln T_0$. Here the temperature perturbation T_1 is in units of $1/m\lambda^2$ and the space variables are in units of λ. The system (3), (4) is similar to the Navier–Stokes system that often appears in fluid and plasma theories (Shukla *et al.* 1995; Shepherd 1990).

The dispersion relation of the MEDV mode can be obtained from (3) and (4) by neglecting the nonlinear terms and assuming that $B(x,y,t)$ and $T_1(x,y,t)$ are of the form $\exp[i(k_x x + k_y y - \omega t)]$, where ω is the wave frequency and $\mathbf{k}$ is the wave vector. When the scale length of the equilibrium inhomogeneities is much larger than the wavelength $2\pi/|\mathbf{k}|$, we have (Yu and Stenflo 1985; Stenflo and Yu 1986)

$$\omega^2 = \frac{k_y^2 \alpha\beta}{1 + k_\perp^2}, \tag{5}$$

where $k_\perp^2 = k_x^2 + k_y^2$, and k_x and k_y are the components of $\mathbf{k}$ along the x and y axes respectively. Thus the linear system is unstable if $\alpha\beta < 0$.

We are interested in the possibility of quasistationary vortex solutions for $p \geqslant 0$. Accordingly, we introduce the moving coordinate $\xi = y - ut$, where u represents the speed of the vortex chain, and look for steady-state solutions in the ξ frame. Equations (3) and (4) can be combined to yield

$$D_\xi(\nabla_\perp^2 B - pB) = 0 \,, \tag{6}$$

where

$$D_\xi = \partial_\xi - \frac{1}{u}[(\partial_x B)\partial_\xi - (\partial_\xi B)\partial_x], \qquad p = 1 - \frac{\alpha\beta}{u^2}.$$

For $p > 0$, (6) yields the usual dipolar vortex solution (Yu and Xiao 1987). One can also obtain a modon-like solution by introducing

$$B = \begin{cases} u_1 x \cos(k_0 \xi) + u_2 x & (|\xi| < y_0), \\ u_3 x \exp(-\kappa|\xi|) & (|\xi| \geqslant y_0), \end{cases}$$

where u_1, u_2, u_3, k_0 and κ are constants (Stenflo 1996*a*, *b*). Here the continuity condition on B at $y = y_0$ leads to $1 + (\kappa/k_0)\tan(k_0 y_0) = 0$, which determines κ.

We now show that there also exists a stationary solution for $p = 0$ (or $u^2 = \alpha\beta$). For $d_x \ln n_0,\ d_x \ln T_0 \ll K$, (6) is satisfied by

$$\boldsymbol{\nabla}_\perp^2 B = \frac{4AK^2}{a^2} \exp\left[-\frac{2}{A}(B - ux)\right], \tag{7}$$

where A, K and a are constants. Equation (7) then yields

$$B = ux + A \ln\left[2\cosh Kx + 2\left(1 - \frac{1}{a^2}\right)^{1/2} \cos K\xi\right], \tag{8}$$

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for $a^2 > 1$. Equation (8) describes a vortex street or chain (Stenflo 1994; Pavlenko and Uby 1993; Shukla *et al.* 1995), with A characterizing the amplitude and $2\pi/K$ the size. Note that the vortex dimensions in the x and y directions are of the same order.

The stationary states discussed above and in the literature can be the result of self-organization from turbulence states associated with linear MEDV mode instabilities. We now show that when dissipation is included, chaotic behaviour can occur through bifurcations of finite-amplitude MEDV waves. Specifically, we show that (1) and (2) can be reduced to the Lorenz (1963) equations. The latter have been used to study deterministic chaos in atmospheric wave dynamics.

Accordingly, following Lorenz (1963) and Stenflo (1996*a*, *b*), we seek a class of solutions of the form

$$B = g(t)\sin(K_x x)\sin(K_y y)\ , \tag{9}$$

$$T_1 = h_1(t)\sin(K_x x)\cos(K_y y) - h_2(t)\sin(2K_x x), \tag{10}$$

where K_x and K_y are constants, and $g(t)$, $h_1(t)$ and $h_2(t)$ depend on time only. Substituting (9) and (10) into (1) and (2) and restricting ourselves to three-wave interaction (Stenflo 1996*a*, *b*), we obtain the system of ordinary differential equations

$$d_\tau X = -\sigma X + \sigma Y, \tag{11}$$

$$d_\tau Y = -Y + rX - XZ, \tag{12}$$

$$d_\tau Z = XY - bZ, \tag{13}$$

where

$$\sigma = \frac{C_2 + C_1 K_0^2}{D(1+\lambda^2 K_0^2)}, \qquad r = \frac{C_3 K_y^2 d_x n_0}{m n_0 D K_0^4 (C_2 + C_1 K_0^2)},$$

$$b = \frac{4K_x^2}{K_0^2}, \qquad K_0^2 = K_x^2 + K_y^2,$$

$$C_1 = \frac{\eta_0 \lambda^2}{m n_0}, \qquad C_2 = 0.51\nu\lambda^2, \qquad C_3 = \lambda^2\left(d_x T_0 - \frac{2T_0 d_x n_0}{3n_0}\right),$$

$$\tau = \frac{t}{t_0}, \qquad t_0 = \frac{1}{DK_0^2}.$$

Furthermore, we have introduced

$$g = \pm\frac{\sqrt{2}DK_0^2}{K_x K_y \lambda^2} X,$$

$$h_1 = \pm\frac{\sqrt{2}DK_0^4 m n_0 (C_2 + C_1 K_0^2)}{K_x K_y^2 \lambda^2 d_x n_0} Y,$$

$$h_2 = -\frac{DK_0^4 m n_0 (C_2 + C_1 K_0^2)}{K_x K_y^2 \lambda^2 d_x n_0} Z,$$

in obtaining (11)–(13), which are valid for small but finite-amplitude waves since the higher spatial harmonics are excluded, but they can also be exact under specific conditions (Stenflo 1996*a*, *b*).

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Equations (11)–(13) are the Lorenz (1963) equations, whose properties are now well understood. In particular, the stability of the stationary states can be studied by a simple linear analysis. Letting $X = X_s + X_1$, $Y = Y_s + Y_1$ and $Z = Z_s + Z_1$, the linearized system is

$$\begin{pmatrix} d_\tau X_1 \\ d_\tau Y_1 \\ d_\tau Z_1 \end{pmatrix} = \begin{pmatrix} -\sigma & \sigma & 0 \\ r - Z_s & -1 & -X_s \\ Y_s & X_s & -b \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}, \tag{14}$$

where $X_1 \ll X_s$, $Y_1 \ll Y_s$ and $Z_1 \ll Z_s$, and (X_s, Y_s, Z_s) represents a stationary state. The corresponding characteristic equation is thus

$$(\lambda + b)[(\lambda^2 + (1 + \sigma)\lambda + (1 - r)\sigma)] = 0 \,, \tag{15}$$

which governs the linear stability of the stationary state. For example, if we take

$$\left(d_x T_0 - \frac{2T_0 d_x n_0}{3n_0}\right) K_y^2 d_x n_0 < D K_0^4 (0.51 m n_0 \nu + \eta K_0^2) \qquad (\text{or } r < 1),$$

the origin is a hyperbolic sink and thus is stable. On the other hand, for $r = 1$, the eigenvalues are $\lambda = -b$ and $\lambda = -(1 + \sigma)$, which are always negative. Finally, for $r > 1$, the nontrivial stationary points are $X_s^{\pm} = Y_s^{\pm} = \pm[b(r-1)]^{1/2}$ and $Z_s = r - 1$. The eigenvalues of (11)–(13) are $\lambda = -(\sigma + b + 1)$ and $\pm i[2\sigma(\sigma + 1)/(\sigma - b - 1)]^{1/2}$, so that the stationary states $(X_s^{\pm}, Y_s^{\pm}, Z_s)$ are sinks for $r \in (1, r_H)$, where $r_H \equiv \sigma(\sigma + b + 3)/(\sigma - b - 1)$. A Hopf bifurcation occurs at r_H. For $\sigma > 1 + b$, imaginary roots are possible, and for $r > r_H$, the nontrivial fixed points are saddles with two-dimensional unstable manifolds. Thus, for $r > r_H$, all the three fixed points are unstable, but the attractor set still exists (Lorenz 1963). Further bifurcations at larger r values eventually lead to chaotic behaviour (Lorenz 1963).

In summary, we have investigated stationary and non-stationary solutions of the nonlinear equations governing the finite-amplitude MEDV mode in dissipationless as well as dissipative media. It is found that in the collisionless case, nonlinear MEDV waves can self-organize into dipolar vortices and vortex streets. In particular, for $u^2 > \alpha\beta$, dipolar vortices exist, whereas for $u^2 = \alpha\beta$, only vortex chains appear. The latter have arbitrary amplitudes and sizes but well-defined translational speeds. Thus the vortex chains are found in the regime where the dipolar vortices are forbidden. Furthermore, we have shown, by reducing the governing equations to those of Lorenz, that temporal chaos can appear if dissipation is included. The latter can lead to chaotic behaviour, depending on the density and temperature gradients as well as the electron collision and diffusion rates. The results here should be useful in understanding ordered and chaotic behaviour of the MEDV mode occurring in space and inertial-confinement-fusion plasmas, as well as in plasma switches (Gordeev *et al.* 1994).

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References

Gordeev, A. V., Kingsep, A. S. and Rudakov, L. I. 1994 *Phys. Rep.* **243**, 215.
Jones, R. D. 1983 *Phys. Rev. Lett.* **51**, 1269.
Jones, R. D. 1988 *Phys. Fluids* **31**, 535.
Lorenz, E. N. 1963 *J. Atmos. Sci.* **20**, 130.
Nycander, J. and Pavlenko, V. P. 1991 *Phys Fluids* **B3**, 1386.
Nycander, J., Pavlenko, V. P. and Stenflo, L. 1987 *Phys. Fluids* **30**, 1367.
Pavlenko, V. P. and Uby, L. 1993 *Phys. Fluids* **B5**, 1980.
Petviashvili, V. I. and Pokhotelov, O. A. 1992 *Solitary Waves in Plasmas and in the Atmosphere*. Gordon and Breach, London.
Shepherd, T. G. 1990 *Adv. Geophys.* **32**, 287.
Shukla, P. K. 1989 *Phys. Rev.* **A39**, 4758.
Shukla, P. K. 1991 *A Variety of Plasmas. Proceedings of the 1989 International Conference on Plasma Physics* (ed. A. Sen and P. K. Kaw), p. 297. Indian Academy of Sciences, Bangalore.
Shukla, P. K., Birk, G. T. and Bingham, R. 1995 *Geophys. Res. Lett.* **22**, 671.
Stenflo, L. 1994 *Phys. Lett.* **186A**, 133.
Stenflo, L. 1996*a* *Phys. Lett.* **222A**, 378.
Stenflo, L. 1996*b* *Physica Scripta* **53**, 83.
Stenflo, L. and Yu, M. Y. 1986 *Phys. Fluids* **29**, 2335.
Yu, M. Y. and Stenflo, L. 1985 *Phys. Fluids* **28**, 3447
Yu, M. Y. and Xiao, C. 1987 *Phys. Fluids* **30**, 3631.

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