The number of unknown paradoxes

ROY SORENSEN AND MARK SAINSBURY

> 'A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science'.[1]

How many paradoxes are there? By 1920, Bertrand Russell's star student had concluded that there are few or zero paradoxes in philosophy. Most philosophical propositions 'are not false but nonsensical'.[2]

The number of paradoxes one will allow depends on whether one adopts a more or a less inclusive definition.[3] On a very inclusive definition, any puzzle or riddle counts as a paradox. A more restricted view is that a paradox is an apparently inconsistent minimal set of apparent truths.[4] We could count how many sets of apparently true sentences have at some past time appeared inconsistent to a reasonable number of people. Some of these paradoxes are now 'dead' paradoxes: they no longer strike us as paradoxical.[5] A standard example is the

[1] Bertrand Russell 'On denoting' *Mind* 14 (1905) 479–93.

[2] Ludwig Wittgenstein *Tractatus Logico-Philosophicus* (London: Trench, Trubner & Co., Ltd., 1922), 4.003. Almost all contemporary definers of 'paradox' agree that a paradox is composed of meaningful statements. An outlier is Roy Sorensen who 'allows for the possibility of meaningless paradoxes'; Roy Sorensen, *A Brief History of the Paradox* (Oxford: Oxford University Press, 2003).

[3] The number of paradoxes discussed in general books on paradoxes varies. For example, Michael Clark *Paradoxes from A to Z*, 3rd ed. (London: Routledge, 2012) discusses 93; Nicholas Rescher *Paradoxes: Their Roots, Range, and Resolution* (Chicago: Open Court, 2001) discusses 146; Mark Sainsbury *Paradoxes*, 3rd ed. (Cambridge: Cambridge University Press, 2009) discusses 34; Roy Sorensen, op. cit. note 1, discusses 29. Of course, these authors do not claim that the paradoxes they discuss are all there are, and they make no claims for what the future holds.

[4] This proposal is similar to, though distinct from, Rescher's, op. cit. note 1, 6. 'Minimal' is to prevent inflating the count of paradoxes by adding irrelevant sentences to a paradoxical set.

[5] The graveyard includes paradoxes that now count as scientific corpses such as the Copernican paradox that the earth orbits the sun. Russell thought progress in 'philosophy' is disguised by the requirement that

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Barber 'paradox', which we predict would not seem paradoxical to readers of this journal. More controversial are Zeno's paradoxes. Many generations of thinkers regarded them as involving apparent inconsistences, but nowadays we might be more inclined to side with C. S. Peirce, regarding these appearances as illusory. He says of the paradox of Achilles and the tortoise that 'this ridiculous little catch presents no difficulty at all to a mind adequately trained in mathematics and logic'.[6] Presumably Peirce would think that Zeno's other paradoxes of space and motion are no less ridiculous, and so are not paradoxes for trained minds. A paradox may be gained thanks to the imagination of a Zeno, and then lost thanks to the acumen of minds 'adequately trained in mathematics and logic'.

If Peirce is right, there are dead paradoxes just as there are dead metaphors. Whether or not something counts as a paradox is relative to a time or a culture. In considering how many paradoxes lie ahead, we should include even those doomed to die, for, with paradoxes, death is progress: the satisfying resolution of apparently conflicting appearances.

Our search for future paradoxes should not be confined to philosophy. Wittgenstein's skepticism about philosophical paradoxes did not extend to physics, because the physicist's subject-matter contains genuine propositions that may conflict with propositions from other sciences, so scientific debates can be regimented as paradoxes. For instance, in the nineteenth century the completeness of physics implied that the earth was younger than evolutionary theory required. Yet Lyell's uniformitarian geology reinforced Darwin's case that the sun has powered life forms for far longer than the twenty million year ceiling calculated by Lord Kelvin. This interdisciplinary paradox was formulated in much the way we have considered: a set of propositions that are individually plausible but jointly inconsistent. Kelvin contended that physics corrects biology (and proposed the alternative hypothesis that life was seeded from an extra-terrestrial source). A generation later, Ernest Rutherford concluded the reverse:

> 'I came into the room, which was half dark, and presently spotted Lord Kelvin in the audience and realized that I was in trouble at the last part of my speech dealing with the age of the Earth, where

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philosophy be speculative rather than settled. We leave the body count to historians.

[6] C.S. Peirce, *The Collected Papers of Charles Sanders Peirce*, ed. Charles Hartshorne and Paul Weiss, 8 vols. (Harvard University Press: Cambridge MA, 1935) vol VI: 122.

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> my views conflicted with his. To my relief, Kelvin fell fast asleep, but as I came to the important point, I saw the old bird sit up, open an eye, and cock a baleful glance at me! Then a sudden inspiration came, and I said, 'Lord Kelvin had limited the age of the Earth, provided no new source was discovered. That prophetic utterance refers to what we are now considering tonight, radium!' Behold! the old boy beamed upon me.'[7]

This is an example of a paradox arising from a conflict between scientific disciplines. There is also scope for paradox within a discipline, as the EPR paradox shows.

As a logical positivist, Rudolph Carnap agreed with Wittgenstein that philosophers had radically over-counted their paradoxes.[8] Carnap rejected metaphysics as a pseudo-science comprised of meaningless utterances, and what is without meaning cannot be paradoxical. The 'propositions' of ethics and aesthetics were emotional utterances misleadingly expressed as descriptions rather than imperatives or optatives. Most of epistemology was as meaningless as metaphysics because it tries to answer skeptical questions such as 'How do you know the universe is older than five minutes?'. However, Carnap believed there were genuine paradoxes of confirmation. These penetrated semantics. Solving these paradoxes would make language more precise. 'Paradoxes' that turn on ambiguity and vagueness will be engineered out in a fashion now familiar from computer languages. The sorites, noted Carnap, requires vague predicates and so will be unthinkable to our descendants.[9] On this view, intellectual progress can be measured by the diminution of the number of paradoxes.

With 2020 hindsight and contemporary search engines, the reverse trend is manifest. The very efforts to dissolve paradoxes led to the discovery of further paradoxes, such as the new riddle of induction, the Kripkenstein rule following paradox, the lottery paradox, Yablo's paradox, etc. How many more paradoxes exist undiscovered? Are we merely seeing increased efficiency in search, perhaps finishing off the paradoxes in the way scientific fishing trawlers exhaust fish species? Or does the increased pace of discovery show that philosophy

[7] Arthur Stewart Eve, *Rutherford: Being the life and letters of the Rt. Hon. Lord Rutherford, O. M.* (Cambridge: Cambridge University Press, 1939).

[8] Rudolf Carnap, *Scheinprobleme in der Philosophie* (Berlin: Weltkreis-Verlag, 1928); translated as *Pseudoproblems in Philosophy* by R. A. George (Berkeley and Los Angeles: University of California Press, 1967).

[9] Rudolf Carnap, *Logical Foundations of Probability* (Chicago: University of Chicago Press, 1950).

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has far more data than imagined? (Most of the paradoxes deployed to test Bertrand Russell's theory of descriptions date from before the fourteenth century.)

Some methods of estimating quantities are not available to us. For example, during World War II, the allies inferred the total number of German tanks from the serial numbers of observed tanks. Since these vehicles were labeled consecutively, the absence of tanks numbered above 1000 suggested a total not far beyond 1000. This rough estimate was refined by studying the distribution of observed numbers below the observed maximum. But paradoxes come with no such potentially revealing labels.

Statistical techniques for estimating population size do not require numeric sequencing. Bayesian biologists infer the number of unknown species by asking which total number would make least surprising their pattern of species discovery. The less frequently an ornithologist re-captures tagged jackdaws, the larger the population of jackdaws. The methodology is topic neutral. Biologists extend capture/re-capture methodology to estimate the size of Shakespeare's vocabulary.[10] Might this method be applied to predict how many novel paradoxes will be mentioned in the 2120 volume of *Philosophy*? The problem is that, for paradoxes, there seems to be no suitable analog for the distinction between tagged and untagged.

Paradoxes emerge organically from the growth of new beliefs. As the total number of beliefs increases, the number of opportunities for inconsistency grows exponentially. Since paradoxes prod the development of further beliefs, there is a positive feedback loop. It might launch the total into the transfinite: as many paradoxes as there are sentences in a language. If paradoxes are indeed 'atoms of philosophy' this is good news for the size of our subject.[11]

In addition to the bare number of future paradoxes, one might wish to make a qualitative assessment. We are familiar with the idea that the Barber 'paradox' is less deep that the Liar (and perhaps should not be classified as a paradox). A plentiful supply of superficial paradoxes might be of less total interest than a much smaller number of deep paradoxes. A rough measure of depth might be life-span: how long does a paradox survive before it counts as dead? This measure is rough, since accidents of culture can leave a superficial paradox ill-explored, giving it a longer life than its superficiality merits; and

[10] Bradley Efron and Ronald Thisted 'Estimating the Number of Unseen Species: How Many Words Did Shakespeare Know?' *Biometrika* 63:3 (1976) 435–447.

[11] Roy Sorensen, op. cit. note 1, xi.

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such accidents might grant a short life to a deep paradox, if it drew a huge amount of concentrated high-level attention.

An alternative way to rank paradoxes is by the fertility of the intellectual tools drawn upon for their solution. This fertility was negligible for the Barber, but considerable in the case of Russell's paradox, and the Liar also has generated a great deal of very important work in semantics. We incline to think that the application of structural features of grounding, as described by Martin Pleitz in this issue, is also a valuable addition to the intellectual toolkit of those wishing to understand paradoxes, and so an addition to the fertility of those paradoxes to which it can be applied.

Let's assume that, one way or another we can assign a qualitative measure to paradoxes. Now we can ask not merely how many paradoxes can be expected for the future, but how much paradoxicality, defined as the number of paradoxes adjusted for their quality. (One might measure quality on a scale from 0 to 1, in which case our target is the sum of the result of multiplying each paradox by its measure.)

Now there is an unsettling thought. We need to make room for the possibility that our intellectual progress may in part consist in becoming better at resolving paradoxes, giving them a shorter life, and reducing the fertility of new paradoxes. If our toolbox has very powerful weapons that can be used to destroy paradoxes, less effort will be required, paradoxes will die sooner, and will seem less fertile, requiring fewer additions to our already well-stocked toolbox. We can envisage an arms race between paradoxes and their resolutions. If the resolutions get ahead of the game, then the total paradoxicality of our future will be significantly less than the total number of paradoxes. Paradoxes that would now strike us as very deep would then, with our new intellectual tools, seem much less deep. The bleak view is that paradoxicality grows faster than the number of paradoxes. The rosy view is that more paradoxes are linked to less paradoxicality.

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