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Density effects in harmonic emission from intense laser interactions with overdense plasmas

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Abstract. Although the harmonics generated when intense laser light interacts with solid targets are broadly understood in terms of their dependence on the incident intensity, less attention has been paid to the ways in which their behaviour depends on plasma density. In this paper the conversion efficiencies of the odd harmonics generated when light is incident normally on the target surface are shown to be strongly resonant in character. The number of resonances is governed by harmonic order and their behaviour with density is further complicated by Langmuir wave emission at the corresponding plasma resonant densities for successive harmonic orders. Only at densities well above these resonances does the emission decay monotonically with density. The density dependence predicted by theory is confirmed by particle-in-cell simulations.

1. Introduction

Broadly speaking the generation of multiple harmonics of intense laser light incident on an overdense plasma slab is well understood and its characteristics have been resolved both experimentally and by particle-in-cell (PIC) simulations to high harmonic orders [1–6]. In the most general terms, the spectrum is governed by two parameters, namely the photon and electron densities. The incident irradiance measured by $I\lambda_L^2$, where I denotes the intensity of light of wavelength λ_L, has to be strong enough to generate high harmonic numbers while the plasma density not only has to be significantly overdense but to be characterized by a steep gradient at the surface for the harmonics themselves to be sufficiently intense. A convenient index characterizing the interaction of laser light with the plasma electrons is the dimensionless quiver velocity $a_0 = 0.85(I_{18}\lambda_L^2)^{1/2}$, where I_{18} denotes the intensity in units of 10^{18} W cm^{-2} and λ_L is the wavelength of the incident light in micrometres.

The harmonic dependence on a_0 is straightforward and broadly understood. For $a_0 \ll 1$ only low harmonic orders are excited but as $a_0 \to 1$ the picture changes with a greatly extended harmonic range. There is no generally valid theoretical interpretation of high harmonic generation. An empirical expression due to Gibbon [2]

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suggests that, for sufficiently large harmonic index m, the efficiency of harmonic generation scales as $(2\,a_0)^4/m^5$. One shortcoming of this empirical scaling is that it is determined solely as a function of laser intensity, although Gibbon elsewhere [7] suggests that one should expect some weak dependence on density and angle of incidence. Recent work by Gordienko et al. [8] presents an alternative scaling, namely a power-law harmonic spectrum decaying as m^{-p} where $p = 5/2$ for a quasi-monochromatic incident laser pulse or $p = 3$ for a broadband pulse. However, this scaling is limited to a harmonic range that satisfies the condition $m < 4\gamma^2$, where γ is the maximum relativistic factor at the plasma boundary. Consequently, there is little overlap between this work and that presented here which is restricted to a quasi-relativistic regime.

The purpose of this paper is to draw attention to the fact that harmonic emission is in fact rather strongly dependent on density. This dependence comes from two distinct sources. The first of these derives from a sequence of cold plasma resonances [9] in the perturbed surface plasma density at $n_e/n_c = 4, 16, \ldots, (2\,m)^2$, where n_c denotes the critical density. The second involves emission from Langmuir waves excited at the plasma surface. The Langmuir waves couple to the radiation field in the steep density gradient at the plasma surface and contribute to the harmonic spectrum for densities $n_e = m^2\, n_c$. Boyd and Ondarza-Rovira [10] showed that this Langmuir wave radiation could affect the line spectrum significantly, especially in the region of low harmonics where the blue wing of the plasma line may be strong enough to envelop the harmonic lines in its neighbourhood.

2. Model and simulations

In this work we confine our interest to low harmonic numbers and examine the dependence on density in the simplest of models. By considering light normally incident on a plane plasma slab we avoid geometrical complications associated with oblique incidence even though the harmonic spectrum now contains just odd harmonics. On top of this we choose to limit a_0 to values at most 1, which allows us to make use of a quasi-relativistic model that is adequate for the purpose in hand. In any event this restriction is needed if our assumption of a plane plasma surface is to remain valid for the duration of the pulse. For high enough laser intensities ($a_0 \geqslant 1$) channel formation within the target becomes a factor, often with rippled plasma surfaces, at variance with our assumption of normally incident light.

Our model follows one first used by Wilks et al. [11] in which linearly polarized light propagates in the z-direction and is incident on a plasma slab in which the ions are taken as stationary, providing a uniform neutralizing background for the electrons. The electron density $n_e > n_c$ and the electron dynamics are described by the cold fluid equations, since the electron quiver velocity $\beta_\perp = a_\perp/\gamma$ is much greater than the thermal velocity. Here $a_\perp(z,t) = eA_\perp/m_0c^2$ is the normalized vector potential and e and m_0 are the electron charge and rest mass, respectively. The normalized vector potential is determined by the wave equation

$$\Box_z^2 \mathbf{a}_\perp = k_p^2 \frac{n_e}{n_0} \boldsymbol{\beta}_\perp = k_p^2 \frac{n_e}{n_0} \frac{\mathbf{a}_\perp}{\gamma}, \tag{1}$$

in which $k_p = \omega_p/c, \omega_p^2 = 4\pi n_0 e^2/m_0$ and $\boldsymbol{\beta}_\perp = \mathbf{v}_\perp/c = \mathbf{a}_\perp/\gamma$ is the transverse electron fluid velocity normalized to the velocity of light; $\Box_z^2 \equiv \partial^2/\partial z^2 - 1/c^2\partial^2/\partial t^2$. The relativistic mass factor γ is defined by $\gamma = (1-\beta_\perp^2-\beta_z^2)^{-1/2} = (1+a_\perp^2)^{1/2}(1-\beta_z^2)^{-1/2}$.

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Harmonic generation is governed by the source term in (1) in which nonlinearity is introduced through the density n_e, on the one hand, and the relativistic mass factor, on the other. The oscillating density is driven by the $\mathbf{v}\times\mathbf{B}$ force, which varies as $a_\perp^2$ and so introduces only even harmonics of $a_\perp$. The same is true for γ and hence the source term in (1) generates just odd harmonics. Even harmonics only appear in the spectrum if we relax the constraint of normal incidence. Since our objective here is limited to highlighting the effect of the plasma density on the harmonic spectrum we confine our attention to low-order odd harmonics ($m = 3, 5, 7, 9$) excited at normal incidence.

The wave equation (1) together with Poisson's equation and the relativistic cold fluid equations of continuity and motion are solved to first and second order using the perturbation expansion $a_\perp = a_0 + \epsilon a_3 + \epsilon^2 a_5 + O(\epsilon^3)$, where a_3, a_5 denote the vector potentials associated with the third and fifth harmonics. Our perturbation analysis differs from that of Wilks et al. [11] in some points of detail and in being carried to second order. One distinction appears in the representation of the fields at the plasma–vacuum boundary. Since we are concerned with overdense plasmas such that $\omega_p \gg \omega_0$, by far the greater part of the incident light is reflected from the plasma surface. The transmitted fraction of the vector potential $\mathbf{a}_\perp$ decays within a skin depth of extent δ that is much smaller than λ_L. The sources generating the spectrum of harmonics are localized at the surface so that the form of the transmitted vector potential within this surface layer is needed. In determining this we made use of the result found by Rozmus and Tikhonchuk [12] and independently by Gamaliy and Dragila [13]. To first order we find that P_3 (normalized to the incident power) is given by

$$\frac{P_3}{P_1} = \left(\frac{9}{8}\right)^2 \left(\frac{\omega_0}{\omega_p}\right)^2 \frac{a_0^4}{(1 - 4(\omega_0^2/\omega_p^2))^2}. \tag{2}$$

This result reproduces the behaviour with a_0 found in [11] but shows differences in its dependence on plasma density. Disregarding the resonant denominator in (2), $P_3/P_1 \sim n_c/n_e$, in contrast to the equivalent expression found by Wilks et al. that shows a quadratic dependence. Since the emphasis in this paper is on density dependence we plot in Fig. 1 the expression in (2) along with the PIC data for a choice of $a_0 = 0.6$ across a density range up to $n_e = 100\ n_c$. A purely formal damping was applied to resolve the cold plasma resonance at $n_e = 4\ n_c$. We note the good agreement between the density dependence in (2) with the PIC data in the range $n_e \geqslant 20\ n_c$, showing approximately n_e^{-1} behaviour. Physically this dependence on density is to be expected, having its roots in the shielding of the electric field of the incident light pulse by the overdense plasma. This effectively reduces the electron quiver velocity by a factor $(n_c/n_e)^{1/2}$. For densities below $\sim 20\ n_c$ the results from the model and from PIC simulations are quite distinct. The theoretical curve peaks at the cold plasma resonance $n_e/n_c = 4$, whereas the PIC emission clearly shows a peak corresponding to the plasma resonant density $n_e/n_c = 9$ as highlighted by Boyd and Ondarza-Rovira [10]. This results from the coupling of the Langmuir waves to the radiation field and is not described by the cold plasma model used to arrive at (2). Indeed plasma emission, which is not included in our theoretical model, in all likelihood contributes to the P_3 emission in Fig. 2 at densities above $9\ n_c$. There is, for example, some suggestion of P_3 enhancement at $n_e \sim 16\ n_c, n_e \sim 25\ n_c$ and $n_e \sim 36\ n_c$ for which the corresponding Langmuir frequencies are $\omega_L \sim 4\omega_0, 5\omega_0$ and $6\omega_0$. Plasma emission at these densities could then contribute to P_3 emission by means of parametric combinations $(\omega_L, \omega_0), (\omega_L, 2\omega_0)$ and $(\omega_L, 3\omega_0)$. Note that,

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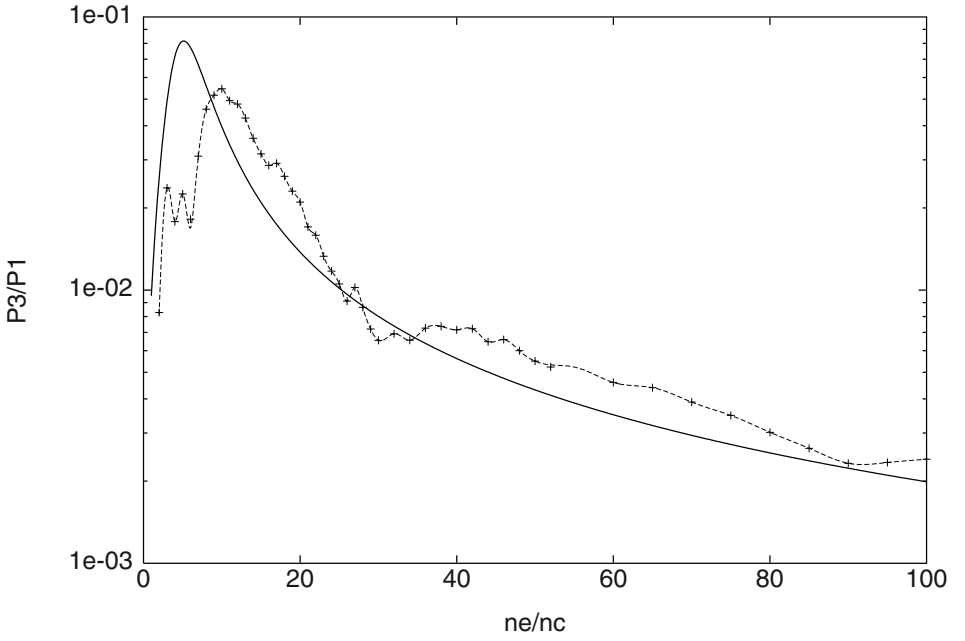


Figure 1. Reflected power in the third harmonic as a function of n_e/n_c, for $a_0 = 0.6$. The solid curve denotes the cold plasma model result from (2) (with phenomenological damping added). The points + were obtained from PIC simulations.

above the region dominated by plasma line emission, the weak dependence of the third harmonic power on density found in (2) agrees reasonably well with the PIC result.

Continuing the perturbation scheme to next order allows us to estimate the power emitted in the fifth harmonic. To curtail the unwieldy algebra of second-order perturbation theory we have imposed some further approximations (amounting to the neglect of non-resonant terms in the density perturbations) to arrive at a simplified expression for P_5/P_1, which nevertheless keeps the essential physics. The expression found from second-order perturbation theory then takes the form

$$\frac{P_5}{P_1} \simeq 0.6 \frac{(\omega_0/\omega_p)^2(1 - \frac{9}{4}\omega_0^2/\omega_p^2)^2}{(1 - 4\omega_0^2/\omega_p^2)^2(1 - 16\omega_0^2/\omega_p^2)^2} a_0^8. \tag{3}$$

No physical significance is to be attached to the apparent cutoff introduced by the term in the numerator; this is fictitious and is prevented by a lower-order contribution neglected for the convenience of writing a compact expression for P_5.

Figure 2 shows P_5/P_1 as a function of plasma density for a choice of $a_0 = 0.6$. The PIC output is characterized by two density resonances, the lower of which extends from $n_e \sim 4\ n_c$ to about $8\ n_c$ while the upper falls in the range (16–25) n_c, in broad agreement with the resonances predicted by (3). In reality the lower of the two resonances in Fig. 2 shows a multiplet structure with peaks at $4\ n_c$ and $7\ n_c$. Note too that the range of the upper resonance extends as far as the plasma resonance at $\sim 25\ n_c$. Overall the result in (3) is a fair representation over the density range (20–60) n_c but thereafter underestimates the contribution to P_5.

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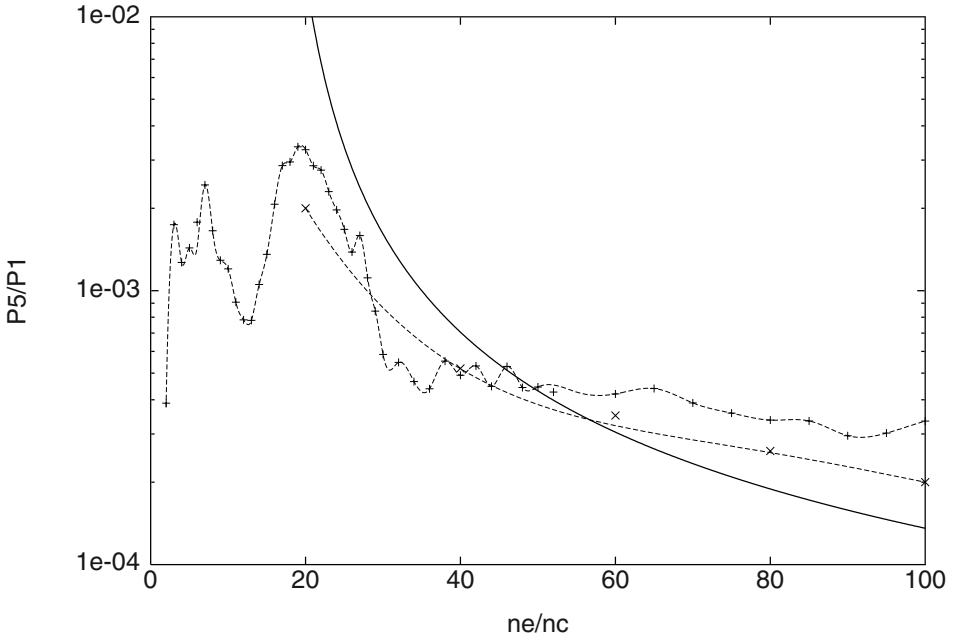


Figure 2. Reflected power in the fifth harmonic as a function of n_e/n_c, for $a_0 = 0.6$. The solid curve denotes the cold plasma model result from (3) while the dashed curve shows the estimated result from (4). The points + were obtained from PIC simulations.

This underestimate can be attributed at least in part to our neglect of non-resonant terms in the approximation to P_5 in (3). In the case of the third harmonic we introduced artificial damping into the cold plasma model to resolve the resonant denominator. In the same way the result in (3) can be made to mimic the PIC (P_5–n_e) plot with its dual resonance structure with finite widths. However, this by itself is physically meaningless. Only by resorting to a more complete model might we be able to reproduce consistently the resonant structures in Fig. 2.

Given the impracticality of perturbation theory as the order increases it is of interest to estimate P_{2m+1} by recognizing that the dominant contribution comes from the nonlinear combination of the neighbouring lower-order harmonic amplitude a_{2m-1} with a_0^2. This suggests that P_{2m+1} should scale as

$$P_{2m+1} \simeq \left(\frac{2m+1}{2m-1}a_0^2\right)^2 P_{2m-1}. \tag{4}$$

By using the PIC data for P_3 from Fig. 1 in (4) we get estimates for P_5 emission across the density range extending beyond the resonances. From Fig. 2 we see that the scaling in (4) provides a fair representation of fifth harmonic power above the resonant densities.

This quasi-empirical scaling proves useful as we proceed to higher harmonic orders. That said, we know from the first- and second-order perturbation results (2) and (3) how to extrapolate to third order to obtain an expression that is at least structurally correct in that it shows the resonant dependence of P_7 on plasma

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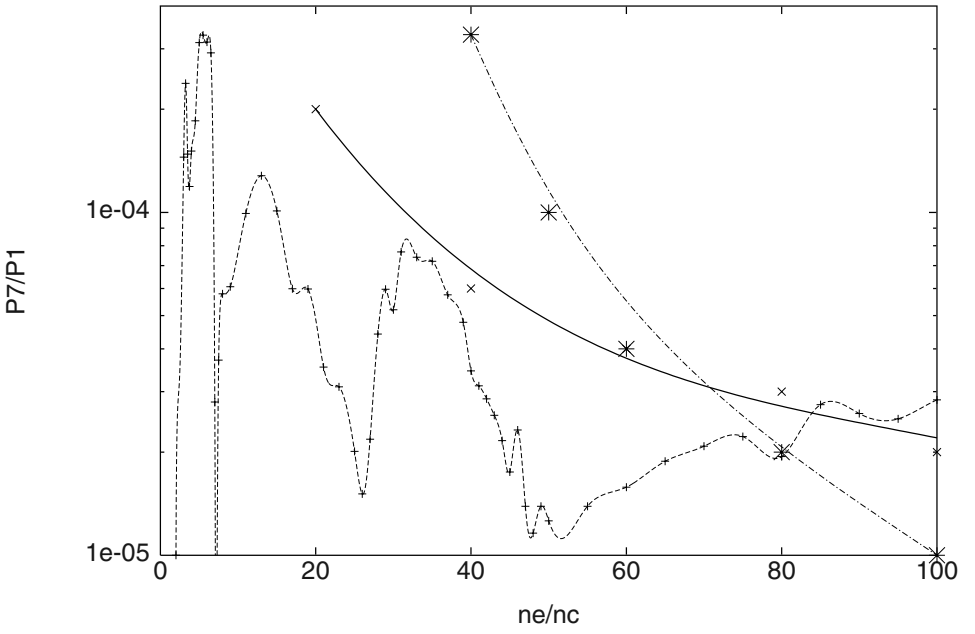


Figure 3. Reflected power in the seventh harmonic as a function of n_e/n_c, for $a_0 = 0.5$. The solid curve denotes the estimate result from (4), while the dashed curve shows the cold plasma model result from (5). The points + were obtained from PIC simulations.

density and on laser intensity. On that basis we expect P_7 to have the form

$$\frac{P_7}{P_1} \simeq C_7 \frac{(\omega_0/\omega_p)^2}{(1-4\omega_0^2/\omega_p^2)^2(1-16\omega_0^2/\omega_p^2)^2(1-36\omega_0^2/\omega_p^2)^2} a_0^{12}, \tag{5}$$

in which C_7 is an undetermined coefficient, only weakly dependent on n_e. It is clear from (5) that to this order there is the expected additional resonant density in the denominator, behaviour that is once again confirmed by the PIC simulations.

Figure 3 shows PIC data for P_7/P_1 with clear resonant structures broadly in line with the predictions of (5). As we have noted already for P_5, resonant densities appear around 6 n_c and (13–16) n_c, with now an additional strong, relatively broad resonance over densities (32–36) n_c in keeping with (5). Included in Fig. 3 are estimates of P_7 from (4). While less compelling than the empirical results for P_5, they at least provide a measure of P_7 that is of the right order. In addition, we show the density dependence represented by (5) with damping included as an artifice to cure the cold plasma resonance at $n_e = 36\ n_c$. This mirrors the behaviour of the perturbation result for the power in the fifth harmonic seen in Fig. 2, showing a relatively steep decay with density. In contrast the PIC data show a modest increase in P_7 power levels at densities above $\sim 50\ n_c$, behaviour that is not readily interpreted.

However, since the extent of the resonance region is determined by $(2m)^2$, it is clear that when we come to P_9 virtually the entire range is now governed by resonant behaviour, with a new resonance a little above 64 n_c. Nevertheless, even here the estimates for P_9 using (4) are surprisingly robust, predicting levels of reflected power in the ninth harmonic to better than an order of magnitude.

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3. Conclusions

We conclude that while the behaviour of the power reflected in the various harmonic orders as a function of laser intensity is relatively uncomplicated, to a degree that allows us to write down a simple scaling with intensity, seen as a function of plasma density its behaviour is altogether more involved. The most striking aspect of the dependence on density is the sequence of resonances predicted by the cold plasma model used in this work and borne out by simulations with a PIC code. Within the resonant range the reflected power varies by up to an order of magnitude, depending on the density. Only above this range does emission decay with density. Physically this is to be expected given the corresponding reduction in electron quiver velocity as the density increases. For low harmonics in the intensity range considered, Langmuir wave-generated radiation contributes to the emission overall. However, at higher densities its contribution is relatively less significant, at least in this weakly driven regime, decreasing approximately as n_e^{-2}.

Crucial to all these considerations is the issue of whether or not the predicted density dependence is observable. The difficulty faced here is that of creating target plasmas over a sufficient range of density to explore the behaviour of, for example, P_5 in the range (10–30) n_c to determine whether or not the variation in intensity predicted by our analysis does indeed occur. One should also bear in mind that these predictions are based on the assumption that the target is a plasma slab with a step in density at the boundary. The presence of a finite density gradient at the plasma surface might be expected to diminish the scale of the variation in the intensities of particular harmonics with density.

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