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Contribution to the dynamic study of humanoid robots interacting with dynamic environment

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SUMMARY
The questions when and why one needs to use mathematical models, and especially the models of dynamics, represent still an unresolved issue. A general answer would be that dynamic modelling is needed as a tool when designing structure of the system and its control unit. In this case we talk about simulation. The other application is in on-line control of the system – the so-called dynamic control. While in simulation one generally uses the best available model, the control can be based on a reduced dynamics, depending on a particular task. The aim of this paper was to highlight the problems important for the dynamics of humanoids and their dynamic environment.

KEYWORDS: Biped mechanisms; Dynamic balance; Zero-Moment Point.

1. INTRODUCTION

Robot dynamics has been used first to analyze the behaviour of active anthropomorphic mechanisms and that was the initial stimulus to the development of robot theory. In view of its many specific features one can say that robot dynamic constituted a new field of applied mechanics. Modelling a biped gait resulted in the mathematical description of kinematics, dynamics, dynamic balance and stability of two-leg walking robots.[1,2] It was then that "Zero-Moment Point" (ZMP) was first recognized as being essential for the synthesis of a stable gait. This research prepared theoretical background for humanoid robotics. As the first practical output a lower-extremities exoskeleton was made to allow paraplegics to walk.[3] Research in humanoids did not have a so fast progress as industrial robotics had. This has been a consequence of the fact that industrial robots were recognized as a key system in an advanced manufacturing automation and hence were supported by high investments. At the same time, work on humanoids has been almost considered as mere enthusiast's concern. Among a few centers we point out Waseda University. The efforts resulted in building the famous WABOT and WABIAN humanoids.[4] It was not long ago when the researchers and robot society, in general, came to the conclusion that humanoids represented a highly prospective direction of work. It was finally "discovered" that humanoids open a huge field of work, potentially very profitable service and personal robotics. For its impact on the way people look at robotics, it is necessary to mention also the HONDA humanoids.[5]

The focus in this paper is on the active environment interacting with humanoid robot. Thus, a mobile platform, that has its own dynamics, is introduced. The platform should have up to six DOFs. It is clear that the platform can be modeled in different ways of which we mention only two. A sequential (linkage) structure may be used to construct the platform. In this case, well known algorithms used for linkages can be applied to form a dynamic model. The other constructive approach refers to the use of special platform structure, like Stewart platform type.[6] The mentioned two options may be applied even if the moving support is of some different kind (boat for instance). In this case the suggested platform mechanisms are just models of a true system. One such experimental system should consist basically of: (i) a biped robot, and (ii) a mobile platform with actively controlled hydro-cylinders, as presented in Fig. 1. This figure is just a global sketch of the experimental setup considered in the paper.

The proposed system enables studying and analysis of the biped's dynamics under the different conditions of possible robot walking/balancing. The benefits of the proposed experimental system is that it ensures different testing conditions without re-configuration of the robot's environment. Before implementation of the experimental system it is possible to check the system performance in the simulation circumstances in order to determine the dynamic system limitations as well as the critical conditions (extreme impacts, powerful inertial effects, etc.). For that purpose, a spatial model of the experimental system robot + platform has been proposed. The objective of the paper is to highlight the influence of "new" phenomena (up to now in-sufficiently elaborated in the literature) on the humanoids behavior. In that sense the dynamic models of appropriate complexity are chosen in consideration.

2. MODELLING OF THE SYSTEM

The dynamic system presented in Fig. 1 is modeled taking into account certain elasticity existing in the system. The goal of such system modeling is to design the appropriate research tool that should ensure the analysis of biped dynamics interacting with dynamic environment as well as simulation of the system's behavior in various operation conditions such as anthropomorphic walking, maintaining dynamic balance, etc. In order to take into account all physical characteristics of

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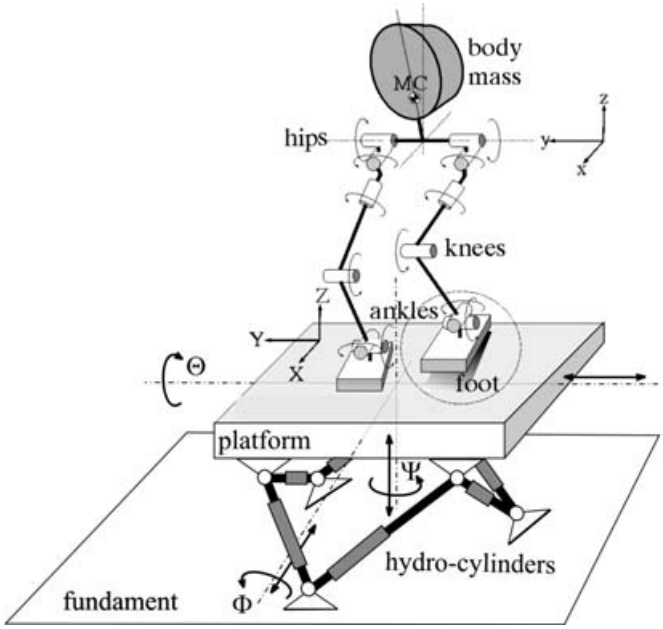


Fig. 1. Global presentation of the experimental system.

interactions between the foot and the supporting system during robot's walking, the model should include the mentioned elastic phenomena. Under the notion of supporting system (SS) the active platform with an elastic covering, i.e. elastic lamination and elastic footpad of biped robot is considered. Concerning the physical effects of interaction, the inertia, damping and stiffness of the SS are taken into account. The Coulomb friction forces are also encompassed by the model equations. In this study, the elasticity of the robot's links and joints are not taken into consideration.

2.1. System analysis

The considered system, presented in Fig. 1, consists of: (i) a biped robot (BR), (ii) an active platform (AP), (iii) an elastic footpad (FP), and (iv) an elastic lamination (EL) placed on the rigid platform. The elastic elements give the system better adaptability to the impacts during operation. On the other hand, the existence of elastic elements in the system increases its order because of the increased number of DOFs. The elastic footpad serves to biped robot's softer contact with the supporting ground. The footpad is fixed at the bottom of the rigid feet of the mechanism. The elastic lamination gives to the supporting system characteristics of either linear or non-linear impedance. The functional scheme of the experimental system from Fig. 1, along with the mentioned elastic elements, is presented in Fig. 2a, whereas its symbolic scheme is shown in Fig. 2b. Figures 2a and 2b represent a simplified planar presentation of the considered dynamic system from Fig. 1, to make easier understanding of the system operation. However, the equations that will be defined in the text to follow will concern the 3D-case of the system, as in reality. In that sense, Fig. 2b serves as the basic model of the considered dynamic system biped + platform. The dynamics of the hydraulic system moving the platform is not modeled; only the resultant hydraulic forces acting upon the active platform are taken into account. There are various possibilities of realization of the hydraulic-servo systems

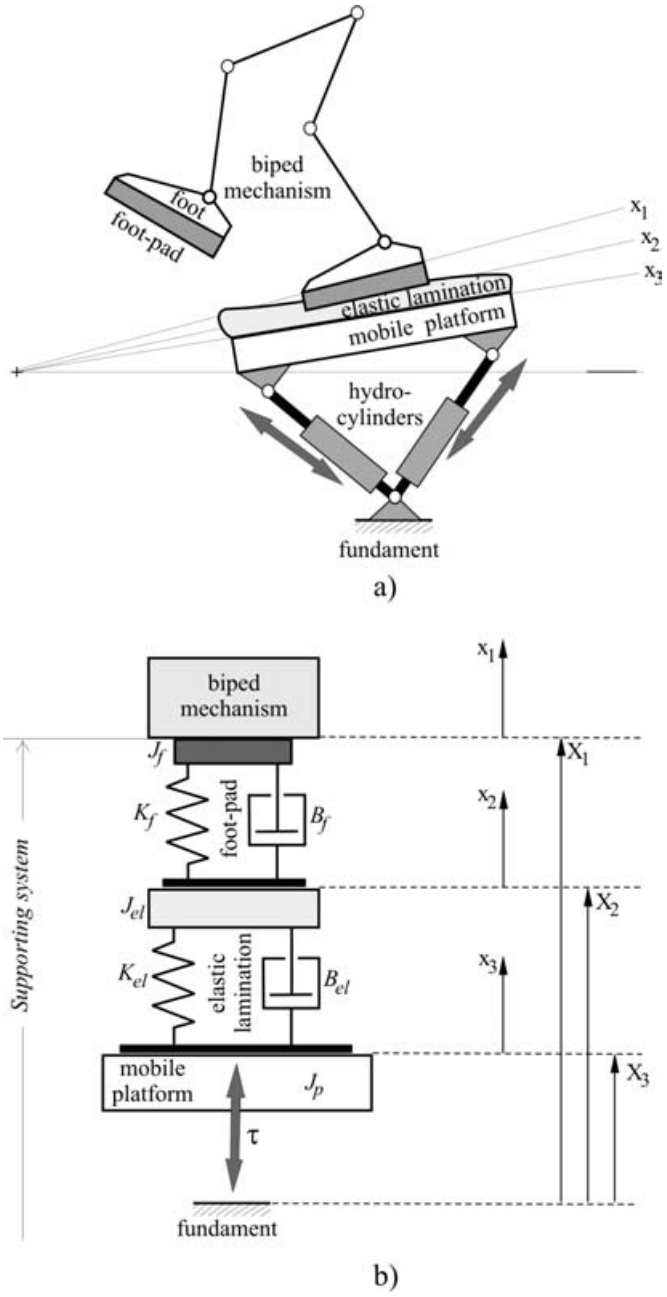


Fig. 2. The simplified model of the experimental system biped + platform shown in Fig. 1: a) the functional, and b) the symbolic scheme.

supporting the hydraulic platform that will not be dealt with in this paper. The characteristics of the hydraulic system are not especially important for our consideration since we focus our attention on the biped dynamics and interaction between the biped and the supporting system. In view of the above discussion and considering Figs. 1 and 2a, it is possible to divide the whole system into two functional modules: (i) the biped mechanism (BM) and, (ii) its supporting system. The BM represents the object of control while the SS can be considered as a perturbation system. Under the notion of the supporting system, the AP, EL as well as the elastic FP is assumed (Fig. 2b). The biped robot stands/walks on the supporting platform while the platform produces perturbations upon it, tending to destabilize the system. The

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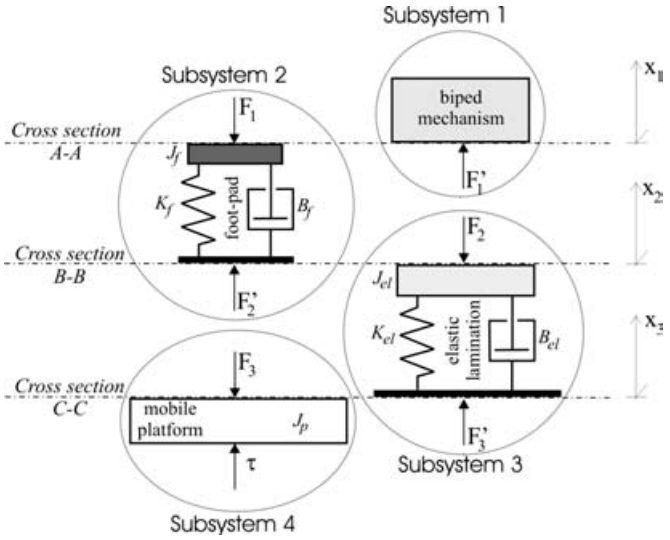


Fig. 3. Splitting the system from Fig. 2b into four functional subsystems.

focus of the present study is on the examination of the new dynamic effects produced by adding the platform dynamics to the robotic system. Besides, its influence upon the dynamic balance of the biped mechanism is analyzed. The control problems will not be the subject of our concern.

2.2. *System equations*

To derive the equations that describe the overall system behavior, we will use the model from Fig. 2b, whereby the three cross sections (A-A, B-B and C-C) of the system (Fig. 3) will be considered. The presented cross sections divide the overall system into four subsystems. Now, it is possible for each of individual subsystem to derive the equations of its behavior. Taken together, these individual equations make a model of the overall considered system biped + platform. The cross section A-A separates Subsystems 1 and 2 one from the other. *Subsystem 1* represents in a symbolic way the biped mechanism (Fig. 1), excluding the elastic footpad fixed to the bottom of its supporting feet. *Subsystem 2* represents the elastic footpad of the biped's leg. The cross section B-B separates the Subsystems 2 and 3. *Subsystem 3* is related to the special elastic lamination that gives to the supporting system of the biped robot characteristics of elasticity and damping. The elastic lamination can be designed as a rubber covering, spoon layer, sandy bag or as an air bag. Finally, the cross section C-C divides the system into the corresponding *Subsystems 3* and *4*. Subsystem 4 represents an active hydraulic platform which has characteristics of a rigid body. In the considered cross sections the active and reactive forces are involved. They are of the same magnitudes but of the opposite directions and, from a physical point of view, they serve as the connections between the particular subsystems of interest. Considering the border *Subsystems 1* and *4* it can be said the following. The reactive force F_1' representing the reaction force of the supporting system of the biped mechanism (Fig. 3) is due to the robot's inertia, centrifugal, gravity and friction forces, as well as to the forces and moments coming from the hydraulic system of the supporting system. It is important to emphasize that this force is not the same in the cases when the robot platform is at rest and when it moves. When moving, the hydraulic platform (Fig. 1) transfers to the robot mechanism a certain transfer motion since the accelerations (linear and angular), as well as the velocities, produce upon it additional forces/moments. That complicates the task of the robot control. Vector τ shown in Fig. 3 represents the vector of hydraulic forces and torques reduced at the platform centre of gravity. These forces/moments introduce the perturbation into the considered system biped + platform.

In the text to follow the equations that describe the dynamic behavior of the particular subsystems will be given sequentially. In these models, the notations used stand for the following quantities:

- n is the order of the robot system, i.e. it is the number of its DOFs,
- $q \in R^n$ is an $(n \times 1)$ vector of biped joint coordinates (angles) q_i,
- $H(q, d)$ is an $(n \times n)$ inertia matrix of the robot mechanism,
- $h(q, \dot{q}, d)$ is an $(n \times 1)$ vector of gravitational, centrifugal and Coriolis forces/moments of the mechanism,
- d is a $(m \times 1)$ vector of kinematical and dynamic parameters of biped mechanism,
- $J(q)$ is the $(6 \times n)$ Jacobian (geometry) matrix that defines the relationship between the robot joint torques and the reaction force F_1' (Fig. 3) acting upon the supporting foot of the biped leg,
- P is a $(n \times 1)$ vector of driving torques acting at the joint centers of the biped mechanism,
- $\mathrm{x}_i \in R^6$ represents the i-th (6×1) vector of state coordinates (Fig. 2b). It has the form: $\mathrm{x}_i = [x_i \ y_i \ z_i \ \phi_i \ \theta_i \ \psi_i]^T, i = 1, \ldots, 3$. The state variables x_i, y_i, z_i represent the translational displacements of the corresponding centers of masses (see Fig. 2b) in the coordinate directions X, Y and Z. Variables ϕ_i, θ_i, ψ_i are the Euler angles of the particular bodies (platform, elastic lamination and footpad) in roll, pitch and yaw directions. Corresponding absolute coordinates of the supporting system shown in Fig. 2b are denoted by the capital letters $\mathrm{X}_i, i = 1, \ldots, 3$,
- e_1 and $\dot{e}_1$ are sequentially the (6×1) vector of the footpad deflection (linear and/or angular) and its derivative in 6 coordinate directions $(X, Y, Z, \phi, \theta, \psi)$. Vector e_1 is defined as the difference $e_1 = \mathrm{x}_1 - \mathrm{x}_2$ (see Fig. 2b),
- e_2 and $\dot{e}_2$ are (6×1) vector of deflection of the elastic lamination (Fig. 2a) as well as its derivative in 6 coordinate directions. The deflection e_2 is defined as the difference $e_2 = \mathrm{x}_2 - \mathrm{x}_3$,
- J_f, J_{el} and J_p are (6×6) matrices of inertia of the footpad, elastic lamination and the mobile platform,
- B_f and B_{el} are (6×6) matrices of damping coefficients of the footpad and elastic lamination,
- K_f and K_{el} are (6×6) matrices of stiffness coefficients of the footpad and elastic lamination,
- g is the gravity constant,
- M_p, M_f and M_{el} are the corresponding masses of the platform, footpad and the elastic lamination,

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- $R_{ZMP} = [X_{ZMP}\ Y_{ZMP}\ 0]^T$ is the 3×1 position vector of the Zero-Moment Point in the fixed coordinate system attached to the contact surface between the footpad and the elastic lamination.,
- Q_p, Q_{el} and Q_f are the generalized forces due to gravity (the forces and torques with respect to the adopted frame). In the simulation example of this paper (Section 3), small inclinations of the platform will be used and the simplification about Z-orientation of gravity effect will be applied.

Now, the equation describing the dynamic behavior of the biped mechanism can be expressed in the following way[7]

$$H(q, d)\,\ddot{q} + h(q, \dot{q}, d) = P \tag{1}$$

Having in mind that the motion of hydraulic platform is transferred to the robot system, and taking into account the absolute motion X_1 of the robot supporting system (Fig. 2b), equation (1) can be written in a similar form:

$$\hat{H}(q, X_1, d)\,\ddot{q} + \hat{h}(q, X_1, \dot{q}, \dot{X}_1, \ddot{X}_1, d) = \hat{P} \tag{2}$$

The new vector $\hat{P}$ from (2) differs from the vector P calculated from the relation (1) because it is determined by taking into account the transfer kinematics (velocity and absolute acceleration) of the robot's supporting plane (*In this example it is the surface between the robot's leg and footpad* (Fig. 2a).). The reaction force F_1' of the supporting system acting between the robotic system and footpad, i.e. acting in the cross section A-A (Fig. 3) can be determined by using the corresponding Jacobian matrix $J(q)$ in the following way:

$$F_1' = J(q) \cdot \hat{P} \tag{3}$$

Relations (2), (3) describe behavior of the *Subsystem 1* as well as the magnitude of the force F_1 acting in the cross section A-A (Fig. 3). Assuming the positive directions of the coordinate systems x_1, x_2 and x_3 given in Fig. 3 the equations that describe the dynamic behavior of the dynamic Subsystems 1–4 can be derived.

Accordingly, the equations that determine behavior of the *Subsystem 2* can be defined in the form:

$$\begin{aligned} F_1 &= -F_1' \quad \text{Subsystem 2/Subsystem 1} \\ -F_1 - Q_f &= J_f \ddot{x}_1 + B_f \dot{e}_1 + K_f e_1 \\ F_2^1 &= -B_f \dot{e}_1 - K_f e_1 \end{aligned} \tag{4}$$

The first relation at the top in the set of equations (4) represents the mathematical description of the physical connection between Subsystems 1 and 2. In the same way we define the relationships between the other subsystems. Relation (4) represents the model of the elastic footpad.

Corresponding equations representing the model of *Subsystem 3* are:

$$\begin{aligned} F_2 &= -F_2' \quad \text{Subsystem 3/Subsystem 2} \\ -F_2 - Q_{el} &= J_{el} \ddot{x}_2 + B_{el} \dot{e}_2 + K_{el} e_2 \\ F_3' &= -B_{el} \dot{e}_2 - K_{el} e_2 \end{aligned} \tag{5}$$

Relation (5) represents the model of the elastic lamination as part of the robot SS.

Finally, the equation that describes the dynamic behavior of *Subsystem 4* can be expressed in the following way:

$$\begin{aligned} F_3 &= -F_3' \quad \text{Subsystem 4/Subsystem 3} \\ \tau - F_3 - Q_p &= J_p \ddot{x}_3 \end{aligned} \tag{6}$$

Relation (6) represents the model of the AP that describes its rigid-body dynamics. The input to this model is the hydraulic forces τ reduced at the centre of gravity.

Taking into account the deflections (e_1, e_2) of the footpad and the elastic lamination, the absolute coordinates can be defined (Fig. 2b) as:

$$X_2 = X_3 + e_2, \quad X_1 = X_2 + e_1 \tag{7}$$

Such definition of deflections (the simple super-position) could be used for small deformations. The derivatives are:

$$\begin{aligned} \dot{X}_3 &= \dot{x}_3, \quad \dot{X}_2 = \dot{X}_3 + \dot{e}_2, \quad \dot{X}_1 = \dot{X}_2 + \dot{e}_1 \\ \ddot{X}_3 &= \ddot{x}_3, \quad \ddot{X}_2 = \ddot{X}_3 + \ddot{x}_2, \quad \ddot{X}_1 = \ddot{X}_2 + \ddot{x}_1 \end{aligned} \tag{8}$$

The magnitudes of the absolute velocities ($\dot{X}_i$, $i = 1, 3$) as well as the absolute accelerations ($\ddot{X}_i$, $i = 1, 3$) of the SS are important for the model calculation, as was pointed out in previous consideration. Besides, the inclination of the platform causes the changes of the components of the gravity forces/moments in 6 particular coordinate directions. In that sense, the transformation matrix, taking the Euler angles of the current angular positions of the particular layers of the system, is taken from.[8] This transformation matrix is used to calculate the dynamic effects in the robotic system arising from inclination of the hydraulic platform (Fig. 1). According to what was previously said, the dynamic behavior of the BM is not the same for the cases of a static and mobile support.

Equations (1)–(8) represent the dynamic and kinematical models of the considered mechanical system. The input variables of the system are $\hat{P}$ and P, whereas the state variables are $q_i, i = 1, n$ and $x_i, i = 1, 3$.

3. SIMULATION EXPERIMENTS

In this section a characteristic simulation example will be described and some characteristic simulation results, obtained by implementation of the Matlab/Simulink toolboxes, will be presented. The goal of this example is to show how the mobile platform with additional elastic lamination influences the dynamic balance of the biped system in the open-loop mode. In that sense, new dynamic effects influencing the biped dynamic balance are examined in order to ensure a better insight into the control task and existing problems. As will be shown, even small attitude variations of the active platform causes the displacement of the ZMP within the contour of the supporting foot. The position of the ZMP can leave this region when the excitation of the platform exceeds certain limits. Simulation was carried out assuming a biped mechanism with

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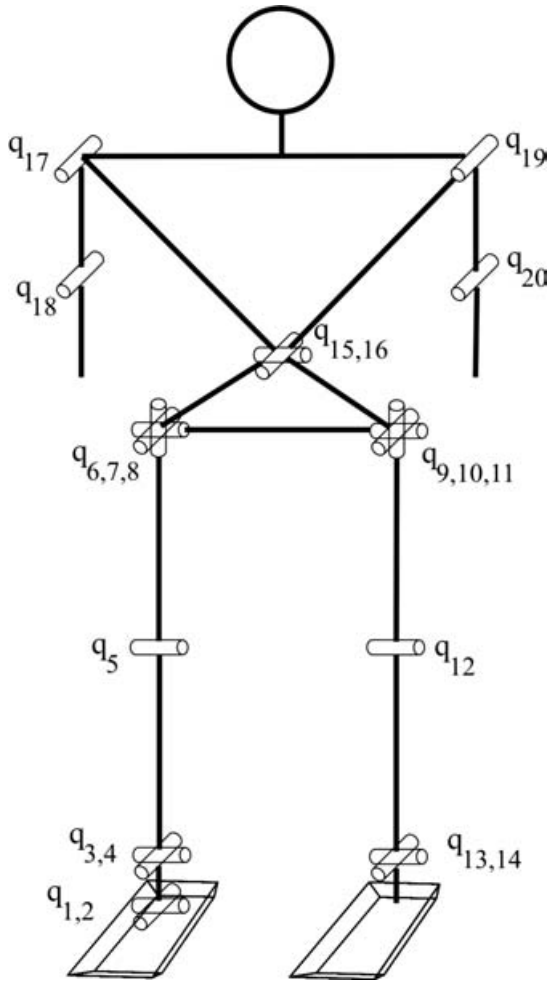


Fig. 4. Kinematic scheme of the biped mechanism model used in the simulation example.

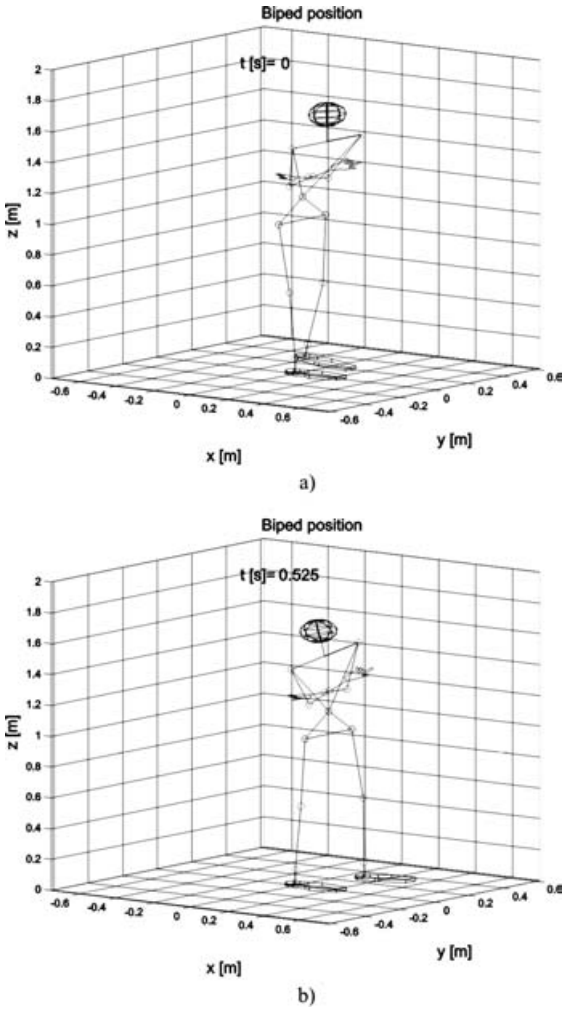


Fig. 5. Computer generated animation of the biped walking in two sampling instants: a) the initial phase of the step, and b) its middle phase.

20 DOFs, the kinematic scheme of which is presented in Fig. 4. The corresponding parameters were taken from reference.[7] The following simulation experiment was carried out. The biped mechanism, with the structure shown in Fig. 4, was put on the mobile platform (Fig. 1).

The biped mechanism is standing on its supporting, right foot making a half-step and changing its joint angles. The half-step motion of the biped robot is shown in Figs. 5a and 5b. Duration of the simulated motion was $t = 0.75\ [s]$. During the robot motion, the active platform was changing its position and attitude causing perturbations upon the biped dynamics. The nominal motion of the biped mechanism was imposed by the angles q_i, $i = 3, 20$ of the biped's joints (Fig. 4). Using the model (2) it is possible to calculate the nominal moments of the biped mechanism acting at its joints. The periodical oscillations of the platform were imposed in the roll ϕ_3 and pitch θ_3 directions, the corresponding Euler angles being presented in Fig. 6a. The pitch and roll angles θ_3 and ϕ_3 are shifted from each other by the phase angle of $\pi/2$. Such angular displacements of the hydraulic platform were imposed in order to enable analysis how the biped robot behaves when its support is mobile in two orthogonal directions. The imposed perturbation causes a corresponding displacement of the ZMP within the foot contour (Fig. 6b). The attitude variation of the active platform with

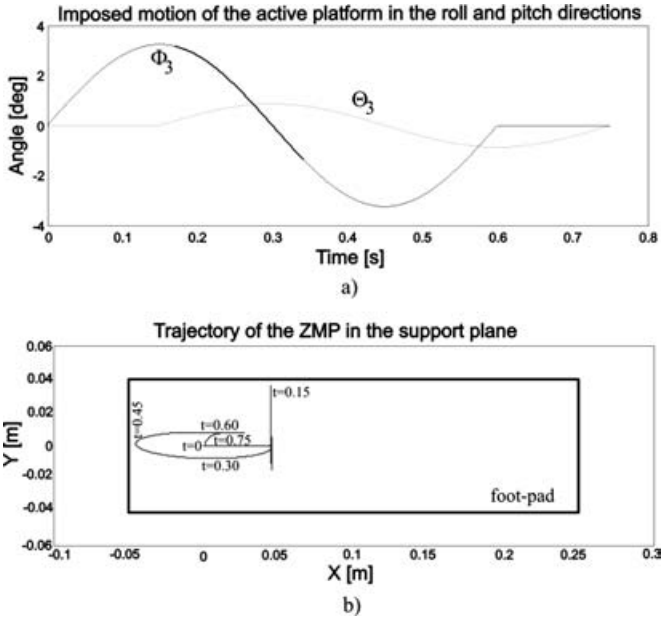


Fig. 6. Simulation results: (a) the imposed motion of active platform, (b) trajectory of the ZMP in the supporting plane.

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the larger amplitudes of periodical signals can cause dynamic imbalance of the biped mechanism and the displacement of the ZMP out of the foot contour. When the imposed motion of the platform has the simple periodical character, the ZMP trajectory within the foot contour has the quasi-circular, i.e. ellipsoidal character (Fig. 6b). Outgoing of the ZMP position from the foot contour implies loosing dynamic balance as well as overturning of the mechanism. In Fig. 6b, the ZMP does not leave the contour, meaning that the robot stays in the stable state. In the considered simulation experiment, the following parameters in the matrix form were used. These matrices were assumed in the diagonal form.

$$J_f = \begin{bmatrix} 0.405 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.405 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.405 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0003 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0003 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0034 \end{bmatrix} \quad (9)$$

$$B_f = 10^3 \times \begin{bmatrix} 1.1384 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.1384 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.1384 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0057 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0169 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0178 \end{bmatrix} \quad (10)$$

$$K_f = 10^5 \times \begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.23529 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.23529 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.23529 \end{bmatrix} \quad (11)$$

$$J_{el} = \begin{bmatrix} 16.86 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16.86 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16.86 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0220 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1344 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1405 \end{bmatrix} \quad (12)$$

$$B_{el} = 10^3 \times \begin{bmatrix} 3.6357 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.6357 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.6357 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0089 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5229 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5194 \end{bmatrix} \quad (13)$$

$$K_{el} = 10^5 \times \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1176 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1176 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1176 \end{bmatrix} \quad (14)$$

$$J_p = \begin{bmatrix} 135 & 0 & 0 & 0 & 0 & 0 \\ 0 & 135 & 0 & 0 & 0 & 0 \\ 0 & 0 & 135 & 0 & 0 & 0 \\ 0 & 0 & 0 & 11.257 & 0 & 0 \\ 0 & 0 & 0 & 0 & 45.007 & 0 \\ 0 & 0 & 0 & 0 & 0 & 56.25 \end{bmatrix} \quad (15)$$

Matrix of the stiffness coefficients K_f was chosen in the simulation experiment in such a way that the mass of 80 $[kg]$ causes linear, elastic deformation of footpad of 1 $[mm]$ in the X, Y and Z directions, as well as the angular deflections of 2 [deg] in the roll, pitch and yaw directions. This stiffness was expressed in $[N/m]$ or $[N/rad]$. Corresponding stiffness matrix of the elastic lamination was assumed to be two times softer than the footpad.

The obtained simulation results describe the dynamic behavior of the considered system. The elasticity of the EL (Subsystem 3, Fig. 3) as well as the damping and stiffness characteristics of the elastic FP (Subsystem 2, Fig. 3) influence significantly the state of the system biped + platform. The absolute positions/attitudes of the particular subsystems (Fig. 3), the elastic deflections of FP and EL as well as the existing forces/moments (inertia, damping and stiffness) in the considered subsystems are presented in Figs. 7–15. In Figs. 7 and 8, the angular (roll, pitch and yaw) as well as the linear (longitudinal, lateral and vertical) displacements of the considered Subsystems 2, 3 and 4 are presented. The oscillatory motion of the platform causes angular and linear displacements of the elastic lamination and the elastic footpad. The roll and the pitch motion of the hydraulic platform cause the changes of the corresponding Euler angles as well as small linear displacements of the elastic layers existing in the supporting system. Figs. 9 and 10 show the corresponding deflections of the elastic lamination and the footpad in 6 main coordinate directions. Also, swinging of the platform in roll and pitch directions (Fig. 6a) causes small elastic deformations of the EL and FP in the X, Y, Z directions due to the perpendicular and tangential forces existing in the elastic layers. The reactive forces F_x, F_y and F_z acting upon the footpad in directions X, Y and Z are presented in Fig. 11. The largest amplitude of the reaction force has the F_z component due to the gravity forces acting upon the biped mechanism links. However, the longitudinal, as well as the lateral, reactive forces exist, too. They are results of the Coulomb friction between the robot mechanism and the supporting system. Besides, there also exist the reactive moments M_x, M_y and M_z, presented in the Fig. 11. The inertia (F_i, M_i), damping (F_d, M_d) and stiffness (F_s, M_s) forces/moments of the footpad as well as of the elastic lamination are

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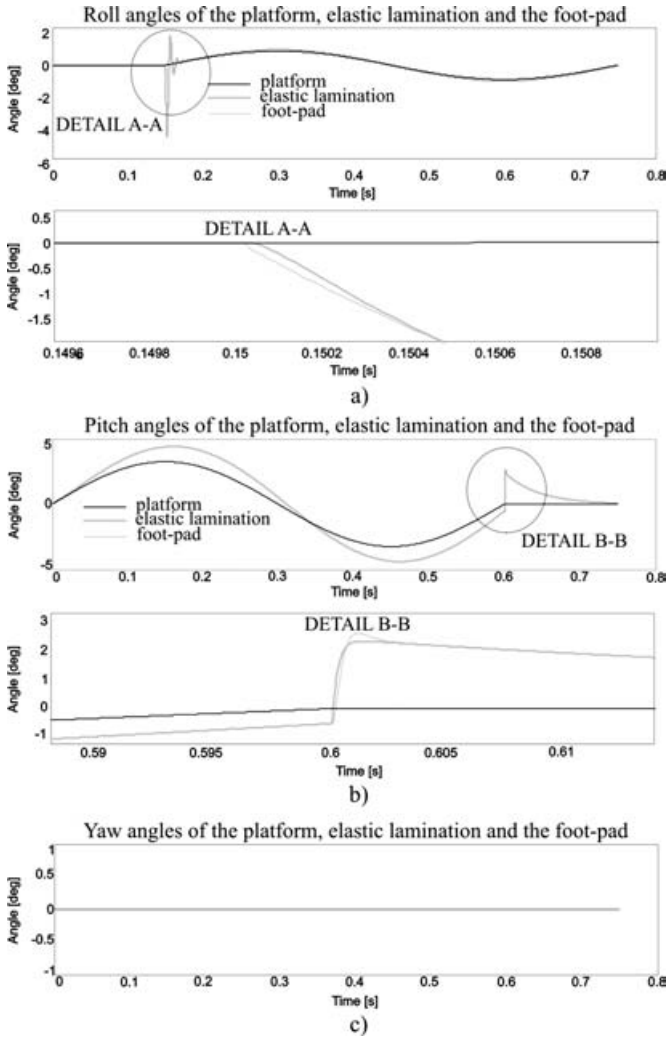


Fig. 7. Simulation results – angular displacements of the platform, elastic lamination and the footpad in the roll ϕ, pitch θ and yaw ψ directions.

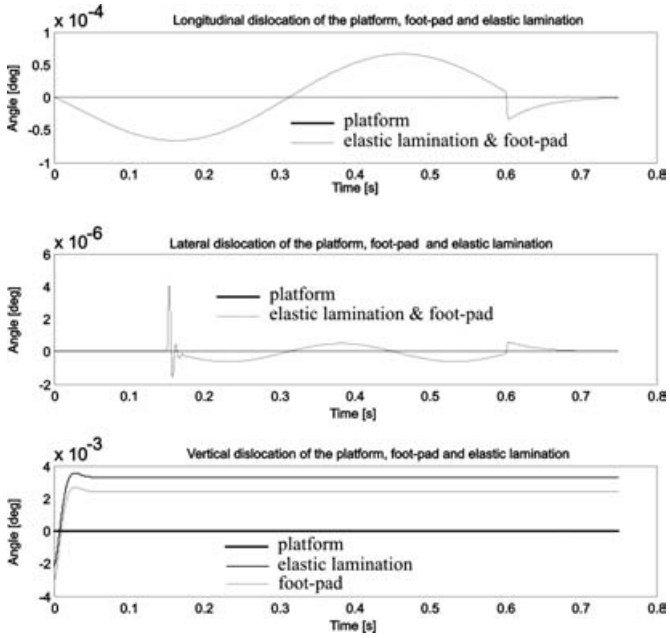


Fig. 8. Simulation results – translational displacements of the platform, elastic lamination and the footpad in the longitudinal X, lateral Y and vertical Z directions.

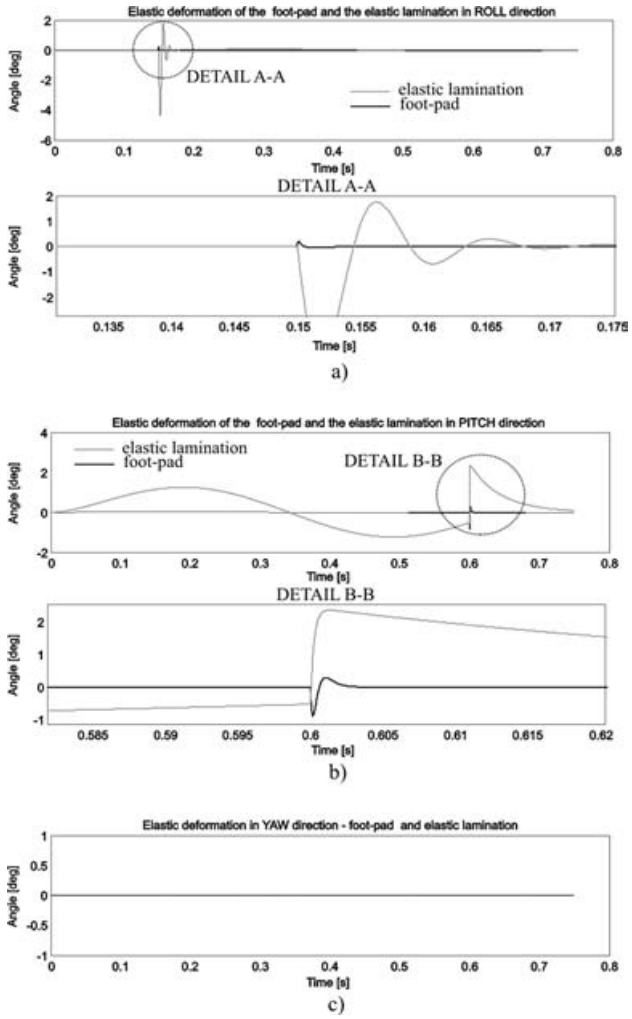


Fig. 9. Simulation results – angular deflections of the elastic lamination and the footpad in the roll ϕ, pitch θ and yaw ψ directions.

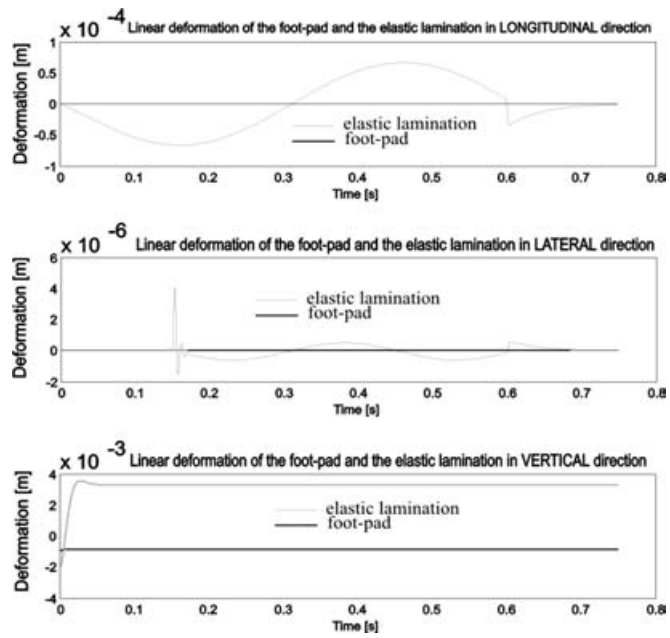


Fig. 10. Simulation results – linear deflections (deformations) of the elastic lamination and the footpad in the longitudinal X, lateral Y and vertical Z directions.

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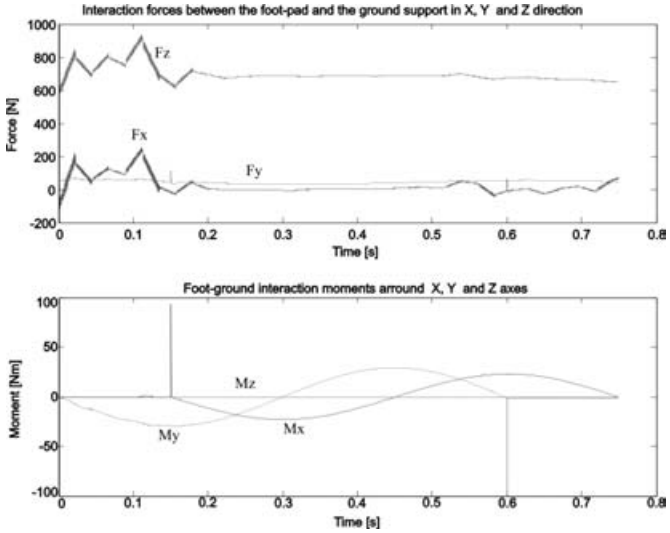


Fig. 11. Simulation results – interaction forces/moments between the robot footpad and the supporting surface.

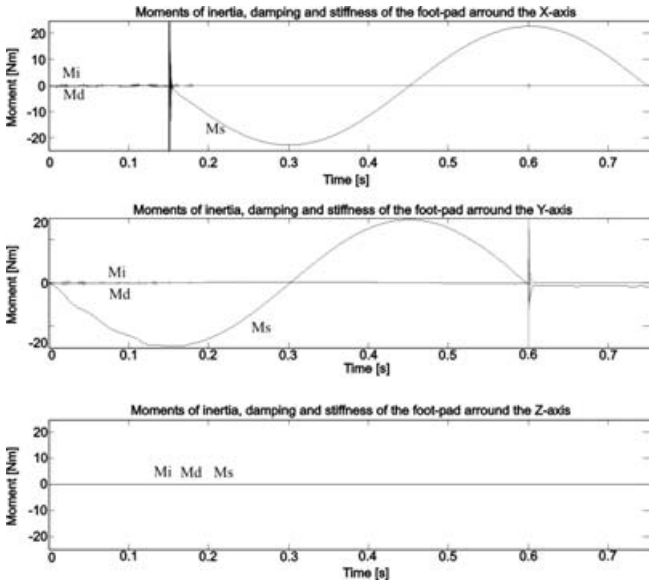


Fig. 13. Simulation results – the inertia, damping and stiffness moments of the footpad produced by the motion of the robot and platform.

presented in Figs. 12–15. By comparing the amplitudes of the obtained forces/moments the following conclusions can be made: (i) the stiffness components of the forces/moments are dominant in comparison with the inertia and damping forces, (ii) the character of the stiffness forces depends on the reactive, i.e. interaction forces/moments between the robot foot and the supporting surface. These forces/moments produce deflections of the elastic layers shown in Figs. 9 and 10.

4. CONCLUSION

This study proved the need for a qualified model for the analysis of biped gait in elastic conditions. Different dynamic effects: moving support and elasto-dynamic contact between the robot and the support were elaborated. The above simulation experiments showed that the biped dynamics is strongly influenced by the behavior of its supporting system. Depending on the amplitude of the displacements the dynamic balance of the robot is changed. The indicator of the robot imbalance is the position of the ZMP. Imposing the perturbation upon the biped mechanism by moving the supporting platform, the position of the ZMP changes its location. In the presented simulation example the displacement of the ZMP (Fig. 6b) was within the foot contour that ensures system to stay on the platform standing.

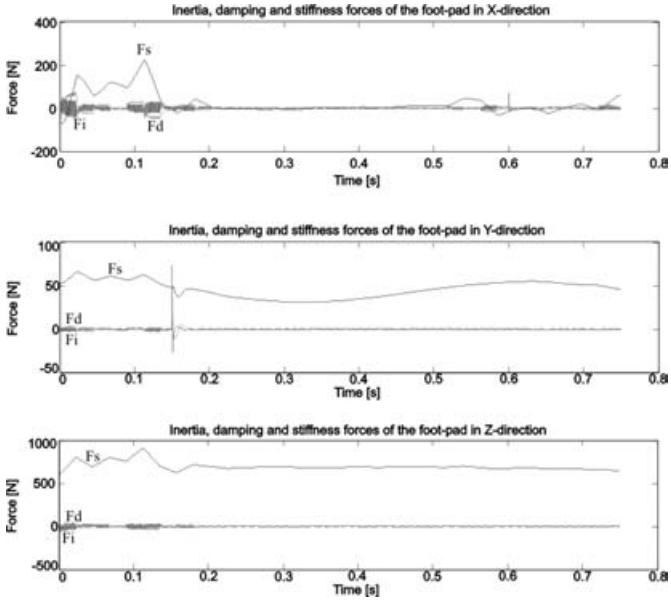


Fig. 12. Simulation results – the inertia, damping and stiffness forces of the footpad produced by the motion of the robot and platform.

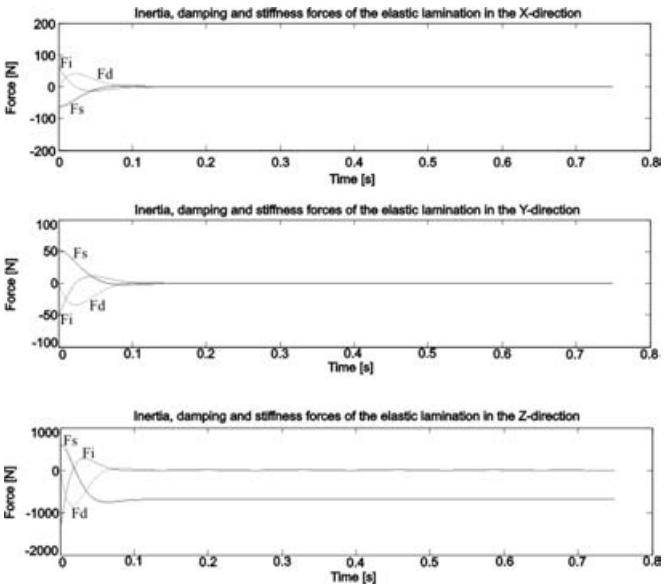


Fig. 14. Simulation results – the inertia, damping and stiffness forces of the elastic lamination produced by the motion of the robot and platform.

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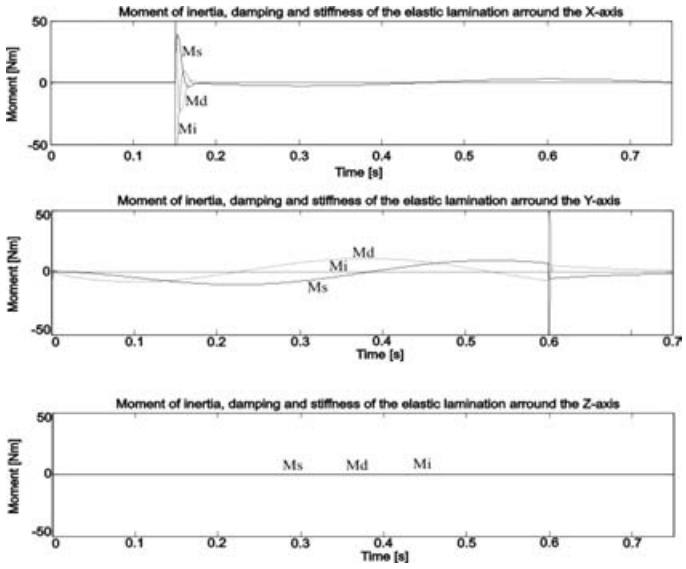


Fig. 15. Simulation results – the inertia, damping and stiffness moments of the elastic lamination produced by the motion of the robot and platform.

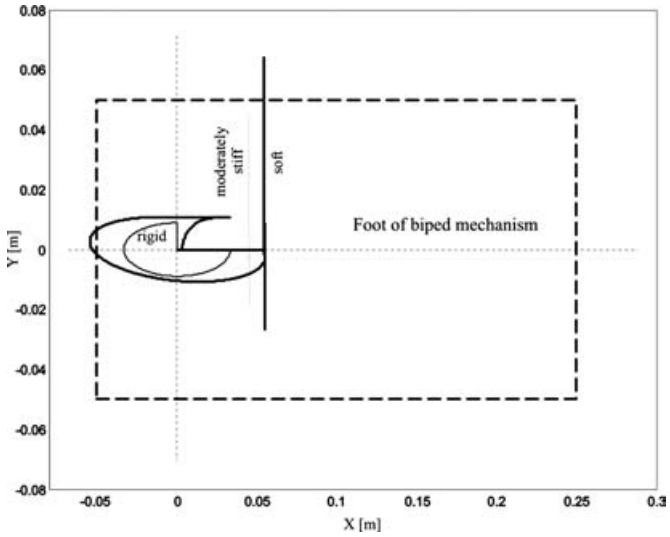


Fig. 16. ZMP trajectories within the foot contour in the case: (i) rigid support, (ii) moderately stiff support and, (iii) soft supporting system.

Beside the platform motion, the elastic elements existing in the SS influence the biped dynamics, too. In Fig. 16, the presented three ZMP trajectories illustrate the case of an absolutely rigid supporting system (no footpad and elastic lamination), moderately stiff support, as well as the quite soft covering of the platform. As a moderately stiff supporting system the one with the parameters given by relations (9)–(15) was assumed, while the soft one was with the two times smaller values of the corresponding parameters. Simulation results showed that the elasticity in the supporting system can negatively influence balance of the biped mechanism. In the case of a soft support, the ZMP trajectory can leave the foot contour and causes overturning of the mechanism. On the other hand, the elasticity in the system enables the system's adaptation upon the impact and peaks of the inertial forces. To solve this functional conflict between better adaptability of the system and a worse dynamical balance, the appropriate control strategy has to be chosen. Problems of control of the human-like walking or keeping the dynamic balance will be the next step of our study.

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