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Dust-acoustic shock waves in a magnetized non-thermal dusty plasma

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A theoretical investigation is carried out to study the basic properties of dust-acoustic (DA) shock waves propagating in a magnetized non-thermal dusty plasma (containing cold viscous dust fluid, non-thermal ions, and non-thermal electrons). The reductive perturbation method is used to derive the Korteweg–de Vries–Burgers equation. It is found that the basic properties of DA shock waves are significantly modified by the combined effects of dust fluid viscosity, external magnetic field, and obliqueness (angle between external magnetic field and DA wave propagation direction). It is shown that the dust fluid viscosity acts as a source of dissipation, and is responsible for the formation of DA shock structures in the dusty plasma system under consideration. The implications of our results in some space and laboratory plasma situations are briefly discussed.

1. Introduction

The dust-acoustic (DA) waves are extremely low frequency electrostatic dust-associated waves in which the mass of dust fluid provides inertia, and the thermal pressures of electrons and ions give rise to the restoring force. Rao et al. (1990) have first predicted theoretically the existence of DA waves, and Barkan et al. (1995) have later verified experimentally this prediction. It is known that in the presence of some sort of dissipation mechanism, the balance between nonlinearity and dissipation may lead to the formation of shock structures. There is an important issue to search for the source of this dissipation, which may be responsible for the formation of shock structures. A number of different mechanisms, which can be responsible for the formation of these shock structures, have been studied by a number of authors in different plasma models (Shukla 2000; Shah and Saeed 2009; Misra et al. 2012). Mamun and Cairns (2009) have shown that a strong correlation among highly charged dust grains can be the source of dissipation and can be responsible for the formation of DA shock waves. Gupta et al. (2001), Asgari et al. (2011), and Shahmansouri and Tribeche (2013) have shown that the effect of dust charge fluctuation can be another source of dissipation, which also leads to the formation of DA shock structures. Merlino et al. (2012) analyzed the effect of dust-neutral collisions on the basic features of DA shock structures. On the other hand, the evolution of shock structures is described by including an additional dissipation term in Korteweg–de Vries (KdV) equation. The KdV equation with the dissipative term (or the dissipative term of the Burgers equation) is known as the KdV–Burgers equation. The KdV–Burgers equation, which elegantly combines the effects of nonlinearity, dispersion, and

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dissipation, has been derived, and the generation and evolution of shock structures have been studied theoretically by many authors (Nakamura et al. 1999; Gupta et al. 2001; Mamun and Shukla 2002; Shah and Saeed 2009; Misra et al. 2012; Shahmansouri and Mamun 2013).

Merlino et al. (2012) experimentally studied the DA solitary and shock waves in the presence of dust-neutral collisions in a dusty plasma. They compared the evolution of experimentally observed shock waves with numerical calculations based on the Riemann solution of the fully nonlinear fluid equations for DA waves. Shukla and Mamun (2001) studied the formation of DA shock waves in a strongly coupled dusty plasma, and found that the effect of dust kinematic viscosity was a source of dissipation and responsible for the formation of shock structures that exhibit either oscillatory or monotonic behavior. Ghosh (2003) and Gupta et al. (2003) studied DA shock waves by deriving the modified KdV–Burgers equation in which the dissipative (Burgers) term arises due to the effect of dust charge fluctuation. Gupta et al. (2003) also presented a numerical investigation for large amplitude DA stationary shocks by using a complete set of nonlinear dust fluid equations coupled with dust charging and Poisson equations. Eliasson and Shukla (2004) have theoretically shown the existence of DA shock waves by numerical solutions of fully nonlinear fluid equations governing the dynamics of collisionless dusty plasmas. The latter have been found to be in good agreement by an experimental observation of DA shock waves by Heinrich et al. (2009). Recently, motivated by the experimental observation of DA shock structures by Heinrich et al. (2009), Bandyopadhyay et al. (2008), Teng et al. (2009), and Shukla and Eliasson (2012) presented a fully nonlinear theory for DA shock and solitary pulses in a strongly coupled dusty plasma by employing a generalized hydrodynamic model, and they related their results to low temperature dusty plasma discharges. The DA shock waves in a dusty plasma with charge fluctuating dust and superthermal plasma particles have also been investigated (Tribeche and Bacha 2010; Shahmansouri and Alinejad 2013a; Shahmansouri and Tribeche 2013). The combined effects of dust charge fluctuation and superthermal electrons on the basic features of DA shock waves are examined by Tribeche and Bacha (2010).

It is well known that the propagation properties of electrostatic solitary structures strongly depend on external magnetic field. The effect of external magnetic field on electrostatic waves has been studied by a number of authors (Cairns et al. 1996; Mamun 1998; Mamun et al. 1998; Zhang and Xue 2005a, b; Chatterjee et al. 2010; Sultana et al. 2010; Alinejad and Mamun 2011; Shahmansouri and Alinejad 2012; Sultana et al. 2012; Tribeche and Bacha 2012; Shahmansouri 2013a, b; Shahmansouri and Alinejad 2013b, c, d). Mamun et al. (1998) studied the obliquely propagating DA solitary structures in a hot magnetized dusty plasma. Mamun (1998) also studied the multi-dimensional instability of the obliquely propagating DA solitary waves in a magnetized dusty plasma with non-thermal ions. The small amplitude DA solitary waves propagating obliquely in a magnetized dusty plasma have been studied by Mamun and Hassan (2000), who have considered the effect of dust charge fluctuation. The DA solitary waves in inhomogeneous magnetized hot dusty plasmas with charge fluctuating dust were investigated by Misra and Chowdhury (2006). They obtained the Zakharov–Kuznnetsov equation for DA solitary waves by employing the standard reductive perturbation technique. Zhang and Xue (2005a) studied the obliquely propagating DA shock waves, which arise due to the effect of dust charge fluctuation, in a magnetized dusty plasma by deriving and analyzing the KdV–Burgers equation. The effects of dust charge fluctuation and a weak magnetic field on small amplitude DA waves in electronegative dusty plasma have been investigated by

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Ghosh et al. (2008). They have shown that a weak magnetic field was responsible for the dispersive term whereas the dust charge fluctuation was responsible for the dissipative term. Tribeche and Bacha (2012) studied the combined effects of dust charge fluctuation and superthermal electrons on the obliquely propagating DA solitary structures in a magnetized dusty plasma. They used a weak nonlinear theory to derive the KdV–Burgers equation. Shahmansouri and Mamun (2013) have investigated the obliquely propagating ion-acoustic shock waves due to the effect of ion kinematic viscosity in a magnetized electron–ion plasma, obtained a monotonic shock profile solution for the KdV–Burger equation via the *tanh* approach, and finally analyzed the stability condition. Shahmansouri and Mamun (2014) also derived the KdV–Burgers equation, and studied the dust ion-acoustic shock waves found to be significantly modified by the effects of obliqueness, external magnetic field strength, non-thermal electron, and dust charge fluctuation.

It is well known that the influence of external magnetic field and obliqueness drastically modify the basic properties of electrostatic structures. Thus, to complete the previously published works, the present authors examine here the properties of DA shock waves in a magnetized non-thermal dusty plasma comprising cold viscous dust fluid, non-thermal ions, and electrons following the distribution of Cairns et al. (1995).

The paper is organized as follows. The basic equations governing the plasma model under consideration are provided in Sec. 2. The KdV–Burgers equation is derived in Sec. 3. The stationary shock wave solution of this KdV–Burgers equation is obtained, and analyzed numerically in Sec. 4. Finally, a brief discussion is given in Sec. 5.

2. Governing equations

We consider a magnetized non-thermal dusty plasma containing cold viscous dust fluid, non-thermal ions, and electrons following the distribution of Cairns et al. (1995). We suppose that an external magnetic field (acting along the z-axis) is present in the system. The dynamics of low phase speed of purely electrostatic perturbation mode (whose phase speed is in between ion and dust thermal speeds) is governed by

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{u}_d) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}_d}{\partial t} + (\mathbf{u}_d \cdot \nabla)\mathbf{u}_d = \nabla\phi - \omega_{cd}\mathbf{u}_d \times \hat{z} + \eta\nabla^2\mathbf{u}_d, \tag{2}$$

$$\nabla^2\phi = n_d + f n_e - (1+f)n_i, \tag{3}$$

where n_d is the dust number density normalized by its equilibrium value, ϕ is the electrostatic wave potential normalized by T_i/e, $\mathbf{u}_d = (u, v, w)$ is the dust fluid speed normalized by $C_s = \sqrt{T_i/m_d}$, t is the time variable normalized by $1/\omega_{pd} = \sqrt{m_d/4\pi n_{d0} Z_{d0}^2 e^2}$, r is the space variable normalized by $\lambda_D = C_s/\omega_{pd}$, η is the dust kinematic viscosity normalized by $\lambda_D^2\omega_{pd}$, $\omega_{cd}$is the ion–cyclotron frequency normalized by ω_{pd}, $f = n_{e0}/Z_d n_{d0}$, and other symbols have their usual meaning.

To model the distribution of non-thermal electrons and ions, we employ the non-thermal distribution function of Cairns et al. (1995). The latter allow us to express electron and ion densities, n_e (normalized by its equilibrium valuen_{e0}) and n_i

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(normalized by its equilibrium value n_{i0}) as (Cairns et al. 1996, 1995) follows:

$$n_e(\phi) = \left\{1 - \beta_e\sigma\phi + \beta_e\sigma^2\phi^2\right\}\exp(\sigma\phi), \tag{4}$$

$$n_i(\phi) = \left\{1 + \beta_i\phi + \beta_i\phi^2\right\}\exp(-\phi), \tag{5}$$

where $\beta_{e,i} = 4\alpha_{e,i}/(1 + 3\alpha_{e,i})$ and α_e (α_i) determine the number of non-thermal electrons (ions). It may be mentioned here that (4) and (5) are correct for the following two assumptions: (i) electrons and ions are assumed to be unmagnetized (Cairns et al. 1996), which is valid for a very strong magnetic field making electron and ion Larmor radii very small in comparison with the size of the structures under consideration, i.e. causing electrons and ions to move almost parallel to a strong magnetic field; (ii) electrons and ions are assumed to follow non-thermal distribution of Cairns et al. (1995) because of the presence of energetic/fast particles (determined by α_e and α_i) that cannot be thermalized (i.e. Maxwellian) on the dust time-scale. It should be noted here that electrons and ions become Maxwellian only when $\alpha_e = 0$ and $\alpha_i = 0$.

3. Derivation of the KdV–Burgers equation

To investigate the small but finite amplitude DA shock structures in magnetized non-thermal dusty plasmas, we employ the standard reductive perturbation method. Thus, we introduce the stretched coordinates (Washimi and Taniuti 1966; Shukla and Yu 1978),

$$\xi = \varepsilon^{1/2}(l_x x + l_y y + l_z z - u_0 t), \quad \tau = \varepsilon^{3/2}t, \tag{6}$$

where ε is a small parameter measuring the weakness of amplitude or dispersion, u_0 is the DA phase speed normalized by C_s, and l_x, l_y, and l_z are, respectively, the directional cosines of the wave vector $\vec{k}$ along the x,y, and z—axes so that $l_x^2 + l_y^2 + l_z^2 = 1$. The dependent variables are then expanded as

$$n_d = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \cdots, \tag{7a}$$

$$u = \varepsilon^{3/2}u_1 + \varepsilon^2 u_2 + \cdots, \tag{7b}$$

$$v = \varepsilon^{3/2}v_1 + \varepsilon^2 v_2 + \cdots, \tag{7c}$$

$$w = \varepsilon w_1 + \varepsilon^2 w_2 + \cdots, \tag{7d}$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots. \tag{7e}$$

It should be noted that the appearance of transverse velocity components at a higher order of ε (relative to the parallel component$w$) comes from an anisotropy induced by the influence of a strong magnetic field (Misra and Bhowmik 2007). We further consider a weak damping situation so that the dust kinematic viscosity can be considered small but finite. This leads us to assume that (Shukla 2000; Shukla and Mamun 2001) $\eta \approx \varepsilon^{1/2}\eta_0$,where η_0 is a finite parameter of the order of unity.

Now, substitute (6) and (7) in (1)–(5), and collect the terms in different powers of ε. Thus, the lowest order of ε leads to the following expressions:

$$w_1 = -\frac{l_z}{u_0}\phi_1, \quad n_1 = \frac{l_z}{u_0}w_1, n_1 + f\sigma(1 - \beta_e)\phi_1 + (1 + f)(1 - \beta_i)\phi_1 = 0. \tag{8}$$

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Now eliminating the first-order variables, (8) leads to the following definition for phase speed:

$$u_0 = \frac{l_z}{\sqrt{f\sigma(1-\beta_e)+(1+f)(1-\beta_i)}}. \tag{9}$$

Again, for the next higher order of ε, x—and y-components of the momentum equation reduces to

$$u_1 = -\frac{l_y}{\omega_{cd}}\frac{\partial\phi_1}{\partial\xi}, \quad v_1 = \frac{l_x}{\omega_{cd}}\frac{\partial\phi_1}{\partial\xi}. \tag{10}$$

The above equations refer to the components of electric field drift. Now the next higher order in ε yields the following set of differential equations:

$$-u_0\frac{\partial n_2}{\partial\xi}+\frac{\partial n_1}{\partial\tau}+l_z\frac{\partial}{\partial\xi}(n_1w_1)+\frac{\partial}{\partial\xi}(l_xu_2+l_yv_2+l_zw_2)=0, \tag{11a}$$

$$\frac{\partial w_1}{\partial\tau}-u_0\frac{\partial w_2}{\partial\xi}+l_zw_1\frac{\partial w_1}{\partial\xi}=l_z\frac{\partial\phi_2}{\partial\xi}+\eta_0\frac{\partial^2w_1}{\partial\xi^2}, \tag{11b}$$

$$\frac{\partial^2\phi_1}{\partial\xi^2}=n_2+f\left(b_1\phi_2+b_2\phi_1^2\right)-(1+f)\left(c_1\phi_2+c_2\phi_1^2\right), \tag{11c}$$

$$u_2=-\frac{u_0l_x}{\omega_{cd}^2}\frac{\partial^2\phi_1}{\partial\xi^2}, \tag{11d}$$

$$v_2=-\frac{u_0l_y}{\omega_{cd}^2}\frac{\partial^2\phi_1}{\partial\xi^2}. \tag{11e}$$

We finally eliminate the second-order perturbed quantities from (8)–(11). This elimination, which needs a few straightforward mathematical steps, leads to the following differential equation:

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=C\frac{\partial^2\phi_1}{\partial\xi^2}. \tag{12}$$

The above equation (12) is a KdV–Burgers equation, which describes the nonlinear evolution of the obliquely propagating DA shock waves in a magnetized non-thermal dusty plasma. The nonlinear, dispersion, and dissipation coefficients of the KdV–Burgers equation are given by

$$A=\frac{u_0^3}{2l_z^2}[1+f-f\sigma^2]-\frac{3l_z^2}{2u_0}, \quad B=\frac{u_0^3}{2l_z^2}\left[1+\frac{1-l_z^2}{\omega_{cd}^2}\right], \quad C=\frac{1}{2}\eta_0. \tag{13}$$

The shock-like solution of (12) depends upon the competition between coefficients$A$, B, and$C$. It is seen that the nonlinear coefficient A and the dispersion coefficient B are influenced by the obliqueness, while the dissipation coefficient is independent of obliqueness. The dissipation coefficient depends only on dust kinematic viscosity. We note that in the absence of dust kinematic viscosity, (12) reduces to the well-known KdV equation.

Therefore, it is clear from the coefficients of the KdV–Burgers equation that the relation between l_z, ω_{cd}, and η_0 can control the behavior of solution in (12). To investigate the effect of obliqueness on nonlinear and dispersion coefficients, we showed the variation of A and B with θ (angle between external magnetic field and wave propagation direction) for different values of α_i (ion non-thermal parameter). The results are depicted in Fig. 1, which shows that the nonlinear coefficient A

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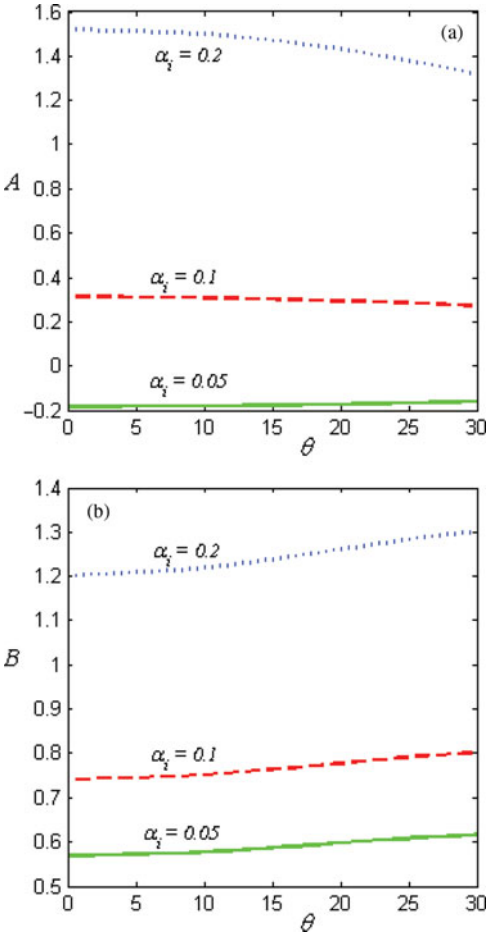


FIGURE 1. (Colour online) Variation of (a) nonlinear and (b) dispersion coefficients of the KdV–Burger equation with obliqueness (θ) for different values of α_i (*viz.* $\alpha_i = 0.05$, $\alpha_i = 0.1$, $\alpha_i = 0.2$) and for $\omega_{cd} = 1$, $\eta = 1$, $f = 0.1$, $\alpha_e = 0.1$, and $\sigma = 0.1$.

decreases with increasing the value of θ. On the other hand, it is seen in Fig. 1(a) that the nonlinear coefficient is a very sensitive function of α_i, and for $\alpha_i < \alpha_{\text{crit}} \approx 0.0688$ the nonlinear coefficient is negative whereas for $\alpha_{\text{crit}} < \alpha_i$, it is positive. The effect of obliqueness on the dispersion coefficient $B$is shown in Fig. 1(b). The dispersion coefficient B increases with increasing the values of θ and α_i.

4. Stationary shock-like solution

To obtain the shock-like solution for (12), we consider a reference frame moving with the shock speed V. This transform allows us to express the independent variables ξ and τ in the form

$$\zeta = (\xi - V\tau')/L \quad \text{and} \quad \tau' = \tau, \tag{14}$$

where V is the shock speed (in the reference frame). Now substituting these new coordinates of (14) into (12), we find the third-order ordinary differential equation for ϕ_1. The latter can be integrated to obtain the second-order ordinary differential

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equation,

$$B\frac{d^2\phi_1}{d\zeta^2} - CL\frac{d\phi_1}{d\zeta} + \frac{AL^2}{2}[\phi_1]^2 - VL^2\phi_1 = 0, \tag{15}$$

where L is the shock width, and the appropriate boundary conditions, namely $\phi_1 \to 0$, $d\phi_1/d\zeta \to 0$, $d^2\phi_1/d\zeta^2 \to 0$ at $\zeta \to \infty$ are used. The differential equation (15) is similar to the equation of motion of a damped inharmonic oscillating particle of a unit mass with speed $d\phi_1/d\zeta$ and position ϕ_1. It is easy to show that the differential equation (15) has two singular points $d\phi_1/d\zeta = 0$ at $\phi_1 = 0$ and $d\phi_1/d\zeta = 0$ at $\phi_1 = 2V/A$. If we assume that the particle was located at $\phi_1 = 0$ for $\zeta \to +\infty$, then for $\zeta \to -\infty$ we have $\phi_1 = 2V/A$. Thus, under the condition that ϕ_1 is bounded at $\zeta \to \pm\infty$, the solution of (15) describes a shock-like structure whose velocity V (in the reference frame) is related by $\phi_1(-\infty) - \phi_1(\infty) = 2V/A$.

So far several methods have been presented to solve nonlinear partial differential equations (for instance, inverse scattering method, Backlund transformation, Hirota bilinear formalism, *tanh* method, et al.), but in the systems, including the combined effects of dispersion and dissipation, the *tanh* method has been the most convenient method. Therefore, by adopting the *tanh* approach (Malfliet 1992, 2004; Kourakis et al. 2012) we can obtain an analytical stationary solution for (15) in the following form:

$$\phi_1(\xi, \tau) = \frac{\Phi_{\max}}{4}\left\{4 - \left[1 + \tanh\left(\frac{\xi - V\tau}{L}\right)\right]^2\right\}, \tag{16}$$

where the shock width, the shock amplitude, and the shock speed are, respectively, defined as

$$L = 10B/C, \quad \Phi_{\max} = 12C^2/25AB, \quad V = 6C^2/25B. \tag{17}$$

The shock speed is supposed to be positive, thus the solution (16) corresponds to a shock traveling toward the $+\zeta$-axis. We see that the width, amplitude, and speed of the shock structures defined by (17) are functions of the parametersβ_e, β_i, l_z, η_0, f, and ω_{cd}.

To study the occurrence of monotonic or oscillatory behavior in shock waves defined by (16), we should investigate the asymptotic behavior of the solution (16). For this purpose we consider a perturbed solution in the form of $\Phi(\xi, \tau) = \phi_1(\xi, \tau) + \varepsilon\Phi_1$, with ε to be a real small parameter. Then we substitute the perturbed solution into (15) and linearize it with respect to Φ_1. The linearized form (16) can be written as (Gupta et al. 2001; Shukla and Mamun 2001; Sultana et al. 2012)

$$B\frac{d^2\Phi_1}{d\zeta^2} - CL\frac{d\Phi_1}{d\zeta} + \Phi_1(A\phi_1 - V)L^2 = 0. \tag{18}$$

It is easy to show that the general solution of the above equation is an exponential function in the form $\exp(p\zeta)$, in which the parameter γ is given by

$$p = \frac{CL}{2B} \pm \sqrt{\frac{C^2L^2}{4B^2} - \frac{L^2}{B}(A\phi_1 - V)}. \tag{19}$$

We note that ϕ_1 in (19) can take any values from 0 to 2 V/A. The nature of parameter p (i.e. real or imaginary) determines the oscillatory or monotonic behavior of shock structures. We further note that for imaginary p, the shock waves will have oscillatory behavior whereas for real$p$ the perturbation around the exact solution (16) presents an exponential behavior. The substitution of ϕ_1 (obtained from (16)) into (19) shows

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that the parameter p always takes the real value. This means that the present dusty plasma model supports the monotonic shock profiles.

We now numerically examine the influence of relevant physical parameters, such as electron and ion non-thermal parameters, magnitude, and obliqueness of external magnetic field and dust kinematic viscosity on DA shock structures.

To find the effect of obliqueness and ion non-thermality on the basic properties (*viz.* amplitude, width, and speed) of the DA shock waves, we numerically analyzed (17). The numerical results are shown in Fig. 2. We observed that the amplitude of shock wave increases with obliqueness, while the deviation of ions from thermodynamic equilibrium shows an inverse effect on the amplitude (shown in Fig. 2(a)). The influence of obliqueness on the shock amplitude is stronger for the lower values of α_i. It should be noted that we restricted the values of θ to the region of $0 < \theta \leqslant 30$ because the perturbation method (which is only valid for the weak amplitude limit) is not valid for the large values of θ. Figure 2(b) indicates that the DA shock width increases with obliqueness and ion non-thermality. The dispersion coefficient B is always positive, thus the shock amplitude defined by (17) indicates that the nature of the shock waves (compressive or rarefactive) depends on the sign of the nonlinear coefficient A. Thus, Fig. 1(a) predicts that the dusty plasma system under consideration supports either compressive or rarefactive shock waves, namely rarefactive shock occurs for $\alpha < \alpha_{\text{crit}}$ and compressive shock occurs for $\alpha_{\text{crit}} < \alpha$. Figure 2(c) indicates that the DA shock speed decreases with both obliqueness and ion non-thermality. Therefore, Fig. 2 shows that when the wave goes more oblique, the dusty plasma model supports the existence of larger, wider, and slower shocks, and that the higher values of ion non-thermality support the existence of smaller, narrower, and slower shock waves.

The effect of different relevant physical parameters on the shock profile has been investigated in Fig. 3. We observed that the dust kinematic viscosity significantly affects the DA shock profile. We observed in Fig. 3(a) that an increase in the dust kinematic viscosity increases the shock wave amplitude. The influence of the magnitude of external magnetic field on the DA shock profile is examined in Fig. 3(b). We observed that the higher values of the magnetic field support the formation of the DA shocks with smaller width and larger amplitude. This means that the magnetic field leads to more spiky shock structures. Figure 3(c) shows how the shock structures are modified by the ion non-thermality. This figure also verifies the occurrence of both compressive and rarefactive DA shock waves. The amplitude of compressive (rarefactive) DA shock waves decreases (increases) with increasing α_i. It can be seen that the effect of ion non-thermality is stronger for rarefactive shock structures. A similar behavior of the compressive shock waves formed in a magnetized non-thermal dusty plasma has been reported by Zhang and Xue (2005a). The influence of obliqueness is also investigated in Fig. 3(d). It is seen that when the wave goes more oblique, the amplitude (thickness) of the DA shock wave increases (decreases). This is similar to the observations reported by others (Zhang and Xue 2005a; Tribeche and Bacha 2012), who studied DA shock waves in magnetized dusty plasmas with charge fluctuating dust.

It should be noted that when the angle of propagation tends to zero ($l_z \to 1$), our results are in good agreement with those obtained by Shukla and Mamun (2001) for DA shock waves in unmagnetized dusty plasmas. All these results are qualitatively similar to the results obtained for DA shock waves in dusty plasmas with Maxwell–Boltzmann or polytrophic electrons and ions. However, there are some quantitative differences arising from the type of distribution function that we have chosen to obtain the number density of electrons and ions. Although the kappa distribution

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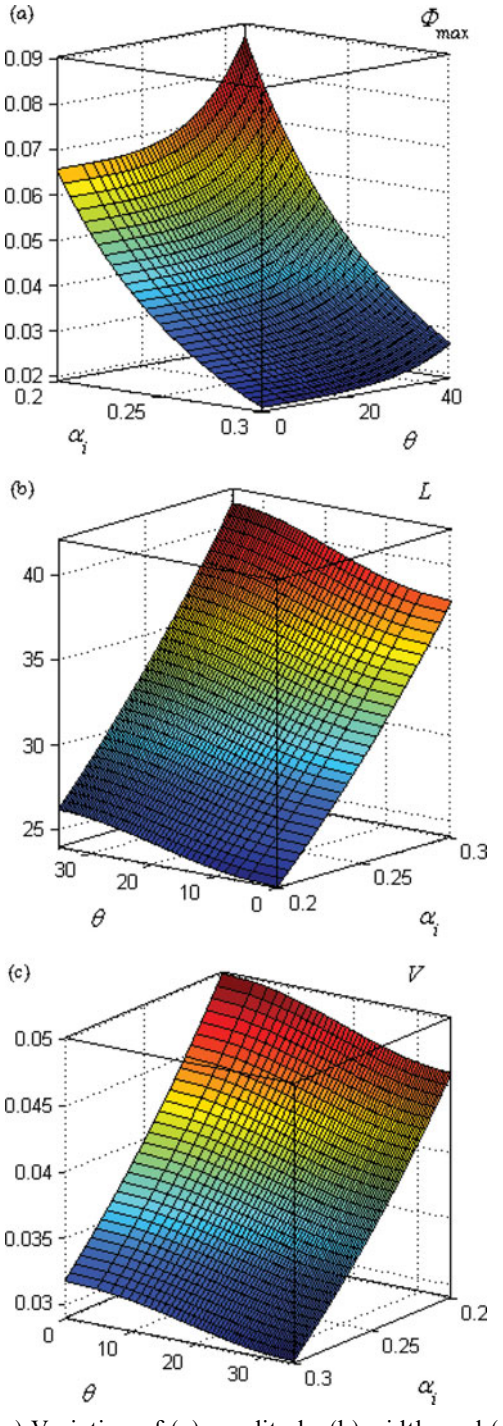


FIGURE 2. (Colour online) Variation of (a) amplitude, (b) width, and (c) velocity of DA shock waves with the obliqueness (θ) and ion non-thermal parameter α_i. The other parameters are the same as in Fig. 1 (*viz.* $\omega_{cd} = 1$, $\eta = 1$, $f = 0.1$, $\alpha_e = 0.1$, and $\sigma = 0.1$).

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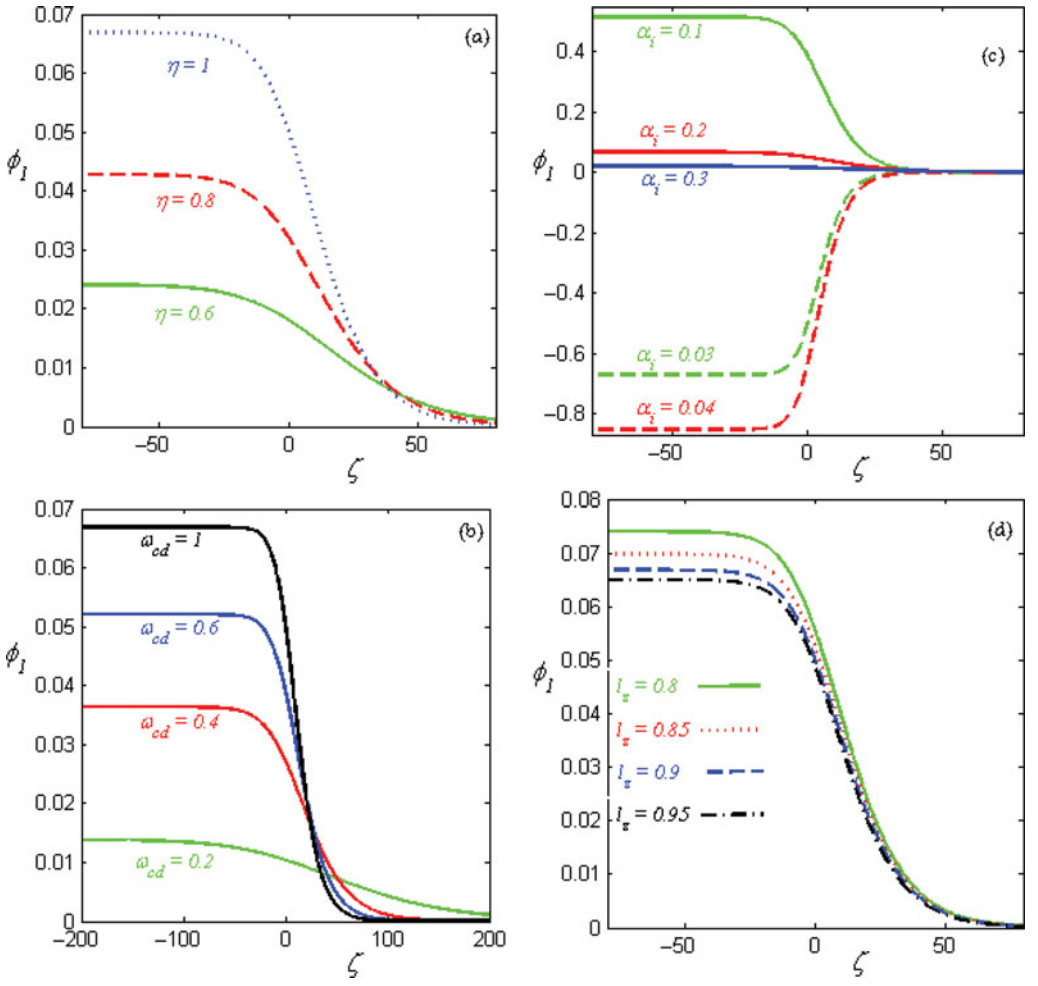


FIGURE 3. (Colour online) DA shock profiles for different values of (a) η with$\omega_{cd} = 1$, $\alpha_i = 0.2$,$l_z = 0.9$; (b) the magnitude of magnetic field with $\eta = 1$, $\alpha_i = 0.2$, $l_z = 0.9$; (c) α_i with $\eta = 1$, $\omega_{cd} = 1$, $l_z = 0.9$; and (d) l_z with $\eta = 1$, $\alpha_i = 0.2$, $\omega_{cd} = 1$. The common parameters are $f = 0.1$, $\alpha_e = 0.2$, $\sigma = 0.1$.

is a non-thermal distribution, the results would be different when one of the hot plasma species has a different non-thermal distribution. However, it is shown that both compressive and rarefactive DA shock waves will be excited in this system relative to other choices for the ion distribution. These differences may depend on the deviation of the distribution function from the Maxwellian one. As for the kappa distribution, the main change with respect to the Maxwellian would be restricted to the 'tail' region, unlike the case of the Cairns distribution (Cairns et al. 1995). A similar behavior also occurs for the DA solitary waves, which have been described by Baluku and Helberg (2008).

The limitation of our present theory is that it is not valid for the waves propagating perpendicularly to the magnetic field, i.e. $\theta \to 90^\circ$ or $l_z \to 0$, since the latter makes the amplitude of the shock waves infinitely large, and thus breaks down the assumption for the perturbation theory. It may be further noted here that even for a large angle,

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the assumption for the waves being electrostatic in nature is no longer valid, and one should then look for electromagnetic shock structures.

5. Discussion

We have considered a magnetized non-thermal dusty plasma containing negatively charged dust fluid and non-thermal ions and electrons following the non-thermal distribution of Cairns et al. (1995), and have theoretically investigated the formation of small but finite amplitude DA shock waves and their basic properties by the reductive perturbation technique. The main points of this investigation are summarized as follows:

1. The nonlinear propagation of DA shock waves in a magnetized non-thermal dusty plasma is governed by the KdV–Burger equation, the coefficients of which are significantly modified by the obliqueness, external magnetic field, non-thermal distribution of electrons and ions, and dust kinematic viscosity coefficient.

2. The shock wave solution of the KdV–Burger equation via the *tanh* approach is obtained, and the stability condition is then analyzed. It implies that the dusty plasma system under consideration supports only monotonic shock structures, which are unstable with respect to external perturbations.

3. The presence of non-thermal ions does not only significantly modify the basic features of DA shock waves but also causes the existence of shock structures with both negative and positive potential. The amplitude of rarefactive (compressive) DA shock waves increases (decreases) with the increase of α_i.

4. The basic properties (amplitude, width, speed, et al.) of DA shock structures are found to be significantly modified by the effects of obliqueness. It is seen that as propagation goes more oblique, i.e. as θ increases, the amplitude and thickness of shock structures increase, while the shock speed decreases. This means that for the larger values of θ, the shock waves are larger, wider, and slower.

5. The amplitude and speed of DA shock waves increase with the increase of the magnitude of external magnetic field. However, the shock width decreases with increase of the magnitude of external magnetic field. Thus, for the higher values of magnetic field, the dusty plasma system under consideration supports the larger, narrower, and faster DA shock structures. This means that the magnetic field leads to more spiky shock structures. It is also found that the dust kinematic viscosity enhances the DA shock height.

We have used a wide range of the dusty plasma parameters in our numerical analysis, i.e. $\omega_{cd} = 0.2 - 1$, $\theta = 0 - 30$, $\alpha_i = 0.04 - 0.3$, and $\eta = 0.6 - 1$, which are within the range of the dusty plasma parameters corresponding to space environments (Dovner et al. 1994; Cairns et al. 1995, 1996; Winske et al. 1995) and laboratory devices (Merlino and Goree 2004; Bandyopadhyay et al. 2008; Heinrich et al. 2009; Teng et al. 2009). Therefore, our results should be useful for understanding localized electrostatic disturbances in space environments and laboratory devices where the magnetized dust fluid (Mamun 1998; D'Angelo 2002) and non-thermal electrons (Dovner et al. 1994; Cairns et al. 1995, 1996; Winske et al. 1995) and ions (Mamun et al. 1996; Mamun 1998) exist.

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