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Defining and Modeling State-Dependent Dynamic Systems

Justin Esarey
Department of Political Science, Rice University, 105 Herzstein Hall, 6100 Main Street, Houston, TX 77005
e-mail: justin@justinesarey.com (corresponding author)

Jacqueline H. R. DeMeritt
Department of Political Science, University of North Texas, 1155 Union Circle #305340, Denton, TX 76203

Edited by Janet Box-Steffenmeier

A state-dependent dynamic system is one in which (1) the marginal effect of x on y at time t ($\partial y_t/\partial x_t$) depends on the prior value of the dependent variable y_{t-1}, and (2) the persistence of the dependent variable ($\partial y_t/\partial y_{t-1}$) depends on x_t. We present a methodological strategy for dealing with state-dependent dynamic systems and demonstrate the consequences of ignoring state-dependence. As an applied example, we find evidence of state-dependence in the relationship between presidential approval and economic performance: high unemployment rates are most damaging to presidential approval among presidents with the highest initial approval ratings.

1 Introduction

It is often observed that monetary policy is effective at regulating growth in good economic times, but ineffective at regulating growth during a recession. Indeed, monetary policy is often analogized to a string: it can be used to pull growth down during a bubble, but not to push it up. Consequently, the relationship between monetary policy (x) and current economic growth (y_t) is a function of the past state of economic growth (y_{t-1}).

This is an example of what we call a *state-dependent dynamic system* (SDDS), one in which (1) the marginal effect of x on y at time t ($\partial y_t/\partial x_t$) depends on the prior value of the dependent variable y_{t-1}, and (2) the persistence of the dependent variable ($\partial y_t/\partial y_{t-1}$) depends on x_t.[1] Some related ideas have been articulated in substantive work, in particular the Markov transition/dynamic probit model introduced by Przeworski et al. (2002) in the study of democratic regime transition and used in other applications (e.g., Powell and Mitchell 2007; Kucik and Reinhardt 2008; Sing 2010).[2] We think that SDDS are often created by political institutions, many of which involve continuous dependent variables not amenable to the dynamic probit, and that developing a methodological strategy to study these systems would enhance our understanding of politics. We demonstrate how simple microfoundational processes can create a state-dependent system and how these microfoundations are likely to be common in politics.

For example, we believe that the relationship between presidential approval and national economic performance in the United States might be such a system.[3] A long tradition of

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[1] Both conditions will be true if either is true in structural models where symmetry of effect is imposed via Young's theorem ($\frac{\partial^2 y_t}{\partial x \partial y_{t-1}} = \frac{\partial^2 y_t}{\partial y_{t-1} \partial x}$), as in typical GLM models.

[2] Additionally, Franzese (2002, chap. 3) studies the relationship between government fractionalization and budget deficits. He finds that fractionalization contributes to fiscal policy inaction, meaning that deficits expand when the accumulated debt is already high and compound interest accumulates, but stay stable or shrink when existing debts are low.

[3] We thank Matt Lebo for suggesting this application to us.

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research has argued that economic conditions have immediate and negative effects on the level of presidential popularity, and this research has found much evidence (but by no means uniformly supportive evidence) for the proposition (Berlemann and Enkelmann 2012). It is also well known[4] that presidential approval tends to erode over the course of a president's term. We believe the speed of this erosion could be a function of both economic performance and prior approval rating. Economic conditions have a direct effect on voters' approval ratings, but might also make efforts to organize opposition more effective and make it easier to convince persuadable voters to oppose the administration. If this is true, then poor economic performance will speed the decline of approval over time as a result of learning and opinion diffusion. The implication is that the relationship between lagged presidential approval (y_{t-1}) and current presidential approval (y_t) depends on the state of the economy (x_t): when the state of the economy is worse, a president's support will erode faster (dy_t/dy_{t-1} will shrink as x_t grows). As a consequence of Young's theorem (presented in footnote 1), this also means that the negative relationship between the state of the economy and current approval (dy_t/dx_t) is larger when lagged presidential approval (y_{t-1}) is higher.

In this article, we lay out a statistical model to study SDDS and consider the model's properties and relationship to existing work. We find that a relatively simple model that interacts y_{t-1} with x_t can recover correct estimates of an SDDS data-generating process (DGP) for a continuous dependent variable in a time-series cross-sectional data set of reasonable length ($T \geq 20$). A notable advantage of this model is that techniques for properly interpreting and presenting results are already well known to the discipline (Brambor, Clark, and Golder 2006). We also propose and verify tests for the presence of state-dependence in an empirical data set. Surprisingly, we find that state-dependent DGPs create complex and unanticipated interactions among the independent variables in their relationship on the dependent variable that can be recovered from a properly specified model. Finally, we use our SDDS model to study the relationship between presidential approval and economic performance. Our extension of an earlier model by Geys (2010) finds that high unemployment rates are most damaging to the approval ratings of presidents with the highest initial approval.

2 A Dynamic Interaction Model for SDDS

To see whether the relationship between economic performance and US presidential approval (or any other relationship of interest to political scientists) is state-dependent, we need a way to quantitatively study state-dependence. This includes articulating a statistical model, describing how to extract substantively important quantities from that model, verifying its reliability at recovering state-dependent DGPs, and finding a way to test for the presence (or absence) of state-dependence. We begin with a model.

2.1 *A Statistical Model of State-Dependence*

Suppose that the relationship between an independent variable x_{it} and a dependent variable y_{it} for a unit i at time t is conditional on the contextual value of y at the previous time $t-1$. Where a linear DGP is appropriate, this suggests the following dynamic interaction model:

$$y_{it} = \beta_0 + \beta_1 y_{i(t-1)} + \beta_2 x_{it} + \beta_3 \left(x_{it} y_{i(t-1)}\right) + \beta_4 z_{it} + u_{it}. \quad (1)$$

Here, i indexes a unit and t indexes time; the data are a time-series cross-section. The relationship between x_{it} and y_{it} depends on the value of the lagged dependent variable $y_{i(t-1)}$. Suppose that β_2 and β_3 are both positive. In this case, changes in x_{it} cause y_{it} to grow, but growth is accelerated in an environment where $y_{i(t-1)}$ is already large. Other forms of state-dependence are possible, but we

[4]For example, MacKuen (1983) notes that "Upon first sight, the striking thing about presidential popularity's movement is that for most presidents it moves ineluctably downward from the first day in office" (p. 178).

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focus on this model because it is simple, has broad application, and the lessons learned from it apply to other models with a similar structure.[5]

The dynamic probit model of Przeworski et al. (2002, 137–9) shares the idea that a DGP can depend on its prior state:

$$\Pr(y_t = 1|y_{t-1} = 0) = \Phi(X\alpha),$$
$$\Pr(y_t = 0|y_{t-1} = 1) = \Phi(X\beta).$$

This structure maps directly onto a Markov model of transition between the two states of y, where the probability of switching from $y = 0 \rightarrow 1$ is determined by a different process than the probability of switching from $y = 1 \rightarrow 0$; for example, economic growth and a high GDP may prevent transition from democracy to autocracy but have little bearing on the transitions from autocracy to democracy. But this model is designed for discrete dependent variables only.[6]

2.2 *Microfoundations of State-Dependence*

The idea of state-dependence is simple: the effect of x on y depends on the prior state of y. But what underlying causal processes would justify using a model like equation (1)? Here we consider three possibilities, each of which we believe has plausible and potentially valuable applications in political science.

2.2.1 Direct state-dependence: Contextual causality

We begin with the simplest case: equation (1) is a direct description of the causal mechanism that is generating the data. That is, the causal power of some independent variable x_t on the dependent variable y_t at time t is directly blunted or enhanced when the lagged dependent variable y_{t-1} gets larger (we drop the unit index i for simplicity). The asymmetric effectiveness of monetary policy on growth may be such a case; we illustrate the argument using a simplified model of investment behavior.

Suppose that individual investors borrow and spend capital when they expect that capital to pay off in profitable returns. We posit that this return is proportional to a monotonic function of the economic growth rate, $f(y_t)$, and an error term ε_j that is fixed for each investor j but stochastic with respect to the population of investors. Further, we suppose that actors will form their expectation of future returns on the basis of past levels of growth, so that $E[f(y_t)] = f(y_{t-1})$. Consequently, an investor j's expected profit per dollar invested will be

$$\begin{aligned} E[\pi_{jt}] &= E[f(y_t) - r + \varepsilon_j] \\ &= f(y_{t-1}) - r + \varepsilon_j, \end{aligned}$$

where r is the current interest rate, set by the central bank. Investors will only spend when $r < f(y_{t-1}) + \varepsilon_j$; alternatively, the proportion of investors who invest $p = F(f(y_{t-1}) - r)$ where F is

[5] For example, Franzese (2002, chap. 3) employs an error-correction model with a lag interaction term:

$$\Delta y_{it} = \beta_0 + \beta_1 \Delta y_{i(t-1)} + \beta_2 \Delta y_{i(t-2)} + \beta_3 y_{i(t-1)} + \beta_4 x_{it} + \beta_5 (x_{it} y_{i(t-1)}) + \beta_6 z_{it} + u_{it}.$$

This model makes the change in y for unit i at time t a function of past changes in y, past levels of y, and exogenous variables, where the effect of certain exogenous variables is contingent on past levels. Based on prior work showing the close relationship between error-correction model and the lagged dependent variable model (De Boef and Keele 2008), we would expect our findings to be highly applicable to the Error Correction Model (ECM) context. An ECM model directly equivalent to ours would be

$$\Delta y_{it} = \beta_0 + \beta_1 y_{i(t-1)} + \beta_2 \Delta x_{it} + \beta_3 x_{i(t-1)} + \beta_4 (x_{it} y_{i(t-1)}) + \beta_5 \Delta z_{it} + \beta_6 z_{i(t-1)} + u_{it},$$

where we obtain this model by subtracting y_{t-1} from both sides of equation (1), then adding and subtracting $x_{i(t-1)}$ and $z_{i(t-1)}$ to the right-hand side. This model lacks the lagged terms of Franzese's model that would change the relationship between y_{it}, $y_{i(t-1)}$ and x_{it} that we describe.

[6] Additionally, dynamic probit imposes state-dependence on all independent variables, leading to a less efficient estimate in cases where only some variables have state-dependent effects.

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the cumulative density of ε. Concordantly, decreasing r will have a smaller marginal effect on investment when y_{t-1} is small because of the shape of the cumulative density function (CDF), but will have a larger marginal effect on investment when y_{t-1} is large. This feeds back to present growth levels if present growth is a function of current investment, $y_t = g(p)$.

The dynamic is straightforward. When investors' expectations of future profits $f(y_{t-1})$ is near zero or negative, lowering interest rates r will not produce additional investment because r is bounded at zero and therefore cannot go low enough to make investment a profitable proposition. Ergo, the relationship between growth and interest rates will be weak when past growth is weak. In contrast, when $f(y_{t-1})$ is large, changes in r can substantially change the profitability of investment and have a significant effect on investment decisions. Thus, the relationship between growth and interest rates will be strong when past growth is strong.

Of course, the actual linkages between interest rates, investment, and growth are considerably more complicated than this model suggests. But the model suffices to show how state-dependence can exist directly in the DGP as a consequence of relatively simple forces that are commonly found in social scientific applications. In this case, profit-seeking by investors combined with feedback between investment and growth creates state-dependence in the form of a low growth/cheap money "liquidity trap" that monetary policy authorities strive to avoid as they manage the macroeconomy.

2.2.2 Direct state-dependence: Changing returns to scale

There are other causal mechanisms through which a DGP might be state-dependent. We would, for example, expect many government actions or programs to face changing returns to scale for a variety of reasons. A United Nations peacekeeping force might be adept at limiting local or small-scale conflicts but ineffective at curbing larger-scale conflicts (e.g., those involving more states or states with greater capabilities) simply because the UN cannot field enough forces to deter in the latter cases. To take another example, a job counseling program might be effective at reducing unemployment when a moderate number of clients are served by a staff of closely supervised professionals, but much less effective when overwhelmed by a huge number of unemployed persons that inundates a small staff or prompts the creation of a larger and less efficient bureaucracy.

In these two cases and others like them, the driving force of state-dependence is *changing returns to scale*. Our two examples are both illustrations of *diminishing* returns to scale: a particular policy intervention is designed to influence the dependent variable (conflict deaths or unemployment rates, in our examples), but these interventions become less effective as the dependent variable grows. There can also be cases of *increasing* returns to scale: interventions to affect the dependent variable become stronger as the dependent variable gets larger.

Put otherwise, changing returns to scale means that the relationship between the dependent variable y_t at time t and an independent variable x_t depends on the prior state of the dependent variable. We can represent this relationship in simple linear fashion:

$$\frac{\partial y_t}{\partial x_t} = \beta_1 + \beta_2 y_{t-1}. \tag{2}$$

Of course, this marginal effect structure is directly implied by our model in equation (1). When $\beta_2 > 0$, there are increasing returns to scale; when $\beta_2 < 0$, there are decreasing or diminishing returns to scale.

Equivalently, some forces may have a proportional effect on the dependent variable instead of a cardinal effect, changing y_{t-1} by a proportion of its size rather than by some absolute number. In principle, economic policies designed as "automatic stabilizers" are designed to work this way (i.e., by cooling inflation more as the economy grows hotter and by injecting more stimulus as growth falls). In cases like these, the marginal effect of the relevant variable is also given by equation (2) and implies a state-dependent model structure.

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2.2.3 Indirect state-dependence: Learning and diffusion

The aggregation of many heterogeneous DGPs can result in a state-dependent system, even when the individual DGPs are not themselves state-dependent, because the individual processes interact with one another. Consider the example of presidential approval ratings in the United States, which we will revisit more closely later in the article. Prior work has argued that voters are not only influenced by external events, such as increased unemployment or the outbreak of war, but also by one another: opinions change via persuasion inside social networks and through new information obtained via the media and personal contacts. External events may mediate the flow of opinions through social networks and enhance voters' receptiveness to opposition arguments. If so, changing macroeconomic conditions will have immediate effects on those directly affected, *plus* longer-term secondary effects as information diffuses through the population and as voters influence each other.

For example, increased unemployment presumably causes decreased presidential approval from those who lose their jobs. But it also provides a sociotropic reason for other, still-employed workers to disapprove and to be more susceptible to the appeals of opposition parties. These secondary effects will not be immediate, as information takes time to spread through social networks, overcome favorable prior beliefs, and motivate coherent opposition movements. These processes are always happening during any presidential administration, but poor economic performance makes them quicker and more efficaceous. In short, increasing unemployment and declining growth should not just provide a shock to presidential approval, but should make its usual downward trend stronger. The statistical model implied by this story resembles equation (1), as we now demonstrate.

As we noted above and in the introduction, presidential approval ratings tend to deteriorate over time. This provides the starting point for a simple model, wherein in the absence of other effects (or noise) the approval rating y at time t is given by

$$y_t = \alpha y_{t-1}$$

for $\alpha \in (0,1)$. Rearranging terms, this model implies that each term a president loses

$$\frac{y_t - y_{t-1}}{y_{t-1}} = -(1-\alpha)$$

proportion of his or her previous supporters. This is equivalent to

$$-\left(1 - \frac{dy_t}{dy_{t-1}}\right) = -(1-\alpha),$$

which makes intuitive sense; the term on the left-hand side is also the proportion of voters that a president expects to lose in each time period. Of course, there is more to presidential approval than simple declines over time, and a more realistic model of approval will include other influences:

$$y_t = \alpha y_{t-1} + \beta x_t + \varepsilon_t,$$

where x_t may represent, for example, the effect of macroeconomic conditions. In these models, changes in an independent variable like x_t have long-term impacts beyond their immediate effect; the initial impact reverberates over time through the lag term (Wilson and Butler 2007, 107; Keele and Kelly 2006, 189; De Boef and Keele 2008). But if our argument above is correct, then it will also be the case that changes in macroeconomic conditions will change the decay of approval:

$$y_t = \alpha y_{t-1} + \beta_1 x_t + \beta_2 x_t y_{t-1} + \varepsilon_t.$$

Using the previous technique, we find that the speed at which presidential approval declines depends on economic conditions:

$$-\left(1 - \frac{dy_t}{dy_{t-1}}\right) = -(1-\alpha-\beta_2 x_t).$$

If x_t measures contemporaneous unemployment, for example, then greater unemployment will cause a president to lose a greater share of supporters at each time period.

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3 Complex Dynamics and Unexpected Conditionalities Created by State-Dependence

Any form of state-dependent relationship between x and y creates interesting temporal dynamics. As we noted earlier, it is already well understood that changes in x have long-term impacts (beyond the effect at time t) on the dependent variable through the lag coefficient in a model with a lagged dependent variable.[7] In our dynamic interaction model (equation (1)), this story is more subtle: both the instantaneous and long-term marginal effects of x on y are highly contextual. Even more surprisingly, the long-term impact of independent variables that are not state-dependent, like z, are contingent on the level of variables with state-dependent effects like x—even without an explicit interaction term between these two variables.

The upshot is that a relatively simple theoretical concept—the idea that the effect of x on y depends on the prior state of y—has complex and important empirical implications for substantive inference that must be carefully teased out by an analyst. In this section, we illustrate how to derive the instantaneous and long-term marginal effects of independent variables, showing how these marginal effects are contextual and suggesting ways to make this contextuality clear to a reader.

3.1 *Instantaneous Marginal Effects of x on y*

In an OLS regression without state-dependence, most marginal effects can be read directly off a coefficient table as a simple β coefficient. A dynamic interaction model has a more complicated marginal effect owing to the interaction term between x_{it} and $y_{i(t-1)}$ (Ai and Norton 2003; Braumoeller 2004; Kam and Franzese 2007). We recommend displaying these effects using the technique of Brambor, Clark, and Golder (2006): calculate the instantaneous marginal effect $\partial y_{it}/\partial dx_{it}$ and its standard error for multiple values of $y_{i(t-1)}$ using simulation, then display a plot of this relationship. Such a plot allows the reader to see how the effects of a change in the independent variable will differ in different contexts. For the model in equation (1), the instantaneous marginal effect is $\beta_2 + \beta_3 y_{i(t-1)}$.

To illustrate the process, we generated a time-series cross-sectional data set out of a DGP with $y_{it} = 0.2y_{i(t-1)} + 0.5x_{it} + 0.3 * x_{it} * y_{i(t-1)} + 0.5z_{it}$; the data set has ten time periods and thirty units, and therefore three hundred total observations.[8] Because the model includes a lag, one time period was discarded for each unit, leading to a final $N = 270$ in the regression. We estimated a correctly specified OLS regression on this data set; the results are shown in Table 1. We then drew one thousand samples out of the multivariate normal distribution of $\hat{\beta}$ using the variance-covariance matrix of the regression; for each draw, we calculated $\partial y_{it}/\partial dx_{it} = \hat{\beta}_2 + \hat{\beta}_3 y_{i(t-1)}$ for every value of $y_{i(t-1)} \in [-6,4]$. We plot the median and 95% confidence interval of this derivative in Fig. 1.

As the figure shows, changes in x_{it} can either increase or decrease y_{it} depending on the state of the world. When $y_{i(t-1)}$ is less than about -3, the marginal effect of increases in x_{it} is negative; otherwise, the marginal effect is positive. Interpreted substantively, the DGP tends to be a self-reinforcing system: when y is already large, increases in x_{it} tend to make it even larger; when y is small, increases in x_{it} tend to have little or even a negative effect on y.

3.2 *Long-term marginal effect of x on y*

The first step for determining the long-term marginal effect of x on y is to determine the steady state of y associated with a level of x. We can then determine how this steady-state y changes as x_{it} changes. We present a technique that analysts can use to visually present long-term marginal effects for easy interpretation.

[7] See, for example, Wilson and Butler (2007, 107), Keele and Kelly (2006, 189), and De Boef and Keele (2008).

[8] x_{it} was drawn from the uniform distribution between -2 and 2. Starting values of y_{i1} were drawn from the uniform distribution between -1 and 1.

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Table 1 Model on simulated data

	β	SE
Intercept	0.02695	0.02492
$y_{i(t-1)}$	0.20391	0.02742
$x_{it}y_{i(t-1)}$	0.31411	0.02572
x_{it}	0.49081	0.02210
z_{it}	0.53688	0.02156

Note. OLS Regression Model. No. of observations = 270. $R^2 = 0.8193$.

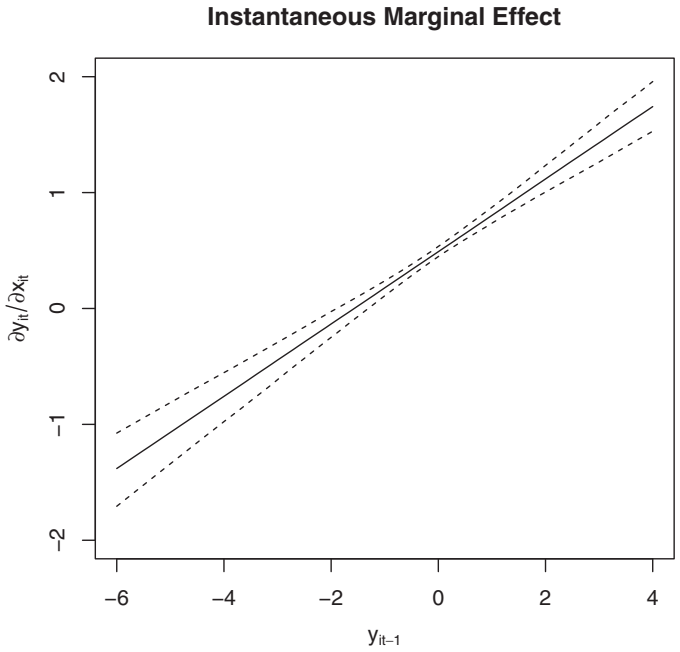


Fig. 1 The effect of x_{it} on y_{it}, from Table 1.

The model in equation (1) establishes a differential equation that must be solved for y in order to determine the steady-state value of y. Start by rearranging terms slightly:

$$y_{it} - y_{i(t-1)} = \beta_0 + \beta_2 x_{it} + \beta_3\left(x_{it}y_{i(t-1)}\right) + \beta_4 z_{it} - (1-\beta_1)y_{i(t-1)} + u_{it}.$$

In a steady state, $y_{it} = y_{i(t-1)}$. So, setting these terms equal to a steady state y, we have

$$0 = \beta_0 + \beta_2 x_{it} + \beta_3(x_{it}y) + \beta_4 z_{it} - (1-\beta_1)y + u_{it}.$$

We may now solve this equation for y (dropping the indices on x and z) and take expectations to eliminate the random term u_{it}:

$$E[y] = \frac{\beta_0 + \beta_2 x + \beta_4 z}{(1-\beta_1-\beta_3 x)}. \tag{3}$$

Equation (3) shows the expected steady state of y associated with a particular value of x. It differs from the standard steady-state calculation for a model with a lagged dependent variable via the presence of the $\beta_3 x$ term in the denominator.

This process of calculating the long-term change in y caused by an instantaneous change in x_{it} is illustrated in Fig. 2. The figure depicts the evolution of y_{it} for forty time periods of a model where $y_{it} = 0.2y_{i(t-1)} + 0.5x_{it} + 0.3 * x_{it} * y_{i(t-1)} + 0.5z_{it}$; we held $z_{it} = 1$. For the first ten time periods,

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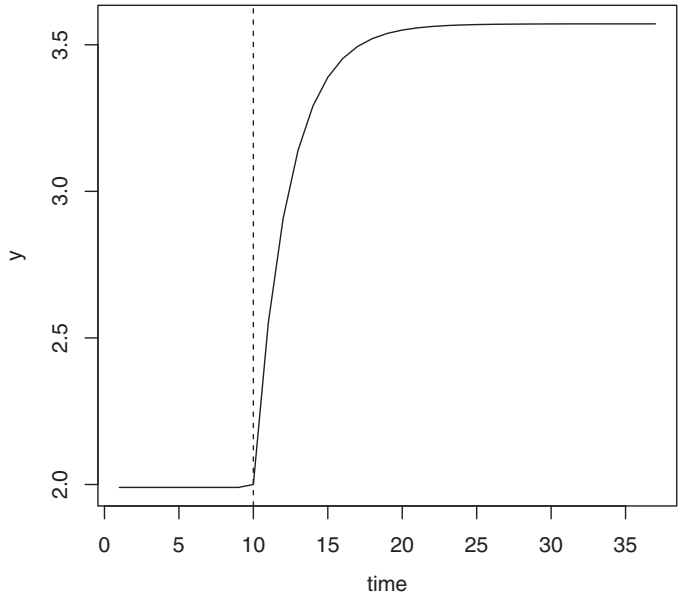


Fig. 2 Long-term change in y after a change in x at $t = 10$.

$x = 1$ and the associated steady-state $y = 2$. At $t = 10$, depicted by a dotted line in the graph, x increases to 1.5. The instantaneous change in y associated with this change in x is $(0.5 + 0.3 * 2) * 0.5 = 0.55$. However, the long-term change in y is far greater: over the next twenty-five periods, y increases to a new steady state of 3.57—a long-term marginal effect of about 1.57.

To account for the change in the steady-state x, we can use equation (3) to determine the change in the steady state of y associated with a change in x. The process is reasonably simple: calculate the steady-state y for both values of x, then subtract the two. The change in steady-state calculation for the change in x from 1 to 1.5 is

$$\frac{0.5(1.5) + 1(0.5)}{(1 - 0.2 - 0.3(1.5)))} - \frac{0.5(1) + 1(0.5)}{(1 - 0.2 - 0.3(1)))} \approx 1.57. \quad (4)$$

As equations (3) and (4) make clear, the long-term marginal effect of x_{it} on y depends on both the starting value of x_{it} and its ending value $x_{i(t+1)}$, not just the gap between them. The values of variables that are not state-dependent also matter, as the presence of x in the denominator does not allow the z values to cancel. Marginal effects must be calculated for changes of interest; they cannot be universally calculated for all possible changes. To illustrate this, we revisit an earlier example where we generated a time-series cross-sectional data set ($N = 30$, $T = 10$) out of a DGP with $y_{it} = 0.2y_{i(t-1)} + 0.5x_{it} + 0.3 * x_{it} * y_{i(t-1)} + 0.5z_{it}$. Using the results from Table 1, what can we predict about the change in steady-state y as x changes? We used the one thousand draws of β that we simulated from the variance-covariance model to calculate the 95% confidence interval for the long-term marginal effect using equation (3).

For example, suppose that a case starts at $x = 0$ and then moves to x_{new}. The marginal effect and its 95% confidence interval are presented in Fig. 3 for values of x_{new} between -1.5 and 1.5. The figure shows that the marginal effect of a change in x varies greatly depending on the value of x_{new}. When x changes from 0 to -1, y declines by about 0.5. But when x changes from 0 to 1, y rises by nearly double that amount. In fact, a sufficiently large change in x can cause explosive growth and cause the series to become nonstationary; note that equation (3) for this DGP would indicate a steady-state $y = \infty$ for $x \geq 2.6\bar{6}$.

3.3 *Stationarity*

The potential for explosive growth in y in the prior example raises an important question: how do we know when a state-dependent time series is stationary? Given that our model (equation (1)) is an

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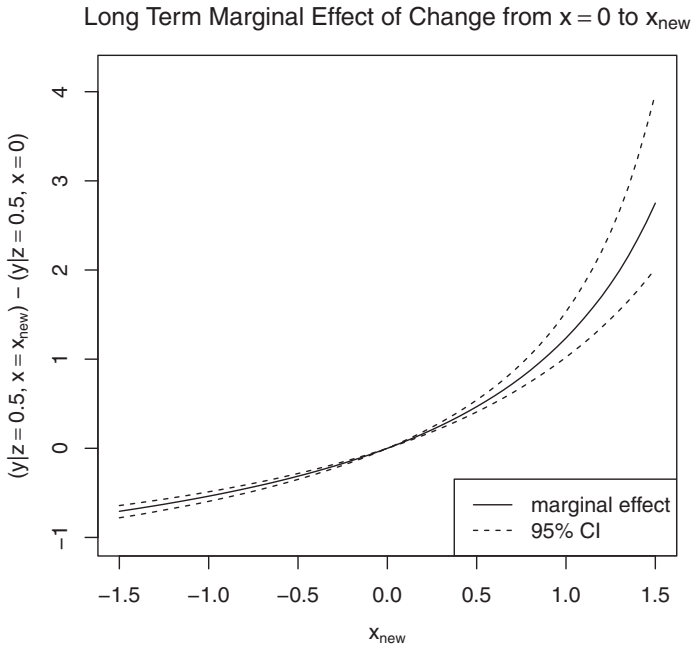


Fig. 3 Long-term marginal effect plot for x with 95% confidence interval, $z = 0.5$.

AR(1) process, a lag coefficient with an absolute value less than 1 indicates stationarity (Keele and Kelly 2006). In our model, the relationship between y_t and y_{t-1} varies depending on the value of x. Imitating the typical procedure for a standard time series[9] without state-dependence, we might look for the roots in L of

$$[1 - (\beta_2 + \beta_3 x_t)L]y_t = 0,$$

where L is the lag operator (and dropping the panel index for simplicity). These roots are as follows:

$$L = \frac{1}{(\beta_2 + \beta_3 x_t)}$$

and are greater than 1 whenever $(\beta_2 + \beta_3 x_t) < 1$. Ergo, the series becomes nonstationary whenever $(\beta_2 + \beta_3 x_t) \geq 1$, or equivalently whenever $x_t \geq (1 - \beta_2)/\beta_3$.

But what if x_t is only slightly greater than $(1 - \beta_2)/\beta_3$, and only for a short time? If x stays at or above $(1 - \beta_2)/\beta_3$ permanently, it will produce an integrated time series and all the typical problems of modeling it (without, e.g., appropriately differencing the series) will apply. But it is not clear that the series will become explosive when $x_t \approx (1 - \beta_2)/\beta_3$ for intermittent spells. Indeed, such series may not even have a particularly long memory: if the value of x_t drops, the total lag coefficient $(\beta_2 + \beta_3 x_t)$ declines accordingly and past periods of cumulative growth in y are forgotten. Are dynamic interaction models of y_t appropriate in these situations?

Our simulation study (presented in the next section) more deeply examines the properties of state-dependent DGPs with intermittent periods of nonstationarity. To preview our results, such excursions usually pose no problem for inference with a dynamic interaction model. When they do, standard tests for a unit root are useful for detecting the problem.

[9]For an overview, see Chatfield (2004).

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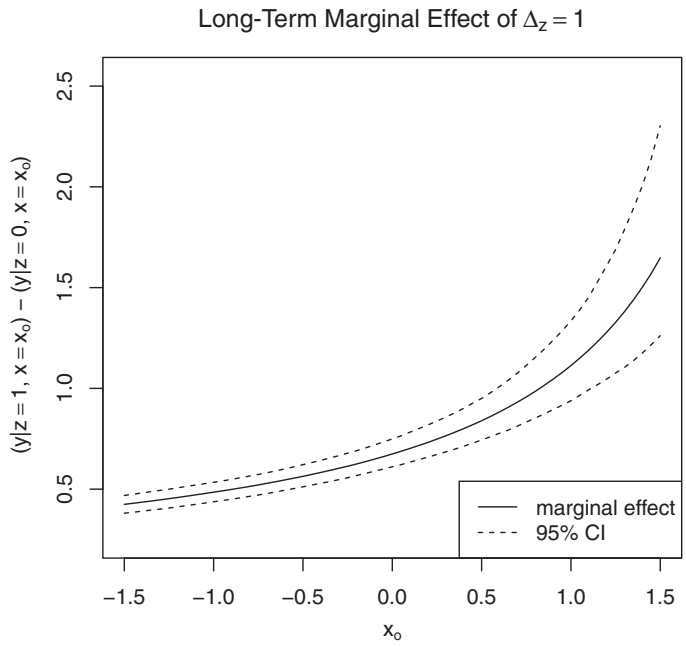


Fig. 4 Long-term marginal effect plot for z with 95% confidence interval.

3.4 *Unexpected Interaction among Independent Variables*

One interesting and subtle implication of state-dependence is that independent variables (such as x and z in equation (1)) have interactive effects on the steady-state y even in the absence of a product term between them in the model. That is, the long-term marginal effect of z on y is contingent on the value of x and cannot be read directly off a coefficient table. The consequence is that marginal effects for z must be estimated using the steady state technique above, *even though no product term between x and z is present in the model*.

Equation 3 indicates that variables that are not interacted with y, which we labeled z in the previous example, are nevertheless a factor in determining the steady-state value of y. If z changes from z_{lo} to z_{hi}, where $\Delta_z = z_{hi} - z_{lo}$, the change in steady state is

$$\frac{\beta_0 + \beta_2 x + \beta_4 z_{hi}}{(1 - \beta_1 - \beta_3 x)} - \frac{\beta_0 + \beta_2 x + \beta_4 z_{lo}}{(1 - \beta_1 - \beta_3 x)} = \frac{\beta_4 \Delta_z}{(1 - \beta_1 - \beta_3 x)}. \quad (5)$$

What this implies is that the long-term marginal effect of z is contingent on the value of the state-dependent variable x—that is, that the effect of a change in z on y depends on the level of x. In terms of their effects on y, then, z and x are indirectly interacted. The equation also shows that changes in the steady state are contingent not on specific values of z but on the magnitude of change in that variable, which we call Δ_z. In short, the marginal effect of z on y depends both on the level of the state-dependent variable x, and on the degree of change in the ordinary variable z. Thus, we should plot the long-term marginal effect for a fixed change in z at different values of x.

To illustrate the procedure, we continue using the data set and estimated model from Table 1. Using this information, we estimated the long-term marginal effect by simulating one thousand draws of β from the variance-covariance model, setting $\Delta_z = 1$, then calculating the 95% confidence interval for equation (5) for values of $x \in [-1.5, 1.5]$. The result is shown in Fig. 4. As the figure indicates, the long-term marginal effect of a change in z on y depends on the value of x_{it}; larger values of x_{it} are associated with a larger marginal effect of changes in z_{it}.

4 Model Performance and the Consequences of Misspecification

Based on the previous section, we can already conclude that ignoring state-dependence is substantively harmful. When state-dependence is present, the effect of changing independent

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variables—including variables that are not state-dependent—is highly contextual. Furthermore, independent variables have both short-term and long-term effects on the dependent variable y. Without a properly specified dynamic interaction model, all these subtleties are lost. The result is that an empirical model may not be able to capture the complexities of a dynamic theory of political interaction.

In this section, we use simulation analysis to assess: (1) whether an appropriately specified dynamic interaction model can accurately recover the DGP, especially in the presence of unit-specific effects; (2) the consequences of a likely misspecification; (3) how an analyst can determine whether state-dependence is present; and (4) whether brief excursions out of stationarity are a problem for modeling state-dependent DGPs. First, past evidence suggests that models that include both a unit-specific intercept and a lagged dependent variable are intrinsically biased, though the bias is negligible in many cases (Wilson and Butler 2007). Our simulation evidence shows that our model can correctly recover the DGP in both fixed and random effects models when there are enough temporal observations to work with ($T \geq 20$), regardless of the number of units N. Second, we need to investigate whether the problem of ignoring state-dependence is merely one of neglecting subtlety; it may be that a misspecified model can still accurately predict the dependent variable y_{it}. Simulations show that this is not the case: a misspecified model is much less capable of predicting the dependent variable. Third, we investigate methods of detecting state-dependence in a data set. Our simulations confirm that a t-test on the product term is a reasonable indicator of state-dependence, whereas the Bayesian Information Criterion (BIC) does even better in this role. Finally, we determine that the dynamic interaction model is useful for analyzing intermittently nonstationary series. The model breaks down when these periods of nonstationarity become too long, but standard unit root tests are useful for determining when this is the case.

4.1 *Simulation Details*

For each simulation, one thousand data sets are generated out of the model:

$$y_{it} = \beta_0 + \beta_1 y_{i(t-1)} + \beta_2 x_{it} + \beta_3 \left(x_{it} y_{i(t-1)}\right) + \beta_4 z_{it} + \alpha_i + u_{it}. \tag{6}$$

The resulting simulated data is a time-series cross-section (TSCS). The β_1 coefficient on y_{t-1} and the interaction coefficient β_3 are drawn from the uniform distribution between -0.4 and 0.4, whereas the β_2 coefficient on x_{it} and the β_4 coefficient on z_{it} are both drawn from the uniform distribution between -2 and 2. The error term u_{it} is normally distributed with zero mean and a standard deviation of 3. x_{it} and z_{it} are drawn from the uniform distribution between -3 and 3. Note that this structure implies that some simulated data sets will be nonstationary for short periods of large x_{it}; when $\beta_1 = \beta_3 = 0.3$, for example, the series is temporarily nonstationary whenever $x_{it} \geq 7/3 \approx 2.33$; we examine this issue more closely in a later subsection.

Two types of simulation are run. One includes unit-specific effects α_i and no common intercept ($\beta_0 = 0$); unit effects are drawn from the uniform distribution between -3 and 3. We systematically vary the number of units $N \in \{10, 20, 50\}$ and the number of time periods $T \in \{5, 10, 20, 30, 40, 50\}$. The other type of simulation, a simple time series with $N = 1$ and $T \in \{5, 10, 20, 50\}$, has a common intercept β_0 drawn from the uniform distribution between -3 and 3 and sets all $\alpha_i = 0$. The results from the simple time-series simulations are all consistent with the results for panel models, and so we concentrate on presenting the panel results; the time-series results are reported separately in a supplementary appendix.

For each of the one thousand data sets, two models are fitted: one with an accurate specification, and one with a specification that drops only the interaction between x_{it} and $y_{i(t-1)}$. When unit effects are present, we consider two approaches to capturing them: a random effects model, and a simple dummy variable specification. Results were substantively similar for simulations with and without unit-specific effects except where noted. Thus, we focus on the results from simulations with unit-specific effects.

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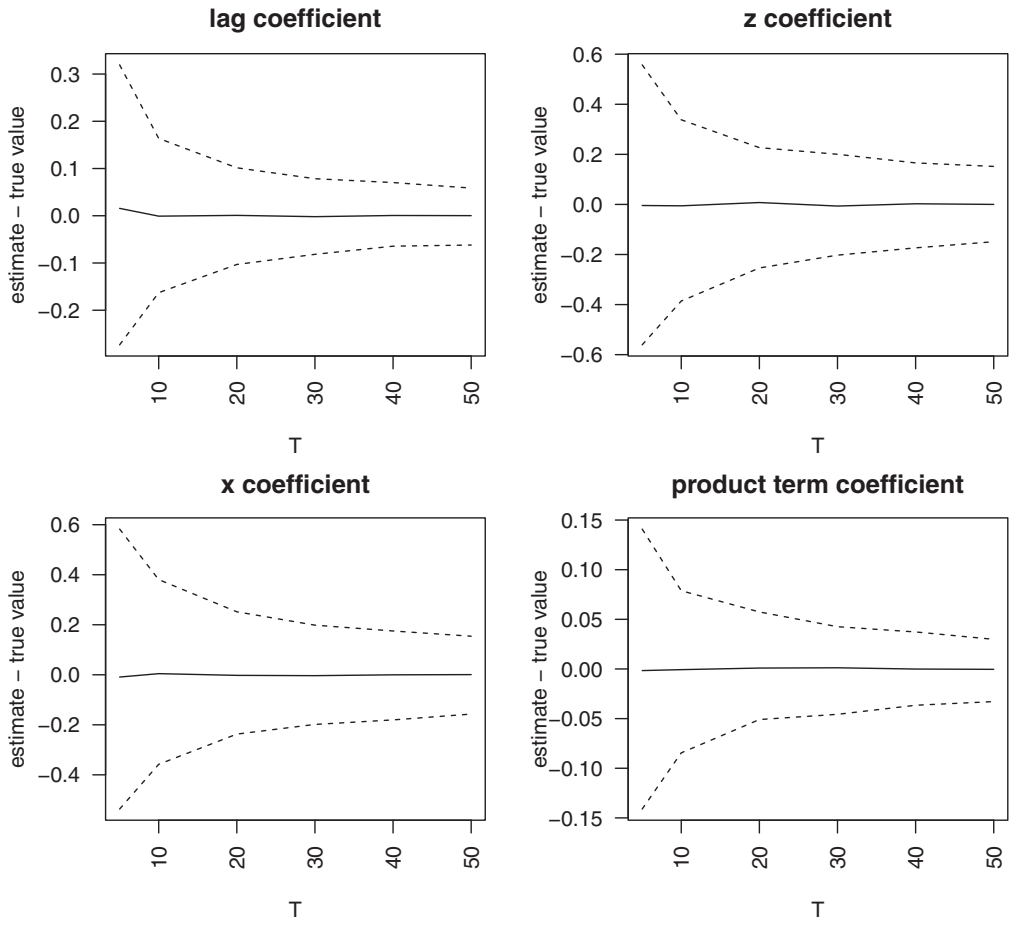


Fig. 5 Simulation results from a correctly specified random effects model, $N = 10$.

4.2 *Accurate Recovery of the DGP*

Our first concern is whether a properly specified model can accurately recover the DGP. We begin by assessing a random effects model, which is appropriate for the simulated DGP because the unit effects are uncorrelated with other independent variables. Our simulation results for the smallest data sets ($N = 10$) are depicted in Fig. 5; the figure shows the median bias of our one thousand simulations along with a 95% confidence interval. The simulations reveal that coefficient estimation is unbiased for all coefficients, even for the very shortest values of T, though estimate variability is considerably reduced for $T \geq 20$. This is good news for our model: when a random effects model is appropriate, dynamic interaction models can accurately recover the DGP structure from a TSCS data set.

But random effects models are not always appropriate, and coefficient bias is a special concern in the presence of a least-squares dummy variable model (Keele and Kelly 2006; Wilson and Butler 2007). Our simulation results for fixed effects models are similar to the results in Fig. 5, except for the lag coefficient. We focus on results for the lag coefficient in Fig. 6, showing results for three different values of N. The simulations indicate that the bias of the lag coefficient shrinks in increasing T and is negligible for $T \geq 20$, but is the same for different values of N. These results very closely conform to the earlier findings of Judson and Owen (1999), who investigate dynamic panel models and find that estimates of nonlag coefficients are generally unbiased and that bias in the lag coefficient diminishes with T. [10]

[10] Judson and Owen recommend $T \geq 30$, rather than $T \geq 20$.

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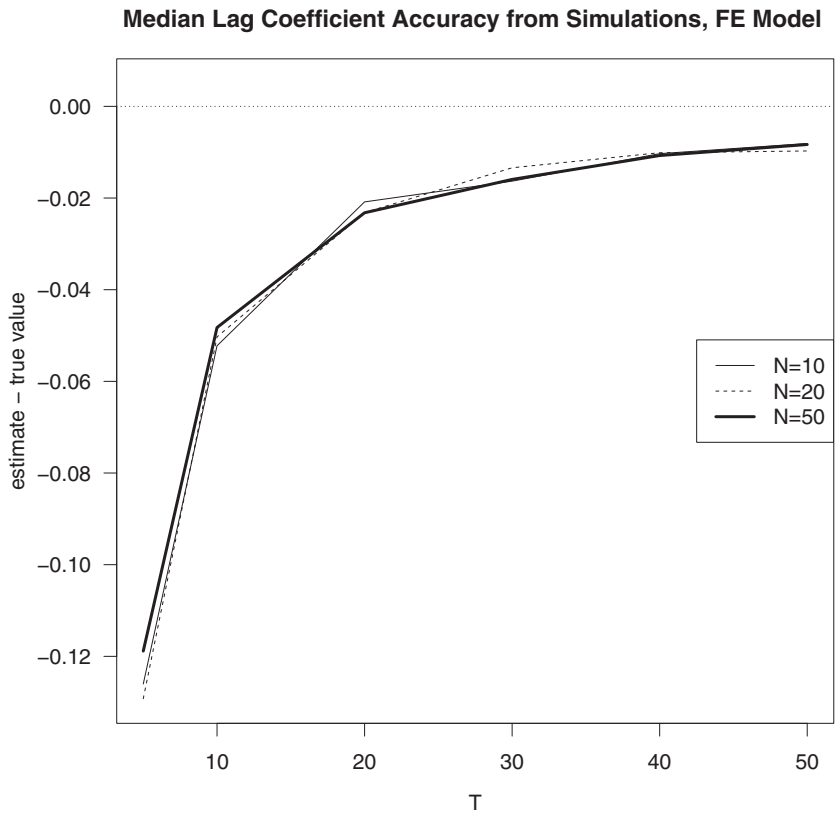


Fig. 6 Lag coefficient bias in fixed effects models, simulation results varying N and T.

Taken as a whole, our results indicate that properly specified dynamic interaction models can recover the DGP, but are most reliable with enough temporal observations ($T \geq 20$) to allow dynamics to be properly observed. This property of the model makes intuitive sense, given the discussion of the previous section. SDDS are associated with subtle, long-term dynamics; the full effects of a change in a variable may not be felt for many periods into the future. A data set must be "long" enough to see these effects unfold and model them properly.[11] This is especially important when a fixed effects model is used: "short" data sets will probably result in a biased estimate of the lag coefficient and subsequently biased marginal effects estimates.

4.3 *The Predictive Accuracy of Correct and Misspecified Models*

Do properly specified dynamic interaction models outperform models that ignore state-dependence? To answer this question, we measured the in-sample predictive capability of models that include a product term $\left(x_{it}y_{i(t-1)}\right)$ to those that do not but are otherwise correctly specified. For each of the one thousand data sets, we calculate the root mean square error (RMSE) of the estimated model's prediction:

$$RMSE = \sqrt{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it}^{o} - \hat{y}_{it}\right)^2},$$

where $y_{it}^{o} = \beta_0 + \beta_1 y_{i(t-1)} + \beta_2 x_{it} + \beta_3\left(x_{it}y_{i(t-1)}\right) + \beta_4 z_{it} + \alpha_i$, the prediction of the true model excepting the error term, and $\hat{y}_{it}$ is the estimated model's prediction of the same quantity.

[11] This is not universally true for all dynamic panel models; see Keele and Kelly (2006) for details.

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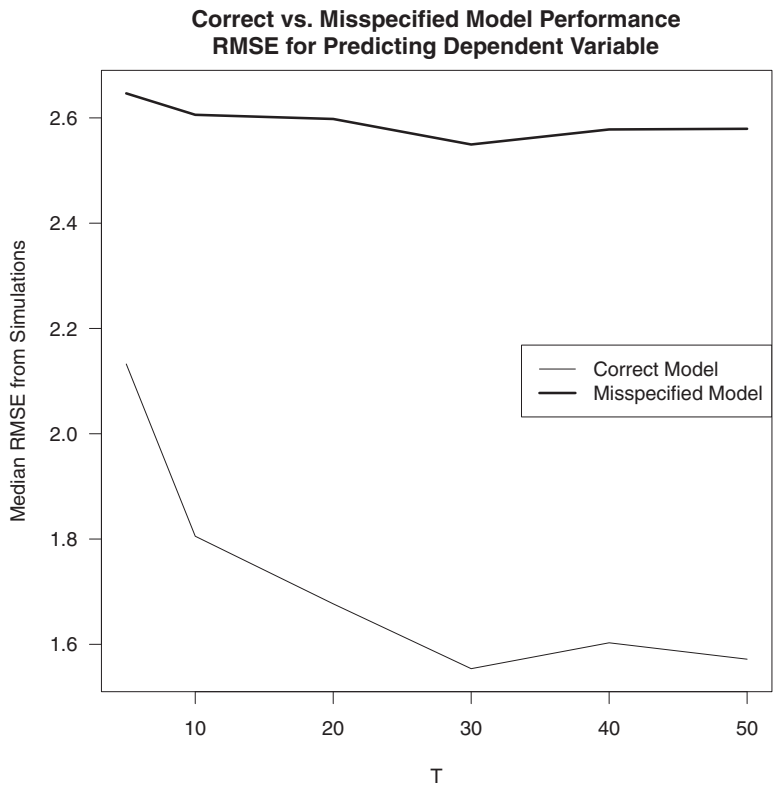


Fig. 7 Predictive performance for correctly and incorrectly specified, RE models, $N = 10$.

Figure 7 displays a comparison of the RMSE for random effects models that include the product term ("correct" models) against those that do not ("misspecified" models).[12] As the figure shows, the consequence of misspecification is poorer performance in predicting the dependent variable. For all values of T, the product term model outperforms the no-product term model. Furthermore, although the misspecified model's predictive performance remains constant for all levels of T, the correctly specified model with a product term gets better as T increases (presumably because of more efficient estimates).

It is also informative to examine the RMSE of correctly specified models for different values of N, as shown in Fig. 8. The figure reiterates a lesson from the previous section: increasing the number of units N does not appreciably improve the performance of a dynamic interaction model, but increasing T does. Consequently, investigating a state-dependent relationship is inadvisable with data sets shorter than $T = 20$, regardless of N.

4.4 *Stationarity Revisited*

As we discussed above, state-dependent DGPs may become intermittently nonstationary whenever x_{it} gets large enough to allow $(\beta_1 + \beta_3 x_{it})$ to exceed 1. To determine the inferential consequences of temporary departures from stationarity, we examined some time-series simulations (with $N = 1$, $T = 200$) designed to manipulate how often the stationarity boundary $x_t < (1 - \beta_2)/\beta_3$ is broken. Specifically, we altered the previous simulation parameters to set the lag coefficient $\beta_2 = 0.4$ and let the interaction coefficient $\beta_3 \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$; we drew x_t from the uniform distribution between 1 and 3. Under these simulations, the series spends progressively more time out of a

[12] Results are very similar for fixed effect models.

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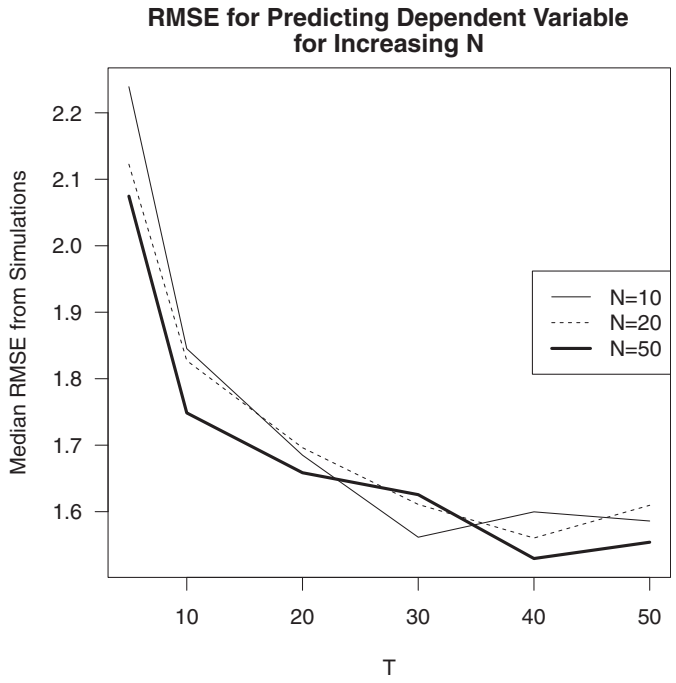


Fig. 8 Predictive performance of correctly specified dynamic interaction models, varying N.

stationary state as β_3 grows; at the highest levels, the series is nonstationary whenever $x \geq 1.2$, about 90% of the time. We also assessed the resulting series using the augmented Dickey-Fuller and Phillips-Perron unit root tests under the null hypothesis that the series is nonstationary (as implemented in the tseries package in R). The one thousand new samples were drawn and analyzed under these new conditions; the results are shown in Fig. 9a and b.

As the figures indicate, neither the coefficient accuracy nor the RMSE is substantially impacted by increasing frequency of excursions outside of stationarity—until $\beta_3 = 0.4$, when the RMSE begins to rise. At $\beta = 0.5$, the RMSE explodes and the accuracy of β_1 and β_4 estimates is dramatically decreased. Interestingly, the lag and product term coefficients continue to be accurately estimated for all values of β_3. The Phillps-Perron test increasingly fails to reject the null of a unit root as β_3 rises, and never rejects the null of a unit root (in favor of the alternative that the series is stationary) for $\beta_3 \geq 0.4$. The augmented Dickey-Fuller follows the same pattern, although rejection rates begin to rise again for extreme values of β_3.

This set of results suggests a strategy for determining whether nonstationarity in a state-dependent series will hinder inference from a dynamic interaction model. First, passing the Phillips-Perron test for a unit root appears to be a useful and favorable indicator for model performance. Second, an analyst can use a model's estimated lag and interaction coefficients to conduct a Monte Carlo study tailored to assess model accuracy under the conditions of the DGP in question. Based on our results, these coefficients should be estimated accurately enough to assess whether a dynamic interaction model is suitable for the sample at hand.

4.5 *How Can Analysts Decide Whether a Relationship Is State-Dependent?*

When state-dependence is suspected, the prior subsection shows that including an interaction term between the lagged dependent variable and the relevant independent variable is essential. As a result, when asking and answering substantive questions, it is important to have a reliable procedure to determine whether state-dependence is present. The statistical significance of the product term and the BIC are both reasonable indicators of state-dependence, but the BIC is the best overall performer.

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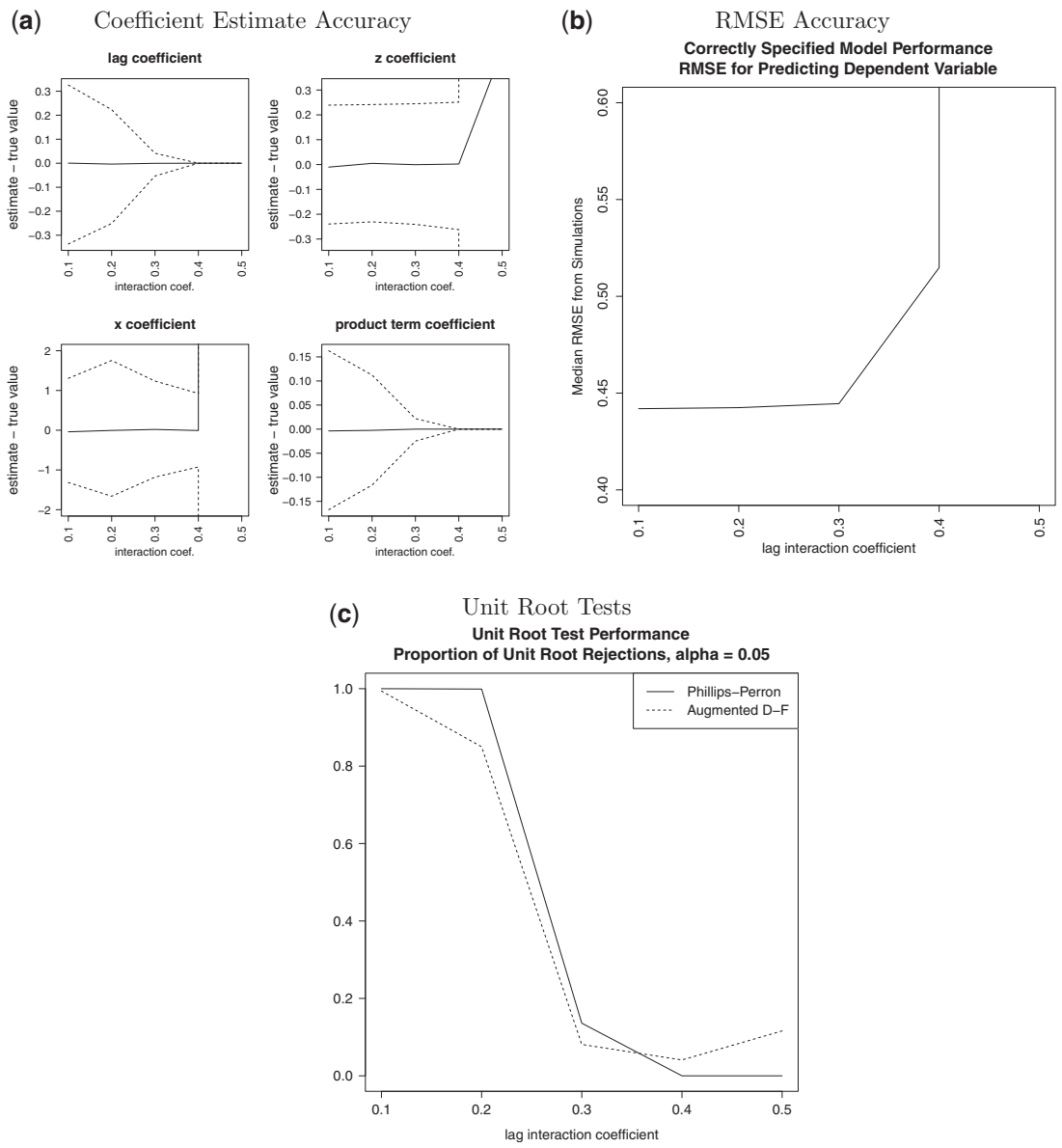


Fig. 9 Simulations when stationarity is temporarily violated. (a) Coefficient estimate accuracy. (b) RMSE accuracy. (c) Unit root tests.

Our analysis is shown in Table 2. To generate this table, we use the simulation framework described in the previous section (and using all the same values for key parameters[13]) to generate one thousand data sets with $N = 10$ and $T = 20$. We then estimate a correctly specified model and determine whether the p-value on the interaction term ($\hat{\beta}_3$) is statistically significant (at $\alpha = 0.05$, two-tailed). We also estimate a random effects model without an interaction term, and compare this misspecified model to the model *with* the product term using the AIC and the BIC. This set of simulations allows us to assess the false negative rate of each of these two procedures. To assess their false positive rate, we repeat the simulations for DGPs that included no interaction term

[13]We bound the absolute value of the product term (β_3) to be between 0.1 and 0.4 to ensure a nonzero level of state-dependence.

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Table 2 Specification test performance, $N = 10$ and $T = 20$ for RE and FE models

	DGP WITH state-dependence (%)	*DGP WITHOUT state-dependence (%)*
	Random effects models	
BIC prefers model w/ product term	96.8	0.1
AIC prefers model w/ product term	98.8	0.4
Product term is statistically significant	99.5	4.1
	Fixed effects models	
BIC prefers model w/ product term	99.3	1.8
AIC prefers model w/ product term	99.8	15.9
Product term is statistically significant	99.4	4.3

Table 3 List of variables (adapted from Geys 2010, 77–78)

Variable	Description
DV: approval	Quarterly average of Gallup Poll respondents expressing approval of the job the incumbent is doing as president
Growth	Real growth rate of GDP in the current quarter
Inflation	Change in inflation rate between the previous and present quarter
Unemployment	Current quarter unemployment (annualized)
Casualties	Natural log of war deaths in the prior quarter for the war indicated

between y_{it} and x_{it} ($\beta_3 = 0$), examine the statistical significance of the product term in a model that includes one, and then compare this product term model to a no-product term model with the AIC and BIC. The entire analysis is repeated for random and fixed effects models.

As Table 2 shows, examining the statistical significance of a product term between y_{it} and x_{it} has a reasonably low false positive rate (4.1% for RE models, 4.3% for FE models) and an extremely high true positive rate (over 99% in both cases). The BIC is much more resistant to false positives (0.1% for RE models, 1.8% for FE models) but slightly less likely to detect a true positive (the true positive rate is 96.8% for RE models, 99.3% for FE models). The AIC model has an excessive false positive rate for FE models (15.9%) and in our time-series simulations without unit effects,[14] and hence we do not recommend its use in this context. On the basis of this evidence, we believe that the BIC is the best test for state-dependence, with the statistical significance of the product term useful as an alternative indicator.

5 Does Economic Performance Affect US Presidential Approval Ratings?

Demonstrating the methodological usefulness of a dynamic interaction model is important, but we think that state-dependence is also an important theoretical concept that can enrich our understanding of substantive issues. To that end, we undertake a replication of a recent study of influences on US presidential approval by Geys (2010). The data set[15] is a time series of quarterly presidential approval ratings collected by the Gallup polling firm between 1948 and 2008. Key independent variables include measures of present economic conditions (GDP growth, unemployment rate, and inflation) and the number of war casualties in three wars that took place during the coverage of the data set; more details about the variables are listed in Table 3. The presidential approval rating dependent variable passes standard tests for stationarity.[16]

[14]See the supplementary appendix for more details.
[15]Thanks to Benny Geys for generously providing the original data and analysis scripts for this replication study.
[16]The Dickey-Fuller test yields a statistic of -4.113, $p \approx 0.001$. The Phillips-Perron test statistic is -4.246, $p \approx 0.001$. A correlogram reveals negligible autocorrelation after ≈ 7 quarters.

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Table 4 State-dependence in Geys (2010)

dv = Gallup approval rating	1	2	3	4
Lag approval	0.901***	0.892***	1.099***	1.092***
	0.026	0.035	0.099	0.098
Growth	0.180	0.034	0.188*	−0.068
	0.092	0.371	0.092	0.375
Lag approval*growth		0.003		0.005
		0.006		0.007
Unemployment	−0.723**	−0.719**	1.135	1.224
	0.256	0.257	0.921	0.924
Lag approval*unemployment			−0.037*	−0.039*
			0.018	0.018
Inflation (first difference)	−1.230*	−1.233*	−1.333**	−1.345**
	0.492	0.497	0.465	0.471
Korea casualties	−0.442*	−0.415	−0.354	−0.304
	0.216	0.236	0.213	0.232
Vietnam casualties	−0.375**	−0.378**	−0.377**	−0.383**
	0.119	0.120	0.119	0.120
Afg/Iraq casualties	−0.403*	−0.407*	−0.354	−0.360
	0.204	0.205	0.206	0.207
Constant	8.967***	9.408***	−0.930	−0.590
	2.427	2.797	5.402	5.421
Adj. R^2	0.8515	0.8509	0.8533	0.8529
BIC	1439.797	1445.103	1441.282	1446.306

Note. Main entries are OLS coefficients; HC3 robust standard errors in parentheses. $N = 231$. ***$p \leq 0.001$, **$p \leq 0.01$, and *$p \leq 0.05$ (two-tailed test).
Column 1 replicates Column 1 of Table 2 in Geys (2010).

Geys's original specification is replicated in Column 1 of Table 4.[17] To this, we add interaction terms between lagged approval and economic growth (Column 2), unemployment (Column 3), or both (Column 4). The substantive interpretation is straightforward: present economic conditions can affect the present level of presidential approval, but they might also affect its trajectory over time. To put it another way, poor economic conditions may cause a faster erosion of past popularity levels (as represented by a decline in $\partial\ \text{approval}_t / \partial\ \text{approval}_{t-1}$). As we argued before, this can happen because high unemployment and low growth speed the diffusion of negative opinions through the population and make opposition appeals more effective, making the usual downward slope of presidential approval ratings even steeper. To model the relationship between economic performance and the trajectory of presidential support, we interact measures of economic performance with lagged approval rating.

The BIC most prefers the baseline Model 1, but Models 3 and 4 have a statistically significant interaction between lagged approval and unemployment. Additionally, the adjusted R^2 value of Model 3 is the highest (by a small margin). We therefore proceed with a further analysis of Model 3 as an interesting candidate.

5.1 *Instantaneous Marginal Effect of Unemployment on Presidential Approval*

We start by assessing the speed of presidential approval decline by examining the marginal effect of the lag coefficient at different levels of unemployment; this is plotted in Fig. 10.[18] As the figure

[17]Geys also estimated many other models including additional controls, such as administration dummies and temporal adjustments for the "honeymoon" period at the beginning of an administration and the period preceding an election. To maintain the simplicity of our analysis, we omit these complicating factors.

[18]We determined the 90% confidence interval of the marginal effect by drawing one thousand samples from the asymptotic (normal) distribution of β coefficients, calculating the marginal effect for each draw, and then plotting the 5th and 95th quantiles of this distribution at each level of unemployment.

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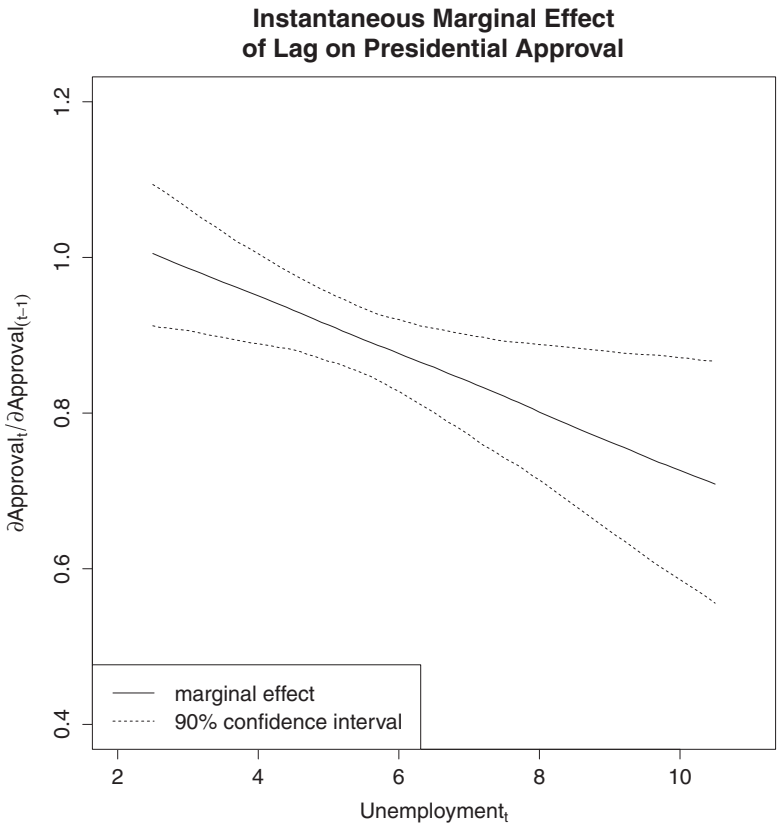


Fig. 10 The instantaneous effect of presidential approval$_{t-1}$ on presidential approval$_t$ (Column 3 of Table 4).

shows, at the very lowest levels of unemployment observed in the data set ($\approx 2.5\%$), presidential approval barely erodes at all.[19] At higher levels of unemployment, however, presidential approval declines more quickly. At the very highest levels of unemployment, $\approx 10.5\%$, only $\approx 70\%$ of existing levels of approval are carried forward into the next quarter. This does not necessarily mean that approval declines at 30% per quarter, because other contemporaneous factors in the model also contribute to approval ratings.

Another, symmetrical view of this phenomenon is shown in Fig. 11, which shows the marginal effect of unemployment on presidential approval at different levels of past approval rating. When past approval is low, unemployment has a comparatively small relationship with contemporaneous approval. But as past approval gets larger, the negative impact of unemployment on current presidential approval also becomes larger.

5.2 *Long-Term Marginal Effect of Unemployment on Presidential Approval*

How would high unemployment influence the trajectory of presidential approval over time? To answer this question, we simulate how presidential approval would change over two years (eight quarters) under two different unemployment rates, 7% and 8% unemployment. We set each variable to its in-sample mean, set initial and lagged approval rating at 50%, then predicted a series of eight quarters of approval ratings. The 90% confidence intervals were obtained by drawing one thousand samples from the asymptotic (normal) distribution of the coefficients of Model 3, predicting eight quarters of approval for each draw, and then plotting the 5th and 95th quantiles of the predictions for each quarter. The result is shown in Fig. 12a. We also calculated the

[19]Indeed, if unemployment stayed below 3% for a long time, the series may not be stationary. However, there are only five quarters in the data set with unemployment less than 3%.

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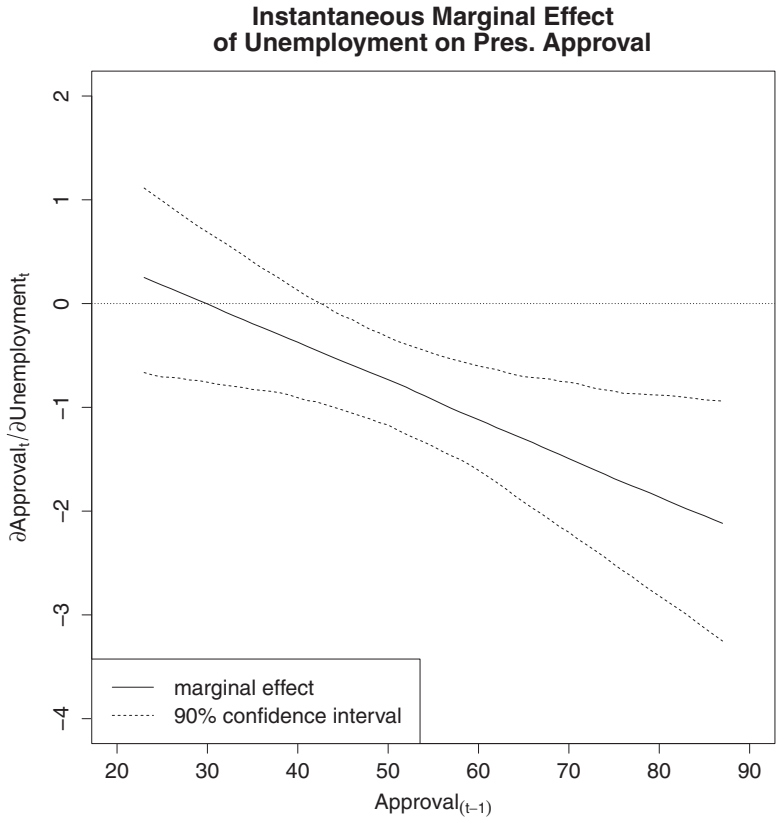


Fig. 11 The instantaneous effect of unemployment$_t$ on presidential approval$_t$ (Column 3 of Table 4).

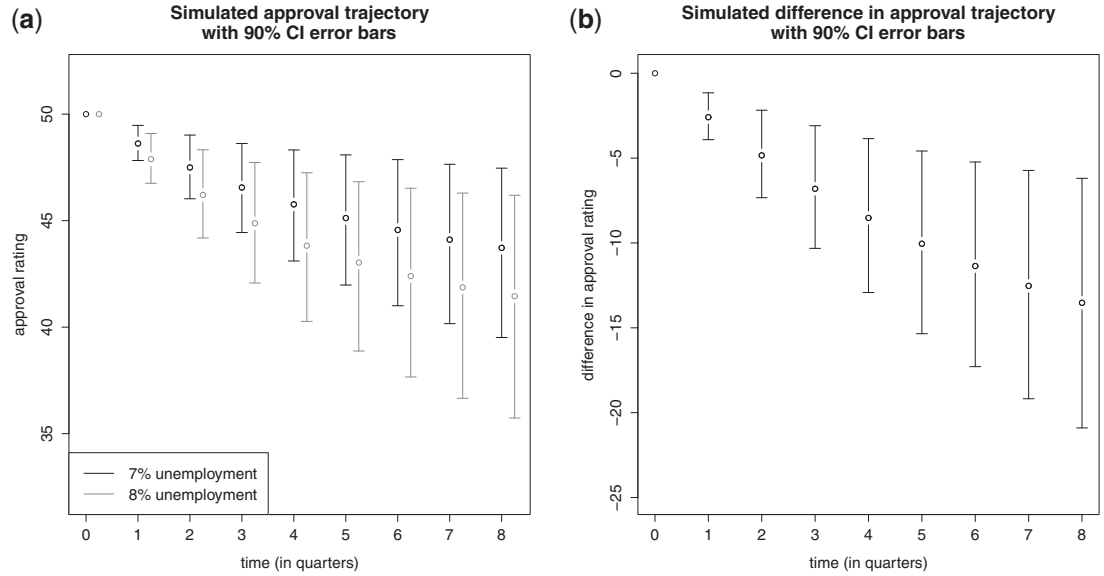


Fig. 12 Presidential approval trajectory under 7% and 8% unemployment (based on Model 3 of Table 4). (a) Approval trajectories at 7% and 8% unemployment. (b) Difference in approval trajectory (approval 8% unemployment) – (approval 7% unemployment).

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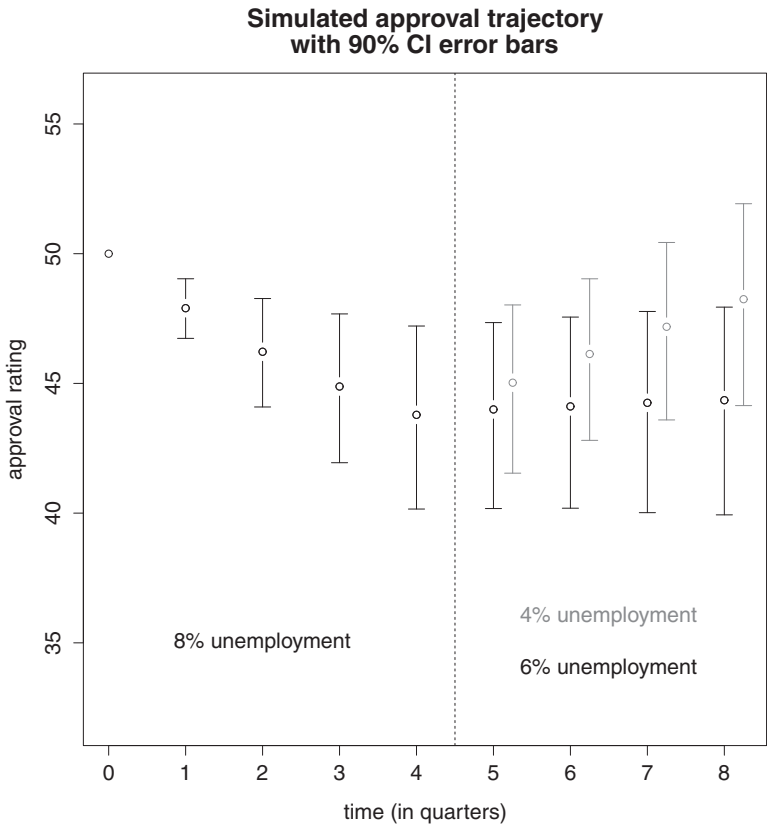


Fig. 13 Presidential approval trajectory 8% unemployment that improves to 6% or 4% unemployment after four quarters (based on Model 3 of Table 4).

difference in trajectories for 7% and 8% unemployment rates and plotted this difference (and its 90% confidence interval) in Fig. 12b.

As the figure shows, even a relatively small (1%) change in unemployment rates results in a gradually increasing degree of difference in presidential approval. Although a president facing 7% unemployment is initially only a few percentage points more popular than a president under 8% unemployment, the difference gradually increases over time such that after two years we would expect the former to be between 5 and 20 percentage points more popular than the latter.

We also simulated the trajectory of presidential approval under initially poor (8%) unemployment that improves after a year (to either 6% or 4% unemployment). The results are shown in Fig. 13. The initially high approval ratings degrade substantially under 8% unemployment. If the economy cuts unemployment by two percentage points—a substantial improvement in economic performance—approval ratings stabilize at the new, lower level but do *not* improve. Only a comparatively miraculous recovery of a four-percentage-point reduction in unemployment reverses the previous trend and results in increased approval ratings.

Substantively speaking, the result is striking: high unemployment has a cumulative negative effect on presidential popularity, one that presidents may stave off but are hard-pressed to reverse.

5.3 *Latent Interaction between Unemployment and Casualties*

The state-dependence of our model creates unanticipated interaction effects among the independent variables. To illustrate this, we examine the trajectory of presidential approval at 4% and 8% unemployment under two scenarios: one in which there have been no Vietnam-related casualties

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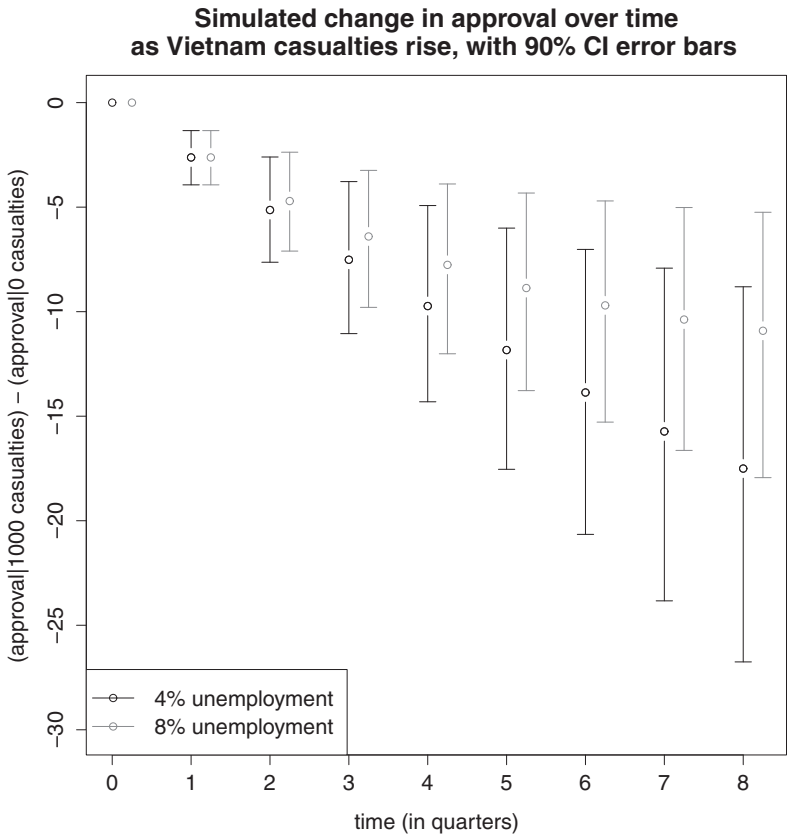


Fig. 14 Difference in presidential approval, with and without Vietnam casualties, under 4% and 8% unemployment.

in the previous quarter, and one in which there have been one thousand such casualties.[20] We then calculate the difference between the high-casualty and no-casualty case for each level of unemployment; the result is depicted in Fig. 14.

The figure shows that war casualties harm presidential approval ratings: in the presence of Vietnam-related casualties, ratings are anywhere from five to twenty-five percentage points lower at the end of eight quarters than they would have been in the absence of those casualties. However, the relative effect of war casualties is predicted to be worse when unemployment is *lower*. Stated another way, when economic conditions are better, the effect of war weariness on presidential popularity is larger. This is a product of the latent interaction between unemployment rates and war casualties that is created by the lag interaction term. Put simply, the relative penalty of war casualties is greater when unemployment is low than when it is high, because high unemployment is already so damaging to presidential approval.

This is not to say that war casualties do not make a high unemployment situation worse for a sitting president: they do. This can be seen in Fig. 15, where we plot the predicted approval ratings for 4% and 8% unemployment under 0 and one thousand Vietnam war casualties.[21] According to this plot, suffering one thousand casualties per quarter under 8% unemployment is considerably worse than suffering one thousand casualties under 4% unemployment. However, under 4% unemployment, moving casualties to 0 results in substantially larger gains in approval than moving casualties to 0 under 8% unemployment.

[20]For this analysis, casualties from other wars were set at zero for the simulation. Initial approval was 50%.
[21]Other variables are set as in Fig. 14.

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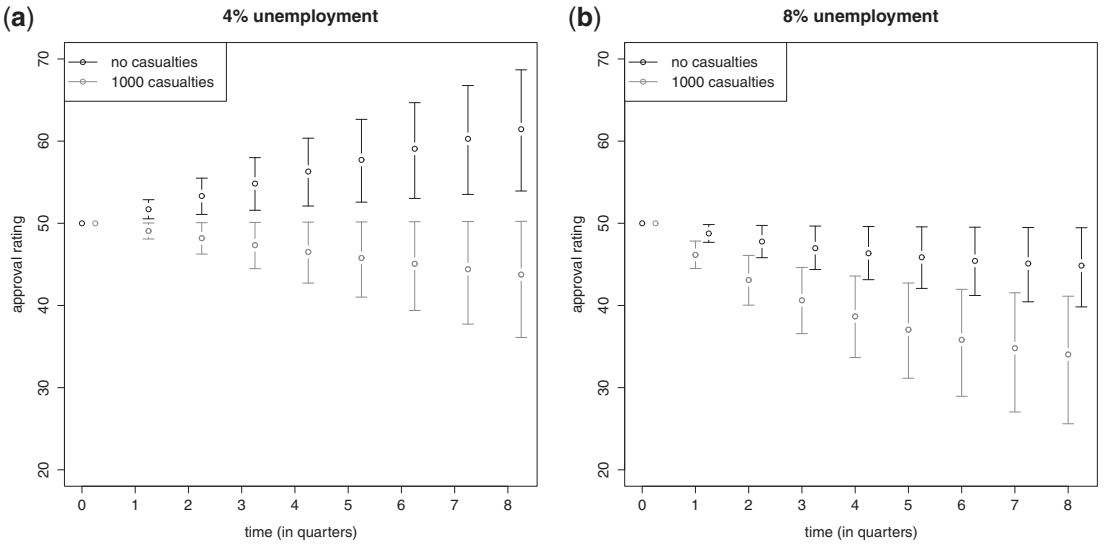


Fig. 15 Presidential approval trajectories with and without Vietnam casualties, low and high unemployment (based on Model 3 of Table 4). (a) Approval trajectories under 4% unemployment. (b) Approval trajectories under 8% unemployment.

The latent interaction between war casualties and unemployment might be substantively interpretable through the lens of learning and opinion diffusion. When unemployment is low, prevailing public opinion is likely in the president's favor and good times tend to put him/her on a pathway to stable or increasing popularity. War casualties offset these positive pressures, pushing approval onto an overall downward trajectory as mounting death tolls sour the public on the conflict and on the president's leadership. The change is substantial, and growing over time. In contrast, when unemployment is high, learning and information diffusion are already working against presidential approval. Voters are getting unfavorable information about the state of the economy that may well make them more skeptical. Increased war casualties speed this process further, but the difference is smaller because there are likely limits to how fast information can spread and how rapidly voters will update their beliefs based on new information.

6 Conclusions and Implications

We hope that our article convinces readers that modeling state-dependence is possible, practical, and substantively illuminating. Our simulations indicate that state-dependence can be identified using appropriately structured tests. The simulations also show that we can accurately model the degree to which empirical relationships are dependent on past outcomes, enhancing our substantive understanding and improving our ability to predict the dependent variable. Through our application to the economic determinants of presidential approval, we uncover evidence for the state-dependence of an important political phenomenon: unemployment rates have a cumulative effect on presidential popularity and disproportionately hurt already-popular presidents. We also find evidence that other influences on presidential approval are implicitly context-dependent: war casualties will have a smaller effect on presidential approval during times of economic hardship.

Consequently, we want political scientists to consider the possibility of state-dependence in their research in the same way that they consider the possibility of hierarchical structure, endogeneity, or interaction. We therefore conclude by proposing two applications that we find especially promising. The first concerns the practical consequences of moral condemnation by the international community.

Do foreign aid donors cut aid to recipients who abuse human rights? The evidence is, at best, inconsistent (Cingranelli and Pasquarello 1985; Carleton and Stohl 1987; Poe 1992; Abrams and Lewis 1993; Apodaca and Stohl 1999; Neumayer 2003a, 2003b). A recent study by Nielsen (2013)

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finds that human rights abuses are punished by donors not allied with the recipient state, but ignored or even *rewarded* by allied donors. This finding is consistent with a larger tradition of research arguing that the donor's political interests are the primary determinant of foreign aid (Lebovic 1988; Meernik, Kreuger, and Poe 1998; Schraeder, Hook, and Taylor 1998; Alesina and Dollar 2000; Alesina and Weder 2002; Lebovic 2005).

This pattern of findings bears on the question of whether international condemnation for human rights abuses (so-called "naming and shaming") will have a measurable impact on the condemned state. One way to approach the question is to accept a key assumption of the political self-interest approach—that donors give more money to recipients in whom their political, military, economic, or other interests are intertwined—and then ask whether donors react differently to human rights abuses by large aid recipients compared to smaller aid recipients. If so, we would expect foreign aid to constitute a SDDS wherein larger aid receipts in the past reduce the negative impact of "naming and shaming" on present aid levels. We pursue this question in another paper (Esarey and DeMeritt 2013), and find supportive evidence in both the aggregate aid levels of recipients and dyadic aid flows between donors and recipients over time.

Our second suggestion concerns the influence of government partisanship on changes in government spending. Conventional wisdom holds that "the basic criterion distinguishing the left from the right concerns the role of government versus that of the market" (Blais, Blake, and Dion 1993, 43). Leftist parties traditionally favor large government, whereas rightist parties privilege the market and minimize government intervention. As a result, left parties in power are characterized by high levels of and increases in spending, whereas right parties in power spend less and reduce government size. Evidence of a statistical relationship between these forces is robust, but conditioned by a series of factors including (e.g.) majoritarian government and outstanding debt (Blais, Blake, and Dion 1993, 1996; see also Cameron 1978; Cusack 1997).

But the influence a party exerts on spending may be conditioned on previous spending changes, and particularly on the party's satisfaction with those changes. A party in power will vary spending only if it is dissatisfied with the status quo. Left-leaning governments will expand only to the extent that previous expansion was low. If previous expansion was high, left parties may have already implemented or begun to implement their preference for big government. Similarly, right governments will contract only to the extent that previous contraction was low. If previous contraction was high, right parties may have already implemented or begun to implement their preference for small government.

In short, changing government size is an ongoing process, such that a party's preferences may be implemented only to the extent that implementation has not already occurred and hence dissatisfaction with the status quo persists. More generally, the effect of partisan preferences on government size may depend on the previous change in spending. If so, then the system is state-dependent; explicitly modeling this feature may reveal that the government's ideological commitments are a greater influence on spending than previously believed.

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