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Algorithm for topological design of multi-loop hybrid mechanisms via logical proposition

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SUMMARY

This paper introduces the mathematical logical proposition into kinematics and presents a novel method for topological design and representation of mechanisms. The six-value non-classical logical matrix in three orders is proposed to represent spatial basic motions, relations, and selection of moving reference frame. The combinatorial logical sets of mechanisms are built and axiomatized to analyze serial–parallel hybrid mechanisms. In accordance to the logical relations, the mapping between topological arrangement and topological geometry is developed, and algorithm of decomposition synthesis is presented. Furthermore, the higher dimensional topological arrangement of multi-loop hybrid mechanism can be obtained by mapping of higher dimensional geometry. At last, four typical examples are developed to illustrate application of method of logical proposition in topological design of mechanisms. The method is helpful for topological design and analysis of multi-loop hybrid mechanisms.

KEYWORDS: Logical proposition; Logical matrix; Topological design; Multi-loop kinematic chains; Hybrid mechanisms.

Nomenclature

T_P:	translation induced by prismatic joint
R_R:	rotation induced by revolute joint
T_R:	derivative translation induced by revolute joint
T_H:	translation induced by screw joint
R_H:	rotation induced by screw joint
T_{HR}:	derivative translation induced by screw joint
L:	common line
C:	common point
P:	common plane
N:	parallel
I:	perpendicular
G:	general
r:	revolute joint
t:	prismatic joint
h:	screw joint
$\mathbf{a}$, $\mathbf{b}$:	axial vectors
$R(r)$:	rotation output of r
$T(r)$:	derivative translation output of r
$R(h)$:	rotation output of h
$T(h(r))$:	derivative translation output of h
$T(h)$:	translation output of h
$T(t)$:	translation output of t
$(\mathbf{a}, \mathbf{b})$:	constraint relation of axial vectors
$O(o)$:	origin of moving reference frame on end-axial vector
d, l, p, s:	the number of vertex, edge, face, and body in geometry
I($\oplus L$):	one-dimensional topology of adding edges logic
II($\oplus P$):	two-dimensional topology of adding faces logic
III($\oplus S$):	three-dimensional topology of adding bodies logic
B:	decomposition of loop
In:	including path
O':	origin of moving reference frame
m:	end-effector
P_F:	fixed platform
P_O:	end-output moving platform
P_i:	platform i
S_i:	subchain i
Z:	output motions
$\mapsto$:	serial operator
$\wedge$:	parallel operator
$\rightarrow$:	if. . . then operator
$\neg$:	not operator
$\vee$:	or operator
$\forall$:	all operators
$\exists$:	exist operator
/:	equivalent operator

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1. Introduction

There are many theories and methods for topological design of mechanisms have being developed. As far as the structural syntheses of serial or parallel mechanisms are concerned, it was mainly focused on screw theory[1–3] or group theory.[4–6] Kong and Gosselin[2,3] also put forward the concept of virtual chains to synthesize parallel mechanisms. For the type synthesis of parallel mechanisms, Jin and Yang[7] utilized output characteristic equation of parallel manipulators to design topological structures. Moreover, Yang *et al.*[8] considered the mapping relations between structure and position and orientation character of mechanisms output

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motion, and utilized method of symbolic operation to synthesize structure of parallel mechanisms.

This paper develops topological design via method of mathematical logical proposition. It is necessary to consider the representation of hybrid mechanisms before building topological structures. Wang *et al.*[9] analyzed and represented serial and parallel mechanisms based on the topological diagram. Ding and Huang[10] represented and synthesized planar multi-loop kinematic chains in the light of the perimeter topological graph. As far as the logical representation of mechanisms is concerned, Gogu[11] represented constituent elements of kinematic chains in "genetic code." For the spatial linear elements, Bribiesca[12] represented connection relations of spatial curves by the method of logical binary chain code. Dai and Rees[13] represented topological configuration transformation of metamorphic mechanisms in matrix form. In order to introduce logical proposition into topological design, this paper presents six-value non-classical logical matrix in three orders to represent basic motions, relations, and selection of moving reference frame. Furthermore, the combinatorial logical sets of mechanisms are built based on the combinatorial relations of motions, which can be used in topological design and analysis.

As far as the topological arrangement is concerned, the forms were mainly serial kinematic chains and parallel structure. For the hybrid topological arrangement, Tanev[14] presented a type of hybrid mechanism, which consists of two parallel mechanisms connected serially. Moreover, Tsai and Joshi[15] introduced a hybrid mechanism with dual end-effectors to realize the cooperative operation. Campos *et al.*[16] considered Assur groups as the simplest basic blocks to build various hybrid mechanisms with spanning tree forms. Alizade and Bayram[17] synthesized hybrid topological arrangement with multi-moving platforms based on the presented new criterion of mobility. After that, Alizade *et al.*[18] presented a new method for the structure synthesis of Euclidean platform hybrid mechanisms with variable connected forms between different moving platforms.

This paper develops hybrid mechanisms with multi-subchains and multi-moving platforms in two types of hybrid topological forms, namely, spanning tree structures and non-spanning tree-netted structures. During the process of the topological design, first it is necessary to analyze motion relations between subchains and moving platforms in multi-loop topological arrangement. For the motion analysis of a single loop, Rico and Ravani[19,20] disassembled single-loop kinematic chains into two subchains and put forward criterion for mobility of a single loop. For the motion analysis of a multi-loop, Chaudhary and Saha[21] utilized cutting points to disassemble closed loop kinematic chains into spanning tree systems, and analyzed kinematics based on recursive formulations. Gogu[22] presented criterion for global mobility calculations of multi-loop mechanisms via the theory of linear transformations. Other researches about multi-loop mechanisms were mainly adopting method of recursions.[23–27] This paper presents a method of multi-loop decomposition in different logical orders to analyze motion relations between subchains and moving platforms for topological arrangement.

In this paper, first the six-value logical matrix in three orders is proposed to represent basic motions in Section 2, and combinatorial logical sets and combinatorial logical axioms are built based on the predicate logic. In Section 3, mapping relations of topological arrangements and method of multi-loop decomposition and configuration are presented to build logical algorithm for topological design of hybrid mechanisms. Section 4 deals with hybrid topological design with higher dimensional topological arrangement. Finally, four typical examples are illustrated in Section 5. The algorithm considers application of logical proposition in topological design. It is helpful for the analysis and synthesis of hybrid mechanisms.

2. Logical Representation and Logical Reasoning of Mechanisms

The type synthesis and analysis of mechanisms cannot only adopt the screw theory or group theory but can also adopt mathematical logical proposition based on the displacement group theory.[26,27] The advantage of this approach is that it can be formulized easily. The general spatial hybrid mechanisms can be represented and analyzed by the logical formula, especially for the complicate spatial mechanisms such as non-spanning tree hybrid mechanisms. The traditional topological graphs can represent general parallel mechanisms by symbols, whereas for the complicate hybrid form of mechanisms, it will appear over-complicate and hard to be analyzed. This approach is not only representing the structure of mechanisms but also analyzing the mechanisms in logical formula. It is convenient for the structure synthesis and analysis of mechanisms. Zeng and Fang[26,27] used mathematical logic to develop the synthesis and analysis of mechanisms, but only operated in binary classical logic for spanning tree forms. This paper develops above contributions and operates the method in three-order, six-value non-classical logic for non-spanning tree forms.

2.1. Non-classical logical representation of mechanisms

The kinematic analysis of mechanisms depends on the basic motions, constraint relations, and selection of frame. The basic motions of mechanisms include two extreme ways, namely, translation and rotation, while screw is compounded by translation and rotation. For the basic motions of mechanisms, rotation includes derivative translation.[8] It can be defined as instantaneous motion of moving reference frame relative to fixed coordinates when origin of moving reference frame, O', is not on the axis of revolute joint, and the direction of the derivative translation is perpendicular to the axis of rotation. Therefore, the basic motions of mechanisms can be divided into six kinds, which are as follows: (1) translation, T_P induced by prismatic joints, P; (2) rotation, R_R; (3) derivative translation, T_R induced by revolute joints, R; (4) translation induced by screw joint, T_H; (5) rotation induced by screw joint, R_H; and (6) derivative translation, T_{HR} induced by screw joints, H. It can be arranged in sequence from translation to rotation as T_P, T_H, T_{HR}, T_R, R_H, and R_R.

The constraint relations of spatial mechanisms can be defined as the six kinds in the light of the relative relations

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of axial vectors, namely, common line L, common point C, common plane P, parallel N, perpendicular I, and general G. The constraint relations between the prismatic joints do not include common line and common point. For the selection of moving reference frame's origin O', revolute joint does not generate derivative translation relative to the fixed coordinates when the O' is on the axis of the revolute joint and vice versa.

The classical logic was mainly to judge true and false in binary logic. In order to build logical analysis about basic motions and relations of mechanisms, the six-value non-classical logical matrix in three orders can be developed to represent basic motions, relations, and selection of moving reference frame. The six-value logical matrix of the basic motions can be represented as

$$[T_P \quad T_H \quad T_{HR} \quad T_R \quad R_H \quad R_R]^{\mathrm{T}},$$

while the six-value logical matrix of the topological relations is

$$[L \quad C \quad P \quad N \quad I \quad G]^{\mathrm{T}}.$$

The six-value logical matrix of the selection of frame is

$$[on \quad 0 \quad 0 \quad 0 \quad 0 \quad off]^{\mathrm{T}},$$

where the any element is 1, which can represent the corresponding motions or relations, while *on* is 1, which can represent origin O' and is on the axis of end-joint and vice versa. The columns of the logical matrix $M_{6\times n}$ represent three-order logic respectively, in which the odd columns represent motions, the even columns represent relations of adjacent motions, and the last column represents the selection of end-output moving reference frame's logic, namely,

$$M_{6\times n} = \begin{bmatrix} T_P & L & T_P & L & \cdots & on \\ T_H & C & T_H & C & \cdots & 0 \\ T_{HR} & P & T_{HR} & P & \cdots & 0 \\ T_R & N & T_R & N & \cdots & 0 \\ R_H & I & R_H & I & \cdots & 0 \\ R_R & G & R_R & G & \cdots & off \end{bmatrix}. \quad (1)$$

2.2. *Logical proposition reasoning of serial and parallel mechanisms*

In accordance to the above logical representation of mechanisms, the logical range of motion synthesis and analysis should be developed in the six-value logic. The proposition of analysis can be defined, and the logical level of proposition is judged by logical reasoning to build topological design and analysis of mechanisms. In order to represent the logic of serial kinematic chains, the predicate logic can be adopted and defined as follows: The R, P, and H represent revolute joint, prismatic joint, and screw joint, respectively, which the individual domain is joint. The **a** and **b** are defined as axial vectors. These are sliding vectors lying on the axis of the revolute joint, while the direction of a prismatic joint is a free vector parallel to the translation direction. The o is defined as logical object of origin O' in end-output moving reference frame. Thus, the three-order predicate logic of mechanisms is as follows:

$R(r)$ represents "rotation output induced by r"; and $T(r)$ represents "derivative translation output perpendicular to rotation axis induced by r."

$R(h)$ represents "rotation output induced by h"; $T(h(r))$ represents "derivative translation output perpendicular to rotation axis induced by h"; and $T(h)$ represents "translation output along rotation axis induced by h."

$T(t)$ represents "translation output induced by t."

$L(\mathbf{a}, \mathbf{b})$ represents "the constraint relation between **a** and **b** in common line," similar to the $C(\mathbf{a}, \mathbf{b})$, $P(\mathbf{a}, \mathbf{b})$, $N(\mathbf{a}, \mathbf{b})$, $I(\mathbf{a}, \mathbf{b})$, and $G(\mathbf{a}, \mathbf{b})$.

$O(o)$ represents "origin O' of end-output moving reference frame on the axis of end-joint."

Meanwhile, logical proposition of motion results is defined as Z to represent "output motions of moving reference frame relative to fixed coordinates." In the light of the above predicate and proposition, the combinatorial logical sets of serial and parallel mechanisms can be built.[8] For the serial kinematic chains, it includes different combinations of the constraint relations among R, P, and H to generate various logical output motions, as shown in Table I.

The above logic sets can be explained as $N(t, r) \wedge \neg O(o) \rightarrow Z = I(N(T(t), R(r)), T(r))$, which represents "if t is parallel to r, and the origin of end-output moving reference frame is not on the axis of r, then end-output Z of the serial combination is $T(t)$, $R(r)$, and $T(r)$, where $T(t)$ is parallel to $R(r)$ and $T(r)$ is perpendicular to formers." The serial combinatorial logical axiom of spatial motions can be obtained according to Table I.

Logical axiom of serial combination:

1. The same kind of joints' motion in the identical orientation is consistent, namely, it will only generate one rotation or translation motion when many revolute joints or prismatic joints are connected in the same direction. Otherwise that is inconsistent in different orientations.
2. There is no derivative translation if end-output moving reference frame's origin is on the axis of end-revolute joint or screw joint.
3. There are only two translation outputs when two or more translations are parallel to a common plane.

The logical axiom can be represented as follows:

$$\begin{cases} N(r/t/h) \wedge (O(o) \vee \neg O(o)) \rightarrow Z \\ \quad = R(r)/T(t)/R(h) \wedge T(h) \\ O(o) \wedge (r/h) \rightarrow Z = R(r)/R(h) \wedge T(h) \\ P(t, \cdots/T(r), \cdots/T(h), \cdots/T(h(r)), \cdots) \rightarrow Z \\ \quad = P(T(t_1/r_1/h_1), T(t_2/r_2/h_2)) \end{cases}. \quad (2)$$

Example 1. Taking spatial serial kinematic chain $PRR\underline{RR}$ as an example, as shown in Fig. 1.

In Fig. 1, t_1 is perpendicular to r_1 whereas r_1 and r_2 are parallel to each other, and r_3 and r_4 intersect at a common point and are general with r_2. The moving reference frame's origin O' is selected on the intersection point of end-joints. In order to analyze the output motions, it can be developed according to the combinatorial logical sets of the serial

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Table I. Combinatorial logical sets of serial kinematic chains.

Combinatorial type	Combinatorial logic
$r_1 \mapsto r_2, \ldots$	$N(r_1, r_2) \wedge O(o) \rightarrow Z = I(R(r_1), T(r_1))$
	$N(r_1, r_2, \ldots) \wedge \neg O(o) \rightarrow Z = I(R(r_1), P(T(r_1), T(r_2)))$
	$L(r_1, r_2, \ldots) \wedge O(o) \rightarrow Z = R(r_1)$
	$L(r_1, r_2, \ldots) \wedge \neg O(o) \rightarrow Z = I(R(r_1), P(T(r_1), T(r_2)))$
	$C(r_1, r_2, r_3, \ldots) \wedge O(o) \rightarrow Z = G(R(r_1), R(r_2), R(r_3))$
	$C(r_1, r_2, r_3, \ldots) \wedge \neg O(o) \rightarrow Z = G(G(R(r_1), R(r_2), R(r_3)), P(T(r_1), T(r_2)))$
	$G(r_1, r_2) \wedge O(o) \rightarrow Z = G(I(R(r_1), T(r_1)), R(r_2))$
	$G(r_1, r_2, r_3 \ldots) \wedge \neg O(o) \rightarrow Z = G(I(R(r_1), P(R(r_2), R(r_3))), P(T(r_1), T(r_2)))$
$t_1 \mapsto t_2, \ldots$	$N(t_1, t_2, \ldots) \rightarrow Z = T(t_1)$
	$P(t_1, t_2, \ldots) \rightarrow Z = P(T(t_1), T(t_2))$
	$\neg P(t_1, t_2, t_3, \ldots) \rightarrow Z = I(T(t_1), P(T(t_2), T(t_3)))$
$h_1 \mapsto h_2, \ldots$	$N(h_1, h_2) \wedge O(o) \rightarrow Z = I(N(R(h_1), T(h_1)), T(h_1(r)))$
	$N(h_1, h_2, \ldots) \wedge \neg O(o) \rightarrow Z = I(N(R(h_1), T(h_1)), P(T(h_1(r)), T(h_2(r))))$
	$L(h_1, h_2, \ldots) \wedge O(o) \rightarrow Z = N(R(h_1), T(h_1))$
	$L(h_1, h_2, \ldots) \wedge \neg O(o) \rightarrow Z = I(N(R(h_1), T(h_1)), P(T(h_1(r)), T(h_2(r))))$
$t \mapsto r$	$N(t, r) \wedge O(o) \rightarrow Z = N(T(t), R(r))$
	$N(t, r) \wedge \neg O(o) \rightarrow Z = I(N(T(t), R(r)), T(r))$
	$I(t, r) \wedge O(o) \rightarrow Z = I(T(t), R(r))$
	$I(t, r) \wedge \neg O(o) \rightarrow Z = I(R(r), P(T(t), T(r)))$
$t \mapsto h$	$N(t, h) \wedge O(o) \rightarrow Z = N(T(t), R(h))$
	$N(t, h) \wedge \neg O(o) \rightarrow Z = I(N(T(t), R(h)), T(h(r)))$
	$I(t, h) \wedge O(o) \rightarrow Z = I(T(t), N(T(h), R(h)))$
	$I(t, h) \wedge \neg O(o) \rightarrow Z = I(P(T(t), T(h(r))), N(T(h), R(h)))$
$h \mapsto r$	$N(h, r) \wedge O(o) \rightarrow Z = I(N(R(r), T(h)), T(h(r)))$
	$N(h, r) \wedge \neg O(o) \rightarrow Z = I(N(R(r), T(h)), P(T(h(r)), T(r)))$
	$L(h, r) \wedge O(o) \rightarrow Z = N(R(r), T(h))$
	$L(h, r) \wedge \neg O(o) \rightarrow Z = I(N(R(r), T(h)), P(T(h(r)), T(r)))$

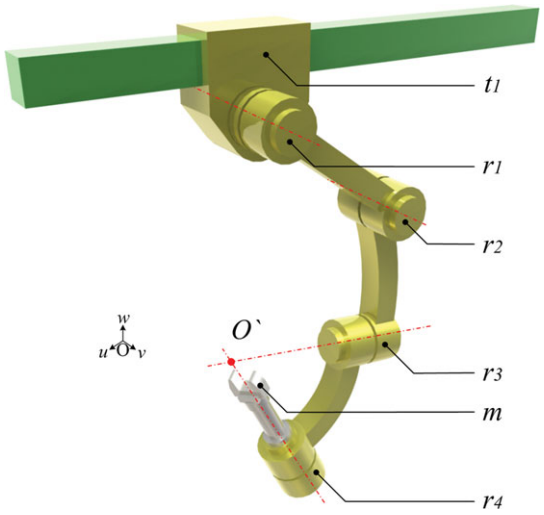


Fig. 1. Serial kinematic chain $PRR\underline{RR}$.

kinematic chains, namely,

$$I(t_1, r_1) \wedge \neg O(o) \rightarrow Z_1 = I(R(r_1), P(T(t_1), T(r_1))), \quad (3)$$

$$N(r_1, r_2) \wedge \neg O(o) \rightarrow Z_2 = I(R(r_1), P(T(r_1), T(r_2))), \quad (4)$$

$$G(r_2, r_3) \wedge O(o) \rightarrow Z_3 = G(I(R(r_2), T(r_2)), R(r_3)), \quad (5)$$

$$C(r_3, r_4) \wedge O(o) \rightarrow Z_4 = G(R(r_3), R(r_4)). \quad (6)$$

The six-value logical matrix in three orders of the structure can be represented as follows:

$$M_{6\times 10} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}. \quad (7)$$

Thus, the end-output motion can be obtained in the light of the logical axiom of serial combination, namely,

$$\begin{aligned} Z_S &= Z_1 \mapsto Z_2 \mapsto Z_3 \mapsto Z_4 \wedge O(o) \\ &= I(R(r_1), P(T(t_1), T(r_1))) \mapsto I(R(r_1), P(T(r_1), \\ &\quad T(r_2))) \mapsto G(I(R(r_2), T(r_2)), R(r_3)) \mapsto G(R(r_3), \\ &\quad R(r_4)) \wedge O(o) \\ &= I(R(r_1), P(T(t_1), T(r_1))) \mapsto G(I(R(r_2), T(r_2)), \\ &\quad R(r_3)) \mapsto G(R(r_3), R(r_4)) \wedge O(o) \\ &= G(I(R(r_1), P(T(t_1), T(r_1))), R(r_3)) \mapsto G(R(r_3), \\ &\quad R(r_4)) \wedge O(o) \\ &= G(G(R(r_1), R(r_3), R(r_4)), P(T(t_1), T(r_1))) \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}. \quad (8)$$

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Table II. Combinatorial logical sets of parallel mechanisms.

Combinatorial type	Combinatorial logic
$\forall RRR \wedge R$	$G(R_1, R_2, R_3) \wedge R_4 \rightarrow Z = R_4$
$\forall RRR \wedge \exists RR$	$G(R_1, R_2, R_3) \wedge G(R_4, R_5) \rightarrow Z = G(R_4, R_5)$
$\forall RRR \wedge \forall RRR$	$G(R_1, R_2, R_3) \wedge G(R_4, R_5, R_6) \rightarrow Z = G(R_1, R_2, R_3)$
$\exists RR \wedge R$	$P(R_1, R_2) \wedge R_3 \wedge N(R_3, P(R_1, R_2)) \rightarrow Z = R_3$ $P(R_1, R_2) \wedge R_3 \wedge \neg N(R_3, P(R_1, R_2)) \rightarrow Z = 0$
$\exists RR \wedge \exists RR$	$P(R_1, R_2) \wedge P(R_3, R_4) \wedge N(P(R_1, R_2), P(R_3, R_4)) \rightarrow Z = P(R_1, R_2)$ $P(R_1, R_2) \wedge P(R_3, R_4) \wedge \neg N(P(R_1, R_2), P(R_3, R_4)) \rightarrow Z$ $= R_5 \wedge P(R_1, R_2, R_5) \wedge P(R_3, R_4, R_5)$
$R \wedge R$	$R_1 \wedge R_2 \wedge N(R_1, R_2) \rightarrow Z = R_1$ $R_1 \wedge R_2 \wedge \neg N(R_1, R_2) \rightarrow Z = 0$
$\forall TTT \wedge T$	$G(T_1, T_2, T_3) \wedge T_4 \rightarrow Z = T_4$
$\forall TTT \wedge \exists TT$	$G(T_1, T_2, T_3) \wedge P(T_4, T_5) \rightarrow Z = P(T_4, T_5)$
$\forall TTT \wedge \forall TTT$	$G(T_1, T_2, T_3) \wedge G(T_4, T_5, T_6) \rightarrow Z = G(T_1, T_2, T_3)$
$\exists TT \wedge T$	$P(T_1, T_2) \wedge T_3 \wedge N(T_3, P(T_1, T_2)) \rightarrow Z = T_3$ $P(T_1, T_2) \wedge T_3 \wedge \neg N(T_3, P(T_1, T_2)) \rightarrow Z = 0$
$\exists TT \wedge \exists TT$	$P(T_1, T_2) \wedge P(T_3, T_4) \wedge N(P(T_1, T_2), P(T_3, T_4)) \rightarrow Z = P(T_1, T_2)$ $P(T_1, T_2) \wedge P(T_3, T_4) \wedge \neg N(P(T_1, T_2), P(T_3, T_4)) \rightarrow Z$ $= T_5 \wedge P(T_1, T_2, T_5) \wedge P(T_3, T_4, T_5)$
$T \wedge T$	$T_1 \wedge T_2 \wedge N(T_1, T_2) \rightarrow Z = T_1$ $T_1 \wedge T_2 \wedge \neg N(T_1, T_2) \rightarrow Z = 0$
$\forall R \cdots \wedge \forall T \cdots$	$R \cdots \wedge T \cdots \rightarrow Z = 0$

For the logical combination of parallel mechanisms, the logical output motion is mainly depended on the intersection of output of serial subchains. The combinatorial logical sets include various logical outputs that are generated by different combinatorial logical relations between $R(r)/R(h)$ and $T(t)/T(r)/T(h)/T(h/r)$, as shown in Table II. These are not including any output about parallel combination between rotation and translation.

The parallel combinatorial logical axiom of the spatial motions can be obtained according to Table II.

Logical axiom of parallel combination:

1. The same kind of motions in identical orientation is consistent, and the output is selected as motions with identical orientation, otherwise there is no output.
2. There is no output of the intersection between rotation and translation.

The logical axiom can be represented as

$$\begin{cases} \forall R/T \wedge \exists R/T \wedge N(R/T, R/T) \rightarrow Z = \exists R/T \\ \forall R \cdots \wedge \forall T \cdots \rightarrow Z = 0 \end{cases}. \quad (9)$$

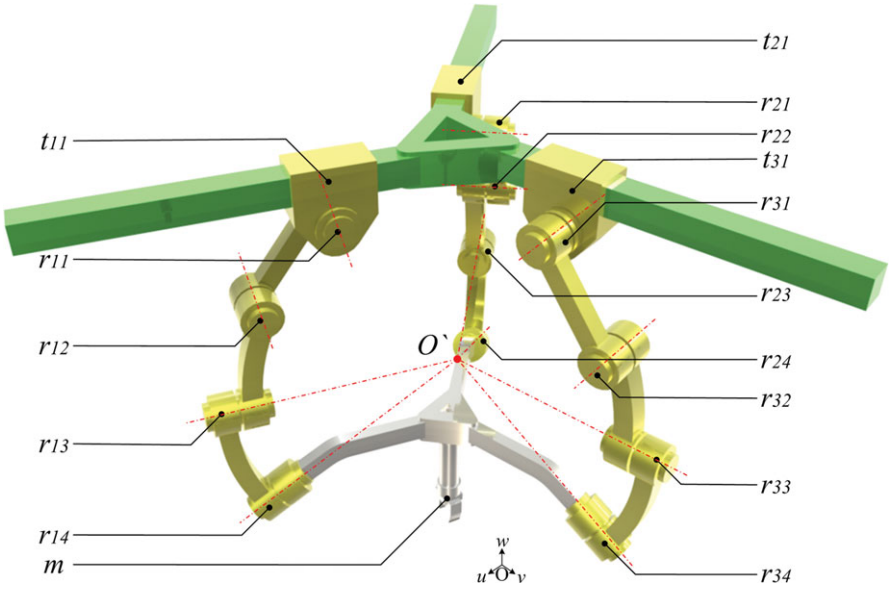


Fig. 2. Parallel mechanism 3 – $PRR\underline{RR}$.

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Example 2. Taking spatial parallel mechanism $3-PRR\underline{RR}$ as an example, as shown in Fig. 2.

In Fig. 2, the three serial kinematic subchains are identical with Example 1, and the moving reference frame's origin O' is selected on the intersection point of end-six revolute joints. In the light of results of Example 1, the output of each subchains are as follows:

$$Z_{S1} = G(G(R(r_{11}), R(r_{13}), R(r_{14})), P(T(t_{11}), T(r_{11}))), \tag{10}$$

$$Z_{S2} = G(G(R(r_{21}), R(r_{23}), R(r_{24})), P(T(t_{21}), T(r_{21}))), \tag{11}$$

$$Z_{S3} = G(G(R(r_{31}), R(r_{33}), R(r_{34})), P(T(t_{31}), T(r_{31}))), \tag{12}$$

where the relations between subchains are $P(P(T(t_{11}), T(r_{11})), P(T(t_{21}), T(r_{21})), P(T(t_{31}), T(r_{31})))$, namely, all prismatic joints and r_{12}, r_{22}, and r_{32} are parallel to a common plane, while the rest of joints are intersected at origin O'. Thus, the end-output motion can be obtained in the light of the logical axiom of parallel combination, namely,

$$\begin{aligned} Z_P &= Z_{S1} \wedge Z_{S2} \wedge Z_{S3} \wedge \neg N(P(T(t_{11}), T(r_{11})), P(T(t_{21}), T(r_{21})), P(T(t_{31}), T(r_{31}))) \wedge O(o) \\ &= G(R_1, R_2, R_3) \wedge T_1 \wedge I(T_1, P(P(T(t_{11}), T(r_{11})), P(T(t_{21}), T(r_{21})), P(T(t_{31}), T(r_{31})))) \wedge O(o) \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{13}$$

The end-effector motion has 4-DOF, namely, three rotations around origin O' and one translation along the axis of w.

3. Topological Design of Multi-Loop Hybrid Mechanisms

There are two types of basic elements in hybrid mechanisms, namely, serial subchains and platforms. For the synthesis of hybrid mechanisms, first it is necessary to determine the topological arrangement. Then it can be disassembled and configured to obtain the required hybrid mechanisms.

3.1. Mapping combination of a multi-loop

The serial subchain and platform are corresponding with the edge and vertex of geometry. Thus, the arbitrary topological arrangements can be obtained via the kind of mapping. In accordance to the geometrical characters of one-dimensional chain, two-dimensional face, and three-dimensional body, the corresponding topological arrangements of three types can be defined as follows: I type of serial, parallel, and hybrid topology with spanning tree one-dimensional forms; II type of hybrid topology with spanning tree or non-spanning tree two-dimensional netted forms; III type of hybrid topology with non-spanning tree three-dimensional netted forms.

For the topological arrangements with multi-subchains and multi-platforms, the combinatorial loop can be defined as vertex unit sets with connected vertexes. The topological arrangements can be obtained by the mapping, while various geometries can be derived from the original geometry, as shown in Table III.

In Table III., the d, l, p, and s represent the number of vertex, edge, face, and body, respectively, in geometry. F and O represent fixed platform and end-output moving platform, respectively. The three methods of formative logic, namely, one-dimensional forms of adding edge logic I($\oplus L$), two-dimensional forms of adding face logic II($\oplus P$), and three-dimensional forms of adding body logic III($\oplus S$) can develop various required geometries. In Table III, the I(2, 1) and I(2, 3) are normal serial and parallel topological arrangements, the rest are spanning tree- or non-spanning tree-netted hybrid topological arrangements, which not only includes triangular forms but also includes quadrangular forms of topological arrangement II(6, 8, 3) and multi-edge connected topological arrangements between two vertexes II(3, 5, 1).

As far as the selection of fixed platform and end-output moving platform is concerned, different selections will generate various forms between moving platforms. For the interim non-end-output moving platform, the connected number of subchains is $D \geq 3$, otherwise it should be merged into the part of subchains, such as II(3, 3, 1) is equal to I(2, 2).

3.2. Algorithm for topological design of hybrid mechanisms

In order to determine the motion relations between each subchains and end-output moving platform, it is necessary to classify logical decomposition for topological loops when the topological arrangement is obtained. The predicate of decomposition can be defined as follows:

$B_0(P_i, P_{i+1}, P_{i+2}, \ldots)$ represents "the zero-class topological loop consisting of platforms P_i, P_{i+1}, and P_{i+2}, it is disassembled at platform P_i to obtain motion relations between platform P_i, connected subchains, and rest platforms," which results in

$$Z_{Pi} = (Z_{Si} \mapsto Z_{Pi+1}) \wedge (Z_{Si+1} \mapsto Z_{Pi+2}) \wedge \cdots, \tag{14}$$

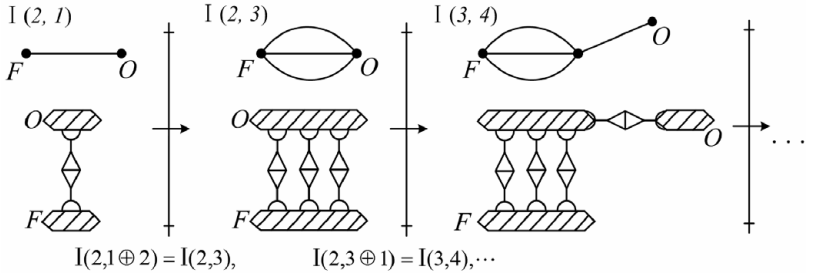
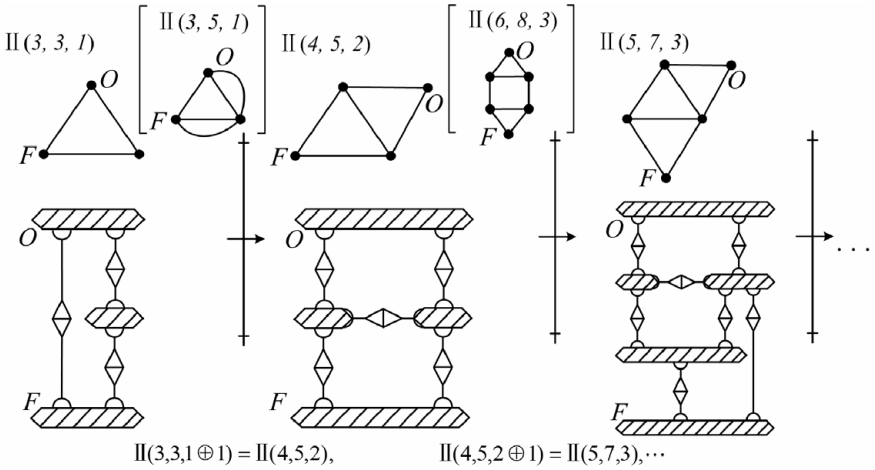
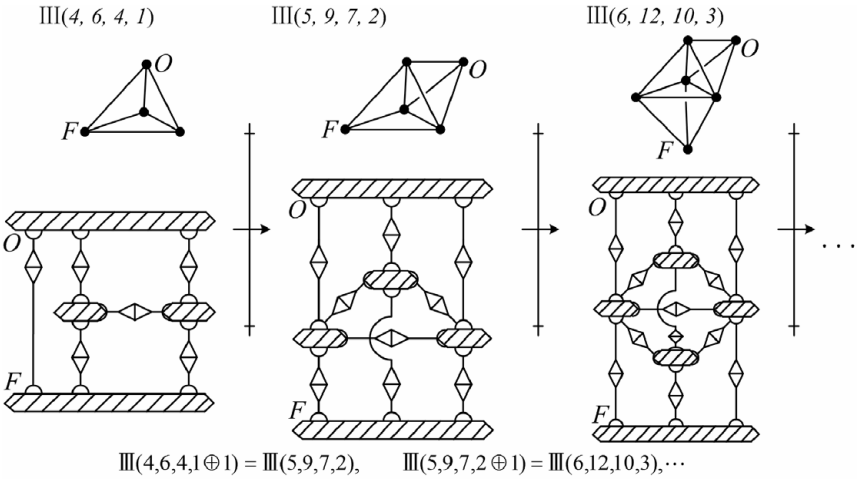
where Z_{Pi} and Z_{Si} represent output motion of platform P_i and subchains S_i, respectively. The motions of Z_{Pi+1} and Z_{Pi+2} are not affected by Z_{Pi} and connected subchains after the decomposition. The sub-class loop can be disassembled by the same rules repeatedly until operation in the ending sub-class. The symbols $\mapsto$ and $\wedge$ represent logic operator for serial and parallel, respectively, and the operation confirms to the distributive property of set operation, namely,

$$Z_{Si} \mapsto (Z_{Sj} \wedge Z_{Sk}) = (Z_{Si} \mapsto Z_{Sj}) \wedge (Z_{Si} \mapsto Z_{Sk}). \tag{15}$$

For the results of decomposition, it can be represented as parallel forms from fixed platform to end-output moving platform according to the distributive property of set operation, which each multi-subchain between fixed platform and end-output moving platform can be defined as including path of the multi-loop, namely, predicate:

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Table III. Formation of geometry and mapping of topological arrangements.

Type and formative logic	Original geometry and topological arrangements
I(d, l) One-dimensional forms of adding edges logic I($\oplus L$)	
II(d, l, p) Two-dimensional forms of adding faces logic II($\oplus P$)	
III(d, l, p, s) Three-dimensional forms of adding bodies logic III($\oplus S$)	

$In(S_i, S_{i+1}, S_{i+2}, \cdots)$ represents "including path of multi-subchains S_i, S_{i+1}, and S_{i+2} from fixed platform to end-output moving platform."

In order to synthesize structure of mechanisms, it is necessary to build configuration of subchains according to the including path and the given output motion. The including path that includes the least number of subchains should be configured first during the process of configuration, and then the structures of other subchains in different including paths can be configured. There are two constraint conditions to be considered, namely,

$$\begin{cases} Z_{In(Si,Sj,\cdots)} = Z_{Si} \mapsto Z_{Sj} \mapsto \cdots \\ Z_{Po} = Z_{In(Si\cdots)} \wedge Z_{In(Sk\cdots)} \wedge \cdots \end{cases}, \tag{16}$$

where Z_{Po} represents the given end-output motions.

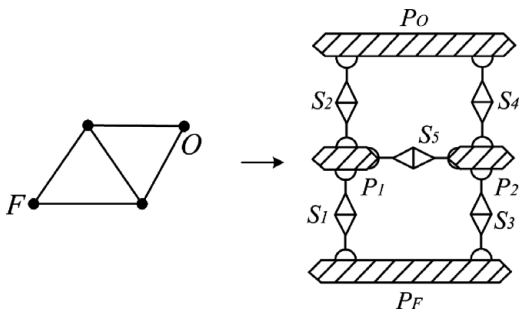


Fig. 3. Topological arrangement II (4, 5, 2).

Example 3. Taking topological arrangement II(4, 5, 2) in Table III as an example to synthesize the structure. Firstly, it is necessary to determine the motion relations between subchains and platforms as shown in Fig. 3.

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In Fig. 3, the P_F and P_O are fixed platform and end-output moving platform, respectively. The two loops and the zero-class loop (P_O, P_1, P_2) can be disassembled as

$$B_0(P_O, P_1, P_2) : Z_{Po} = (Z_{S2} \mapsto Z_{P1}) \wedge (Z_{S4} \mapsto Z_{P2}), \tag{17}$$

and then the first-class loop (P_1, P_F, P_2) is disassembled as

$$\begin{cases} B_1(P_1, P_F, P_2) : Z_{P1} = Z_{S1} \wedge (Z_{S5} \mapsto Z_{S3}) \\ B_1(P_2, P_1, P_F) : Z_{P2} = Z_{S3} \wedge (Z_{S5} \mapsto Z_{S1}) \end{cases}. \tag{18}$$

Substituting Eq. (18) to Eq. (17), the expression of the including path is obtained according to the distributive property of set operation as follows:

$$\begin{aligned} Z_{Po} &= (Z_{S2} \mapsto (Z_{S1} \wedge (Z_{S5} \mapsto Z_{S3}))) \\ &\wedge (Z_{S4} \mapsto (Z_{S3} \wedge (Z_{S5} \mapsto Z_{S1}))) \\ &= (Z_{S2} \mapsto Z_{S1}) \wedge (Z_{S2} \mapsto Z_{S5} \mapsto Z_{S3}) \\ &\wedge (Z_{S4} \mapsto Z_{S3}) \wedge (Z_{S4} \mapsto Z_{S5} \mapsto Z_{S1}) \\ &= Z_{In(S2,S1)} \wedge Z_{In(S2,S5,S3)} \wedge Z_{In(S4,S3)} \wedge Z_{In(S4,S5,S1)} \end{aligned}. \tag{19}$$

Obviously, there are four including paths. The end-output is determined as $3R1T$, and configure from the including path with the least number of subchains. In the light of combinatorial logical sets of serial kinematic chains, the given subchain S_1 is

$$S_1 : I(t_{11}, N(r_{11}, r_{12})) \wedge \neg O(o) \to Z_{S1} = I(R(r_{11}), P(T(t_{11}), T(r_{11}))). \tag{20}$$

In accordance to the constraint conditions, Eq. (16) and combinatorial logical sets of parallel mechanisms, the subchain S_2 can be obtained in including path $In(S_1, S_2)$, namely,

$$S_2 : C(r_{21}, r_{22}) \wedge O(o) \to Z_{S2} = G(R(r_{21}), R(r_{22})). \tag{21}$$

Similarly, the given subchain S_4 is

$$S_4 : N(r_{41}, r_{42}, r_{43}) \wedge \neg O(o) \to Z_{S4} = I(R(r_{41}), P(T(r_{41}), T(r_{42}))). \tag{22}$$

Thus, subchains S_3 and S_5 are obtained, respectively, in including path $In(S_3, S_4)$ and $In(S_2, S_5, S_3)$, namely,

$$S_3 : G(t_{31}, C(r_{31}, r_{32})) \wedge O(o) \to Z_{S3} = G(T(t_{31}), G(R(r_{31}), R(r_{32}))), \tag{23}$$

$$S_5 : C(r_{51}, r_{52}) \wedge O(o) \to Z_{S5} = G(R(r_{51}), R(r_{52})). \tag{24}$$

The relative connected relations between different subchains are

$$I(t_{11}, t_{31}) \wedge I(r_{11}, t_{31}) \wedge C(r_{21}, r_{22}, r_{31}, r_{32}, r_{51}, r_{52}).$$

Substituting Eqs. (20)–(24) to Eq. (19), the end-output motion of non-spanning tree-netted spatial hybrid mechanism II(4, 5, 2) with two loops and four including paths is

$$Z_{Po} = G(R_1, R_2, R_3) \wedge T_1 \wedge N(T_1, T(t_{31})) \wedge O(o) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}. \tag{25}$$

The moving reference frame's motion is $3R$ around origin O' and $1T$ along the axis of w, the structure is shown in Fig. 4.

In accordance to the above analysis, the algorithm for topological design and analysis of spatial multi-loop hybrid mechanisms are as follows:

1. Firstly, the geometry can be obtained by method of formative logic, and then the topological arrangement is developed by the mapping method.

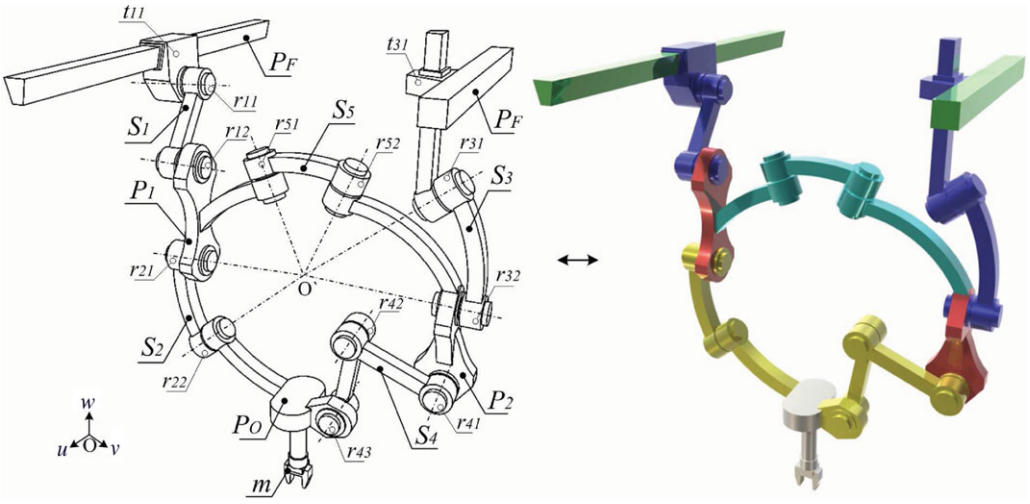


Fig. 4. (Colour online) Spatial multi-loop hybrid mechanism with $3R1T$.

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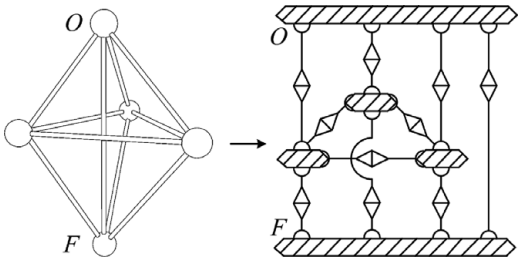


Fig. 5. Four-dimensional geometry IV(5, 10, 10, 5) and topological arrangement.

2. Determination of logical decomposition for the topological loops to determine the motion relations between each subchains and end-output moving platform.
3. Transformation of above relations as an expression of including path to obtain all including paths according to the distributive property of set operation.
4. Determination of the end-output motions and configuration of subchain structures start from the including path with the least number of subchains according to the constraint conditions in including paths.
5. Development of logical reasoning via combinatorial logical sets and relations of including paths to determine the rest motions of the corresponding subchains.
6. Verification of the validity of motions between end-output moving platform and each subchain to determine the structure of the hybrid mechanism.

The algorithm can be concisely described by the logical formulas as follows:

$$
\begin{aligned}
&\rightarrow \mathrm{I}(2, 1 \oplus 1) \vee \mathrm{II}(3, 3, 1 \oplus 1) \vee \mathrm{III}(4, 6, 4, 1 \oplus 1) \cdots \hookleftarrow \\
&\rightarrow B_0(P_i, P_{i+1}, P_{i+2}, \cdots), B_1(\cdots), \cdots : Z_{Po} \\
&\quad = (Z_{Si} \mapsto Z_{Pi+1}) \wedge (Z_{Si+1} \mapsto Z_{Pi+2}) \wedge \cdots \hookleftarrow \\
&\rightarrow Z_{Si} \mapsto (Z_{Sj} \wedge Z_{Sk}) = (Z_{Si} \mapsto Z_{Sj}) \wedge (Z_{Si} \mapsto Z_{Sk}) : Z_{Po} \\
&\quad = Z_{In(Si,Sj)} \wedge Z_{In(Sk,Sl)} \wedge \cdots \hookleftarrow \\
&\rightarrow \begin{cases} Z_{In(Si,Sj,\cdots)} = Z_{Si} \mapsto Z_{Sj} \mapsto \cdots \\ Z_{Po} = Z_{In(Si\cdots)} \wedge Z_{In(Sk\cdots)} \wedge \cdots \end{cases} : S_i, S_j, S_k, \cdots \hookleftarrow \\
&\rightarrow Z_{Po}
\end{aligned}
\tag{26}
$$

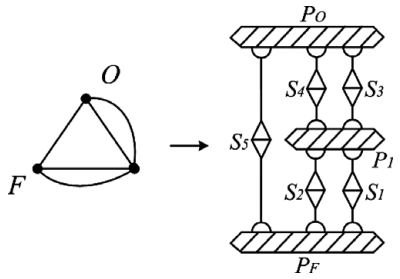


Fig. 7. Topological arrangement II (3, 5, 1).

4. Mapping of Multi-Loop Hybrid Mechanisms in Higher Dimensional Geometry

Besides one-, two-, and three-dimensional original geometry, the four-dimensional original geometry can be obtained by adding edges on tetrahedron by method of induction, namely, "simplex." The four-dimensional geometry includes ten edges, ten faces, and five spatial tetrahedrons, as shown in Fig. 5.

In accordance to the topological mapping, the four-dimensional geometry can also be transformed into topological arrangement. There is a kind of "four-dimensional multi-loop hybrid mechanism," which can be realized by the algorithm of the topological design, as shown in Fig. 6. The end-output motions of non-spanning tree-netted spatial hybrid mechanism IV(5, 10, 10, 5) with 10 loops and 16 including paths is $3T2R$, namely, $3T$ along the axis of u, v, and w, and $2R$ around the axis of u and v.

5. Examples of Topological Design of Spatial Multi-Loop Hybrid Mechanisms

Four examples are developed to illustrate the application of the logical proposition reasoning in topological design of spatial multi-loop hybrid mechanisms according to the algorithm of synthesis and analysis.

Example 4. In the light of the method of two-dimensional topology, topological arrangement II(3, 5, 1) is obtained, as shown in Fig. 7.

The topological arrangement includes three loops. Firstly, the zero-class loop (P_O, P_1, P_F) can be disassembled

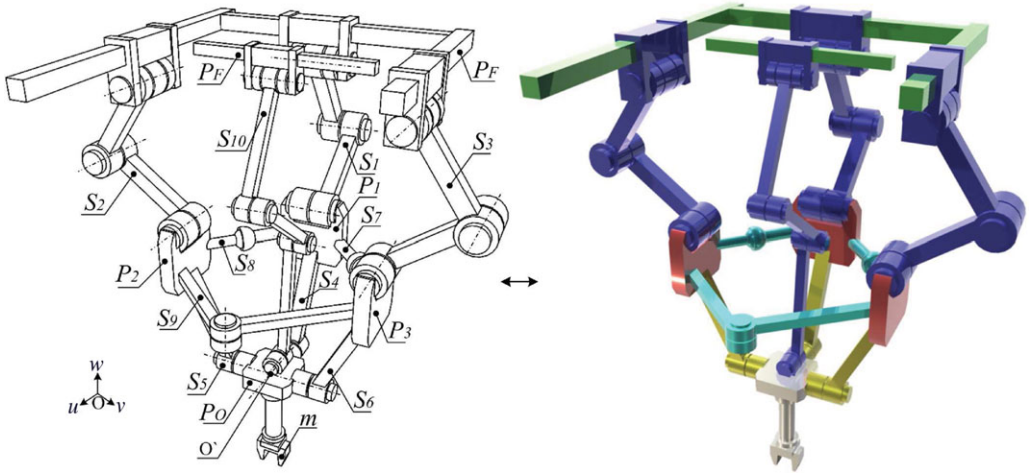


Fig. 6. (Colour online) Four-dimensional multi-loop hybrid mechanism with $3T2R$.

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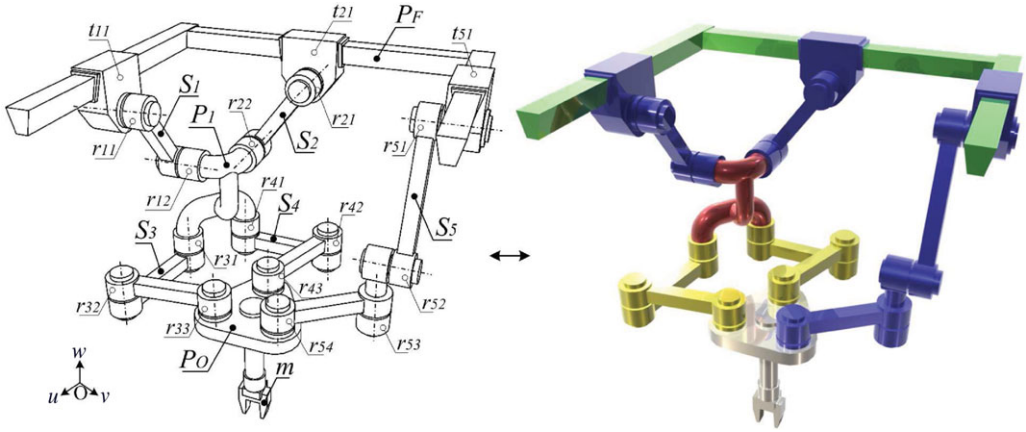


Fig. 8. (Colour online) Spatial multi-loop hybrid mechanism with $3T1R$.

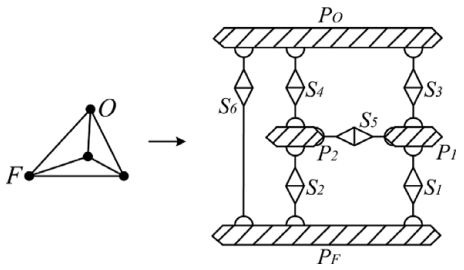


Fig. 9. Topological arrangement III (4, 6, 4, 1).

as

$$B_0(P_O, P_1, P_F) : Z_{Po} = ((3 \wedge 4) \mapsto Z_{P1}) \wedge 5, \tag{27}$$

where numbers 1, 2, 3, ... represent Z_{S1}, Z_{S2}, Z_{S3}, ..., namely, output motion of subchains S_1, S_2, S_3, ... Then the first-class loop (P_1, P_F) is disassembled as

$$B_1(P_1, P_F) : Z_{P1} = 1 \wedge 2. \tag{28}$$

The expression of including path is obtained according to the distributive property of set operation, namely,

$$Z_{Po} = (1 \mapsto 3) \wedge (2 \mapsto 3) \wedge (1 \mapsto 4) \wedge (2 \mapsto 4) \wedge 5. \tag{29}$$

There are four including paths. The end-output is determined as $3T1R$, and configuring from the including path with the least number of subchains. The given subchains $S_5 : I(N(r_{53}, r_{54}), I(t_{51}, N(r_{51}, r_{52})))$, $S_1 : I(t_{11}, N(r_{11}, r_{12}))$, and then subchains $S_3 : N(r_{31}, r_{32}, r_{33})$, $S_4 : N(r_{41}, r_{42}, r_{43})$ are determined in including paths $In(S_1, S_3)$ and $In(S_1, S_4)$. Subchain $S_2 : I(t_{21}, N(r_{21}, r_{22}))$ is determined in $In(S_2, S_4)$.

In accordance to the constraint conditions, the relative connected relations between different subchains are

$$I(t_{21}, N(t_{11}, t_{51})) \wedge I(N(r_{31}, r_{41}), I(r_{12}, r_{22})) \wedge N(r_{33}, r_{43}, r_{54}).$$

The end-output motions of spanning tree spatial hybrid mechanism II(3, 5, 1) with three loops and five including paths can be deduced according to the combinatorial logical sets, namely,

$$Z_{po} = G(T_1, T_2, T_3) \wedge R_1 \wedge N(R_1, R(r_{31})) \wedge \neg O(o) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}. \tag{30}$$

The moving reference frame's motion is $1R$ around the axis of w and $3T$ along the axis of u, v, and w, the structure is shown in Fig. 8.

Example 5. In the light of the method of three-dimensional topology, topological arrangement III(4, 6, 4, 1) is obtained, as shown in Fig. 9.

The topological arrangement includes four loops, and the zero-class loop (P_O, P_1, P_2) can be disassembled as

$$B_0(P_O, P_1, P_2) : Z_{Po} = (3 \mapsto Z_{P1}) \wedge (4 \mapsto Z_{P2}) \wedge 6. \tag{31}$$

Then the first-class loop (P_1, P_2, P_F) is disassembled as

$$\begin{cases} B_1(P_1, P_2, P_F) : Z_{P1} = 1 \wedge (5 \mapsto 2) \\ B_1(P_2, P_1, P_F) : Z_{P1} = 2 \wedge (5 \mapsto 1) \end{cases}. \tag{32}$$

The expression of including path is obtained according to the distributive property of set operation, namely,

$$Z_{Po} = (3 \mapsto 1) \wedge (3 \mapsto 5 \mapsto 2) \wedge (4 \mapsto 2) \wedge (4 \mapsto 5 \mapsto 1) \wedge 6. \tag{33}$$

There are five including paths. The end-output is determined as $3R2T$, and configuring from the including path with the least number of subchains. The given subchains $S_6 : G(C(r_{63}, r_{64}), I(t_{61}, N(r_{61}, r_{62})))$, $S_1 : I(t_{11}, N(r_{11}, r_{12}))$, $S_2 : I(t_{21}, r_{21})$, and then subchains $S_3 : C(r_{31}, r_{32})$, $S_4 : G(t_{41}, C(r_{41}, r_{42}))$ are determined in including paths $In(S_3, S_1)$ and $In(S_4, S_2)$. Subchain $S_5 : (t_{51})$ is determined in $In(S_3, S_5, S_2)$. In accordance to

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the constraint conditions, the relative connected relations between different subchains are

$$I(t_{21}, N(t_{11}, t_{61})) \wedge N(t_{11}, t_{41}, t_{51}) \wedge C(r_{31}, r_{32}, r_{41}, r_{42}, r_{63}, r_{64}).$$

The end-output motions of non-spanning tree-netted spatial hybrid mechanism III(4, 6, 4, 1) with four loops and five including paths can be deduced according to the combinatorial logical sets, namely,

$$Z_{po} = G(R_1, R_2, R_3) \wedge P(T_1, T_2) \wedge I(R(r_{11}), P(T_1, T_2)) \wedge O(o) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (34)$$

The moving reference frame's motion is $3R$ around origin O' and $2T$ along the axis of u and v, the structure is shown in Fig. 10.

Example 6. In the light of the method of two-dimensional topology, topological arrangement II(5, 8, 4) is obtained. The zero-class loop is disassembled, as shown in Fig. 11.

The topological arrangement includes four loops, and the zero-class loop (P_O, P_2, P_3, P_F) can be disassembled as

$$B_0(P_O, P_2, P_3, P_F) : Z_{Po} = (6 \mapsto Z_{P2}) \wedge (7 \mapsto Z_{P3}) \wedge 8. \quad (35)$$

Then the first-class loops (P_2, P_1, P_3) and (P_3, P_2, P_1, P_F) are disassembled as

$$\begin{cases} B_1(P_2, P_1, P_3) : Z_{P2} = (3 \mapsto Z_{P1}) \wedge (4 \mapsto Z_{P3}) \\ B_1(P_3, P_2, P_1, P_F) : Z_{P3} = (4 \mapsto 3 \mapsto 1) \wedge (5 \mapsto 1) \wedge 2 \end{cases}. \quad (36)$$

The decomposition of the second-class loop (P_1, P_3, P_F) is

$$\begin{cases} B_2(P_1, P_3, P_F) : Z_{P1} = 1 \wedge (5 \mapsto 2) \\ B_2(P_3, P_1, P_F) : Z_{P3} = 2 \wedge (5 \mapsto 1) \end{cases}. \quad (37)$$

The expression of the including path is obtained according to the distributive property of set operation, namely,

$$\begin{aligned} Z_{Po} = {} & (6 \mapsto 3 \mapsto 1) \wedge (6 \mapsto 3 \mapsto 5 \mapsto 2) \\ & \wedge (6 \mapsto 4 \mapsto 2) \wedge (6 \mapsto 4 \mapsto 5 \mapsto 1) \\ & \wedge (7 \mapsto 4 \mapsto 3 \mapsto 1) \wedge (7 \mapsto 5 \mapsto 1) \\ & \wedge (7 \mapsto 2) \wedge 8 \end{aligned}. \quad (38)$$

There are eight including paths. The end-output is determined as $2R2T$, and configuring from the including path with the least number of subchains. The given subchains $S_8 : I(N(t_{81}, r_{83}), N(r_{81}, r_{82}))$, $S_1 : I(t_{11}, r_{11})$, and $S_3 : N(r_{31}, r_{32}, r_{33})$, and then subchain $S_6 : (r_{61})$ are determined

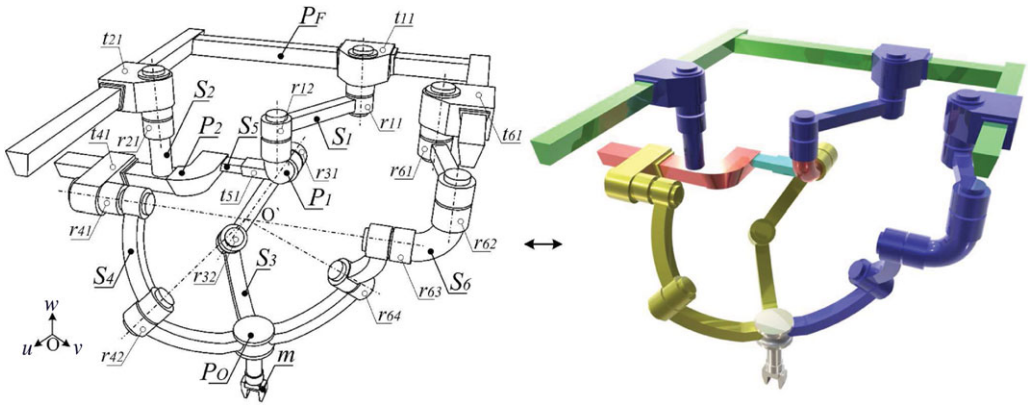


Fig. 10. (Colour online) Spatial multi-loop hybrid mechanism with $3R2T$.

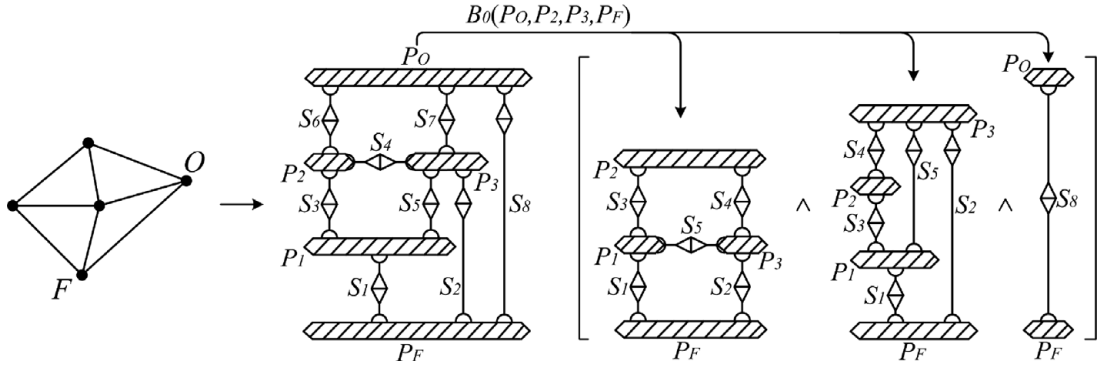


Fig. 11. Topological arrangement II(5, 8, 4) and decomposition of the zero class loop.

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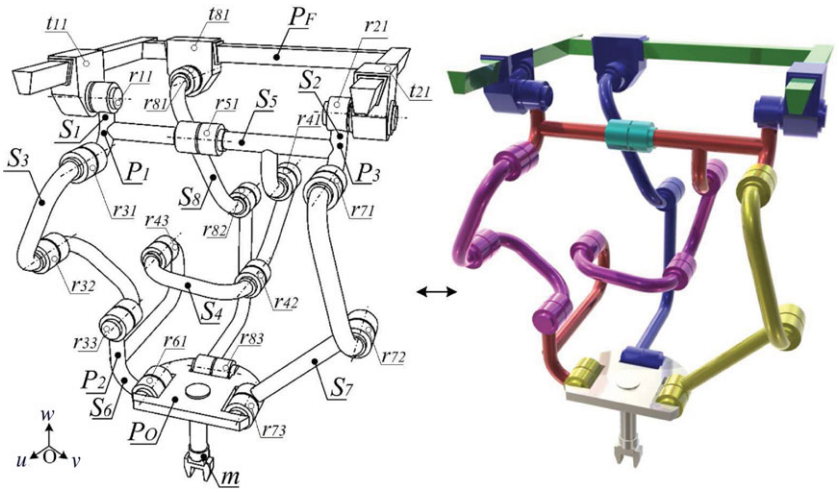


Fig. 12. (Colour online) Spatial multi-loop hybrid mechanism with $2R2T$.

in including path $In(S_6, S_3, S_1)$. The given subchains $S_2 : I(t_{21}, r_{21})$, $S_4 : N(r_{41}, r_{42}, r_{43})$, and $S_7 : N(r_{71}, r_{72}, r_{73})$ are determined in including paths $In(S_6, S_4, S_2)$ and $In(S_7, S_2)$. Subchain $S_5 : (r_{51})$ is determined in $In(S_7, S_5, S_1)$. In accordance to the constraint conditions, the relative connected relations between different subchains are

$$I(t_{81}, N(t_{11}, t_{21})) \wedge N(r_{11}, r_{21}, r_{51}) \wedge N(r_{31}, r_{41}, r_{61}, r_{71}, t_{11}).$$

The end-output motions of non-spanning tree-netted spatial hybrid mechanism II(5, 8, 4) with four loops and eight including paths can be deduced according to the combinatorial logical sets, namely,

$$\begin{aligned} Z_{po} &= I(R_1, R_2) \wedge P(T_1, T_2) \wedge N(R_1, R(r_{11})) \\ &\quad \wedge N(R_2, R(r_{81})) \wedge P(P(T_1, T_2), P(T(t_{11}), T(t_{81}))) \\ &\quad \wedge \neg O(o) \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{39}$$

The moving reference frame's motion is $2R$ around the axis of u and v, and $2T$ along the axis of v and w, the structure is shown in Fig. 12.

Example 7. In the light of the method of three-dimensional topology, topological arrangement III(6, 12, 10, 3) is obtained. The zero-class loop is disassembled, as shown in Fig. 13.

The topological arrangement includes 10 loops, and the zero-class loop (P_O, P_1, P_2, P_4) can be disassembled as

$$\begin{aligned} B_0(P_O, P_1, P_2, P_4) : Z_{Po} &= (10 \mapsto Z_{P1}) \wedge (12 \mapsto Z_{P2}) \\ &\quad \wedge (11 \mapsto Z_{P4}). \end{aligned} \tag{40}$$

Then the first-class loops $(P_1, P_F, P_3, P_2, P_4)$ and (P_4, P_1, P_2, P_3) are disassembled as

$$\begin{cases} B_1(P_1, P_F, P_3, P_2, P_4) : Z_{P1} = 1 \wedge (4 \mapsto Z_{P3}) \\ \quad \wedge (8 \mapsto Z_{P2}) \wedge (5 \mapsto Z_{P4}) \\ B_1(P_2, P_F, P_1, P_3, P_4) : Z_{P2} = 3 \wedge (8 \mapsto Z_{P1}) \\ \quad \wedge (7 \mapsto Z_{P3}) \wedge (6 \mapsto Z_{P4}) \\ B_1(P_4, P_1, P_2, P_3) : Z_{P4} = (5 \mapsto Z_{P1}) \wedge (6 \mapsto Z_{P2}) \\ \quad \wedge (9 \mapsto Z_{P3}) \end{cases}. \tag{41}$$

Similarly, the decomposition of the second-class loops (P_3, P_F, P_2, P_4), (P_4, P_3, P_2), (P_1, P_F, P_3, P_4), (P_4, P_1, P_3), and (P_1, P_F, P_2, P_3) and the third-class loops

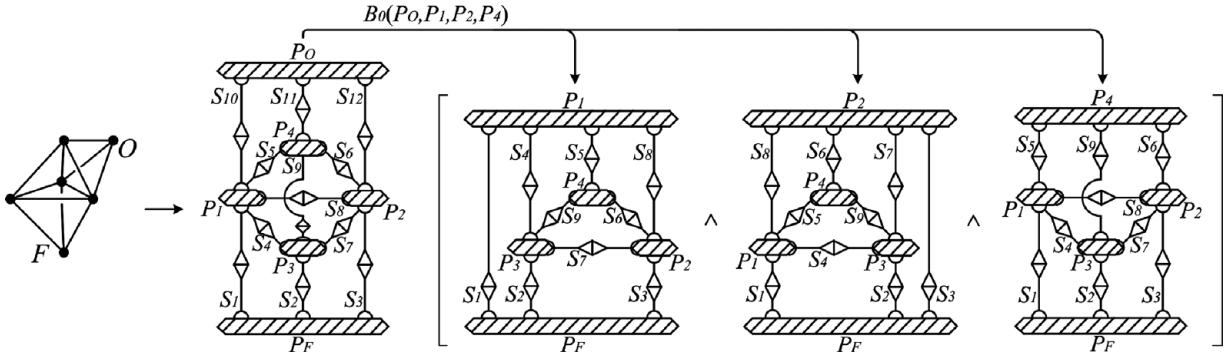


Fig. 13. Topological arrangement III(6, 12, 10, 3) and decomposition of the zero class loop.

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(P_3, P_F, P_2), (P_3, P_F, P_1), and (P_2, P_F, P_1) are

$$\begin{cases} B_2(P_3, P_F, P_2, P_4) : Z_{P3} = 2 \wedge (7 \mapsto 3) \wedge (9 \mapsto 6 \mapsto 3) \\ B_2(P_2, P_F, P_3, P_4) : Z_{P2} = 3 \wedge (7 \mapsto 2) \wedge (6 \mapsto 9 \mapsto 2) \\ B_2(P_4, P_3, P_2) : Z_{P4} = (9 \mapsto 2) \wedge (9 \mapsto 7 \mapsto 3) \\ \quad \wedge (6 \mapsto 3) \wedge (6 \mapsto 7 \mapsto 2) \\ B_2(P_1, P_F, P_3, P_4) : Z_{P1} = 1 \wedge (4 \mapsto 2) \wedge (5 \mapsto 9 \mapsto 2) \\ B_2(P_3, P_F, P_1, P_4) : Z_{P3} = 2 \wedge (4 \mapsto 1) \wedge (9 \mapsto 5 \mapsto 1) \\ B_2(P_4, P_1, P_2) : Z_{P4} = (9 \mapsto 2) \wedge (9 \mapsto 4 \mapsto 1) \\ \quad \wedge (5 \mapsto 1) \wedge (5 \mapsto 4 \mapsto 2) \\ B_2(P_1, P_F, P_2, P_3) : Z_{P1} = 1 \wedge (8 \mapsto 3) \wedge (8 \mapsto 7 \mapsto 2) \\ \quad \wedge (4 \mapsto 2) \wedge (4 \mapsto 7 \mapsto 3) \\ B_2(P_2, P_F, P_1, P_3) : Z_{P2} = 3 \wedge (8 \mapsto 1) \wedge (8 \mapsto 4 \mapsto 2) \\ \quad \wedge (7 \mapsto 2) \wedge (7 \mapsto 4 \mapsto 1) \\ B_2(P_3, P_F, P_1, P_2) : Z_{P3} = 2 \wedge (4 \mapsto 1) \wedge (4 \mapsto 8 \mapsto 3) \\ \quad \wedge (7 \mapsto 3) \wedge (7 \mapsto 8 \mapsto 1) \end{cases} \tag{42}$$

The expression of including path is obtained according to the distributive property of set operation, namely,

$$\begin{aligned} Z_{Po} = {} & (10 \mapsto 1) \wedge (10 \mapsto 4 \mapsto 2) \wedge (10 \mapsto 4 \mapsto 7 \mapsto 3) \\ & \wedge (10 \mapsto 4 \mapsto 9 \mapsto 6 \mapsto 3) \wedge (10 \mapsto 8 \mapsto 3) \\ & \wedge (10 \mapsto 8 \mapsto 7 \mapsto 2) \wedge (10 \mapsto 8 \mapsto 6 \mapsto 9 \mapsto 2) \\ & \wedge (10 \mapsto 5 \mapsto 9 \mapsto 2) \wedge (10 \mapsto 5 \mapsto 9 \mapsto 7 \mapsto 3) \\ & \wedge (10 \mapsto 5 \mapsto 6 \mapsto 3) \wedge (10 \mapsto 5 \mapsto 6 \mapsto 7 \mapsto 2) \\ & \wedge (12 \mapsto 3) \wedge (12 \mapsto 8 \mapsto 1) \wedge (12 \mapsto 8 \mapsto 4 \mapsto 2) \\ & \wedge (12 \mapsto 8 \mapsto 5 \mapsto 9 \mapsto 2) \wedge (12 \mapsto 7 \mapsto 2) \\ & \wedge (12 \mapsto 7 \mapsto 4 \mapsto 1) \wedge (12 \mapsto 7 \mapsto 9 \mapsto 5 \mapsto 1) \\ & \wedge (12 \mapsto 6 \mapsto 9 \mapsto 2) \wedge (12 \mapsto 6 \mapsto 9 \mapsto 4 \mapsto 1) \\ & \wedge (12 \mapsto 6 \mapsto 5 \mapsto 1) \wedge (12 \mapsto 6 \mapsto 5 \mapsto 4 \mapsto 2) \\ & \wedge (11 \mapsto 5 \mapsto 1) \wedge (11 \mapsto 5 \mapsto 8 \mapsto 3) \\ & \wedge (11 \mapsto 5 \mapsto 8 \mapsto 7 \mapsto 2) \wedge (11 \mapsto 5 \mapsto 4 \mapsto 2) \\ & \wedge (11 \mapsto 5 \mapsto 4 \mapsto 7 \mapsto 3) \wedge (11 \mapsto 6 \mapsto 3) \\ & \wedge (11 \mapsto 6 \mapsto 8 \mapsto 1) \wedge (11 \mapsto 6 \mapsto 8 \mapsto 4 \mapsto 2) \\ & \wedge (11 \mapsto 6 \mapsto 7 \mapsto 2) \wedge (11 \mapsto 6 \mapsto 7 \mapsto 4 \mapsto 1) \\ & \wedge (11 \mapsto 9 \mapsto 2) \wedge (11 \mapsto 9 \mapsto 4 \mapsto 1) \\ & \wedge (11 \mapsto 9 \mapsto 4 \mapsto 8 \mapsto 3) \wedge (11 \mapsto 9 \mapsto 7 \mapsto 3) \\ & \wedge (11 \mapsto 9 \mapsto 7 \mapsto 8 \mapsto 1) \end{aligned} \tag{43}$$

There are 37 including paths. The end-output is determined as $3T1R$, and configuring from the including path with the least number of subchains. The given subchains $S_1 : I(t_{11}, r_{11})$, $S_3 : I(t_{31}, r_{31})$, $S_2 : (t_{21})$, and $S_{11} : N(r_{111}, r_{112}, r_{113})$, and then subchains $S_{10} : N(r_{101}, r_{102}, r_{103})$, $S_{12} : N(r_{121}, r_{122}, r_{123})$, and $S_9 : N(r_{91}, r_{92}, r_{93})$ are determined in including paths $In(S_{10}, S_1)$, $In(S_{12}, S_3)$, and $In(S_{11}, S_9, S_2)$. Furthermore, subchain $S_8 : N(r_{81}, r_{82}, r_{83})$ is determined in $In(S_{10}, S_8, S_3)$. Subchain $S_7 : N(r_{71}, r_{72})$ is determined in $In(S_{12}, S_7, S_2)$. Subchain $S_4 : N(r_{41}, r_{42})$ is determined in $In(S_{10}, S_4, S_2)$. Subchain $S_6 : N(r_{61}, r_{62})$ is determined in $In(S_{11}, S_6, S_3)$. Subchain $S_5 : N(r_{51}, r_{52})$ is determined in $In(S_{11}, S_5, S_1)$. In accordance to the constraint conditions, the relative connected relations between different subchains are

$$\begin{aligned} & I(t_{21}, N(t_{11}, t_{31})) \wedge N(r_{31}, r_{41}, r_{51}, r_{61}, r_{71}, r_{81}, r_{91}) \\ & \wedge N(t_{11}, r_{101}, r_{121}) \wedge N(t_{21}, r_{111}). \end{aligned}$$

The end-output motions of non-spanning tree-netted spatial hybrid mechanism III(6, 12, 10, 3) with 10 loops and 37 including paths can be deduced according to the combinatorial logical sets, namely,

$$\begin{aligned} Z_{po} &= G(T_1, T_2, T_3) \wedge R_1 \wedge N(R_1, R(r_{11})) \wedge \neg O(o) \\ &= \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \end{aligned} \tag{44}$$

The moving reference frame's motion is $1R$ around the axis of w, and $3T$ along the axis of u, v, and w, the structure is shown in Fig. 14.

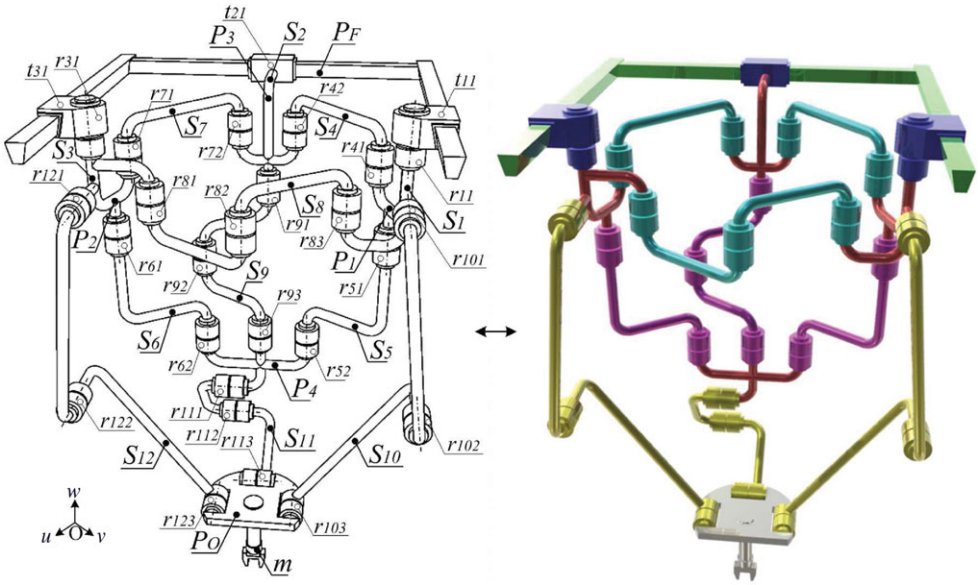


Fig. 14. (Colour online) Spatial multi-loop hybrid mechanism with $3T1R$.

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6. Conclusion

This paper presents an algorithm for topological design of multi-loop hybrid mechanisms via non-classic logical proposition. The six-value logical matrix in three orders can represent basic motions and relations. The combinatorial logical sets and logical axioms of the serial and parallel mechanisms are built on the basis of given predicate logic and propositions. The topological arrangement, topological mapping, multi-loop decomposition, and configuration are developed to obtain the algorithm of the logical design. After the determination of different output motions, arbitrary hybrid mechanisms can be obtained by the algorithm. The logical decomposition of spatial multi-loop and method of configuration solve the critical problem of topological design and motion analysis for non-spanning tree-netted spatial hybrid mechanisms. This contribution is helpful to the design of hybrid mechanisms in the future work.

Acknowledgments

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