Wave forms, and a special problem. By Mr R. HARGREAVES, St John's College.

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I think that some light may be thrown on a recent paper of Professor Whittaker's by giving concrete examples. The first example considered belongs to a transformation, which I gave some years ago, whereby a constant acceleration is incorporated in Einstein's form. Opportunity is taken to develop some features of the kinetic form of solution, in which all space coordinates are relative to a source, and the time of the latter is the only time appearing. This is carried as far as the treatment of energy.

§ 1. The form

$$dx^2 + dy^2 + \frac{dz^2}{1-2fz} - (1-2fz)\,dt^2$$

is derivable from $dx'^2 + dy'^2 + dz'^2 - dt'^2$ by using a transformation

$$x' = x, \quad y' = y, \quad 1 - fz' = \sqrt{1-2fz}\cosh ft, \quad ft' = \sqrt{1-2fz}\sinh ft. \qquad \ldots\ldots(1)$$

If the source in the original scheme is stationary at the origin, then for the corresponding source in the new scheme we may write

$$1 = \sqrt{1-2fz_0}\cosh ft_0,$$

and therefore

$$\sqrt{1-2fz_0}\sinh ft_0 = \frac{\dot{z}_0\cosh ft_0}{\sqrt{1-2fz_0}} = \frac{\dot{z}_0}{1-2fz_0} = \sqrt{2fz_0}, \quad \ldots(2)$$

where $\dot{z}_0 = \dfrac{dz_0}{dt_0}$.

It will be observed that if

$$ds^2 = (1-2fz)\,dt^2 - dt^2/(1-2fz),$$

then $s = \int\sqrt{1-2fz-\dot{z}^2/(1-2fz)}\,dt$ is made stationary by use of (2).

Now denoting by ρ the distance of a point $\varpi' z'$ from the origin, we have

$$f^2\rho^2 = f^2\varpi^2 + (1-\sqrt{1-2fz}\cosh ft)^2$$
$$= f^2\varpi^2 + (\sqrt{1-2fz_0}\cosh ft_0 - \sqrt{1-2fz}\cosh ft)^2; \quad \ldots(3)$$

and also

$$f\rho = f(t'-t_0') = \sqrt{1-2fz}\sinh ft - \sqrt{1-2fz_0}\sinh ft_0$$
$$= \sqrt{1-2fz}\sinh ft - \sqrt{2fz_0}. \quad \ldots(4)$$

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On eliminating ρ from the second form of (3) and the first form of (4), we obtain

$$f^2\varpi^2 + 1 - 2fz + 1 - 2fz_0 = 2\sqrt{1-2fz}\,\sqrt{1-2fz_0}\cosh f(t-t_0). \quad \ldots\ldots(5)$$

It may be observed that when f is small the terms of zero and first order in f vanish, but the second power of f gives the equality $\varpi^2 + (z-z_0)^2 = (t-t_0)^2$. Thus there is no doubt that (5) represents the wave equation for the new motion.

Again, eliminating t from (3) and (4) in the forms

$$\sqrt{1-2fz}\cosh ft = 1 - f\sqrt{\rho^2-\varpi^2},$$

and

$$\sqrt{1-2fz}\sinh ft = f\rho + \sqrt{2fz_0},$$

we obtain

$$\sqrt{\rho^2-\varpi^2} + \rho\sqrt{2fz_0} = z - z_0 - f\varpi^2/2,$$

from which

$$\rho(1-2fz_0) + \sqrt{2fz_0}(z - z_0 - t_0\varpi^2) = \sqrt{\varpi^2(1-2fz_0) + (z - z_0 - f\varpi^2/2)^2},$$

$$\rho = \sqrt{\frac{\varpi^2}{1-2fz_0} + \frac{(z-z_0-f\varpi^2/2)^2}{(1-2fz_0)^2}} - \frac{\dot{z}_0(z-z_0-f\varpi^2/2)}{(1-2fz_0)^2}$$

$$\equiv R - \frac{\dot{z}_0(z-z_0-f\varpi^2/2)}{(1-2fz_0)^2} \quad \ldots.(6)$$

This gives for ρ, a substantive quantity in the new scheme, viz. the denominator of potential functions, an expression in terms of relative coordinates into which t does not enter. We proceed to demonstrate that t_0 satisfies the characteristic equation

$$\left(\frac{\partial t_0}{\partial \varpi}\right)^2 + (1-2fz)\left(\frac{\partial t_0}{\partial z}\right)^2 = \frac{1}{1-2fz}\left(\frac{\partial t_0}{\partial t}\right)^2, \quad \ldots\ldots\ldots(7)$$

and first note that if $\sinh f(t-t_0)$ is found from (5) the result is

$$\sqrt{1-2fz}\,\sqrt{1-2fz_0}\sinh f(t-t_0) = f(1-2fz_0)\,R. \quad \ldots(8)$$

By differentiating (5) we obtain

$$\frac{\partial t_0}{\partial t}\left[\sqrt{1-2fz_0}\,\sqrt{1-2fz}\sinh f(t-t_0) - \dot{z}_0 + \dot{z}_0\sqrt{1-2fz}\cosh f(t-t_0)/\sqrt{1-2fz_0}\right] = \sqrt{1-2fz}\,\sqrt{1-2fz_0}\sinh f(t-t_0),$$

and on using the value of $\sinh f(t-t_0)$, this is

$$\left.\begin{aligned} &\frac{\partial t_0}{\partial t}\left[R - \frac{\dot{z}_0(z-z_0-f\varpi^2/2)}{(1-2fz_0)^2}\right] = R, \text{ or } \frac{\partial t_0}{\partial t} = \frac{R}{\rho}; \\ &\text{and similarly} \\ &\frac{\partial t_0}{\partial \varpi} = \frac{-\varpi}{\rho(1-2fz_0)}, \text{ and } \frac{\partial t_0}{\partial z} = -\frac{z-z_0+f\varpi^2/2}{\rho(1-2fz_0)}. \end{aligned}\right\} \quad \ldots\ldots(9)$$

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From these values follows (7); the forms

$$g^{pq}\frac{\partial t}{\partial x_p}\frac{\partial t}{\partial x_q}=0, \text{ and } g^{pq}\frac{\partial F}{\partial x_p}\frac{\partial F}{\partial x_q}=0,$$

the latter used by Whittaker, are as readily proved.

§ 2. We next examine the null geodesic, or time geodesic, for $d\varpi^2+dz^2/(1-2fz)=(1-2fz)\,dt^2$, where

$$t=\int\sqrt{\frac{1}{1-2fz}+\frac{1}{(1-2fz)^2}\left(\frac{dz}{d\varpi}\right)^2}\,d\varpi$$

is to be made stationary. The equation of energy type is

$$\left(\frac{dz}{d\varpi}\right)^2+1-2fz=C$$

in which we secure symmetry and an origin $\varpi=0$, $z=z_0$ by writing $C=1-2fz_0$, which gives

$$\left(\frac{dz}{d\varpi}\right)^2=2f(z-z_0), \text{ or } 2(z-z_0)=f\varpi^2.$$

Thus

$$t=\int\sqrt{1-2fz+\left(\frac{dz}{d\varpi}\right)^2}\,\frac{d\varpi}{1-2fz}=\sqrt{1-2fz_0}\int\frac{d\varpi}{1-2fz}$$

$$=\sqrt{1-2fz_0}\int\frac{d\varpi}{1-2fz_0-f^2\varpi^2},$$

or
$$f\varpi=\sqrt{1-2fz_0}\tanh f(t-t_0)=\sqrt{2f(z-z_0)}, \quad \ldots\ldots(10)$$

are the equations of a system of rays emerging from the source $\varpi=0$, $z=z_0$.

Now (5) gives

$$\sqrt{1-2fz}\,\sqrt{1-2fz_0}\sinh f(t-t_0)\frac{\partial t}{\partial\varpi}=f\varpi,$$

and

$$\sqrt{1-2fz}\,\sqrt{1-2fz_0}\sinh f(t-t_0)\frac{\partial t}{\partial z}(1-2fz)$$

$$=-(1-2fz)+\sqrt{1-2fz}\,\sqrt{1-2fz_0}\cosh f(t-t_0)$$

$$=\tfrac{1}{2}\{f^2\varpi^2+2f(z-z_0)\},$$

or at the crossing where ray meets wave, this is $f^2\varpi^2$. But here we have for an arc of the geodesic $dz=f\varpi\,d\varpi$, and it follows that $\frac{\partial t}{\partial\varpi}dz-\frac{\partial t}{\partial z}(1-2fz)\,d\varpi=0$, which is the condition that the arc of the geodesic should cut the wave surface at right angles. The

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relation (10) makes R and ρ agree, and therefore also $\frac{\partial t_0}{\partial t}=1$; thus the paraboloid (10) divides the ranges of the wave-surface for which $\frac{\partial t_0}{\partial t}>1$ from those for which $\frac{\partial t_0}{\partial t}<1$. In this respect it resembles the plane surface $z=z_0$, or more generally

$$(x-x_0)\dot{x}_0+(y-y_0)\dot{y}_0+(z-z_0)\dot{z}_0=0$$

of Euclidean metric.

It will be observed that we have now made use of the characteristic wave-surface and two geodesics, one for the path of the source and another for the time geodesic.

§ 3. The potentials in this problem may be readily found as follows:

$$\int\phi_4'dt'=\int-\frac{e}{\rho}dt'=-\int\frac{e}{\rho}\,d\left(f^{-1}\sqrt{1-2fz}\sinh ft\right)$$

$$=\int\frac{e}{\rho}\left(\frac{\sinh ft}{\sqrt{1-2fz}}\,dz-\sqrt{1-2fz}\cosh ft\,dt\right),$$

and so $$\phi_3=\frac{e\sinh ft}{\rho\sqrt{1-2fz}},\quad \phi_4=-\frac{e}{\rho}\sqrt{1-2fz}\cosh ft \qquad \text{.....(11)}$$

In particular the kinetic potential may be found from the form

$$L_0=\frac{e}{2}(\phi_3\dot{z}_0+\phi_4),$$

in which ϕ_3 and ϕ_4 have values at the surface corresponding to that of the original sphere of small radius b which constitutes the statical charge. Thus we have

$$L_0=\frac{e^2}{2b}\left\{\frac{\dot{z}_0\sinh ft_0}{\sqrt{1-2fz_0}}-\sqrt{1-2fz_0}\cosh ft_0\right\}=\frac{e^2}{2b}\left\{\frac{\dot{z}_0^2}{(1-2fz_0)^2}-1\right\}; \qquad \text{.....(12)}$$

and thence $\frac{d}{dt}\frac{\partial L_0}{\partial\dot{z}_0}-\frac{\partial L_0}{\partial z_0}=\frac{e^2f}{b}$, or more properly $\frac{e^2f}{bV^2}$, which is equal to mf if $e^2/b=mV^2$, V being velocity of light.

The first of the six vectors needed to specify the electromagnetic position is given by

$$\xi_1=\frac{\partial\phi_4}{\partial x},\quad \xi_2=\frac{\partial\phi_4}{\partial y},\quad \xi_3=\frac{\partial\phi_4}{\partial z}-\frac{\partial\phi_3}{\partial t};\quad B_1=\frac{\partial\phi_3}{\partial y},\quad B_2=-\frac{\partial\phi_3}{\partial x},\quad B_3=0,$$

while the second* is connected with it by

$$\xi_1=(1-2fz)X_1,\quad \xi_2=(1-2fz)X_2,\quad \xi_3=X_3,$$

* "The Electromagnetic Equations as Basis of Einstein's Quadratic Form", *Proc. Camb. Phil. Soc.*, 22 (1924), 154.

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and similar connexions for B and β. Thus we obtain

$$\left.\begin{aligned}&\Sigma X\xi - \Sigma B\beta\\ &= \frac{1}{1-2fz}\left(\frac{\partial\phi_4}{\partial\varpi}\right)^2 - (1-2fz)\left(\frac{\partial\phi_3}{\partial\varpi}\right)^2 + \left(\frac{\partial\phi_4}{\partial z} - \frac{\partial\phi_3}{\partial t}\right)^2 = \frac{e^2}{\rho^4},\\ \text{and}\qquad &\Sigma B\beta = \frac{e^2\varpi^2\sinh^2 ft}{\rho^6}.\end{aligned}\right\}\ldots(13)$$

The first of these is $-2L$, the second is $L+E$ where L and E are contents of kinetic potential and of energy per unit volume.

We shall also require the components of flux of energy, which are

$$\left.\begin{aligned}&\xi_2\beta_3 - \xi_3\beta_2 = \xi_3(1-2fz)\frac{\partial\phi_3}{\partial\varpi}\cos\phi\\ &\qquad = (1-2fz)\frac{\partial\phi_3}{\partial\varpi}\left(\frac{\partial\phi_4}{\partial z} - \frac{\partial\phi_3}{\partial t}\right)\cos\phi \equiv P\cos\phi,\\ &\xi_3\beta_1 - \xi_1\beta_3 = P\sin\phi,\\ \text{and}\quad &\xi_1\beta_2 - \xi_2\beta_1 = -(1-2fz)\frac{\partial\phi_3}{\partial\varpi}\frac{\partial\phi_4}{\partial\varpi} \equiv Q.\end{aligned}\right\}\qquad(14)$$

§ 4. The method of dealing with the energy equation will be explained in connexion with Euclidean metric. The innovation proposed is to transform four variables x, y, z, t to x', y', z', t_0, where $x' = x - x_0$. Thus a scheme

$$\Sigma\rho uX + \frac{\partial E}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 0,$$

becomes

$$\rho\Sigma uX + \left(\frac{\partial E}{\partial t_0} - u\frac{\partial E}{\partial x'} - \ldots\right)\frac{\partial t_0}{\partial t} + \frac{\partial P}{\partial x'} + \frac{\partial t_0}{\partial x'}\left(\frac{\partial P}{\partial t_0} - u\frac{\partial P}{\partial x'} - \ldots\right) = 0,$$

or

$$\begin{aligned}&\rho\Sigma uX + \frac{\partial E}{\partial t_0} - u\frac{\partial E}{\partial x'} - v\frac{\partial E}{\partial y'} - w\frac{\partial E}{\partial z'}\\ &+ \left(1 - \frac{\Sigma ux'}{r'}\right)\left(\frac{\partial P}{\partial x'} + \frac{\partial Q}{\partial y'} + \frac{\partial R}{\partial z'}\right) - \frac{x'}{r'}\frac{\partial P}{\partial t_0} - \frac{y'}{r'}\frac{\partial Q}{\partial t_0} - \frac{z'}{r'}\frac{\partial R}{\partial t_0}\\ &+ \frac{x'}{r'}\left(u\frac{\partial P}{\partial x'} + v\frac{\partial P}{\partial y'} + w\frac{\partial P}{\partial z'}\right) + \frac{y'}{r'}\left(u\frac{\partial Q}{\partial x'} + \ldots\right) + \frac{z'}{r'}\left(u\frac{\partial R}{\partial x'} + \ldots\right) = 0,\end{aligned}$$

the term $\rho\Sigma uX$ being unaltered by division by $\frac{\partial t_0}{\partial t}$, which is unity where charge exists. This is

$$\begin{aligned}&\rho\Sigma uX + \frac{\partial}{\partial t_0}(E - \Sigma x'P/r')\\ &+ \frac{\partial}{\partial x'}\{u\Sigma x'P/r' - uE + P(1 - \Sigma ux'/r')\} + \ldots = 0. \quad\ldots\ldots(15)\end{aligned}$$

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Now with this metric we have relations* such as

$$u = (y'Z - z'Y)/r'$$

between magnetic and electric vectors, and therefore

$$\Sigma x'P/r' = \Sigma x'(Yc - Zb)/r' = \Sigma a(y'Z - z'Y)/r' = \Sigma u^2,$$

twice the magnetic energy, so that

$$\Sigma x'P/r' - E = L = \tfrac{1}{2}(\Sigma u^2 - \Sigma X^2).$$

Therefore equation (15) stands

$$\left.\begin{aligned}\frac{\partial L}{\partial t_0} = \rho\Sigma uX + \frac{\partial}{\partial x'}\left(uL + P\Big/\frac{\partial t_0}{\partial t}\right) \\ + \frac{\partial}{\partial y'}\left(vL + Q\Big/\frac{\partial t_0}{\partial t}\right) + \frac{\partial}{\partial z'}\left(wL + R\Big/\frac{\partial t_0}{\partial t}\right),\end{aligned}\right\} \quad \ldots(16)$$

and the volume integral is

$$\frac{dL_0}{dt_0} = e\Sigma uX + \iiint\left\{\frac{\partial}{\partial x'}\left(uL + P\Big/\frac{\partial t_0}{\partial t}\right) + \ldots\right\} dx'dy'dz'.$$

In our special problem (16) becomes

$$\varpi\frac{\partial L}{\partial t_0} = \rho\dot{z}_0 Z + \frac{\partial}{\partial\varpi}\frac{\rho\varpi P}{R} + \frac{\partial}{\partial z'}\left(\frac{\rho\varpi Q}{R} + \dot{z}_0\varpi L\right),$$

to be multiplied by $d\varpi\, dz'\, d\phi$ for integration, so that

$$\frac{dL_0}{dt_0} = mf\dot{z}_0 + 2\pi\iint\left\{\frac{\partial}{\partial\varpi}\left(\frac{\rho\varpi P}{R}\right) + \frac{\partial}{\partial z'}\left(\frac{\rho\varpi Q}{R} + \dot{z}_0\varpi L\right)\right\} d\varpi\, dz', \quad \ldots(17)$$

in which the integral of $\rho\dot{z}Z$ is replaced by $mf\dot{z}_0$.

We deal first with the integration of

$$2\pi\iint\left\{\frac{\partial}{\partial\varpi}\frac{\rho\varpi P}{R} + \frac{\partial}{\partial z'}\frac{\rho\varpi Q}{R}\right\} d\varpi\, dz'$$

$$= 2\pi\int\frac{\rho\varpi}{R}(-P\,dz' + Q\,d\varpi),$$

or

$$= 2\pi\int\frac{\rho\varpi}{R}\{-P\,d\zeta + (Q - f\varpi P)\,d\varpi\},$$

when $z' - f\varpi^2/2 = \zeta$; and this integral is to be carried round the contour of $\rho =$ constant. But

$$\zeta = (1 - 2fz_0)\,R\cos\theta = (1 - 2fz_0)\,\rho\cos\theta/(1 - \sqrt{2fz_0}\cos\theta),$$

and

$$\varpi = \sqrt{1 - 2fz_0}\,\rho\sin\theta/(1 - \sqrt{2fz_0}\cos\theta);$$

* But note that $r'X = (z'b - y'c) + ex'(1 - \Sigma u^2)/(r' - \Sigma ux')$.

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so that we have to deal with

$$2\pi \int \frac{\rho^2 \varpi \sqrt{1-2fz_0}}{R(1-\sqrt{2fz}\cos\theta)^2} \times [\sqrt{1-2fz_0}\, P \sin\theta + (Q - f\varpi P)(\cos\theta - \sqrt{2fz_0})]\, d\theta,$$

and proceed to show that the bracket vanishes. We have in fact

$$P = (1-2fz)\frac{\partial\phi_3}{\partial\varpi}\left(\frac{\partial\phi_4}{\partial z} - \frac{\partial\phi_3}{\partial t}\right), \quad Q = -(1-2fz)\frac{\partial\phi_3}{\partial\varpi}\frac{\partial\phi_4}{\partial\varpi},$$

$$Q - f\varpi P = -(1-2fz)\frac{\partial\phi_3}{\partial\varpi}\left\{\frac{\partial\phi_4}{\partial\varpi} + f\varpi\left(\frac{\partial\phi_4}{\partial z} - \frac{\partial\phi_3}{\partial t}\right)\right\},$$

with

$$\frac{\partial\phi_4}{\partial z} - \frac{\partial\phi_3}{\partial t} = \frac{1-\sqrt{1-2fz}\cosh ft}{f\rho^3}, \quad \frac{\partial\phi_4}{\partial\varpi} = \frac{\varpi\sqrt{1-2fz}\cosh ft}{\rho^3},$$

and therefore

$$\frac{\partial\phi_4}{\partial\varpi} + f\varpi\left(\frac{\partial\phi_4}{\partial z} - \frac{\partial\phi_3}{\partial t}\right) = \frac{\varpi}{\rho^3} = \frac{\sqrt{1-2fz_0}\sin\theta}{\rho^2(1-\sqrt{2fz_0}\cos\theta)^2};$$

while the value of $\dfrac{\partial\phi_4}{\partial z} - \dfrac{\partial\phi_3}{\partial t}$ may be written as $\dfrac{\sqrt{\rho^2-\varpi^2}}{\rho^3}$ or

$$\frac{\cos\theta - \sqrt{2fz_0}}{\rho^2(1-\sqrt{2fz_0}\cos\theta)^2},$$

so that in fact

$$\sqrt{1-2fz_0}\, P\sin\theta + (Q - f\varpi P)(\cos\theta - \sqrt{2fz_0})$$

vanishes.

Also the total flow of $\dot{z}\varpi L$ varies as

$$\int \varpi\, d\varpi \text{ or as } \int_0^\pi \frac{\sin\theta(\cos\theta - \sqrt{2fz_0})\, d\theta}{(1-\sqrt{2fz_0}\cos\theta)^3},$$

which again vanishes. Thus there is no flux in (17) across any part of a surface for which ρ is constant. Now I find that the volume integral of L, viz. I_0, agrees with the value in (12), so that

$$\frac{dL_0}{dt_0} = \frac{d}{dt_0}\frac{e^2}{2b}\left\{\frac{\dot{z}_0^2}{(1-2fz_0)^2} - 1\right\} = \frac{e^2}{b}\left\{\frac{\dot{z}_0\ddot{z}_0}{(1-2fz_0)^2} + \frac{2f\dot{z}_0^3}{(1-2fz_0)^3}\right\}$$

$$= \frac{e^2 f\dot{z}_0(1-6fz_0+4fz_0)}{b(1-2fz_0)} = \frac{e^2 f\dot{z}_0}{b} = \frac{e^2 f\dot{z}_0}{bV^2} = mf\dot{z}_0$$

represents the outcome of (17).

No immediate consequence as regards total energy appears to follow, on the ground, I think, of the pre-existing relation (2) between kinetic and potential elements.

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§ 5. Another example is furnished by Einstein's central form. Thus for instance if we only take one space-variable, then the wave equation is given by

$$\left(\frac{dt}{dr}\right)^2\left(1-\frac{2m}{r}\right)=1\Big/\left(1-\frac{2m}{r}\right),$$

while for the time geodesic we have

$$\left(\frac{dr}{dt}\right)^2\Big/\left(1-\frac{2m}{r}\right)=1-\frac{2m}{r}.$$

These equations are identical, and their solution is

$$(r-2m)^2\,e^{r/m}+(r_0-2m)^2\,e^{r_0/2m}$$
$$=2\,(r-2m)\,(r_0-2m)\,e^{(r+r_0)/2m}\cosh\,(t-t_0)/2m,$$

or $(V^2r-2m)^2\,e^{V^2r/m}+\ldots$

$$=2\,(V^2r-2m)\,(V^2r_0-2m)\,e^{V^2(r+r_0)/2m}\cosh\,V^3\,(t-t_0)/2m.$$

It is also possible to give a general solution of the whole form attaching to Einstein's gravitational quadratic, under the limitation that $2m/r$ is a small quantity. When that quantity is neglected, the equation is

$$V\,(t-t_0)=\sqrt{r^2+r_0^2-2rr_0\cos\gamma},$$

where $\cos\gamma=\cos\theta\cos\theta_0+\sin\theta\sin\theta_0\cos(\phi-\phi_0)$, in which θ_0, ϕ_0 refer to the source. With the correction, the formula stands

$$V\,(t-t_0)=\sqrt{r^2+r_0^2-2rr_0\cos\gamma}$$
$$+\frac{2m}{V^2}\log\frac{r}{r_0}\left(1+\frac{r-r_0\cos\gamma}{\sqrt{r^2+r_0^2-2rr_0\cos\gamma}}\right)$$
$$-\frac{m\,(r-r_0\cos\gamma)}{V^2\sqrt{r^2+r_0^2-2rr_0\cos\gamma}}+\frac{m}{V^2}.$$

When the motion of the source is confined to one plane, we may write $\cos\gamma=\sin\theta\cos(\phi-\phi_0)$.

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