By Pythagoras' theorem in the main triangle: $k^2s^2 = k^2h^2 + (k-1)^2t^2$. Substituting from (1), (2) and simplifying: $t = \sqrt{k}$, whence $s = \sqrt{k-1}$.

Our triple $(kh, (k-1)t, ks)$ becomes

$$\left(\sqrt{k(k-1)}, (k-1)\sqrt{k}, k\sqrt{k-1}\right).$$

(Note in passing that the triangle cannot be Pythagorean as it would require consecutive integers to be squares.) Finally, dividing through by $\sqrt{k(k-1)}$, we have the triple $\left(1, \sqrt{k-1}, \sqrt{k}\right)$ our Theodoran triangle.

An additional property

We drop a perpendicular on to the hypotenuse from the right angle vertex. By the equal intercept theorem we have the divisions marked on Figure 2B.

Readers can readily confirm that the triangle T' is the triangle T scaled by $\sqrt{k-1}$ and the areas of T, T', the trapezium between, and the whole triangle, stand in the ratio $1 : k-1 : k(k-2) : k(k-1)$.

Reference

1. Tony Crilly, A family of discrete spirals, *Math. Gaz.* **104** (July 2020) pp. 215-224.

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On 105.13: Martin Lukarevski writes: The derivation of the triangle identity in this Note

$$\sum \frac{1}{a(s-a)} = \frac{s^2 + (4R+r)^2}{sabc}$$

is rather cumbersome. Using the previously shown identities

$$\sum \frac{1}{s-a} = \frac{4R+r}{rs}$$

and $ab + bc + ca = s^2 + r(4R+r)$, we let the decomposition

$$\frac{1}{a(s-a)} = \left(\frac{1}{a} + \frac{1}{s-a}\right)\frac{1}{s}$$

do the trick:

$$\sum \frac{1}{a(s-a)} = \frac{1}{s}\left(\sum \frac{1}{a} + \sum \frac{1}{s-a}\right)$$

$$= \frac{1}{s}\frac{s^2 + r(4R+r) + 4R(4R+r)}{abc} = \frac{s^2 + (4R+r)^2}{sabc}.$$

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