*Hankel Transforms**. By E. C. TITCHMARSH. (Communicated by Mr G. H. HARDY.)

[*Received* 26 January 1924.]

I have to correct the proof of the assertion on p. 471 of my paper that if $f(t)$ is integrable over $(0, a)$ and $\nu \geqq -\frac{1}{2}$, then

$$\lim_{N\to\infty} \int_0^a t^{\frac{1}{2}} f(t)\, dt \int_N^\infty J_{\nu+1}(ux)\, J_\nu(ut)\, du = 0.$$

This follows from Lebesgue's convergence theorem if

$$t^{\frac{1}{2}} \int_N^\infty J_{\nu+1}(ux)\, J_\nu(ut)\, du = t^{-\frac{1}{2}} \int_{Nt}^\infty J_{\nu+1}\left(\frac{ux}{t}\right) J_\nu(u)\, du$$

is bounded for $0 < t < a$, $N > N_0$. The result is immediate if $\nu = -\frac{1}{2}$, so it may be assumed that $\nu > -\frac{1}{2}$. Since the argument of the first Bessel function remains greater than a constant we may use the asymptotic form

$$J_{\nu+1}(z) = \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{1}{2}\nu\pi - \frac{3\pi}{4}\right) + O(z^{-\frac{3}{2}}).$$

Since $|J_\nu(u)| < Au^{-\frac{1}{2}}$ for all values of ν, the error term is

$$O\left\{t^{-\frac{1}{2}} \int_{Nt}^\infty (u/t)^{-\frac{3}{2}} u^{-\frac{1}{2}}\, du\right\} = O\left(t \int_{Nt}^\infty u^{-2}\, du\right) = O(1).$$

The main term is a constant multiple of

$$\int_{Nt}^\infty \cos\left(\frac{ux}{t} - \frac{1}{2}\nu\pi - \frac{3\pi}{4}\right) J_\nu(u)\, u^{-\frac{1}{2}}\, du.$$

Any part of the integral for which $u < k$ is

$$O\left(\int_{Nt}^k u^{\nu-\frac{1}{2}}\, du\right) = O(1).$$

In the remainder we may use the asymptotic formula for the other Bessel function. This gives an error term

$$O\left(\int_k^\infty u^{-2}\, du\right) = O(1),$$

and the main term

$$\int_k^\infty \cos\left(\frac{ux}{t} - \frac{1}{2}\nu\pi - \frac{3\pi}{4}\right) \cos\left(u - \frac{1}{2}\nu\pi - \frac{\pi}{4}\right) \frac{du}{u}$$

$$= \frac{1}{2}\int_k^\infty \sin u \left(\frac{x}{t} - 1\right) \frac{du}{u} - \frac{1}{2}\int_k^\infty \cos\left\{u\left(\frac{x}{t} + 1\right) - \nu\pi\right\} \frac{du}{u},$$

which is easily seen to be bounded. This completes the proof.

* These *Proceedings*, vol. XXI, pp. 463–473.

https://doi.org/10.1017/S0305004100000141 Published online by Cambridge University Press