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Dispersion relations in relativistic astrophysical outflows

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Abstract. In this work we delineate the physical picture for the propagation of electromagnetic oscillations in a moving plasma. We present calculations devoted to describing the dispersion features of the radiation that propagates within a magnetoactive plasma. Both classical and relativistic treatments have been employed. We have concluded that if the outflow is moving with small velocities, the general motion of the plasma does not affect the propagation of transverse electromagnetic waves. The waves are refracted according to the ordinary refraction law deduced for non-moving magnetoactive plasmas. The global movement does not change the dispersion relations, and the shapes of the corresponding curves remain unaltered. In the relativistic formulation, we have derived the equations that determine the electromagnetic wave spectrum for a moving magnetoactive plasma. In this case, it is found that the flow velocity affects the plasma dispersion properties and is responsible for new dispersion branches: transverse and longitudinal perturbations. The ordinary and extraordinary electromagnetic modes are modified and described by dispersion relations whose points of resonance are also in explicit dependence on the plasma velocity. Furthermore, the relativistic motion causes the split of electromagnetic modes into two independent waves that propagate at different rates. This effect is remarkable for highly relativistic velocities. The results presented in this work may be applied in order to incorporate suitable corrections to the dispersion models, developed to explain the anomalous features of observed lines or continuum radiation, associated with astrophysical relativistic outflows which are present in gamma-ray bursts, blazars, relativistic jets in AGNs (active galactic nuclei), quasars and microquasars.

1. Introduction

Collective processes related to the interaction between beams and electron plasmas have been investigated by several authors starting probably with the work by Clemmow and Wilson (1956). The study of instabilities of this stream interaction, using linear and nonlinear models for unmagnetized and homogeneous plasmas, as was performed by Matsiborko et al. (1972), Bludman et al. (1960) and Fainberg et al. (1969), has been quite intensive. In most cases, relativistic beams have been injected into a static plasma. The effects of an external magnetic field, the determination of the electromagnetic spectrum and the study of the stability of the modes are some

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of the topics related to these stream interactions (see, for example, Godfrey et al. (1975)). Owing to the complicated nature of the physical processes involved, the electromagnetic dispersion relations are established using some simplified assumptions, whereas their solutions are obtained numerically (e.g. Godfrey et al. 1975; Hardee and Rose 1976).

Previous works, devoted to the analysis of the wave propagation in isotropic moving media (Lopez 1996a; 1996b), have shown that when the global movement of the plasma is included in the calculations, the dispersion symmetry of the system is changed and the medium acquires new optical properties. For instance, an isotropic cold plasma becomes anisotropic. However, for an isotropic medium the anisotropies introduced are so weak that the dispersion relations are covariant (i.e. the dispersion relations have the same form in any inertial frame of reference). Consequently, if the plasma is moving with small velocity, the global movement does not affect the plasma response, and we see the usual dispersion shapes for the electromagnetic waves as in a static medium. On the other hand, if this motion is relativistic the propagation of new dispersion electromagnetic branches is possible.

In this paper, we present the study of the dispersion properties of a moving magnetoactive plasma. The dielectric tensor of permittivity has been established in the classical and relativistic approaches. The dispersion equations obtained have been solved and the expressions that describe the electromagnetic response of the medium have been derived. The reduction of the results obtained to the small velocity regime shows that our results are in agreement with the well-known classical dispersion theory, previously developed for a non-moving magnetoactive plasma.

Some enigmatic galactic and extragalatic objects have bipolar relativistic outflows apparently collimated by a magnetic field. We consider these objects as excellent samples where our conjectures and results might be tested in order to investigate the properties of the observed radiation.

2. Relativistic treatment of a magnetoactive plasma

We consider a magnetized, low-temperature plasma consisting of electrons and ions only, and let both components move relative to each other in thermal equilibrium. Thus, the particles can be treated classically, i.e. $v_{\mathrm{t}}/c \ll 1$ (where v_{t} is the electron thermal velocity). We consider, in addition, a collisionless regime where all parameters involved have been transformed to the laboratory frame. We begin by finding the suitable expression for the transformation of the dielectric tensor of permittivity ϵ_{ij} for a magnetoactive plasma, which is moving with respect to the observer placed within the framework of the laboratory. As usual, we use Lorentz's transformations and Maxwell's equations to find the relations between the vectors $\mathbf{E}$, $\mathbf{E}'$ and the electron currents $\mathbf{j}$, $\mathbf{j}'$ in two different inertial frames. Thus, it is not difficult to derive the expression for the transformation of the conductivity tensor σ_{ij} and, consequently, to obtain the relation for the transformation of the dielectric tensor of permittivity ϵ_{ij}. The final result can be written in the following form, as it has been presented in previous works (Lopez 1996a; 1996c):

$$\epsilon_{ij}(w,\mathbf{k}) = \delta_{ij}\left(1-\frac{w'^2}{w^2}\right) + \frac{w'^2}{w^2}\epsilon'_{ij} - \frac{w'}{w}\gamma(1-\beta)\frac{\epsilon'_{i\nu}v_\nu v_j}{v^2} + \frac{w'}{w^2}\gamma\epsilon'_{i\nu}k_\nu v_j$$

$$+ \frac{w'}{w}\gamma\left[(1-\beta)\frac{v_i v_j}{v^2} - \frac{v_j k_i}{w}\right] + \frac{w'^2}{w^2}\frac{\gamma(1-\beta)}{v^2}(v_i\epsilon'_{\mu\mathrm{j}}v_\mu - v_i v_j)$$

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$$+\frac{w'}{w}\frac{\gamma^2}{v^4}(1-\beta)^2 v_i v_j (v^2 - v_\mu \epsilon'_{\mu\nu} v_\nu)$$

$$+\frac{w'}{w^2}\frac{\gamma^2(1-\beta)}{v^2} v_i v_j (v_\mu \epsilon'_{\mu\nu} k_\nu - v_\mu k_\mu)$$

$$+\frac{w'}{w^2}\gamma v_i (k'_\mu \epsilon'_{\mu j} - k'_j) - \frac{\gamma^2 v_i v_j}{w v^2}(1-\beta)(k'_\mu \epsilon'_{\mu\nu} v_\nu - v_\mu k'_\mu)$$

$$+\frac{\gamma^2}{w^2} v_i v_j (k'_\mu \epsilon'_{\mu\nu} k_\nu - k'_\mu k_\mu), \tag{1}$$

where $\epsilon_{ij}(w, \mathbf{k})$ is the dielectric permittivity tensor in the xyz laboratory frame, $\epsilon'_{ij}(w', \mathbf{k}')$ the dielectric permittivity tensor in the rest frame of the plasma $x'y'z'$, w is the radiation frequency in the xyz frame, w' is the radiation frequency in the $x'y'z'$ frame, $\mathbf{k}$ and $\mathbf{k}'$ are the wavevectors in the xyz and $x'y'z'$ frames, respectively, $v(x, y, z)$ is the plasma velocity, and

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad \beta = \frac{1}{\gamma}$$

$$w' = \gamma(w - \mathbf{k}\cdot\mathbf{v}), \quad \mathbf{k}' = \mathbf{k} + \frac{\gamma}{v^2}\left[(1-\beta)\mathbf{k}\cdot\mathbf{v} - w\frac{v^2}{c^2}\right]\mathbf{v}.$$

Here the product $\mathbf{k}\cdot\mathbf{v} = kv\cos\psi$ is expressed using the dispersion angle ψ, i.e. the angle formed between the outflow velocity vector and the direction of propagation of the wave.

We restrict the analysis to the case of an electron plasma flowing along the magnetic field lines, the so-called longitudinal flow, which corresponds to the situation typically observed in astrophysical outflows. Therefore, the global plasma velocity will be $\mathbf{v} = v_z \mathbf{i}_3$ (where $\mathbf{i}_3$ is the unitary vector along the z-axis), and the above tensor can be written in simple matrix form as

$$\epsilon = I + \frac{w'}{w}\alpha(\epsilon' - I)\beta, \tag{2}$$

where I is the unitary matrix,

$$\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \dfrac{\gamma v k'_x}{w'} & 0 & 1 + \gamma\left[(1-\beta) + \dfrac{v k'_z}{w'}\right] \end{pmatrix},$$

$$\beta = \begin{pmatrix} \dfrac{w'}{w} & 0 & \dfrac{\gamma v k'_x}{w} \\ 0 & \dfrac{w'}{w} & 0 \\ 0 & 0 & \dfrac{w'}{w} - \gamma\left[(1-\beta) - \dfrac{v k_z}{w}\right] \end{pmatrix},$$

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ϵ' is the dielectric permittivity tensor in a comoving system for a magnetoactive plasma

$$\epsilon' = \begin{pmatrix} \epsilon_\perp & ig & 0 \\ -ig & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_\parallel \end{pmatrix}, \tag{3}$$

$$\epsilon_\perp = 1 - \frac{w_\mathrm{p}^2}{w'^2 - w_\mathrm{b}'^2}, \quad g = \frac{-w_\mathrm{p}^2 w_\mathrm{b}'}{w'(w'^2 - w_\mathrm{b}'^2)}, \quad \epsilon_\parallel = 1 - \frac{w_\mathrm{p}^2}{w'^2},$$

$w_\mathrm{p} = 4\pi e^2 n_\mathrm{e}/m_\mathrm{e}$ and $w_\mathrm{b}' = eB/m_\mathrm{e}c$ are the plasma frequency and electron gyrofrequency, expressed in the comoving frame, respectively. The electron density is represented by n_e, and the electron mass at the rest frame is m_e.

Substituting (3) into (2), the elements of the dielectric tensor of permittivity in a relativistic moving magnetoactive plasma, after some simplifications, can be written as

$$\epsilon_{xx} = -\frac{c^2k^2v^2(w_\mathrm{p}^2 - w^2)\cos^2\psi + 2c^2kvw(w^2 - w_\mathrm{p}^2)\cos\psi}{w^2[c^2k^2v^2\cos^2\psi - 2c^2kv\cos\psi + c^2(w^2 - w_\mathrm{b}^2) + w_\mathrm{b}^2v^2]}$$
$$+ \frac{w^2[c^2(w_\mathrm{b}^2 + w_\mathrm{p}^2 - w^2) - w_\mathrm{b}^2v^2]}{w^2[c^2k^2v^2\cos^2\psi - 2c^2kv\cos\psi + c^2(w^2 - w_\mathrm{b}^2) + w_\mathrm{b}^2v^2]},$$

$$\epsilon_{zz} = 1 + \frac{w_\mathrm{p}^2v^2}{c^2(kv\cos\psi - w)^2} - \frac{w_\mathrm{p}^2}{(kv\cos\psi - w)^2}$$
$$- \frac{c^2k^2w_\mathrm{p}^2v^2\sin^2\psi}{w^2[c^2k^2v^2\cos^2\psi - 2c^2kv\cos\psi + c^2(w^2 - w_\mathrm{b}^2) + w_\mathrm{b}^2v^2]},$$

$$\epsilon_{xy} = \frac{icw_\mathrm{b}w_\mathrm{p}^2\sqrt{c^2 - v^2}(kv\cos\psi - w)}{w^2[c^2k^2v^2\cos^2\psi - 2c^2kv\cos\psi + c^2(w^2 - w_\mathrm{b}^2) + w_\mathrm{b}^2v^2]},$$

$$\epsilon_{xz} = \frac{c^2kw_\mathrm{p}^2v\sin\psi(kv\cos\psi - w)}{w^2[c^2k^2v^2\cos^2\psi - 2c^2kv\cos\psi + c^2(w^2 - w_\mathrm{b}^2) + w_\mathrm{b}^2v^2]},$$

$$\epsilon_{yz} = \frac{ickw_\mathrm{b}w_\mathrm{p}^2v\sqrt{c^2 - v^2}\sin\psi}{w^2[c^2k^2v^2\cos^2\psi - 2c^2kv\cos\psi + c^2(w^2 - w_\mathrm{b}^2) + w_\mathrm{b}^2v^2]},$$

$$\epsilon_{xx} = \epsilon_{yy}, \quad \epsilon_{xy} = -\epsilon_{yx}, \quad \epsilon_{xz} = \epsilon_{zx},$$

where w is the observed frequency, ψ is the dispersion angle, w_p and w_b are the plasma frequency and electron gyrofrequency in the laboratory frame, respectively, k is the wavevector module, v is the plasma velocity, and c is the speed of light.

These elements define the dielectric tensor of permittivity for a relativistic moving magnetoactive plasma, which is a hermitician matrix for this particular jet configuration:

$$\epsilon = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ -\epsilon_{xy} & \epsilon_{xx} & \epsilon_{yz} \\ \epsilon_{xz} & -\epsilon_{yz} & \epsilon_{zz} \end{pmatrix}. \tag{4}$$

Once again, applying electrodynamics and assuming that the first-order electromagnetic perturbation has the form $\exp[i(\mathbf{k}\cdot\boldsymbol{r} - wt)]$, the algebraic equations that

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describe the wave propagation in the plasma have been derived. The condition for non-singular solutions of those equations is expressed in this paper as

$$\frac{k_i k_j \epsilon_{ij}}{k^2} k^4 + \left[\frac{k_i k_j}{k^2} \epsilon_{i\mu} \epsilon_{\mu j} - \epsilon_{ij} \delta_{ij} \frac{k_i k_j \epsilon_{ij}}{k^2} \right] \left[\frac{w^2}{c^2} k^2 \right] + |\epsilon_{ij}| \frac{w^4}{c^4} = 0. \tag{5}$$

Equation (5), together with the tensor ϵ_{ij} expressed by (4), gives the dispersion relations that define the electromagnetic spectrum in a relativistic moving anisotropic plasma. This is a large and complex equation of tenth order in w containing 123 terms, that does not admit exact solutions except for some particular jet orientations. In the most general case of an arbitrary dispersion angle ψ, (5) is treated numerically for the appropriate geometry and physical conditions of a given flowing plasma.

At this point, we choose an adequate system of coordinates where the propagation of electromagnetic waves occurs in the xz plane, the magnetic field is placed along the z-axis, and the plasma velocity is aligned with the magnetic field. Here, we have considered that the direction of wave propagation is described by the dispersion angle ψ, which is the angle formed by the wavevector $\mathbf{k}$ and the flow velocity vector $\mathbf{v}$. Thus, analytical solutions have only been obtained for two directions of the wave propagation: the first when the wave is propagating in the z direction, along the outflow ($\psi = 0°$), and the second for the wave propagation normal to the moving plasma ($\psi = 90°$).

2.1. Radiation propagation parallel to the outflow ($\psi = 0°$)

In the aforementioned context, we began by analyzing the situation when the wave is propagating in the z direction along the jet ($\psi = 0°$). In this case, the general dispersion equation (5) is separated into three different dispersion branches, two of these corresponding to transverse electromagnetic waves and the third branch to longitudinal oscillations.

The permissible transverse electromagnetic waves that could propagate in a magnetoactive moving plasma are described by the following dispersion relation, which is derived for a rarefied plasma:

$$n_{1,2}^2 = 1 \pm \frac{w_\mathrm{p}^2 [w_\mathrm{b}/\gamma \mp (1 - v/c) w]}{w[(1 - v/c) w^2 - (1 + v/c) w_\mathrm{b}^2]}, \tag{6a}$$

where $\mathbf{n} = (c/w)\mathbf{k}$ is the complex refraction index. The subscript 1 corresponds to the ordinary wave, whereas 2 corresponds to the extraordinary wave.

The curves corresponding to (6a) are similar to those for waves propagating along the magnetic field in a static magnetoactive plasma (see Fig. 1). However, in a moving plasma the extraordinary branch has blue-shifted absorption features due to the relativistic motion of the plasma to the observer, so that

$$w_\mathrm{r} = \pm w_\mathrm{b}' \left(1 + \frac{v}{c} \right). \tag{6b}$$

The second resonance point occurs at $w_\mathrm{r} = 0$, without dependence on the plasma motion. We see from (6b) that the higher the flow velocity the higher the resonant frequency, and in the limit $v \to c$ it tends to the maximum displacement $w_\mathrm{r} = 2w_\mathrm{b}'$. The ordinary wave has only a singular point at $w_\mathrm{r} = 0$. The electromagnetic modes that are propagating along the jet are circularly polarized. The propagation of the modes is possible in the whole electromagnetic spectrum except in the resonance

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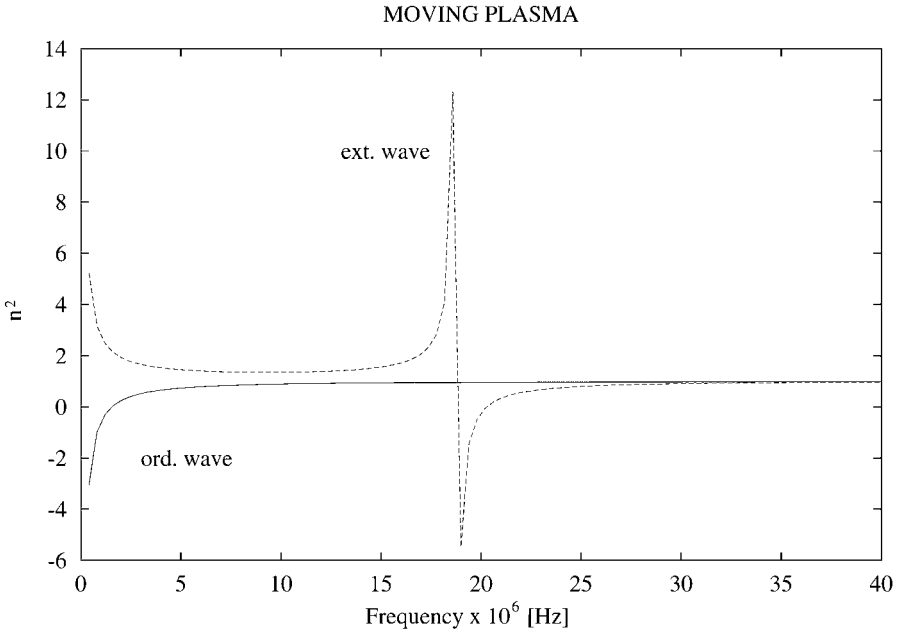


Figure 1. Dispersion curve for the radiation in the superluminal jet of V4641 Sgr ($\psi \sim 0°$): ordinary and extraordinary transverse waves. The plasma frequency $w_{\rm p} = 5.6 \times 10^6$ Hz, the gyrofrequency $w'_{\rm b} = 9.6 \times 10^6$ Hz, the flow velocity is $0.95c$, and the frequency of resonance, for these values, corresponds to $w_{\rm r} = 18.7 \times 10^6$ Hz.

points, their neighborhoods, and when the refraction index becomes imaginary (because of important absorption processes).

We could also estimate the propagation effect due to Faraday rotation on these electromagnetic solutions. The theory predicts slow rotation of the plane of polarization along the path of the ray. We found the angle of rotation $\Delta\varphi$, including the relativistic motion of the plasma, to be expressed as

$$\Delta\varphi = \frac{1}{2}\int_0^L \frac{w'_{\rm b} w_{\rm p}^2 (1+v)}{c[w^2 - (1+v)^2 w'^2_{\rm b}]}\, ds. \tag{6c}$$

When $w'^2_{\rm b} \ll w^2$ (weak magnetic field), the last expression is reduced to a simple linear dependency of the bulk velocity and can be written as

$$\Delta\varphi = (1+v)\Delta\varphi',$$

where $\Delta\varphi'$ is the Faraday rotation angle in the comoving frame. For instance, assuming a density of 10^4 cm^{-3}, a length path of 10 pc, and a magnetic field strength of 0.05 μG, typical values for the quasars 3C273, 3C279, and probably for 3C345, the rotation angle in the comoving frame is about $\Delta\varphi' \sim 5.7$ rad at 8 GHz. Since, the Lorentz factor of the bulk motion of the jets in 3C273 is $\gamma \sim 10.8$ (Abraham and Romero 1999) and $\gamma \sim 13.6$ in 3C345 (Caproni 2002), we would hope to have an observed rotation angle about $\Delta\varphi \sim 2\Delta\varphi' = 11.37$ rad, in both sources. Therefore, this provides a clue to why we think the values of $\langle n_{\rm e} B\| \rangle$, obtained from the observations of RM (rotation measure), could be overestimated.

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The dispersion relations obtained are easily reduced to the classical regime. For small velocities $v^2/c^2 \ll 1$, the above equation (6a) recovers the classical ordinary and extraordinary dispersion forms corresponding to a non-moving plasma (Ginzburg 1960):

$$n_1^2 = 1 - \frac{w_\mathrm{p}^2}{w(w + w_\mathrm{b})}, \quad n_2^2 = 1 - \frac{w_\mathrm{p}^2}{w(w - w_\mathrm{b})}.$$

The recently discovered x-ray transient V4641 Sgr (SAXJ1819.3-2525) (see in't Zand et al. 1999, 2000) is a galactic object of special interest because it has characteristics that resemble those of the extragalactic blazars. It has a highly relativistic jet (with $v \sim 0.95c$) and is nearly aligned with the line of sight. For this reason it has been identified as a galactic *microblazar* as was suggested by Orosz et al. (2001). Since V4641 Sgr is oriented towards the observer, we have applied to this source the dispersion relations obtained for a relativistic plasma with the electromagnetic waves propagating parallel to the jet velocity (see (6a) and (6b)). According to those solutions, two electromagnetic modes with circular polarization arise. A magnetic field for the radio regions of the jet $B \sim 0.54$ G has been assumed, implying a resonance frequency $w_\mathrm{r} = 18.7 \times 10^6$ Hz which is much greater than the gyrofreqency $w'_\mathrm{b} \approx 9.6 \times 10^6$ Hz (6b)(see Fig. 1). In general, for $\psi \simeq 0^\circ$ the influence of the bulk velocity upon the dispersion properties of the medium, is expressed mainly in the displacement of the line absorption from the gyrofrequency w'_b toward higher frequencies values (blue-shift), as the plasma motion is increased.

The other analytical solution obtained from the dispersion equation (5) corresponds to longitudinal waves which are described by

$$n_{3,4} = \frac{c}{v}\left[1 \pm \frac{w_\mathrm{p}}{w}\sqrt{\left(1 - \frac{v^2}{c^2}\right)}\right]. \tag{7}$$

The physics involved in these oscillations is very interesting and it is discussed elsewhere (see Lopez 2003, in preparation).

2.2. Radiation propagation normal to the outflow ($\psi = 90^\circ$)

In the case when the wave is propagating normal to the direction of the jet motion, i.e. perpendicular to the magnetic field ($\psi = 90^\circ$), the Doppler effect becomes quadratic and, once again, we are able to obtain analytical solutions from the general dispersion equation (5). We find two electromagnetic waves linearly polarized: one of them corresponds to an ordinary wave (n_1^2) and the other one to an extraordinary wave (n_2^2). As expected, in this normal direction we do not find any longitudinal oscillation. The transverse electromagnetic modes are characterized by the following dispersion relations:

$$n_{1,2}^2 = \left(1 - \frac{w_\mathrm{p}^2}{w^2}\right) \pm \left\{ \frac{w_\mathrm{b}}{w}\frac{w_\mathrm{p}^2}{w^2}\left[\sqrt{\frac{w_\mathrm{b}^2}{w^2} + 4\frac{v^2}{c^2}\left(1 - \frac{w_\mathrm{b}^2}{w^2} - \frac{w_\mathrm{p}^2}{w^2}\right)} \mp \frac{w_\mathrm{b}^2}{w^2}\frac{w_\mathrm{p}^2}{w^2}\right] \right.$$
$$\left. \times \left[2\left[\left(1 - \frac{w_\mathrm{b}^2}{w^2} - \frac{w_\mathrm{p}^2}{w^2}\right) + \frac{v^2}{c^2}\left(\frac{w_\mathrm{b}^2}{w^2} + \frac{w_\mathrm{p}^2}{w^2}\right)\right]\right]^{-1} \right\}. \tag{8a}$$

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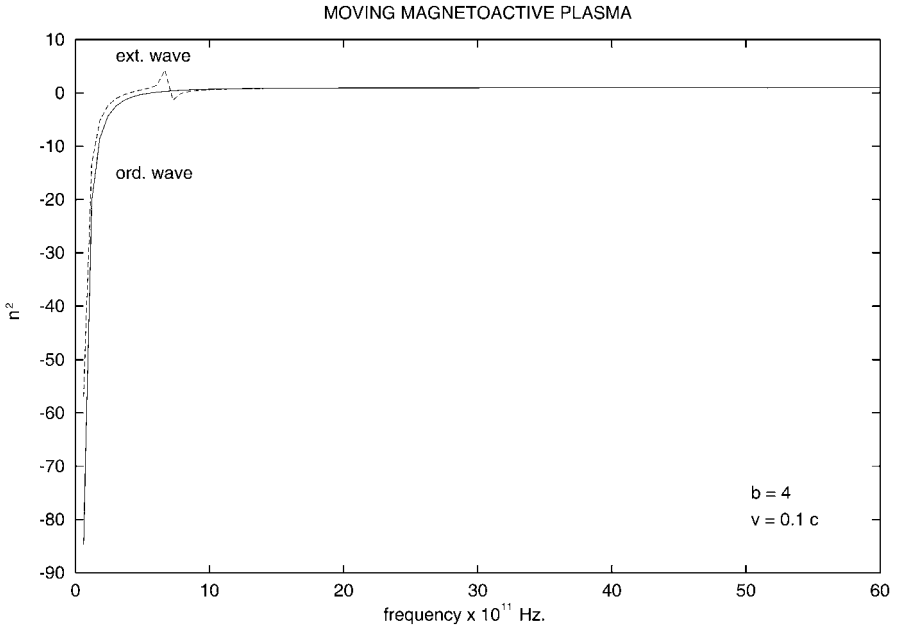


Figure 2. Dispersion curve for radiation propagating normal to the jet velocity ($\psi = 90°$): ordinary (full lines) and extraordinary (dashed lines) transverse waves. $w_p = 5.64 \times 10^{11}$ Hz, $w_b = 4 \times 10^{11}$ Hz, and flow velocity $v = 0.1c$. For these physical parameters the frequency of resonance is placed at $w_r = 6.88 \times 10^{11}$ Hz, as is shown in the figure.

The corresponding frequencies of resonance are given by

$$w_r = 0 \quad \text{and} \quad w_r^2 = a(v)^2 w_b'^2 + a(v) w_p^2, \tag{8b}$$

with $a(v) = (1 - v^2/c^2)$. Here, w_p and w_b' are the plasma and gyrofrequency in the comoving frame, respectively.

The solution given by (8a) represents the transverse electromagnetic waves coming from the relativistic jets to the observer, who is placed in the laboratory frame in the direction perpendicular to the outflow. The local plasma is mainly anisotropic with two waves linearly polarized in mutually perpendicular directions. For mildly relativistic velocities the curves are very similar to those obtained in the non-moving plasma formulation (see Fig. 2). One resonance point appears near to $w_r^2 \sim w_b^2 + w_p^2$ and it is displaced, as the plasma velocity is increased, to regions of low frequencies according to the right-hand equation of (8b). In the limit $v \to c$, w_r tends to zero.

Figure 3 corresponds to the relativistic jets of the microquasar GROJ1655-40, which are oriented nearly perpendicular to the line of sight and are moving with a highly relativistic velocity $\sim 0.92c$ (Orosz and Bailyn 1997).

We note, as in previous works (e.g. Lopez 1996a; 1997), that the relativistic effects become important for highly relativistic plasma velocities. The bulk velocity affects the spectral properties of the medium and introduces substantial modification in the usual dispersion relation of the electromagnetic waves. Furthermore, the higher the flow velocity, the greater the separation between the electromagnetic branches (see Figs 2–4). This means that the anisotropic character of the dispersion medium becomes stronger (the spectral properties of the radiation depend upon the direction of the wave propagation). For higher frequencies ($w \gg w_p$) the curves

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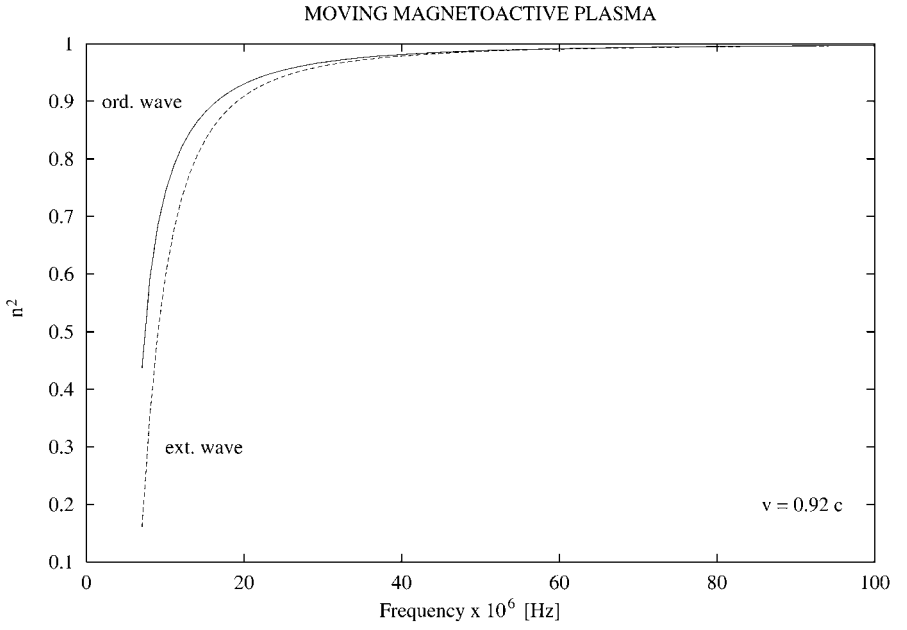


Figure 3. Dispersion curve for radiation in the jets of the superluminal source GROJ1655-40 ($\psi \sim 90°$): ordinary (full line) and extraordinary (dashed line) waves. $w_{\rm p} = 5.6 \times 10^6$ Hz, $w'_{\rm b} = 7.65 \times 10^6$ Hz, and flow velocity $v = 0.92c$. In this case, the frequency of resonance is moved to the position $w_{\rm r} = 2.5 \times 10^6$ Hz, which falls below the plasma frequency into the forbidden region ($w_{\rm c} = 6.14 \times 10^6$ Hz, see (8c)).

tend to the limit where the refraction index $n \to 1$, the separation between the electromagnetic modes disappears, and the local plasma can be considered isotropic (see Fig. 3). Moreover, if one increases the magnetic field the dispersion curves remain separated at higher frequencies (increase of anisotropy), and the minimum values of the refraction index are those shown in Fig. 4, for example.

The dispersion curves given by (8a) have a range of frequencies where the refraction index takes imaginary values, this is the so-called forbidden region where no radiation is expected to propagate. This region corresponds to frequencies $0 \leqslant w \leqslant w_{\rm c}$, where the cut-off frequency $w_{\rm c}$ defines the width of the region and it is introduced only for highly relativistic jets with velocities $v \geqslant 0.5c$. The cut-off frequency is given by the following expression:

$$w_{\rm c} = \sqrt{w_{\rm p}^2 + \left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{1}{4}\frac{c^2}{v^2}\right) w_{\rm b}'^2}, \tag{8c}$$

that shows the $w_{\rm c}$ dependence on the physical parameters of the plasma and the bulk motion.

Once again, the above solutions are reduced to the well-known relations for the perpendicular direction in a slowly moving plasma $v^2/c^2 \ll 1$, in agreement with the non-moving solutions presented by Ginzburg (1960):

$$n_1^2 = 1 - \frac{w_{\rm p}^2}{w^2}, \quad n_2^2 = 1 - \left[\frac{w_{\rm p}^2}{w^2}\left(1 - \frac{w_{\rm p}^2}{w^2}\right)\right]\left[1 - \frac{w_{\rm b}^2}{w^2} - \frac{w_{\rm p}^2}{w^2}\right]^{-1}.$$

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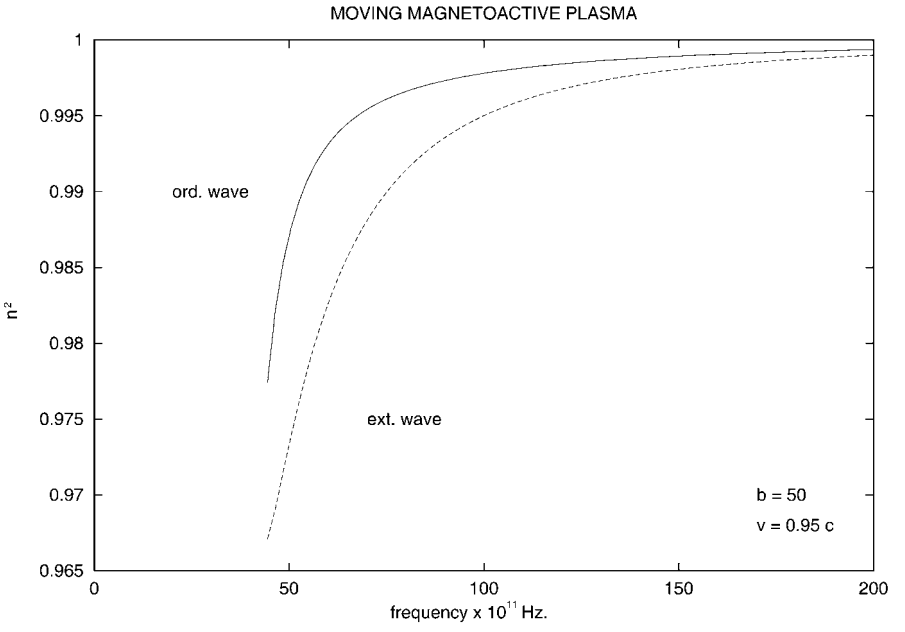


Figure 4. Dispersion curve for radiation propagating perpendicular to the jet velocity ($\psi = 90°$): ordinary and extraordinary transverse waves. $w_p = 5.64 \times 10^{11}$ Hz, $w'_b = 1.6 \times 10^{13}$ Hz, and flow velocity $v = 0.95c$. Then, for these values w_r is located at 15.7×10^{11} Hz, inside the forbidden region ($w_c = 42.9 \times 10^{11}$ Hz, see (8c)).

In the most general case, for an arbitrary angle of dispersion, the equations involved do not admit exact solutions. Numerical methods have been invoked to solve the tensorial relations along with the particular choice of geometry and physical conditions appropriate to a given source.

The next section is devoted to the presentation of the main results that we have derived from the classical treatment of an anisotropic outflow moving with small velocities.

3. Classical approach of a magnetoactive moving plasma

The complicated expression (1) for the dielectric tensor of permittivity presented in the previous section acquires a simplified aspect in the classical approach. This means that, for small velocities of the moving plasma ($v^2/c^2 \ll 1$), the tensorial relation for the dielectric tensor of permittivity ϵ_{ij} in the laboratory frame may be written as

$$\epsilon_{ij}(w, \mathbf{k}) = \delta_{ij}\left(1 - \frac{w'^2}{w^2}\right) + \frac{w'^2}{w^2}\epsilon'_{ij} + \frac{w'^2}{w^2}\frac{v_i k_\mu \epsilon'_{\mu j}}{w'} - \frac{w'^2}{w^2}\frac{v_i k_j}{w'} + \frac{w'}{w}\frac{k_\nu v_j \epsilon'_{i\nu}}{w}$$

$$- \frac{w'}{w}\frac{k_i v_j}{w} + \frac{w'}{w}\frac{v_i v_j}{w' w}(k_\mu \epsilon'_{\mu\nu} k_\nu - k^2). \tag{9}$$

The veracity of this expression may be verified by following in a straightforward way the basic electrodynamics equations and relativistic transformations, as is usually done.

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The dielectric tensor for a magnetoactive plasma in the comoving frame $\epsilon'_{ij}(w', \mathbf{k}')$ is given, as before, by (3). Substituting this tensor into relation (9) the most general expression for the dielectric tensor of permittivity $\epsilon_{ij}(w, \mathbf{k})$ can be found. The components of this tensor for a moving medium are:

$$\epsilon_{xx} = 1 + \frac{w'^2}{w^2}(\epsilon_\perp - 1) + 2\frac{w'^2}{w^2}\frac{v_x k_x}{w'}(\epsilon_\perp - 1) + \frac{w'^2}{w^2}\frac{v_x^2}{w'^2}(\epsilon_\perp k_\perp^2 + \epsilon_\parallel k_z^2 - k^2),$$

$$\epsilon_{yy} = 1 + \frac{w'^2}{w^2}(\epsilon_\perp - 1) + 2\frac{w'^2}{w^2}\frac{v_y k_y}{w'}(\epsilon_\perp - 1) + \frac{w'^2}{w^2}\frac{v_y^2}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],$$

$$\epsilon_{zz} = 1 + \frac{w'^2}{w^2}(\epsilon_\parallel - 1) + 2\frac{w'^2}{w^2}\frac{v_z k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'^2}{w^2}\frac{v_z^2}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],$$

$$\epsilon_{xy} = \frac{w'^2}{w^2} ig + \frac{w'^2}{w^2}\frac{ig}{w'}(v_x k_x + v_y k_y) + \frac{w'^2}{w^2}\frac{(\epsilon_\perp - 1)}{w'}(v_x k_y + v_y k_x)$$
$$+ \frac{w'^2}{w^2}\frac{v_x v_y}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],$$

$$\epsilon_{yx} = -\frac{w'^2}{w^2} ig - \frac{w'^2}{w^2}\frac{ig}{w'}(v_x k_x + v_y k_y) + \frac{w'^2}{w^2}\frac{(\epsilon_\perp - 1)}{w'}(v_x k_y + v_y k_x)$$
$$+ \frac{w'^2}{w^2}\frac{v_x v_y}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],$$

$$\epsilon_{xz} = \frac{w'^2}{w^2}\frac{v_x k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_x v_z}{w}(\epsilon_\perp - 1) + ig\frac{w'}{w}\frac{v_z k_y}{w}$$
$$+ \frac{w'^2}{w^2}\frac{v_x v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],$$

$$\epsilon_{zx} = \frac{w'^2}{w^2}\frac{v_x k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_x v_z}{w}(\epsilon_\perp - 1) - ig\frac{w'}{w}\frac{v_z k_y}{w}$$
$$+ \frac{w'^2}{w^2}\frac{v_x v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],$$

$$\epsilon_{yz} = \frac{w'^2}{w^2}\frac{v_y k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_y v_z}{w}(\epsilon_\perp - 1) - ig\frac{w'}{w}\frac{v_z k_x}{w}$$
$$+ \frac{w'^2}{w^2}\frac{v_y v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],$$

$$\epsilon_{zy} = \frac{w'^2}{w^2}\frac{v_y k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_y v_z}{w}(\epsilon_\perp - 1) + ig\frac{w'}{w}\frac{v_z k_x}{w}$$
$$+ \frac{w'^2}{w^2}\frac{v_y v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2], \tag{10}$$

where $\epsilon_\parallel$, $\epsilon_\perp$, g, and the other parameters maintain the same definitions as in the previous expression (3). The above relations indicate that the anisotropies of the system become more complex than those in the rest frame as a result of the plasma motion. Once again, the dielectric tensor of permittivity in the laboratory frame is a hermitic matrix ($\epsilon = \epsilon^{\mathrm{H}}$).

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Applying condition (5), which defines the electromagnetic oscillations which are able to propagate in the medium, to the components of the permittivity tensor (10), we derive the expressions that characterize the dispersion properties of a slowly moving magnetoactive plasma.

In what follows, we restrict the analysis to two geometric configurations that are frequently involved in astrophysical problems. One corresponds to the longitudinal displacement of the plasma along the magnetic field and the other corresponds to outflows that are moving normal to the external magnetic field. Both situations are of real interest in astrophysics.

3.1. Outflow moving along the magnetic field

With the assumption that the plasma is moving along the magnetic field, which is placed along the z-axis, the flow velocity vector becomes more simple and can be written as $\mathbf{v} = v_z\mathbf{i}_3$ ($\mathbf{i}_3$ is the unitary vector along the z direction). Additionally, we have assumed that the propagation of the electromagnetic wave occurs in the xz plane and the wave forms an angle ψ with the magnetic field (the dispersion angle). Then, the dispersion equations derived from condition (5) can be treated more easily and exact solutions are found for some particular directions of the wave propagation.

The first case, when the solutions are derived analytically, corresponds to the dispersion angle $\psi = 0°$, i.e. when the wave is propagating along the outflow. Therefore, the curves for the refraction index $\mathbf{n} = (c/w)\mathbf{k}$ are defined by the following expressions:

$$\begin{aligned} n_1^2 &= \left(1 - \frac{w_\mathrm{p}^2}{w^2}\right), \quad n_2^2 = \left(1 - \frac{w_\mathrm{p}^2}{w^2}\right), \\ n_{3,4} &= \frac{c}{v}\left(1 \pm \frac{w_\mathrm{b}}{w}\right), \quad n_{5,6} = \frac{c}{v}\left(1 \pm \frac{w_\mathrm{p}}{w}\right), \end{aligned} \tag{11}$$

where n_1^2 and n_2^2 represent the ordinary waves of a magnetoactive moving plasma propagating along the outflow (these are degenerate), whereas $n_{3,4}$ and $n_{5,6}$ represent the solution obtained for the longitudinal oscillations. Here, we identify two separated cut-offs at w_p and w_b, whose values are modulated by the flow velocity v in agreement with the above dispersion relations. The moving plasma along this direction behaves as an isotropic plasma and the medium does not affect the wave dispersion.

The second case, where the solutions are handled analytically, corresponds to the wave propagation in the direction perpendicular to the outflow ($\psi = 90°$). Then, the dispersion curves can be written as

$$n_1^2 = \left(1 - \frac{w_\mathrm{p}^2}{w^2}\right), \quad n_2^2 = \left[1 + \frac{w_\mathrm{p}^4}{w^4} - \frac{w_\mathrm{b}^2}{w^2} - 2\frac{w_\mathrm{p}^2}{w^2}\right]\left[1 - \frac{w_\mathrm{p}^2}{w^2}\left(1 - \frac{v^2}{c^2}\right) - \frac{w_\mathrm{b}^2}{w^2}\right]^{-1}. \tag{12}$$

The ordinary dispersion branch is obtained together with an expression for the extraordinary wave. We do not see longitudinal oscillations. Applying the condition of smallness of the velocity ($v^2/c^2 \ll 1$), the extraordinary electromagnetic mode obtained for a moving plasma and described by the right-hand equation of (12) is

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reduced to the usual dispersion relation for a static anisotropic plasma (Ginzburg 1960):

$$n_2^2 = 1 - \frac{w_{\rm p}^2(1 - w_{\rm p}^2/w^2)}{(w^2 - w_{\rm b}^2 - w_{\rm p}^2)}. \tag{13}$$

Consequently, we note that for the case when the plasma is flowing along the magnetic field lines with small velocities (the classical approach), the dispersion properties of the magnetoactive plasma are not affected. The anisotropy introduced due to the movement of the medium into the tensor ϵ_{ij} is weak and cannot produce an additional separation between the electromagnetic modes (the modes are separated due to the action of the magnetic field only).

3.2. Outflow moving transverse to the magnetic field

Another interesting example is related to the so-called transverse plasma displacement, i.e. when $\mathbf{v} = v_x \mathbf{i}_1$ ($\mathbf{i}_1$ is the unitary vector along the x-axis). In this case, the medium is moving in a transverse way perpendicular to the magnetic field direction. This situation happens less often in astrophysical outflows, but sometimes is invoked to understand, for instance, the physics of material flows in connection with surface and solar atmospheric structures.

We employ a similar procedure as in the previous subsection to obtain the following results.

(a) If the radiation is propagating along the magnetic field ($\psi = 0°$), the waves that are able to propagate in the moving magnetoactive plasma are described by the following expressions:

$$n_1^2 = 1 - \frac{w_{\rm p}^2}{w(w - w_{\rm b})}, \quad n_2^2 = 1 - \frac{w_{\rm p}^2}{w(w + w_{\rm b})}. \tag{14}$$

These solutions have the same form as the solutions for electromagnetic waves in a magnetoactive non-moving plasma (Ginzburg 1960). Therefore, we see the same dispersion shapes in the laboratory frame as we see in the co-moving frame (covariance).

(b) On the other hand, if the wave is propagating along the outflow, i.e. in the direction perpendicular to the magnetic field ($\psi = 90°$), our calculations yield

$$n_1^2 = \left(1 - \frac{w_{\rm p}^2}{w^2}\right), \quad n_2^2 = 1 - \frac{w_{\rm p}^2(1 - w_{\rm p}^2/w^2)}{(w^2 - w_{\rm b}^2 - w_{\rm p}^2)}. \tag{15}$$

Once again, we have obtained the same shapes for the ordinary and non-ordinary waves as they have in a static medium (Ginzburg 1960). It is important to comment that the flow velocity does not appear explicitly in the dispersion relations, however, this dependence is contained in the classical relation used to perform the frequency transformation from one inertial system to other (Doppler shift). The results obtained show that in the classical approximation the dispersion curves for the transversal electromagnetic waves are covariant.

An additional solution appears in this direction corresponding to the longitudinal perturbations and it is written as

$$n_3 = \frac{c}{2v}\left(1 - \frac{w_{\rm b}^2 + w_{\rm p}^2}{w^2}\right). \tag{16}$$

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The bulk motion is responsible for this perturbation, which contains a cut-off at $w \sim (w_p^2 + w_b^2)^{1/2}$. This branch has refraction indices $n \gg 1$ and is able to excite pressure waves and turbulence in the plasma. The longitudinal modes are discussed elsewhere (see Lopez 2003, in preparation).

For other wave orientations we verified a similar behavior. The shapes of the dispersion curves remain unaltered, i.e. the dispersion relations for magnetoactive plasma have the same form in any inertial frame of reference within this classical treatment.

4. Conclusions

(a) The expressions for the dielectric tensor of permittivity for a magnetoactive moving plasma have been obtained in their classical and relativistic approaches. The corresponding dispersion relations have been established and solved for the most typical geometric configurations found in astrophysical problems.

(b) If the outflow is moving with small velocities, the general motion of the plasma does not affect the propagation of transverse electromagnetic waves. The waves are refracted according to the ordinary refraction law deduced for non-moving magnetoactive plasma. The global movement does not change the dispersion relations and the shapes of the corresponding dispersion curves remain unaltered (relativistic covariant). In this case, the flow velocity only induces the existence of longitudinal perturbations.

(c) Under a relativistic formulation we have derived the equations which describe the propagation of electromagnetic waves in a moving magnetoactive plasma. The flow velocity substantially affects the plasma dispersion properties and it is responsible for new dispersion transverse branches and longitudinal perturbations.

(d) The dispersion equations only have analytic solutions for some particular dispersion angles. One of these is when the radiation is propagating along the jet direction. In this case, the extraordinary dispersion curve presents a point of resonance which is dependent on the flow velocity (blue-shifted absorption features). This is an interesting result that might be used to explain the observed radiation in GRBs (gamma-ray bursts) invoking magnetic field strengths that are much smaller than those presently required by the existing models in the literature (Lopez 2003, in preparation).

(e) Another exact solution is obtained for the transverse propagation of the electromagnetic waves with respect to the jet velocity. In this case, the electromagnetic modes are not degenerate (two transverse branches appear) and their anisotropic character depends strongly on the bulk plasma velocity. The wave propagation is not possible inside of a forbidden region which is defined for highly relativistic jet velocities $v \geqslant 0.5c$ (see (8c)), and it is also dependent on the physical parameters and bulk velocity of the plasma.

(f) We have concluded that the observed radiation in highly relativistic jets is strongly dependent on the amount of relativistic beaming, as well as on the intrinsic properties of the jet. The influence on the measured radiation of the angle formed by the jet axis with the line of sight is clear.

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(g) The observations of gamma-ray bursts, blazars, relativistic jets in AGNs, quasars, and microquasars show that the physical nature of the observed radiation is not often completely understood. The models that are usually used do not explain some special features or anomalous behavior observed in the lines and continuum radiation. The results presented in this work can be applied in order to incorporate suitable corrections to the models, and to understand the physical nature of the observed radiation associated with astrophysical relativistic outflows.

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