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Plasma sheath evolution from perturbed electrodes in a negative-ion plasma

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The two-dimensional spatial and temporal evolution of the components of a plasma surrounding an electrode whose potential is suddenly decreased is investigated numerically. The electrode contains a localized convex or a localized concave voltage perturbation. The quasineutral plasma consists of positive ions and various proportions of negative ions and electrons. The results are compared and contrasted with those obtained with a uniform electrode under identical plasma conditions.

1. Introduction

The temporal and spatial plasma evolution from a flat metal object that is inserted in an unmagnetized homogeneous two- or three-component quasineutral plasma and to which a negative step voltage is applied is of much interest at present. Negative ions form the third component in the latter plasma, and we call such a plasma a *negative-ion plasma*. In particular, it has been observed that electrons would initially be expelled into the uniform ambient plasma before either species of the more massive ions have a chance to move. This creates a localized region of net positive charge adjacent to the electrode. As time increases, the positive ions adjacent to the electrode would be absorbed by it, causing the front of a rarefactive positive ion sheath to expand into the plasma with a velocity that is initially greater than the ion acoustic velocity in a two-component plasma and the fast-mode ion acoustic velocity in a three-component negative-ion plasma. The expanding sheath slows down to the respective velocities in these plasmas. Negative ions will be repelled by the negative potential applied to the electrode, and these ions will also expand into the plasma.

This local region of charge non-neutrality adjacent to the pulsed electrode has

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been given the name *transient sheath* in the ion acoustic wave community. The existence of such a sheath was used to initially interpret the actual location of the ion acoustic wave excitation in laboratory experiments (Alexeff *et al.* 1969). The derivation of the sheath dimension L follows from a solution of Poisson's equation in one dimension with the following boundary conditions in planar geometry: the voltage $V = -V_{ex}$ at $x = 0$ and the voltage V and the electric field $E = -dV/dx$ are both zero at $x = L$. The sheath width L for a negative-ion plasma is given by

$$L = \left(\frac{2e\, V_{\rm ex}/M_+}{n_e e^2/M_+\, \epsilon_0} \right)^{1/2} = \left[\frac{2e\, V_{\rm ex}/M_+}{n_0(1-\epsilon)e^2/M_+\, \epsilon_0} \right] = \frac{v_+}{(1-\epsilon)^{1/2}\omega_{p^+}}, \tag{1}$$

where the parameter ϵ = negative-ion density n_0-/positive-ion density n_0. In (1), v_+ and ω_{p^+} are the ion velocity caused by the potential that is applied to the electrode and the ion plasma frequency respectively. The electron density is n_e, with $n_e = (1-\epsilon)n_0$ in a quasineutral negative-ion plasma.

The positive ions are attracted to and are eventually implanted in the electrode to which the negative voltage is applied. These positive ions can change the surface properties of the metal electrode, and this effect has significant practical applications. The electrode shape can also be very complicated in practice. In this community whose interests are directed toward ion implantation, the process is called plasma source ion implantation (PSII) and the sheath is called an *ion matrix sheath* (Conrad 1987).

The temporal and spatial evolution of the various plasma species has been noted in one-dimensional plasma simulations of a two-component plasma (Widner *et al.* 1970) and a three-component plasma (Amin *et al.* 1993). In these simulations, the coordinate of particle motion was chosen to be perpendicular to the flat surface of the electrode. Fluid equations were used to describe the plasma. The results have been observed in laboratory experiments for the two-component plasma (Widner *et al.* 1970; Cooney *et al.* 1991).

There are cases, however, where the surface of the electrode is not flat. An example is a corner or a localized perturbation in an electrode. In these cases, a higher-dimensional numerical code must be developed in order to simulate the plasma response. Two-dimensional codes describing a two-component plasma have recently been written that would describe the plasma behavior surrounding a 90° convex or a 90° concave corner (Hong and Emmert 1994, 1995; Sheridan and Alport 1994*a*; Sheridan 1995). We have recently extended a two-dimensional code to include effects of negative ions in a three-component plasma with the 90° convex and concave corners. In addition, preliminary results for an electrode perturbation were obtained. A summary of the results that focus on the effects that the negative ions have on the sheath expansion and an extensive list of references is reported in El-Zein *et al.* (1996). Using a particle-in-cell (PIC) code, it was shown that the plasma evolution in a two-component plasma near an electrode containing a semi-infinite unbiased region juxtaposed with a semi-infinite negatively biased region would lead to self-similar plasma expansion (Sheridan 1996).

In the present investigation, we focus our attention on the effects that a localized perturbation in a flat electrode will have on the plasma sheath expansion. This paper reports the results of this study. The set of fluid equations comprising the

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model is described in Sec. 2. Section 3 summarizes the results that were obtained from the simulation of a two- and three-component plasma surrounding electrodes that contain either a convex or a concave localized perturbation. The results are compared with those obtained from the uniform electrode that exists on either side of the perturbation. We also summarize the temporal evolution of the positive-ion flux that impinges on these electrodes. Concluding comments are given in Sec. 4.

2. Model

The fraction of negative ions in the plasma is a variable that is adjustable in the simulation. The parameter e is defined as the uniform negative ion concentration, $\epsilon \equiv n_0\text{-}/n_0$, and all perturbed densities are normalized with respect to the uniform positive-ion density n_0. We shall choose the value of $\epsilon = 0.75$ in order to demonstrate effects that are attributable to the presence of negative ions. Negligible qualitative differences from those presented here were noted using different values for the parameter ϵ.

The set of normalized equations we use to model a three-component plasma with a quasineutral density of n_o positive ions with charge $+e$, negative ions with charge $-e$, and $(1-\epsilon)n_0$ electrons with charge $-e$ are as follows:

the 2D nonlinear equations of continuity and motion for the cold positive ions

$$\frac{\partial n^+}{\partial t} + \nabla \cdot (n^+ \mathbf{u}^+) = 0, \tag{2}$$

$$\frac{\partial \mathbf{u}^+}{\partial t} + \mathbf{u}^+ \cdot \nabla \mathbf{u}^+ = -\nabla\phi; \tag{3}$$

the 2D nonlinear equations of continuity and motion for the cold negative ions

$$\frac{\partial n^-}{\partial t} + \nabla \cdot (n^- \mathbf{u}^-) = 0, \tag{4}$$

$$\frac{\partial \mathbf{u}^-}{\partial t} + \mathbf{u}^- \cdot \nabla \mathbf{u}^- = +\frac{1}{M}\nabla\phi; \tag{5}$$

a Boltzmann assumption for the electron distribution

$$n_e = n_{eo} e\phi = (1-\epsilon)e^{\phi}; \tag{6}$$

Poisson's equation to determine the electric potential ϕ in terms of the densities of the three species

$$\nabla^2\phi = (1-\epsilon)e^{\phi} + n^- - n^+. \tag{7}$$

The laboratory variables (capital letters) are related to the normalized variables in this set of equations by

$$x = \frac{X}{\lambda_D}, \quad y = \frac{Y}{\lambda_D}, \quad t = \omega_{p+}T, \quad \phi = \frac{eV}{\kappa_B T_e},$$

$$\lambda_D = \left(\frac{\epsilon_0 \kappa_B T_e}{n_{0+} e^2}\right)^{1/2}, \quad u = \frac{U}{c_s} = \frac{U}{(\kappa_B T_e / M^+)^{1/2}}.$$

The parameter $M = M^-/M^+$ is the ion mass ratio. The electron mass is much smaller than either ion mass, and will be neglected here. This choice of normalization parameters that we use is based on our interest in wave–plasma studies.

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The normalization of distance in PSII applications is given with respect to the size of the ion matrix sheath dimension $L = (2e\epsilon_0 V_{\rm ex}/n_{0+}e^2)^{1/2}$. Hence the results presented here can be easily converted to that application by using a factor of $\lambda_D/L = (\kappa_B T_e/2e\epsilon_0 V_{\rm ex})^{1/2}$. For the results to be reported here, this factor is $\frac{1}{4}\sqrt{\frac{1}{3}}$. The simulations are performed in the region defined by $y > 0$. The electrodes are assumed to be uniform in one of the dimensions, which will be taken to be along the z axis. This implies that in this simulation, $\partial/\partial z \equiv 0$.

The initial plasma surrounding the electrode was assumed to be uniform in space and electrically charge-neutral. The initial velocity of either species of ion and the potential were set equal to zero at all positions. Standard finite-difference techniques were employed in order to solve the model (Sheridan and Alport 1994*a*). The model equations were solved with a finite-difference method using a grid spacing $\Delta X = \Delta Y = \lambda_D$ and a time step of $\Delta t = \frac{1}{64}$. Other parameters had the following numerical values: the ion mass ratio $M = 4$ and the size of the simulation space was 65×65. All plasma species remained within this region during the simulation. A negative step potential with zero rise time was applied at time $t = 0$ to the electrode that had been inserted in the plasma. The code that we use in this study is a modification of a two-dimensional code developed earlier (El-Zein *et al.* 1996).

The normalized value of the excitation voltage signal applied to the unperturbed electrode was

$$\phi_{\rm ex} = -24. \tag{8}$$

With this value for the excitation voltage, the expanding sheath edge was run to such a time that it would remain within the region of the plasma simulation. Perturbations in the electrode were simulated by choosing an inhomogeneous but symmetrical distribution of the excitation potential. For the work shown here, we chose a linear variation of the potential for the value that was applied to the unperturbed electrode (8). In addition, we assumed that the potential changed in Debye length increments. We used

$$\phi_{\rm ex} = -\phi_0 + \zeta\,\Delta x \tag{9}$$

for the concave perturbation and

$$\phi_{\rm ex} = -\phi_0 - \zeta\,\Delta x \tag{10}$$

for the convex perturbation. We used a linear symmetrical variation about a central value at $x = 32$. In (9) and (10), $\Delta x = 1$ and $\zeta = [0, 3, 6, \ldots, 24]$. The calculation was repeated with negligible differences in the results for a flatter perturbation using the sequence $\zeta = [0, 1, 2, \ldots, 8]$. In one simulation to be described below, we used a larger voltage $\phi = 96$. In this case, the sequence $\zeta = [0, 12, 24, \ldots, 96]$. This ensured that $\phi_{\rm ex}$ at $x = 32$ would equal zero.

3. Results

As an initial indication of the effects that a perturbation in the electrode surface will have on the creation and the subsequent evolution of the sheath surrounding an electrode, a summary of the electron density profiles computed at the time $t = \Delta t = \frac{1}{64}$ is displayed in Fig. 1. The equidensity contours are taken at an electron density $\frac{1}{2}n_e$ for both the two- and three-component plasmas. With our choice of

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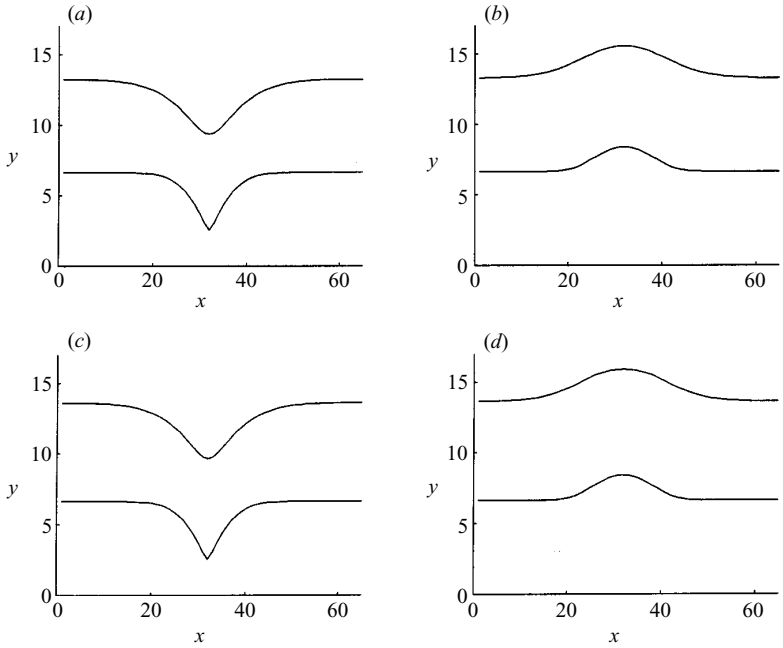


Figure 1. Equidensity contours of electron density after the first iteration time after the application of the voltage step ($t = 1/64$). The contours correspond to the half-maximum electron density profiles. The bottom trace in each of the four figures corresponds to the case of $\phi = 24$ and $\epsilon = 0$. (*a*) Concave perturbation, $\phi_0 = 24$, and $\epsilon = 0$ and 0.75. (*b*) Convex perturbation, $\phi_0 = 24$ and $\epsilon = 0$ and 0.75. (*c*) Concave perturbation, $\phi_0 = 24$ and 96, and $\epsilon = 0$. (*d*) Convex perturbation, $\phi_0 = 24$ and 96, and $\epsilon = 0$.

$\epsilon = 0$ and $\epsilon = 0.75$, this corresponds to the 50% and 12.5% profiles respectively. During this first time step in the calculation, the electrons respond to the negative voltage step that is applied to the electrode at $t = 0$, and are expelled away from the electrode. However, at this early time in the simulation, neither species of ion has had a chance to move. This, of course, is the condition of applicability of the transient or ion matrix sheath interpretation.

The electron density contours are shown for the concave perturbation in Figs 1(a, c) and the convex perturbation in Figs 1 (b, d). The bottom trace in each of the four figures corresponds to the case of $\phi_0 = 24$ and $\epsilon = 0$, i.e. the normal two-component plasma transient sheath. The normalized value of this sheath L/λ_{D} as calculated from (1) is $4\sqrt{3}$. This is the value that is found from the simulation on either side of the perturbation. In the centre of the concave perturbation at $x = 32$, the applied potential is locally equal to zero, which is the same potential as the quasineutral plasma far from the electrode. However, in the simulation, the electrons adjacent to this narrow isthmus can transversely migrate and fill it, causing the potential to decrease at that point

From (1), we note that the size of the transient sheath depends upon two parameters that can be externally controlled. These are the concentration of negative ions ϵ and the amplitude of the applied voltage ϕ_0. We chose the values (I) $\epsilon = 0.75$, $\phi_0 = 24$ and (II) $\epsilon = 0$, $\phi_0 = 96$. The normalized value of the sheath L/λ_D in both cases should be $8\sqrt{3}$. The results of the simulation of Case (I) are shown in the top

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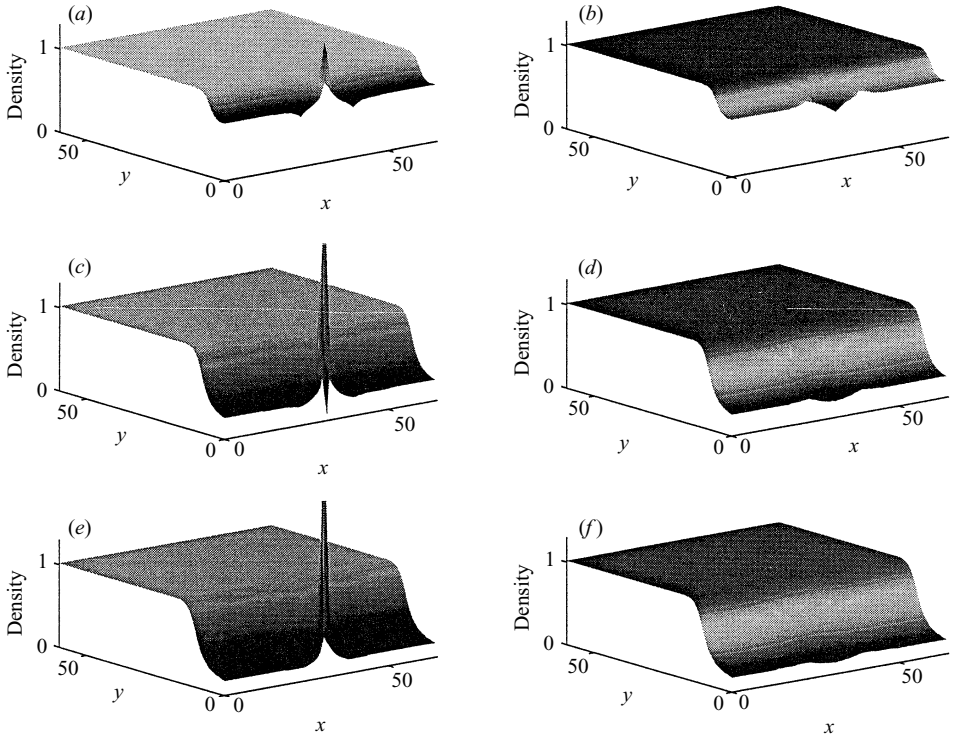


Figure 2. Evolution of the positive-ion density surrounding the two electrodes after the application of the voltage step $\phi_0 = 24$ in a two-component plasma ($\epsilon = 0$). (*a*) Concave perturbation, $t = 1$. (*b*) Convex perturbation, $t = 1$. (*c*) Concave perturbation, $t = 3$. (*d*) Convex perturbation, $t = 3$. (*e*) Concave perturbation, $t = 5$. (*f*) Convex perturbation, $t = 5$.

curves of Figs 1(*a*, *b*). The results of the simulation of Case (II) are shown in the top curves of Figs 1(*c*, *d*). The expected increase by a factor of two is evident on both sides of either perturbation. In addition, the depth and the sharpness of the perturbed region can be reduced with either the addition of negative ions to the plasma or an increase in the amplitude of the applied negative voltage step. We recall that the distribution of the local applied voltage was changed in the latter case.

The temporal and spatial evolution of the positive ions in a two-component plasma ($\epsilon = 0$) is illustrated in the panorama in Fig. 2, which shows the positive ion density at the three normalized times: $t = 1$, 3 and 5. The electrode with the concave perturbation appears in the sequence of three plots on the left-hand side. The electrode with the convex perturbation appears in the sequence of three plots on the right-hand side. In the region closest to the electrode on either side of either perturbation, the positive-ion density decreases in time. These ions are absorbed by the electrode. This causes a rarefactive positive-ion perturbation that expands into the surrounding plasma. It has been noted previously in one-dimensional simulations that the front of this expanding rarefactive perturbation has a velocity that is initially greater than the ion acoustic velocity. It eventually slows down to this value (Widner *et al.* 1970).

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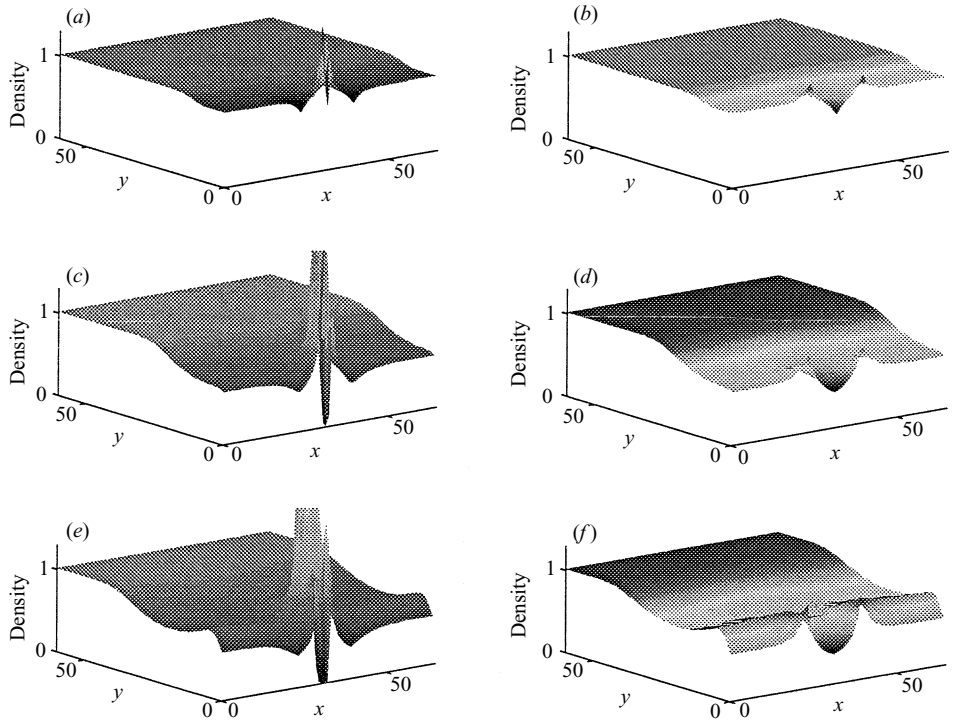


Figure 3. Evolution of the positive-ion density surrounding the two electrodes after the application of the voltage step $\phi_0 = 24$ in a three-component plasma ($\epsilon = 0.75$). (*a*) Concave perturbation, $t = 1$. (*b*) Convex perturbation, $t = 1$. (*c*) Concave perturbation, $t = 3$. (*d*) Convex perturbation, $t = 3$. (*e*) Concave perturbation, $t = 5$. (*f*) Convex perturbation, $t = 5$.

At the centre of the concave perturbation, positive-ion density accumulates. The peak values in Figs 2(*c*, *e*) have been 'capped' for display purposes. The normalized peak values of the positive-ion density in Figs 2(*a*, *c*), and (*e*) are 1.4, 2.8 and 3.0 respectively. This localized peak in the positive ion density has also been noted in the simulation of a 90° concave corner (El-Zein *et al.* 1996). The local electric field at this corner is decreased, since the equipotential contours are 'pushed away' from it. This was also noted in numerical and experimental studies of the transient sheath at 90° corners. (Donnelly and Waterson 1989; Lim et al. 1991; Sheridan and Alport 1994*b*).

The temporal and spatial evolution of the positive ions in a three component plasma ($\epsilon = 0.75$) is illustrated in the panorama in Fig. 3, which shows the positive-ion density at the same three normalized times that were previously used: $t = 1$, 3 and 5. The electrode with the concave perturbation appears in the sequence of three plots on the left-hand side. The electrode with the convex perturbation appears in the sequence of three plots on the right-hand side. At the centre of the concave perturbation, positive-ion density accumulates. The peak values in Figs 3(*c*, *e*) have been 'capped' for display purposes. The normalized peak values of the positive-ion density in Figs 3(*a*, *c*), and (*e*) are 1.7, 3.1 and 3.2 respectively.

A small increase in the positive-ion density on either side of the convex perturba-

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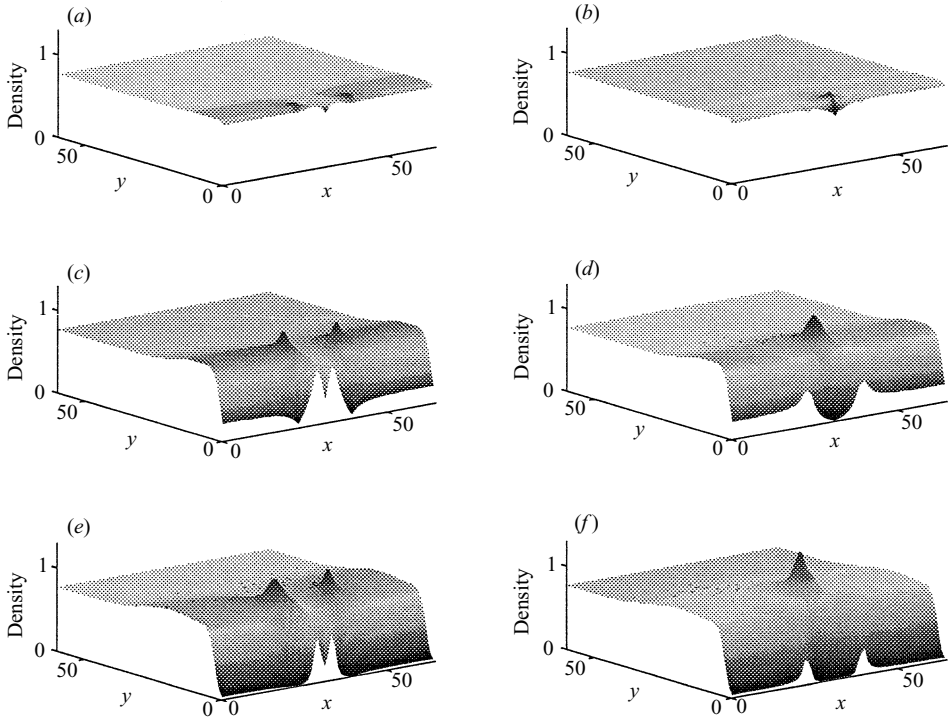


Figure 4. Evolution of the negative-ion density surrounding the two electrodes after the application of the voltage step $\phi_0 = 24$ in a three-component plasma ($\epsilon = 0.75$). (*a*) Concave perturbation, $t = 1$. (*b*) Convex perturbation, $t = 1$. (*c*) Concave perturbation, $t = 3$. (*d*) Convex perturbation, $t = 3$. (*e*) Concave perturbation, $t = 5$. (*f*) Convex perturbation, $t = 5$.

tion is noted. We interpret this small increase in density as being due to the effect of a small 'concave' corner joining the convex perturbation to the flat electrode. The depth of the rarefactive perturbation is decreased in the three-component plasma. This decrease is, however, not monatomic in time since it appears that the local positive-ion density increases close to the electrode.

In Fig. 4, the corresponding negative-ion density profiles are shown. The temporal response of these ions should be slower than that of the positive ions, since their mass is larger. The mass ratio M in the simulation was chosen to be 4. The application of the negative potential to the electrode will also 'expel' these ions into the background plasma. This expulsion process can be followed on the ion time scales that are used here. We might expect similar behaviour for the electrons in a simulation performed on the electron time scale. The negative ions that were initially adjacent to the electrode at $t = 0$ are slowly bunched together and accumulate in front of it. These negative ions will attract positive ions, locally raising their density. This local accumulation was noted in Fig. 3.

If we compare the results for the identical times between the results for the two- and three-component plasmas shown in Figs 2 and 3 along with additional results, we note that the front of the rarefactive sheath expands faster into the ambient plasma in the three-component plasma. The ratio of the fast mode ion acoustic velocity c_{s+} in a three-component plasma to the ion acoustic velocity c_{s+} in a two-

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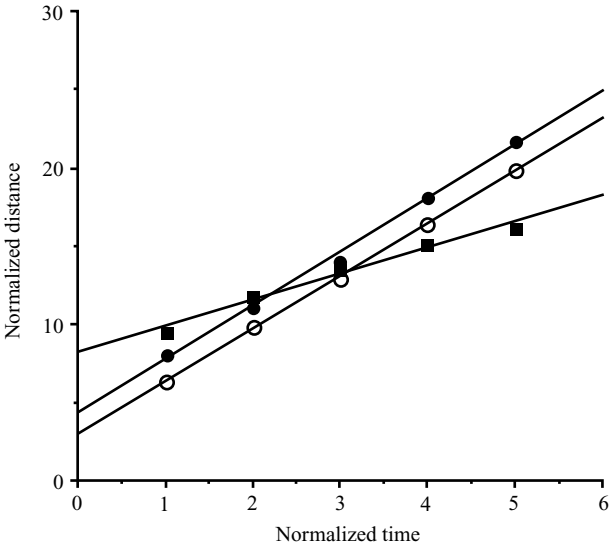


Figure 5. Trajectories of the front of the expanding rarefactive positive-ion density in a two-component plasma ($\epsilon = 0$) (■) and in a three component plasma ($\epsilon = 0.75$) (●), and trajectory of the front of the expanding compressive negative-ion density in a three-component plasma ($\epsilon = 0.75$) (○).

component plasma is given in laboratory variables by

$$\frac{c_{s+}}{c_s}\left(\frac{1+\epsilon/M}{1-\epsilon}\right)^{1/2}. \tag{11}$$

For the negative-ion concentration used here ($\epsilon = 0.75$), this ratio is 2.2. The trajectories of the front as measured at the amplitude of 2% are shown in Fig. 5. These trajectories were measured in front of the unperturbed region of an electrode. If we convert the normalizations that were employed for the ion data in Figs 2–4 to the *electron Debye length* $\lambda_{De} = \lambda_D/(1-\epsilon)^{1/2}$ then the ratio of velocities is 2.1.

We also compared the temporal evolution of the positive ion flux that would be absorbed be either electrode in the two plasmas. This is illustrated in Fig. 6. The flux in the two-component plasma is shown in Figs 6(a, b). The flux in the three-component plasma is shown in Figs 6(c, d). From these results, we conclude that in the region of the uniform electrode, the peak of the ion flux occurs earlier in time in the two-component plasma. This peak is also larger in the two-component plasma. However, the duration of higher positive-ion flux occurs over a longer time in the three-component plasma. A large positive-ion flux will be incident upon the center of the convex perturbation in both plasmas as shown in Figs 6(c, d). In addition, there is a slight increase at the edges of the concave perturbation in Fig. 6(c).

4. Conclusions

We have summarized the expected two-dimensional responses of two- and three-component plasmas to the application of a negative voltage step to an electrode that contains a concave or a convex perturbation in the applied voltage signal. The

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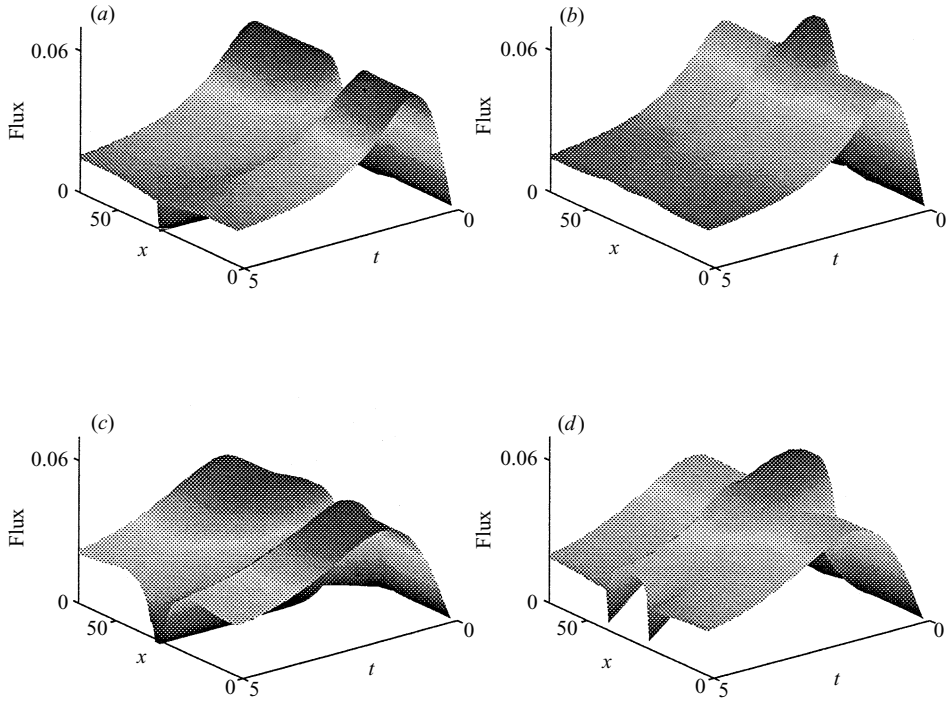


Figure 6. Temporal and spatial evolution of the positive-ion flux absorbed by the electrodes after the application of the voltage step $\phi_0 = 24$ in two- and three-component plasmas. (*a*) Concave perturbation, $\epsilon = 0$. (*b*) Convex perturbation, $\epsilon = 0$. (*c*) Concave perturbation, $\epsilon = 0.75$. (*d*) Convex perturbation, $\epsilon = 0.75$.

temporal and spatial evolution of the sheath and the positive-ion flux incident upon the electrode are affected by the presence of a perturbation.

The authors wish their friend, their colleague and their teacher David Montgomery a happy 60th birthday.

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