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Criteria for the evaluation of linear two-port stability using two geometrically derived parameters

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Two geometrically derived stability parameters, μ' (or μ) and ν'(or ν), are used to analyze the stability of a two-port network. The magnitudes of both μ' (or μ) and ν' (or ν) parameters determine geometrical relations between the stability circle and the unit Smith chart in the Γ_S (or Γ_L) plane. $|\mu'|$ (or $|\mu|$) is defined as the minimum distance from the stability circle to the origin of the unit Smith chart in the Γ_S (or Γ_L) plane, while $|\nu'|$(or $|\nu|$) is defined as the maximum distance. Moreover, the signs of ν' (or ν) and μ' (or μ) parameters determine which region of the stability circle is stable (inside or outside) and whether the stable region includes the origin of the Smith chart, respectively. There are totally 12 situations between the stability circle and the unit Smith chart in the Γ_S (or Γ_L) plane by using μ' (or μ) and ν' (or ν) parameters. This paper classifies 12 situations in the Γ_S plane and also presents the corresponding practical two-port networks.

Keywords: Stability criteria, S parameters, Linear circuit, Two-port network

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I. INTRODUCTION

Two-port network stability has been discussed in a great deal of literature [1–5]. Conventional stability parameters such as Rollet *K* and *B* parameters were used to discuss the unconditionally stable cases [4]. The *K* and *B* parameters are algebraic combinations of *S* parameters and can be expressed as follows:

$$K = \frac{E_1 - D_1}{2|S_{12}S_{21}|}, \tag{1}$$

$$B_1 = D_1 + E_1, \tag{2}$$

where D_1 and E_1 are defined as

$$D_1 = |S_{11}|^2 - |\Delta|^2, \tag{3}$$

$$E_1 = 1 - |S_{22}|^2, \tag{4}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}. \tag{5}$$

$K > 1$ and the auxiliary condition $B1 > 0$ were used to identify unconditional stability in the Γ_S plane. It can be shown that $K > 1$, $B1 > 0$ is equivalent to $K > 1$, $B2 > 0$ [5, 6]; thus unconditional stability occurs in both the Γ_S plane and the Γ_L plane. The B_2, D_2, and E_2 parameters used in the Γ_L plane can be formed by interchanging the subscripts ($1 \rightarrow 2$ and $2 \rightarrow 1$). Edwards and Sinsky [5] used a single geometrically derived parameter μ' (or μ) to assess unconditional stability instead of the conventional *K* and *B* factors. They showed that $\mu' > 1$ (or $\mu > 1$) is necessary and sufficient for a two-port network to be unconditionally stable, where

$$\mu' = \frac{E_1}{|S_{12}S_{21}| + |C_1|} = \frac{|C_1| - |S_{12}S_{21}|}{D_1}, \tag{6}$$

$$\mu = \frac{E_2}{|S_{12}S_{21}| + |C_2|} = \frac{|C_2| - |S_{12}S_{21}|}{D_2} \tag{7}$$

and where C_1 and C_2 are defined as

$$C_1 = S_{11} - \Delta S_{22}^*, \tag{8}$$

$$C_2 = S_{22} - \Delta S_{11}^*. \tag{9}$$

The formulae [6] below are used to obtain the expressions in equaions (6)–(7):

$$|C_1|^2 = |S_{12}S_{21}|^2 + D_1E_1, \tag{10}$$

$$|C_2|^2 = |S_{12}S_{21}|^2 + D_2E_2. \tag{11}$$

Unconditionally stable $\mu' > 1$ (or $\mu > 1$) indicates that the stability circle (SC) is outside the unit smith chart (USC) in the Γ_S (or Γ_L) plane. Although $\mu' > 1$ (or $\mu > 1$) provides a useful single stability parameter to replace the dual Rollet ($K > 1$) plus auxiliary condition ($B_1 > 0$), there exist other situations between SC and USC. As shown in Fig. 1, the condition of $\mu' > 1$ only determines that the maximum distance of SC is inside USC ($|C_{IN}| + R_{IN} < 1$), but the minimum distance is still unknown (in the Γ_{IN} plane). One is unable to

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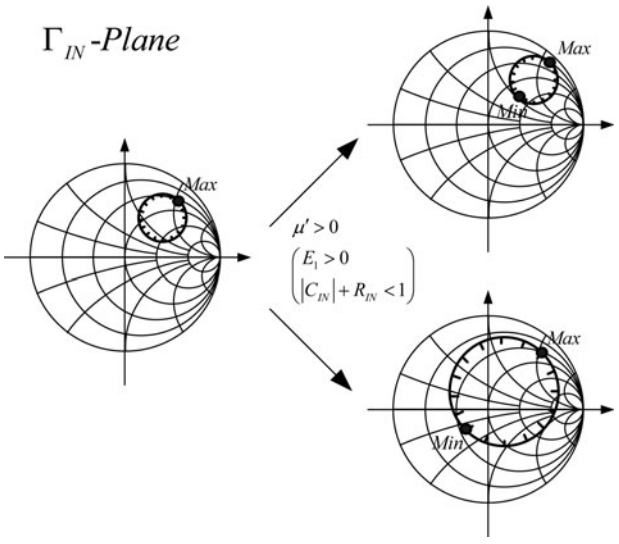


Fig. 1. Two different stability conditions with only the discriminative parameter of μ'.

determine whether the origin of USC is inside SC or not. Therefore, another geometrically derived parameter ν' (or ν) is presented in this paper to include all possible situations between SC and USC in the Γ_S (or Γ_{IN}) plane.
ν' (or ν) is defined as

$$\nu' = \frac{E_1}{|S_{12}S_{21}| - |C_1|} = \frac{|C_1| + |S_{12}S_{21}|}{-D_1}, \tag{12}$$

$$\nu = \frac{E_2}{|S_{12}S_{21}| - |C_2|} = \frac{|C_2| + |S_{12}S_{21}|}{-D_2}. \tag{13}$$

R_S and C_S can be expressed in terms of S parameters as explained in the Appendix. On substituting Equation (41) into Equation (6), the expression of μ' becomes

$$\mu' = |C_S| - R_S \quad \text{if} \quad D_1 > 0, \quad \text{i.e.} \quad \nu' < 0, \tag{14}$$

$$\mu' = R_S - |C_S| \quad \text{if} \quad D_1 < 0, \quad \text{i.e.} \quad \nu' > 0. \tag{15}$$

We can also substitute Equation (A.13) into Equation (12). Thus, the expression for ν' becomes

$$\nu' = -(R_S + |C_S|) \quad \text{if} \quad D_1 > 0, \tag{16}$$

$$\nu' = R_S + |C_S| \quad \text{if} \quad D_1 < 0. \tag{17}$$

It is straightforward that $|\mu'|$ and $|\nu'|$ have geometrical meanings in the Γ_S plane, and $|\nu'|$ is obviously larger than $|\mu'| \cdot |\nu'|$ is the maximum distance from SC to the origin of USC and $|\mu'|$ is the minimum distance from SC to the origin of USC in the Γ_S plane, respectively. Moreover, the sign of μ' determines whether the origin of USC is in the stable region or not. From Equation (12), ν' is the opposite sign of D_1. It implies that the sign of ν' can also determine whether the internal region of the SC (disk) is stable or the external region of the SC (disk complement) is stable. Therefore, two geometrically derived parameters, μ' and ν', have the concepts of position and stable region.

II. USING TWO GEOMETRICALLY DERIVED PARAMETERS TO CLASSIFY 12 SITUATIONS IN THE Γ_S AND Γ_{IN} PLANES

According to the μ' and ν' geometrical meanings, the geometrical relationship between SC and USC in the Γ_S plane can be classified into 12 situations as illustrated in Figs 2–4. The region inside SC is stable if the tick goes inwards and the region outside SC is stable if the tick goes outwards for all the figures in this paper.

In the Appendix, the stable region ($|\Gamma_{OUT}| < 1$) maps into a disk in the Γ_S plane if $D_1 < 0$ and into a disk complement in the Γ_S plane if $D_1 > 0$. From Equation (12), ν' is positive if $D1 < 0$ and negative if $D_1 > 0$. It is easy to see from Equations (14) and (15) that in the case of $\nu' > 0$ (or $\nu' < 0$), the distance from the origin of USC to the center of SC is larger than the radius of SC in the Γ_S plane when $\mu' > 0$ (or $\mu' < 0$) and the distance from the origin of USC to the center of SC is smaller than the radius of SC in the Γ_S plane when $\mu' < 0$ (or $\mu' > 0$). The 12 situations can be grouped into three groups as follows:

1) If $|\nu'| > 1 > |\mu'|$, SC intersects USC in two points in the Γ_S plane. As illustrated in Fig. 2, the four situations according to the signs of μ' and ν' correspond to $|K| < 1$ [7]. The geometrical relation between SC and USC provides useful insights and information for the conditionally stable design [7–9].
2) If $1 > |\nu'| > |\mu'|$, SC is inside USC as illustrated in Fig. 3. All situations correspond to conditionally stable cases with $|K| > 1$. The K factor is less than -1 in the cases of Figs 3(a) and 3(b), while the K factor is larger than one in the cases of Figs 3(c) and 3(d).
3) If $|\nu'| > |\mu'| > 1$, SC is outside USC. Figure 4 also shows four situations according to the signs of ν' and μ'. Figs 4(a) and 4(c) correspond to unconditionally stable cases, which is equivalent to $K > 1$ and $B_1 > 0$. However, all matching passive terminations located inside the USC of Figs 4(b) and 4(d) will result in absolutely unstable conditions.

By a bilinear transformation (see the Appendix), μ' and ν' can also have geometrical meanings in the Γ_{IN} plane.We can substitute (A.9) into (6). Thus, the expressions for μ' become

$$\mu' = \frac{1}{R_{in} + |C_{in}|} \quad \text{if} \quad E_1 > 0, \tag{18}$$

$$\mu' = -\frac{1}{R_{in} + |C_{in}|} \quad \text{if} \quad E_1 < 0. \tag{19}$$

We can also substitute (A.9) into (12). Thus, the expressions for ν' become

$$\nu' = \frac{1}{R_{in} - |C_{in}|} \quad \text{if} \quad E_1 > 0, \quad \text{i.e.} \quad \mu' > 0, \tag{20}$$

$$\nu' = \frac{1}{|C_{in}| - R_{in}} \quad \text{if} \quad E_1 < 0, \quad \text{i.e.} \quad \mu' < 0. \tag{21}$$

Thus, we can find the one-to-one correlation for μ' and ν' parameters in the Γ_{IN} and Γ_S planes. The meaning of μ' and ν' signs in the Γ_{IN} plane is opposite to that of μ' and ν' signs in the Γ_S plane. It has been shown in the Appendix that the

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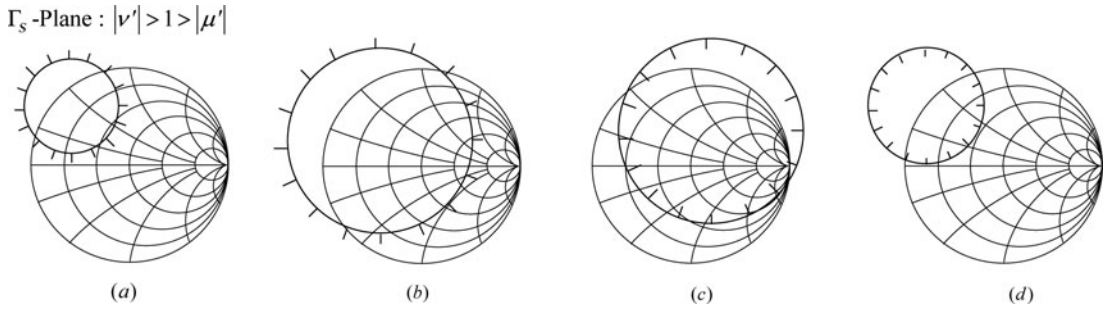


Fig. 2. SC and USC in the Γ_S plane when $|\nu'| > 1 > |\mu'|$. (a) $\nu' < 0$, $\mu' > 0$. (b) $\nu' < 0$, $\mu' < 0$. (c) $\nu' > 0$, $\mu' > 0$. (d) $\nu' > 0$, $\mu' < 0$ (tick marks indicate the stable region).

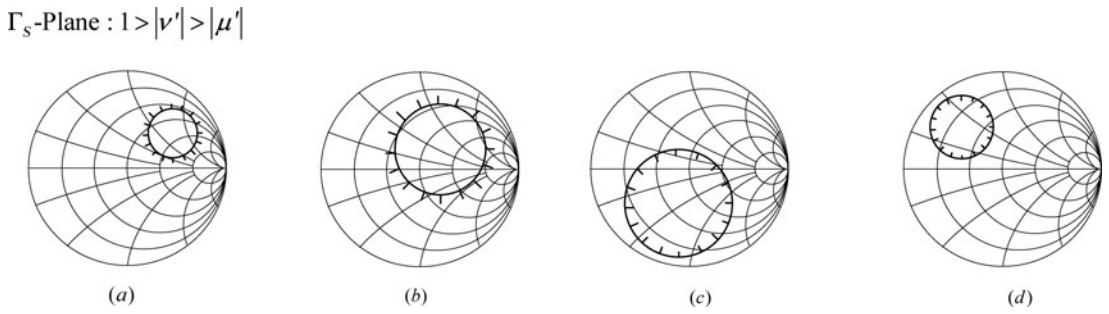


Fig. 3. SC and USC in the Γ_S plane when $1 > |\nu'| > |\mu'|$. (a) $\nu' < 0$, $\mu' > 0$. (b) $\nu' < 0$, $\mu' < 0$. (c) $\nu' > 0$, $\mu' > 0$. (d) $\nu' > 0$, $\mu' < 0$ (tick marks indicate the stable region).

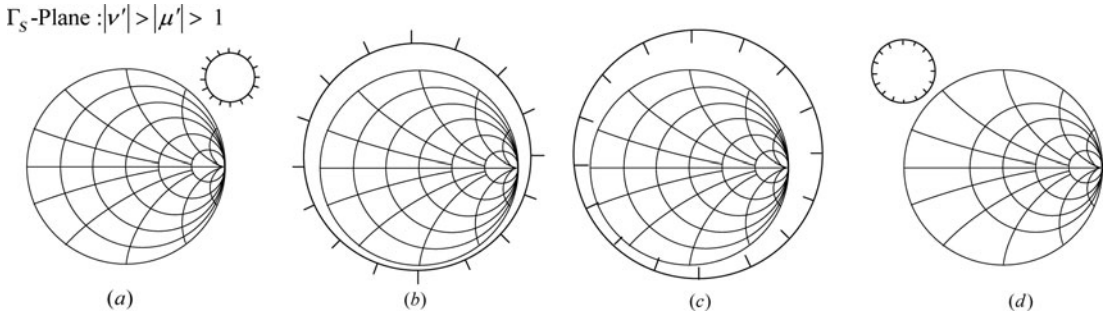


Fig. 4. SC and USC in the Γ_S plane when $|\nu'| > |\mu'| > 1$. (a) $\nu' < 0$, $\mu' > 0$. (b) $\nu' < 0$, $\mu' < 0$. (c) $\nu' > 0$, $\mu' > 0$. (d) $\nu' > 0$, $\mu' < 0$ (tick marks indicate the stable region).

stable region ($|\Gamma_L| < 1$) maps into a disk in the Γ_{IN} plane if $E_1 > 0$, and maps into a disk complement in the Γ_{IN} plane if $E_1 < 0$. μ' and E_1 have the same sign from Equation (6). In other words, the sign of μ' determines whether the stable region in the Γ_{IN} plane is a disk or a disk complement while the sign of ν' determines whether the origin of USC in the Γ_{IN} plane is stable or not. Certainly, the same criteria for the one-to-one correlation can be applied to μ and ν parameters in the Γ_{OUT} and Γ_L planes. The one-to-one correlation for μ' and ν' parameters in the Γ_{IN} and Γ_S planes is established in the next section.

III. RELATIONS BETWEEN THE Γ_{IN} AND Γ_S PLANES FOR UNCONDITIONALLY STABLE CASES

It can be understood from Equations (14)–(19) that the μ' parameter has geometrical meaning in both the Γ_S and Γ_{IN} planes. A two-port network is unconditionally stable if the unit disk in the Γ_L plane maps into a disk inside the unit disk of the Γ_{IN} plane. This is equivalent to saying that the unit disk in the Γ_{IN} plane maps onto a region that contains the unit disk in the Γ_L plane. $|\mu'| > 1$ means that $R_{IN} + |C_{IN}| < 1$ as illustrated in Equation (18) and the unit disk in the Γ_L plane maps into a disk in the Γ_{IN} plane when $E_1 > 0$ as explained in the Appendix, that is to say, μ' is positive as revealed in Equation (6). In other words, $\mu' > 1$ is necessary and sufficient for unconditional stability as illustrated by Edwards and Sinsky [5]. Edwards and Sinsky have also shown that $\mu' > 1$ is equivalent to $\mu > 1$. Thus, an unconditionally stable two-port network means that there is no combination of passive source ($|\Gamma_S| < 1$) and load ($|\Gamma_L| < 1$) that could make the two-port network oscillate.

Figure 5 illustrates the one-to-one correlation between the Γ_{IN} and Γ_S planes under an unconditionally stable condition. Both situations in Figs 5(a) and 5(c) correspond to $\mu' > 1$ and the SCs in the Γ_{IN} plane are similar. According to the bilinear transformation, the unit disk in the Γ_L plane maps into a disk in the

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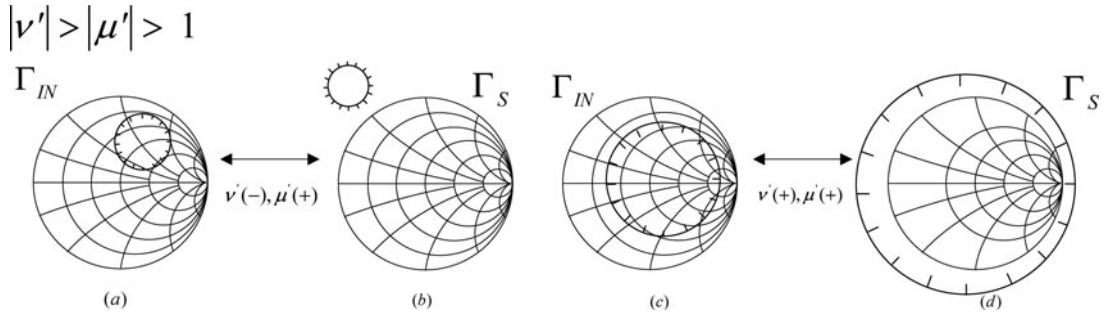


Fig. 5. One-to-one correlation between Γ_{IN} plane and Γ_S plane for unconditional stability in (a) Γ_{IN} plane with $\nu' < 0$, $\mu' > 0$, (b) Γ_S plane with $\nu' < 0$, $\mu' > 0$, (c) Γ_{IN} plane with $\nu' > 0$, $\mu' > 0$, and (d) Γ_S plane with $\nu' > 0$, $\mu' > 0$.

Γ_{IN} plane if $E_1 > 0$ (see the Appendix). The geometrical description for SC in Fig. 5(a) can be mathematically expressed as

$$E_1 = 1 - |S_{22}|^2 > 0 \quad \text{(disk)}, \tag{22}$$

$$|C_{IN}| + R_{IN} < 1 \quad \text{(SC is inside USC)}, \tag{23}$$

$$R_{IN} < |C_{IN}| \quad \text{(SC does not enclose USC origin)}. \tag{24}$$

Considering Equations (12), (20), (22), and (24), we obtain

$$D_1 > 0. \tag{25}$$

Substituting (A.9) and (A.13) into (A.8) and considering $D_1 > 0$ yields

$$(|C_S| - R_S)(|C_{IN}| + R_{IN}) = 1. \tag{26}$$

From inequality (23), we obtain

$$|C_S| - R_S > 1 \tag{27}$$

and thus

$$|C_S| > R_S. \tag{28}$$

From the Appendix, $D_1 > 0$ in Equation (25) means that the stable region is a disk complement in the Γ_S plane. The geometrical description of Fig. 5(b) coincides with Equations (25), (27), and (28). Obviously, Fig. 5(b) corresponds to unconditional stability in the Γ_S plane. The one-to-one correlation between Figs 5(c) and 5(d) can also be obtained in a similar way. This explains the advantage of using μ' (or μ) and ν' (or ν) to discuss the stability of a two-port network.

IV. GENERATING 12 SITUATIONS IN THE Γ_S PLANE BY PRACTICAL TWO-PORT NETWORKS

Relations between SC and USC are classified into $|\nu'| > 1 > |\mu'|$, $1 > |\nu'| > |\mu'|$, and $|\nu'| > |\mu'| > 1$. Each case has four situations between SC and USC in the Γ_S plane. *S* parameters of a commercial available device (NEC n128040a MESFET) at $V_{DS} = 5$ V and $I_{DS} = 240$ mA are used here to synthesize all the 12 situations in Figs 2–4. The corresponding two-port networks are discussed in this section, and the corresponding S parameters and stability factors will be listed at the end of each example.

The sign of the ν' (or ν) parameter determines that the region inside SC is stable if ν' (or ν) > 0, or the region outside SC is stable if ν' (or ν) < 0. The stable region includes USC origin if the μ' (or μ) parameter is positive. Otherwise, the stable region excludes USC origin when the μ' (or μ) parameter is negative.

A) Case I: $|\nu'| > 1 > |\mu'|$ (conditionally stable)

SC intersects USC in two distinct points when $|\nu'| > 1 > |\mu'|$. Edwards *et al.* [17] have proved that $-1 < k < 1$ occurs if and only if SCs intersect USC. The four conditions are conditionally stable, and could oscillate under a certain matching network.

1) I-1. ν' IS NEGATIVE AND μ' IS POSITIVE

A two-port network containing only a single device as shown in Fig. 6(a) can generate the SC condition of Fig. 2(a). At f = 1.8 GHz, this two-port network has the state of $\nu' = -1.41$ and $\mu' = 0.77$. The stable region is outside SC (disk complement, $\nu' < 0$) and the origin of USC is included in the stable region ($\mu' > 0$). Like most practical microwave transistors, this case is potentially unstable and coincides with the factors of K and B_1 ($K < 1$: potential unstable cases [6, 10]). On collocating with the factor of D_1, the unstable region occupied by SC can be estimated from the maximum and minimum distance between SC and the origin of USC, so as to initially evaluate the percentage of the available-matching region.

$S_{11} = 0.91\angle -144.69^\circ$	$S_{12} = 0.03\angle 12.08^\circ$
$S_{21} = 8.31\angle 95.99^\circ$	$S_{22} = 0.41\angle -147.07^\circ$
$K = 0.11$ $B_1 = 1.6$	$D_1 = 0.77$ $E_1 = 0.83$
$\nu' = -1.41 = -(\lvert C_S\rvert + R_S)$	$\mu' = 0.77 = (\lvert C_S\rvert - R_S)$

2) I-2. ν' IS NEGATIVE AND μ' IS NEGATIVE

A device with a series-connected capacitor of 10 pF between source and ground is illustrated in Fig. 6(b), and the 1.9 GHz corresponding SC is plotted in Fig. 2(b). This capacitor enlarges the values of $|S_{11}|$ and $|S_{22}|$, and then increases the possibility of oscillation [6]. The difference between Figs 2(a) and 2(b) is that the stable region in Fig. 2(b) excludes the origin of USC because $\mu' = -0.36 < 0$. $\nu' < 0$ and $\mu' < 0$ imply a negative value of K from -1 to 0. It is conditionally stable.

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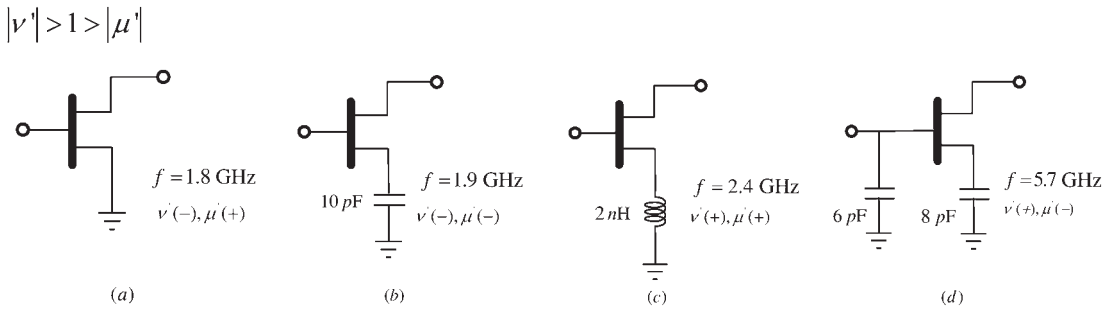


Fig. 6. Schematics of two-port networks when $|\nu'| > 1 > |\mu'|$. (a) $\nu' < 0$, $\mu' > 0$. (b) $\nu' < 0$, $\mu' < 0$. (c) $\nu' > 0$, $\mu' > 0$. (d) $\nu' > 0$, $\mu' < 0$ (tick marks indicate the stable region).

$S_{11} = 3.08\angle -118.70^\circ$	$S_{12} = 0.42\angle -24.28^\circ$
$S_{21} = 15.23\angle 132.05^\circ$	$S_{22} = 2.13\angle -123.79^\circ$
$K = -0.92$ $B_1 = 4.71$	$D_1 = 8.25$ $E_1 = -3.54$
$\nu' = -1.19 = -(\|C_S\| + R_S)$	$\mu' = -0.36 = (\|C_S\| - R_S)$

$S_{11} = 1.01\angle -173.78^\circ$	$S_{12} = 0.05\angle -49.07^\circ$
$S_{21} = 1.15\angle 70.46^\circ$	$S_{22} = 1.07\angle -110.20^\circ$
$K = -0.57$ $B_1 = -0.25$	$D_1 = -0.09$ $E_1 = -0.16$
$\nu' = 2.26 = (\|C_S\| + R_S)$	$\mu' = -0.81 = (R_S - \|C_S\|)$

3) I-3. ν' IS POSITIVE AND μ' IS POSITIVE

Figure 6(c) shows another two-port network with source-degeneration inductance. The 2.4 GHz SC in the Γ_S plane is similar to the condition of Fig. 2(c). $\nu' > 0$ and $\mu' > 0$ mean that the stable region is inside SC (disk) and USC origin is in the stable region. The series feedback inductor of 2 nH makes the two-port network more stable. This topology has been commonly used in LNA design to achieve low noise figure and good input return loss simultaneously.

$S_{11} = 0.41\angle -30.40^\circ$	$S_{12} = 0.27\angle 87.86^\circ$
$S_{21} = 2.47\angle 72.26^\circ$	$S_{22} = 0.56\angle -20.12^\circ$
$K = 0.96$ $B_1 = 0.10$	$D_1 = -0.59$ $E_1 = 0.69$
$\nu' = 1.45 = (\|C_S\| + R_S)$	$\mu' = 0.80 = (R_S - \|C_S\|)$

4) I-4. ν' IS POSITIVE AND μ' IS NEGATIVE

In this special case, a shunt capacitor of 6 pF is added at the input of a transistor, which has a series feedback capacitor of 8 pF at the source. SC in the Γ_S plane at 5.7 GHz is illustrated in Fig. 2(d). The operating behavior of this two-port network is similar to that of Fig. 6(b) because most of the region in USC is unstable and the origin of USC is outside the stable region. However, they can be further differentiated by the new derived parameter, ν'. In this case, the stable region is inside SC ($\nu' > 0$).

B) Case II: $1 > |\nu'| > |\mu'|$ (conditionally stable)

SC in the Γ_S plane is situated entirely inside USC for the case of $1 > |\nu'| > |\mu'|$. These four situations are all conditionally stable with $|K| > 1$, and oscillation could occur under some matching networks.

1) II-1. ν' IS NEGATIVE AND μ' IS POSITIVE

A two-port network containing a transistor with a series feedback capacitor of 4 pF and a shunt resistor of 80 Ω at the output is shown in Fig. 7(a). SC in the Γ_S plane is plotted in Fig 3(a). Because $\nu' < 0$, the stable region is located outside SC. The shunt resistor allows the diameter of the SC to shrink obviously, and this phenomenon can be observed by the difference in the absolute values of ν' and μ'. Compared with I-2, a resistor shunted at output can increase the stability of the two-port network.

$S_{11} = 2.02\angle -25.24^\circ$	$S_{12} = 0.27\angle 73.39^\circ$
$S_{21} = 3.81\angle -125.42^\circ$	$S_{22} = 0.64\angle -34.71^\circ$
$K = -1.66$ $B_1 = 4.57$	$D_1 = 3.99$ $E_1 = 0.58$
$\nu' = -0.72 = -(\|C_S\| + R_S)$	$\mu' = 0.21 = (\|C_S\| - R_S)$

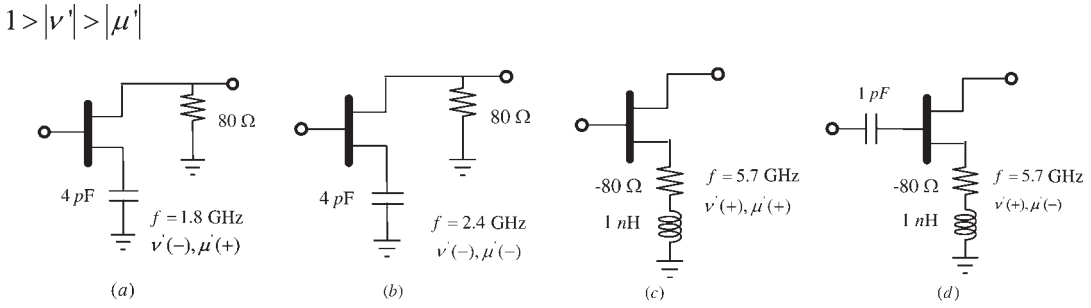


Fig. 7. Schematics of two-port networks when $1 > |\nu'| > |\mu'|$. (a) $\nu' < 0$, $\mu' > 0$. (b) $\nu' < 0$, $\mu' < 0$. (c) $\nu' > 0$, $\mu' > 0$. (d) $\nu' > 0$, $\mu' < 0$ (tick marks indicate the stable region).

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2) II-2. ν' IS NEGATIVE AND μ' IS NEGATIVE

SC in the Γ_S plane, which corresponds to the two-port network of Fig. 7(b) at 2.4 GHz, is plotted in Fig. 3(b). This circuit topology is the same as the previous example. In fact, the diameter of SC gradually increases due to the growing capacitive effect at higher operating frequency. The diameter of the unstable region expands from 0.51 (II-1) to 0.65 (this example). Finally, this network becomes more unstable at $f = 2.4$ GHz.

$S_{11} = 3.38\angle -52.33^\circ$	$S_{12} = 0.51\angle 48.99^\circ$
$S_{21} = 7.10\angle -155.75^\circ$	$S_{22} = 1.21\angle -59.28^\circ$
$K = -1.6$ $B_1 = 10.63$	$D_1 = 11.09$ $E_1 = -0.46$
$\nu' = -0.58 = -(\lvert C_S\rvert + R_S)$	$\mu' = -0.07 = (\lvert C_S\rvert - R_S)$

3) II-3. ν' IS POSITIVE AND μ' IS POSITIVE

A two-port network of Fig. 7(c) originates fromthe topology of Fig. 6(c). In this case, a negative resistance added in the source is equivalent to a positive feedback in the two-port network. When the resistance becomes more negative, the stable area inside SC is entirely shrunk. The relation between SC and USC is illustrated in Fig. 3(c).

$S_{11} = 0.80\angle -17.18^\circ$	$S_{12} = 1.08\angle 82.22^\circ$
$S_{21} = 1.31\angle 23.85^\circ$	$S_{22} = 0.84\angle -30.99^\circ$
$K = 1.35$ $B_1 = -3.22$	$D_1 = -3.53$ $E_1 = 0.31$
$\nu' = 0.68 = (\lvert C_S\rvert + R_S)$	$\mu' = 0.12 = (R_S - \lvert C_S\rvert)$

4) II-4. ν' IS POSITIVE AND μ' IS NEGATIVE

After adding a series capacitor at the input of Fig. 7(c), the topology of Fig. 7(d) results in $\mu' = -0.08 < 0$. From Equations (15) and (17), we can directly determine that USC origin is out of the stable region and SC is inside USC by values of ν' and μ'. The stable region becomes smaller when compared with II-3.

$S_{11} = 0.83\angle -16.43^\circ$	$S_{12} = 1.01\angle 85.69^\circ$
$S_{21} = 1.23\angle 27.32^\circ$	$S_{22} = 1.11\angle -16.13^\circ$
$K = 1.35$ $B_1 = -3.81$	$D_1 = -3.59$ $E_1 = -0.22$
$\nu' = 0.78 = (R_S + \lvert C_S\rvert)$	$\mu' = -0.08 = (R_S - \lvert C_S\rvert)$

From the above examples of Section II, the absolute values of μ' and ν' can reveal variations between stable and unstable regions (the diameter of SC).

C) Case III: $|\nu'| > |\mu'| > 1$ (unconditionally stable and absolutely unstable)

SC in the S plane is outside USC for the case of $|\nu'| > |\mu'| > 1$. Two situations are unconditionally stable and the other two situations are absolutely unstable. The corresponding networks for these situations are illustrated here.

1) III-1. ν' IS NEGATIVE AND μ' IS POSITIVE

As illustrated in Fig. 8(a), it is a simple and effective approach to stabilize a potentially unstable amplifier by connecting in series a 10 Ω resistor at the input of the single transistor. However, the method suffers from gain reduction. Compared with I.1, the increasing μ' implies that the point of SC closest to USC origin is gradually removed from the edge of USC. At the same time, the negative ν' determines that all kinds of 2.4 GHz passive matching networks designed in the Γ_S plane are unconditionally stable.

$S_{11} = 0.62\angle -151.64^\circ$	$S_{12} = 0.02\angle 9.05^\circ$
$S_{21} = 5.37\angle 86.40^\circ$	$S_{22} = 0.42\angle -150.83^\circ$
$K = 2.21$ $B_1 = 1.17$	$D_1 = 0.35$ $E_1 = 0.82$
$\nu' = -1.87 = -(R_S + \lvert C_S\rvert)$	$\mu' = 1.26 = (\lvert C_S\rvert - R_S)$

2) III-2. ν' IS NEGATIVE AND μ' IS NEGATIVE

A two-port network with a series feedback capacitor of 5 pF at the transistor source and a shunt resistor of 80 Ω at the input is shown in Fig. 8(b). The capacitor could make the two-port network unstable. SC completely encloses USC and the stable region is outside SC in Fig. 4(b) because $\nu' = -2.37 < 0$. It is clear that an absolutely unstable situation occurs and any passive matching network in the Γ_S plane results in $|\Gamma_{OUT}| > 1$. The circuit is absolutely unstable.

$S_{11} = 0.91\angle -32.48^\circ$	$S_{12} = 0.27\angle 68.53^\circ$
$S_{21} = 4.86\angle -130.80^\circ$	$S_{22} = 1.79\angle -34.16^\circ$
$K = -1.11$ $B_1 = -1.50$	$D_1 = 0.71$ $E_1 = -2.21$
$\nu' = -2.37 = -(R_S + \lvert C_S\rvert)$	$\mu' = -1.3 = (\lvert C_S\rvert - R_S)$

3) III-3. ν' IS POSITIVE AND μ' IS POSITIVE

A two-port network containing a common source amplifier with a source degenerated inductor and an additional 15 Ω series resistor at $f = 1.8$ GHz is illustrated in Fig. 8(c). SC in the Γ_S plane encloses USC completely, and the stable region is inside SC because $\nu' = 1.67 > 0$. SC of Fig. 4(c) looks very different from that of Fig. 4(a) even though both circuits have $K > 1$ and $B_1 > 0$. Thus, the parameter helps determine the relation between SC and USC.

$S_{11} = 0.71\angle -15.66^\circ$	$S_{12} = 0.23\angle 84.95^\circ$
$S_{21} = 1.62\angle 73.43^\circ$	$S_{22} = 0.72\angle -13.52^\circ$
$K = 1.01$ $B_1 = 0.20$	$D_1 = -0.27$ $E_1 = 0.47$
$\nu' = 1.67 = (R_S + \lvert C_S\rvert)$	$\mu' = 1.06 = (R_S - \lvert C_S\rvert)$

4) III-4. ν' IS POSITIVE AND μ' IS NEGATIVE

The circuit topology of Fig. 8(d) is identical to that of Fig. 8(b). From $\nu' > 1$ and $\mu' < 1$, SC is completely outside USC and the region inside SC is stable, as shown in Fig. 4(d). Obviously, the circuit is absolutely unstable. The S parameters and stability factors at 1.8 GHz are as follows:

$S_{11} = 0.61\angle -175.27^\circ$	$S_{12} = 0.06\angle 69.41^\circ$
$S_{21} = 2.10\angle -133.10^\circ$	$S_{22} = 1.40\angle -39.84^\circ$
$K = -1.66$ $B_1 = -1.52$	$D_1 = -0.56$ $E_1 = -0.96$
$\nu' = 1.55 = (R_S + \lvert C_S\rvert)$	$\mu' = -1.10 = (R_S - \lvert C_S\rvert)$

Table 1 compares the two geometrically derived parameters and the corresponding conventional stability factors (K, B_1, D_1, and E_1) for each geometrical situation. The signs of ν' and μ' are independent and useful in identifying each

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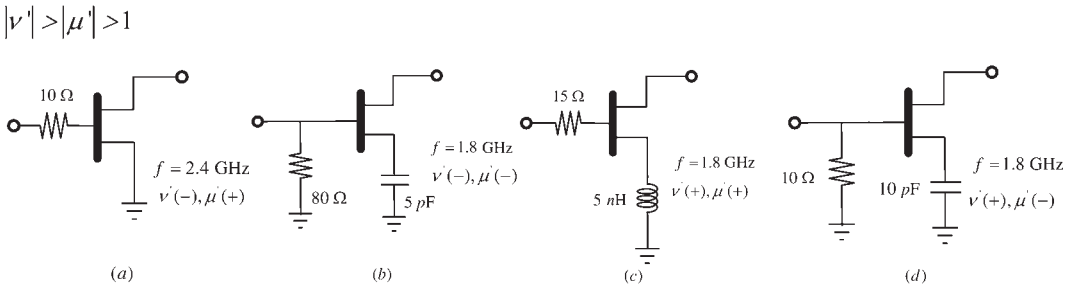


Fig. 8. Schematics of two-port networks when $|\nu'| > |\mu'| > 1$. (a) $\nu' < 0$, $\mu' > 0$. (b) $\nu' < 0$, $\mu' < 0$. (c) $\nu' > 0$, $\mu' > 0$. (d) $\nu' > 0$, $\mu' < 0$ (tick marks indicate the stable region).

Table 1. The comparison of stability parameters

Magnitudes of ν' and μ'	Case	Signs of ν' and μ'	Corresponding conventional stability criteria
$\|\nu'\| > 1 > \|\mu'\|$	I-1	$\nu'(-)$, $\mu'(+)$	$-1 < K < 1$, $B_1 > 0$, $D_1 > 0$, $E_1 > 0$
	I-2	$\nu'(-)$, $\mu'(-)$	$-1 < K < 0$, $B_1 > 0$, $D_1 > 0$, $E_1 < 0$
	I-3	$\nu'(+)$, $\mu'(+)$	$0 < K < 1$, $B_1 > 0$, $D_1 < 0$, $E_1 > 0$
	I-4	$\nu'(+)$, $\mu'(-)$	$-1 < K < 1$, $B_1 < 0$, $D_1 < 0$, $E_1 < 0$
$1 > \|\nu'\| > \|\mu'\|$	II-1	$\nu'(-)$, $\mu'(+)$	$K < -1$, $B_1 > 0$, $D_1 > 0$, $E_1 > 0$
	II-2	$\nu'(-)$, $\mu'(-)$	$K < -1$, $B_1 > 0$, $D_1 > 0$, $E_1 < 0$
	II-3	$\nu'(+)$, $\mu'(+)$	$K > 1$, $B_1 < 0$, $D_1 < 0$, $E_1 > 0$
	II-4	$\nu'(+)$, $\mu'(-)$	$K > 1$, $B_1 < 0$, $D_1 < 0$, $E_1 < 0$
$\|\nu'\| > \|\mu'\| > 1$	III-1*	$\nu'(-)$, $\mu'(+)$	$K > 1$, $B_1 > 0$, $D_1 > 0$, $E_1 > 0$
	III-2	$\nu'(-)$, $\mu'(-)$	$K < -1$, $B_1 < 0$, $D_1 > 0$, $E_1 < 0$
	III-3*	$\nu'(+)$, $\mu'(+)$	$K > 1$, $B_1 > 0$, $D_1 < 0$, $E_1 > 0$
	III-4	$\nu'(+)$, $\mu'(-)$	$K < -1$, $B_1 < 0$, $D_1 < 0$, $E_1 < 0$

*Unconditional stable.

geometrical situation. However, the signs of K, B_1, D_1, and E_1 are dependent and lack geometrical meaning.

V. DISCUSSIONS AND CONCLUSIONS

The complete analysis of stability is an important step in designing an active two-port network. Most discussions about stability in the literature [1, 5] only focus on the unconditionally stable case. There exist other conditions between SC and USC, and this paper has extended the criterion of stability to more situations by two derived parameters, μ' and ν'.

A picture is worth a thousand words. Two stability parameters, μ' and ν', using the geometrical approach to assess the stability problem for a two-port network are established. Both magnitude and signs of them have distinct geometrical meanings in the Γ_S plane, and 12 geometrical situations between SC and USC are classified. Simultaneously, practical two-port networks are also given to demonstrate the 12 conditions. This work provides a straightforward insight into the stability analysis of a two-port network and is helpful for the RF circuit design.

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APPENDIX

The following is a bilinear transformation between the ω and z planes:

$$\omega = \frac{az+b}{cz+d} \quad (ad - bc \neq 0). \tag{A.1}$$

Let the radius of the circle be one in the ω plane. The following expression is to map the unit disk in the ω plane into a disk or a disk complement in the z plane:

$$\left|\frac{az+b}{cz+d}\right|^2 = \left(\frac{az+b}{cz+d}\right)\left(\frac{az+b}{cz+d}\right)^* < 1, \tag{A.2}$$

which can be expanded as

$$\begin{aligned} &|z|^2[|a|^2-|c|^2] - z[cd^* - ab^*] - z^*[c^*d - a^*d] \\ &+ |b|^2-|d|^2 < 0. \end{aligned} \tag{A.3}$$

If $|a|^2-|c|^2 > 0$, Equation (A.3) becomes

$$|z|^2 - z\left[\frac{cd^* - ab^*}{|a|^2-|c|^2}\right] - z^*\left[\frac{c^*d - a^*b}{|a|^2-|c|^2}\right] + \frac{|b|^2-|d|^2}{|a|^2-|c|^2} < 0. \tag{A.4}$$

On the other hand, if $|a|^2 - |c|^2 < 0$, Equation (A.3) becomes

$$|z|^2 - z\left[\frac{cd^* - ab^*}{|a|^2-|c|^2}\right] - z^*\left[\frac{c^*d - a^*b}{|a|^2-|c|^2}\right] + \frac{|b|^2-|d|^2}{|a|^2-|c|^2} > 0. \tag{A.5}$$

This is equivalent to saying that the unit disk in the ω plane ($|\omega| < 1$) maps into a disk in the z plane if $|a|^2 - |c|^2 > 0$ or into a disk complement in the z plane if $|a|^2 - |c|^2 < 0$. A bilinear transformation exists between the load and input reflection coefficients of a two-port network as shown below:

$$\Gamma_L = \frac{-\Gamma_{IN} + S_{11}}{-S_{22}\Gamma_{IN} + \Delta}, \tag{A.6}$$

where

$$a = -1,\ b = S_{11},\ c = -S_{22},\ d = \Delta. \tag{A.7}$$

The radius and center of the relation in the Γ_{IN} plane is

$$|C_{IN}|^2 = R_{IN}{}^2 + \frac{D_1}{E_1}, \tag{A.8}$$

where

$$C_{IN} = \frac{C_1}{E_1} \quad \text{and} \quad R_{IN} = \frac{|S_{12}S_{21}|}{|E_1|}. \tag{A.9}$$

The condition, $|a|^2 - |c|^2 = 1 - |S_{22}|^2 = E_1 > 0$, means that the mapped region of $|\Gamma_L| < 1$ is an inner disk on the Γ_{IN} plane. A bilinear transformation also exists between the output and source reflection coefficients of a two-port network as shown below:

$$\Gamma_{OUT} = \frac{\Delta\Gamma_S - S_{22}}{S_{11}\Gamma_S - 1}, \tag{A.10}$$

where

$$a = \Delta,\ b = -S_{22},\ c = S_{11},\ d = -1. \tag{A.11}$$

Also, the radius and center of the relation in the Γ_S plane is

$$|C_S|^2 = R_S{}^2 + \frac{E_1}{D_1}, \tag{A.12}$$

where

$$C_S = \frac{C_1^*}{D_1} \quad \text{and} \quad R_S = \frac{|S_{12}S_{21}|}{|D_1|}. \tag{A.13}$$

The condition is $|a|^2 - |c|^2 = |\Delta|^2 - |S_{11}|^2 = -D_1 > 0$ to imply $D_1 < 0$ for the mapping of $|\Gamma_{OUT}| < 1$ on the Γ_S plane being disk.



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