THREE LITTLE TRUTH-VALUE PARADOXES

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This is an introduction to some questions about the truth or falsehood of pairs of sentences. Truth is, of course, something philosophers have always been interested in. Finding out the truth can be difficult, but it's usually been thought that truth itself behaves in a very simple way. In particular, every statement is either true or false, and no statement can be both at once.

Look at this pair of sentences and, before reading on, try to decide whether sentence A is true or false:

A: Sentence B is false

B: Oranges are lemons

Whether sentence A is true or false obviously depends on whether sentence B is true or false. Sentence B is obviously false – oranges aren't lemons – that makes sentence A *true*, because sentence A says that sentence B is false and sentence B actually *is* false.

Here's a similar pair – again, try to decide whether sentence C is true or false:

C: Sentence D is false

D: Oranges are oranges

Of course, oranges are indeed oranges, so sentence D is true. Yet sentence C claims it's false; that claim isn't true. This makes sentence C *itself* false. Look again at the sentences and it should be clear to you that sentence C is indeed not true. So far, so good; truth and falsehood seem to be behaving just the way we'd like them to.

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Here are another two harmless-looking sentences:

E: Sentence F is false

F: Sentence E is true

Which, if either, of these sentences are true? Well, if sentence E is true then that means sentence F must be false. But that means sentence F is wrong when it claims that sentence E is true – that can't be right, because we just said sentence E *is* true, and it can't be both true and not true. So Sentence E must be false, right? That means that sentence F is true (since sentence E says it isn't, and sentence E is false). But then if sentence F is true so is sentence E – which can't be right either! So that means sentence E can't be true and it can't be false. This is a bit worrying.

The 'truth value' of a proposition is just whether it's true or not. The truth value of 'oranges are oranges' is 'true', and the truth value of 'oranges are lemons' is 'false'. The truth values of the sentences in the last example, however, can't be determined. They're neither true nor false. This breaks a rule that's really fundamental to the way we usually think. The philosopher Roy Sorensen calls this a 'truth-value gap' – it's as if the truth-values are missing.

Sentences E and F are an example of a general problem known as the 'liar paradox'. Although very ancient, it's never been solved to everyone's satisfaction. We can hardly ban sentences that refer to other sentences: there's no problem with sentences A or C in the earlier examples, and in fact those sentences say something perfectly reasonable and even useful, so that would really be rather extreme.

I'm guessing you might have seen some version of the liar paradox before, so try this one, a variant known as the 'truth-teller':

G: Sentence H is true.

H: Sentence G is true.

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They're both true, right? Or – wait – are they both false? It seems we can assign either truth-value to the two sentences, as long as they both have the same truth-value. So is sentence G true or false? You can't need more information to decide – what information could possibly make the difference? Here it seems as if sentences G and H can be true or false and there's no real way to decide which. This is what Sorensen calls a 'truth-value glut' – we have too many possible truth-values.

Another example Sorensen gives is what he calls the 'no-no' paradox – before reading on try assigning truth-values to the sentences and see whether you can find a combination that works:

J: Sentence K is false.

K: Sentence J is false.

In this case, as you may have discovered, one of the sentences must be true and the other must be false. But which is which? And doesn't it seem wrong to you to say that sentence J is true and sentence K is false even though they're more or less identical sentences, and you have no good reason for choosing that rather than the other assignment of truth values? What on Earth could make you say *which* one was true and which was false?

These are, I'm sure you'll agree, rather artificial sentences. Perhaps in real life we don't need to worry about them; perhaps they're just inventions of the typical philosopher, disconnected from reality and musing over invented problems. There are problems enough in the world, after all, and our time might be better spent worrying about those. Yet I think that at least the second and third of the paradoxes point to quite profound problems that can arise in real life and that can seem to defeat any attempt at a purely rational resolution.

Let's look at sentences G and H. Here each speaker endorses the other's views in a circular manner. It's

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possible that both are right and it's possible that both are wrong. The pair of sentences constitutes a self-affirming set of beliefs. They contain no external references that make a difference to their truth-value, so externally it's impossible to make any judgement about whether the system as a whole contains true statements or false ones. Once again, it's not that we're missing the information we might need to decide one way or another: the system is hermetically sealed from the outside world. There's no information you could possibly ask for that would help make your mind up.

This is a situation we often find in belief systems that contain their own justification. Some religions may be like this and likewise some political doctrines. Much more controversially, the natural sciences could be thought of as a closed system of this kind: the success of a scientific theory is determined by scientific methods, and scientific methods are justified by the success of scientific theories. The system internally asserts its own truth and nothing outside it is relevant. These systems are the kind that the philosopher Jean-François Lyotard saw everywhere in our present age; whether he was right about that is a matter on which reasonable people can disagree, of course.

Lyotard's work is also relevant to the third paradox, sentences J and K. Here each sentence claims the other is false, but again there are not exterior conditions that enable us to tell which is right. This situation is close to what Lyotard calls a 'differend' – an unresolvable dispute, a disagreement so fundamental that it's impossible to arbitrate or negotiate a settlement because there just isn't enough common ground. The only recourse is to power so that, as the economist Lionel Robbins said in a related context, it is just a matter of 'thy blood or mine'.

This situation is familiar in religious and political disputes. When belief systems that are self-justifying – systems that contain their own foundations – come into conflict, it may well happen that this kind of paradox is at work. Take the debate over the teaching of creationism in schools.

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Creationists believe this is important because creation is the correct theory, and they believe this because it is what the Bible says and the Bible's revealed truth trumps any scientific theory. Evolutionary biologists, on the other hand, think quite differently: they say that the empirical evidence points to evolution as the best theory available, and rules out the account given in the Bible.

The dispute has become a standoff because the justifications on one side don't count as justifications on the other. In effect each of the two systems points at the other and declares it false. Certainly one side is right about the other being wrong, at least as long as we want our logic to work in the classical way. Yet we have literally no way to work out which is which unless we do one of two things. First, we can take one side or the other and become combatants rather than observers. Second, we can take up some third belief system that enables us to make a decision. Yet this third belief system may not be recognised by any of the people involved in either side of the debate, and what good would that do us?

Real-world arguments are always much messier and more complex than the neat, two-sentence examples we started with, and the idea of 'belief system' that I've introduced into this article is pretty crude. Furthermore, Lyotard's work lies somewhat outside mainstream philosophy and is highly controversial. I do want to suggest, though, that amusing logic puzzles like the ones we started with might well be worth taking seriously. Throughout most of the history of philosophy they've been treated as aberrations, special cases that need to be fixed with a bit of tinkering. What if they're not aberrations? What if they're actually typical of how human beings really think, and of how human societies really work?

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