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Linear collisionless Landau damping in Hilbert space

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The equivalence between the Laplace transform (Landau, *J. Phys. USSR* **10** (1946), 25) and Hermite transform (Zocco and Schekochihin, *Phys. Plasmas* **18**, 102 309 (2011)) solutions of the linear collisionless Landau damping problem is proven.

1. Hermite revival

Vibrations in plasmas can be damped even in the absence of collisions. This phenomenon is known as Landau damping (Landau 1946). Landau damping acts on different types of waves: Langmuir waves, sound waves, kinetic Alfén waves (KAW), drift waves, and many more. It can basically occur any time the momentum associated to a wave propagating in a plasma can sample regions of velocity-space where the plasma distribution function is prone to the formation of a singularity. A particularly interesting case is that of KAW (Hasegawa and Chen 1975), and will be studied here. These waves are of pivotal importance in many physical phenomena in magnetized plasmas such as magnetic reconnection (Yamada et al. 2010), auroral electromagnetic turbulence (Louarn et al. 1994), and astrophysical gyrokinetics (Schekochihin et al. 2009), for instance.

A simple hybrid fluid-kinetic model that supports KAW *and* their Landau damping was introduced by Zocco and Schekochihin (2011). Therefore, an efficient way to simulate the model equations numerically via the Hermite representation of velocity-space was proposed. This proved to be useful to study electron heating and nonlinear Landau damping of KAW in collisionless magnetic reconnection (Loureiro et al. 2013).

While some physical insight on nonlinear Landau damping (Loureiro et al. 2013; Parker and Dellar 2014) can be obtained by 'brute force' numerical simulations that make use of a Hermite transform, it is still unclear what is the relation between the original linear result of Landau (1946) and the Hermite representation of velocity-space (Hammett et al. 1993; Smith 1997; Sugama et al. 2001; Zocco and Schekochihin 2011). In this work, we address this issue, and prove the equivalence of the two treatments.

2. Equations

We briefly review the system of equations studied. Details can be found in Zocco and Schekochihin (2011). We start with the collisionless electron drift-kinetic equation

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(Frieman and Chen 1982) for $h_{\mathrm{e}} = F_{\mathrm{e}} - F_{0\mathrm{e}}(1 + e\varphi/T_{0\mathrm{e}})$

$$\frac{\partial h_{\mathrm{e}}}{\partial t} + \mathbf{v}_E \cdot \nabla h_{\mathrm{e}} + v_{\parallel}\hat{\mathbf{b}} \cdot \nabla h_{\mathrm{e}} = -\frac{eF_{0\mathrm{e}}}{T_{0\mathrm{e}}}\frac{\partial}{\partial t}\left(\varphi - \frac{v_{\parallel}A_{\parallel}}{c}\right), \tag{2.1}$$

where $\mathbf{v}_E = cB_0^{-1}(-\partial_y\varphi\mathbf{e}_x + \partial_x\varphi\mathbf{e}_y)$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity, $\hat{\mathbf{b}}\cdot\nabla = \partial_z - B_0^{-1}\{A_{\parallel},\}$, φ and $A_{\parallel}$ are the electrostatic and magnetic potential, $\{,\}$ is the Poisson bracket, and

$$F_{0\mathrm{e}} = \frac{n_{0\mathrm{e}}}{\left[\pi v_{\mathrm{the}}^2\right]^{3/2}} e^{-\frac{v_{\parallel}^2 + v_{\perp}^2}{v_{\mathrm{the}}^2}} \tag{2.2}$$

is the Maxwellian equilibrium with temperature $T_{0\mathrm{e}} = m_{\mathrm{e}}v_{\mathrm{the}}^2/2$. Equation (2.1) describes the statistical properties of a magnetized electron species for low-frequency anisotropic fluctuations in the presence of a mean magnetic field. Here, this is a constant straight magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{b}}$, whose direction defines the z-axis.

We introduce a formal mass ratio expansion for the electron perturbed distribution function, so that to zeroth order

$$h_{\mathrm{e}} = \left(-\frac{e\varphi}{T_{0\mathrm{e}}} + \frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}} + \frac{v_{\parallel}u_{\parallel\mathrm{e}}}{T_{0\mathrm{e}}}m_{\mathrm{e}}\right)F_{0\mathrm{e}} + g_{\mathrm{e}} + \mathcal{O}\left(\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\right), \tag{2.3}$$

here $\delta n_{\mathrm{e}} = \int d^3\mathbf{v}h_{\mathrm{e}}$, $u_{\parallel\mathrm{e}} = n_{0\mathrm{e}}^{-1}\int d^3\mathbf{v}v_{\parallel}h_{\mathrm{e}}$, and

$$\int d^3\mathbf{v}(1, v_{\parallel})g_{\mathrm{e}} \equiv 0. \tag{2.4}$$

Using expression (2.3) in (2.1), and taking the zeroth moment we obtain the electron continuity equation

$$\frac{d}{dt}\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}} = -\hat{\mathbf{b}}\cdot\nabla u_{\parallel\mathrm{e}} \tag{2.5}$$

with $d/dt = \partial_t + \mathbf{v}_E\cdot\nabla$.

The first moment of (2.1) yields the generalized Ohm's law

$$\frac{d}{dt}(A_{\parallel} - d_{\mathrm{e}}^2\nabla_{\perp}^2A_{\parallel}) = -c\frac{\partial\varphi}{\partial z} + \frac{T_{0\mathrm{e}}c}{e}\hat{\mathbf{b}}\cdot\nabla\left[\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}} + \frac{\delta T_{\parallel\,\mathrm{e}}}{T_{0\mathrm{e}}}\right] + \frac{m_{\mathrm{e}}c}{e}\frac{1}{n_{0e}}\frac{d}{dt}n_{0\mathrm{e}}u_{\parallel\mathrm{i}}, \tag{2.6}$$

with

$$\frac{\delta T_{\parallel\mathrm{e}}}{T_{0\mathrm{e}}} \equiv \frac{1}{n_{0\mathrm{e}}}\int d^3\mathbf{v}2\frac{v_{\parallel}^2}{v_{\mathrm{the}}^2}g_{\mathrm{e}}. \tag{2.7}$$

In (2.6), we used parallel Ampere's law

$$u_{\parallel\mathrm{e}} = \frac{e}{m_{\mathrm{e}}c}d_{\mathrm{e}}^2\nabla_{\perp}^2A_{\parallel} + u_{\parallel\mathrm{i}}, \tag{2.8}$$

where $d_{\mathrm{e}} = c/\omega_{\mathrm{pe}}$ is the electron skin depth, and ω_{pe} the electron plasma frequency. An equation for g_{e} is derived after using (2.5) and (2.6) in (2.1). The result is

$$\frac{dg_{\mathrm{e}}}{dt} + v_{\parallel}\left[\hat{\mathbf{b}}\cdot\nabla g_{\mathrm{e}} - F_{0\mathrm{e}}\hat{\mathbf{b}}\cdot\nabla\frac{\delta T_{\parallel\mathrm{e}}}{T_{0\mathrm{e}}}\right] = F_{0\mathrm{e}}\left(1 - 2\frac{v_{\parallel}^2}{v_{\mathrm{the}}^2}\right)\hat{\mathbf{b}}\cdot\nabla\left[\left(\frac{e}{m_{\mathrm{e}}c}d_{\mathrm{e}}^2\nabla_{\perp}^2A_{\parallel} + u_{\parallel i}\right)\right]. \tag{2.9}$$

The kinetic information is embedded in the function

$$\frac{\delta T_{\parallel\mathrm{e}}}{T_{0\mathrm{e}}} \equiv \frac{1}{n_{0\mathrm{e}}}\int d^3\mathbf{v}2\frac{v_{\parallel}^2}{v_{\mathrm{the}}^2}g_{\mathrm{e}}. \tag{2.10}$$

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The system of equations is closed by solving for the ion dynamics, imposing quasineutrality (Zocco and Schekochihin 2011)

$$\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}} = \frac{\delta n_{\mathrm{i}}}{n_{0\mathrm{i}}}, \tag{2.11}$$

and using the ion solution(Coppi et al. 1979; Antonsen and Coppi 1981; Crew et al. 1982; Pegoraro et al. 1989; Porcelli 1991; Zocco and Schekochihin 2011; Connor et al. 2012)

$$\frac{\delta n_{\mathrm{i}}}{n_{0\mathrm{i}}} = \int_{-\infty}^{+\infty} dp e^{\mathrm{ipx}} F(p\rho_i) \frac{Ze\varphi}{T_{0\mathrm{i}}} \equiv \hat{F} \frac{Ze\varphi_{\mathbf{k}}}{T_{0\mathrm{i}}}, \tag{2.12}$$

with

$$F(p\rho_{\mathrm{i}}) = -(1 - \Gamma_0), \tag{2.13}$$

$\tau = T_{0\mathrm{i}}/T_{0\mathrm{e}}$, Z is the charge number, $k^2 = k_x^2 + k_y^2$, and $\Gamma_n = \exp[-p^2\rho_{\mathrm{i}}^2/2] I_n(p^2\rho_{\mathrm{i}}^2/2)$, where I_n is the modified Bessel function (Abramowitz and Stegun 1972). The 'hat' on $F(p\rho_i)$ is a short-hand notation for the inverse Fourier transform.The ion solution guarantees that $u_{\parallel i} = 0$. In the truly collisionless case considered here, we can solve the electron kinetic equation (2.9). We linearize (2.9) using perturbations of the form $g_{\mathrm{e}} \propto \exp[-i\omega t + ik_{\parallel} z]$.

After using standard algebra, we find (Zocco and Schekochihin 2011),

$$\frac{\delta T_{\parallel \mathrm{e}}}{T_{0\mathrm{e}}} = -\frac{k_{\parallel}}{|k_{\parallel}|} \frac{u_{\parallel \mathrm{e}}}{v_{\mathrm{the}}} \frac{Z(\zeta) - 2\zeta\,[1 + \zeta Z(\zeta)]}{1 + \zeta Z(\zeta)}, \tag{2.14}$$

where $\zeta = \omega/(|k_{\parallel}|\, v_{\mathrm{the}})$, and

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dt \frac{e^{-t^2}}{t - \zeta}, \quad \Im[\zeta] > 0, \tag{2.15}$$

is the plasma dispersion function (Fried et al. 1968). A representation valid $\forall\ \zeta$ is

$$Z(\zeta) = 2ie^{-\zeta^2} \int_{-\infty}^{i\zeta} dt e^{-t^2}. \tag{2.16}$$

We can replace (2.14) in (2.6) to obtain (Zocco and Schekochihin 2011)

$$\left[\zeta^2 - \frac{\tau}{Z} \frac{k_{\perp}^2 d_{\mathrm{e}}^2}{1 - \Gamma_0\left(k_{\perp}^2\rho_{\mathrm{i}}^2/2\right)}\right] [1 + \zeta Z(\zeta)] = \frac{1}{2} k_{\perp}^2 d_{\mathrm{e}}^2. \tag{2.17}$$

In this context, what is generally known as 'Landau prescription' is a choice for an integration path in the complex t plane, so that the plasma dispersion function, defined via the integral in (2.15), can be analytically continued to $\Im[\zeta] < 0$. Looking for solutions with $\zeta = \omega/(|k_{\parallel}|\, v_{\mathrm{the}}) \ll 1$, one gets the dispersion relation of shear and KAW

$$\omega_0 = \pm k_{\parallel} v_A k_{\perp} \rho_{\mathrm{i}} \sqrt{\frac{1}{2}\left[\frac{Z}{\tau} + \frac{1}{1 - \Gamma_0\left(k_{\perp}^2\rho_{\mathrm{i}}^2/2\right)}\right]}, \tag{2.18}$$

where the damping rate is found by solving (2.17) perturbatively in $\gamma/\omega_0 \ll 1$, seeking for a solution $\omega = \omega_0 + i\gamma$:

$$\gamma = -|k_{\parallel}|\, v_A \frac{k_{\perp}^2\rho_{\mathrm{i}}^2}{4} \sqrt{\pi \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \frac{Z^3}{\tau^2 \beta_{\mathrm{e}}}}. \tag{2.19}$$

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In the following, we solve analytically (2.9) by using Hermite polynomials as a basis in Hilbert space, and prove analytically that the Hilbert space solution converges to (2.14) when we take to infinity the number of Hermite moments kept in the system.

3. Hilbert Space

We introduce the Hermite inverse transform defined as

$$\hat{g}_{\mathrm{e}}(v_{\parallel}) = \sum_{m=2}^{\infty} \frac{H_m(\hat{v}_{\parallel})}{\sqrt{2^m m!}} \hat{g}_m F_{0\mathrm{e}}(\hat{v}_{\parallel}^2), \tag{3.1}$$

with coefficients

$$\hat{g}_m = \frac{1}{n_{0\mathrm{e}}} \int_{-\infty}^{\infty} d\hat{v}_{\parallel} \frac{H_m(\hat{v}_{\parallel})}{\sqrt{2^m m!}} \hat{g}_{\mathrm{e}}(v_{\parallel}), \tag{3.2}$$

where $\hat{v}_{\parallel} = v_{\parallel}/v_{\mathrm{the}}$, and $\hat{g}_{\mathrm{e}} = 2v_{\mathrm{the}}^{-2} \int dv_{\perp} v_{\perp} \exp[-v_{\perp}^2/v_{\mathrm{the}}^2] g_{\mathrm{e}}$. The resulting electron kinetic equation is

$$\begin{aligned} &\frac{d}{dt}\hat{g}_m + v_{\mathrm{the}}\hat{\mathbf{b}} \cdot \nabla \left(\sqrt{\frac{m+1}{2}} \hat{g}_{m+1} + \sqrt{\frac{m}{2}} \hat{g}_{m-1} - \delta_{m,1}\hat{g}_2 \right) \\ &= -\sqrt{2}\delta_{m,2}\hat{\mathbf{b}} \cdot \nabla u_{\parallel \mathrm{e}} \end{aligned} \tag{3.3}$$

Hence, for the first Hermite moments, we obtain

$$\frac{d}{dt}\hat{g}_2 + \sqrt{\frac{3}{2}} v_{\mathrm{the}} \hat{\mathbf{b}} \cdot \nabla \hat{g}_3 = -\sqrt{2}\hat{\mathbf{b}} \cdot \nabla u_{\parallel \mathrm{e}}, \tag{3.4}$$

for $m = 2$, and

$$\frac{d}{dt}\hat{g}_m + v_{\mathrm{the}}\hat{\mathbf{b}} \cdot \nabla \left(\sqrt{\frac{m+1}{2}} \hat{g}_{m+1} + \sqrt{\frac{m}{2}} \hat{g}_{m-1} \right) = 0, \tag{3.5}$$

for $m \geqslant 3$. The moments g_0 and g_1 do not appear in the summation in (3.1) as they must be set to zero in order to satisfy (2.4).

Equation (3.5) seems to suggest we should look for an iterative solution. However, it manifests the typical problem of a kinetic system, where low-order moments are coupled to high-order ones, therefore requiring to solve an infinite number of equations in order to know the full kinetic dynamics. We take advantage of the scaling of Hermite coefficients with the Hermite order, and virtually solve for an infinite number of equations in a very compact form. We then prove that our solution is equivalent to the solution found by using Landau contour integration.

4. Landau–Hermite equivalence

We find useful to consider the following limit

$$\frac{\hat{g}_m}{\hat{g}_{m-1}} \sim \frac{k_{\parallel} v_{\mathrm{the}}}{\sqrt{m}\omega} \ll 1, \tag{4.1}$$

with

$$\frac{k_{\parallel} v_{\mathrm{the}}}{\omega\sqrt{m}} \ll 1, \quad \text{and} \quad \frac{k_{\parallel} v_{\mathrm{the}}}{\omega} \sim 1, \tag{4.2}$$

which is true for large $m \gg 1$.

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If there is an $N \gg 1$ for which $\hat{g}_{N+1} \ll \hat{g}_N$ in the sense of (4.1), then the Nth component of the kinetic equation is

$$\omega \hat{g}_N = k_\parallel v_{\rm the} \sqrt{\frac{N}{2}} \hat{g}_{N-1}. \tag{4.3}$$

We then use this solution for $\hat{g}_N$ in the equation for the $N-1$ component and solve for the $N-1$ component as a function of the $N-2$ component

$$\left[\omega - k_\parallel v_{\rm the} \frac{k_\parallel v_{\rm the} N/2}{\omega}\right] \hat{g}_{N-1} = k_\parallel v_{\rm the} \sqrt{\frac{N-1}{2}} \hat{g}_{N-2}. \tag{4.4}$$

If ω is complex g_{N-1} is not singular, and after n iterations we have

$$\hat{g}_{N-n} = k_\parallel v_{\rm the} \sqrt{\frac{N-n}{2}} \hat{g}_{N-(n+1)} \frac{1}{\omega - k_\parallel v_{\rm the} \frac{k_\parallel v_{\rm the}(N-n+1)/2}{\omega - k_\parallel v_{\rm the} \frac{k_\parallel v_{\rm the}(N-n+2)/2}{\cdots - k_\parallel v_{\rm the} \frac{k_\parallel v_{\rm the} N/2}{\omega}}}}. \tag{4.5}$$

Equation (4.5) is a *finite* continued fraction that can be used to generate the *infinite* one which is the exact solution of the collisionless problem. Effects of finite collisionality within this formalism have been considered somewhere else (Sugama et al. 2001; Hatch et al. 2013; Loureiro et al. 2013; Parker and Dellar 2014; Zocco et al. 2014). Now, when $N - n = 3$, we are able to write $\hat{g}_3$ in (3.4) explicitly as a function of all other $\hat{g}_m$ up to $\hat{g}_N$, and therefore the electron temperature perturbation in terms of the continued fraction, which does not need to be truncated. The result is

$$\frac{v_{\rm the}}{u_{\parallel \rm e}} \frac{\delta T_\parallel}{T_{0\rm e}} = \frac{k_\parallel}{|k_\parallel|} \frac{2}{\zeta} \frac{1}{1 - \frac{1}{\zeta^2} \frac{3/2}{1 - \frac{1}{\zeta^2} \frac{4/2}{1 - \frac{1}{\zeta^2} \frac{5/2}{\ddots}}}}} \equiv \frac{k_\parallel}{|k_\parallel|} \frac{2}{\zeta} \frac{1}{\mathscr{D}^{(2)}}. \tag{4.6}$$

We now prove that (4.6) and (2.14) are the same.

We rewrite (4.6) in the following way

$$\frac{v_{\rm the}}{u_{\parallel \rm e}} \frac{\delta T_\parallel}{T_{0\rm e}} = \frac{k_\parallel}{|k_\parallel|} \frac{1}{\zeta} \frac{1 - \frac{1}{\zeta^2} \frac{2/2}{\mathscr{D}^{(2)}} - 1}{-\frac{1}{2\zeta^2}}, \tag{4.7}$$

and notice that

$$\frac{1}{\zeta} \frac{1 - \frac{1}{\zeta^2} \frac{2/2}{\mathscr{D}^{(2)}} - 1}{-\frac{1}{2\zeta^2}} = \frac{\frac{1}{\zeta} - \frac{1}{\zeta \mathscr{D}^{(1)}}}{-\frac{1}{2\zeta^2 \mathscr{D}^{(1)}}}, \tag{4.8}$$

where

$$\mathscr{D}^{(1)} = \frac{1}{1 - \frac{1}{\zeta^2} \frac{2/2}{1 - \frac{1}{\zeta^2} \frac{3/2}{1 - \frac{1}{\zeta^2} \frac{4/2}{\ddots}}}}. \tag{4.9}$$

Similarly, we rewrite the RHS of (4.8) as

$$\frac{\frac{1}{\zeta} - \frac{1}{\zeta \mathscr{D}^{(1)}}}{-\frac{1}{2\zeta^2 \mathscr{D}^{(1)}}} = \frac{\frac{1}{\zeta} + 2\zeta \left[1 - \frac{1}{2\zeta^2 \mathscr{D}^{(1)}} - 1\right]}{1 - \frac{1}{2\zeta^2 \mathscr{D}^{(1)}} - 1}, \tag{4.10}$$

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which implies

$$\frac{v_{\text{the}}}{u_{\parallel \text{e}}}\frac{\delta T_{\parallel}}{T_{0\text{e}}} = \frac{k_{\parallel}}{\left|k_{\parallel}\right|}\frac{\frac{1}{\zeta\mathscr{D}^{(0)}} + 2\zeta\left[1-\frac{1}{\mathscr{D}^{(0)}}\right]}{1-\frac{1}{\mathscr{D}^{(0)}}}, \tag{4.11}$$

with

$$\mathscr{D}^{(0)} = \frac{1}{1-\frac{1}{\zeta^2}\frac{1/2}{1-\frac{1}{\zeta^2}\frac{2/2}{1-\frac{1}{\zeta^2}\frac{3/2}{\ddots}}}}. \tag{4.12}$$

Since

$$\frac{dZ(\zeta)}{d\zeta} = -2\left[1+\zeta Z(\zeta)\right], \quad \forall\zeta \tag{4.13}$$

with $Z(0) = i\sqrt{\pi}$, successive differentiation of (4.13) yields (McCabe 1984)

$$\frac{Z^{(n)}}{Z^{(n-1)}} = \frac{-2n}{2\zeta + Z^{(n+1)}/Z^{(n)}}, \tag{4.14}$$

and therefore (see also (7.1.15) of Abramowitz and Stegun (1972))

$$Z(\zeta) = -\frac{1}{\zeta\mathscr{D}^{(0)}}. \tag{4.15}$$

Hence, we showed that

$$\frac{2}{\zeta}\frac{1}{\mathscr{D}^{(2)}} = -\frac{Z(\zeta) - 2\zeta\left[1+\zeta Z(\zeta)\right]}{1+\zeta Z(\zeta)}. \tag{4.16}$$

This concludes our proof. The same equivalence is valid for the collisionless damping of electrostatic plasma oscillations originally studied by Landau.

5. Conclusion

We solved the problem of linear collisionless Landau damping for kinetic and shear Alfvén waves by using both the traditional Laplace transform approach and a Hermite transform.

We introduced a recursive formula for the coefficients of the inverse Hermite transform that allowed us to construct a finite continued fraction whose extension to infinite elements gave a new exact solution for the electron distribution function. We proved that this new solution is equivalent to the solution found by using Landau contour integration.

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