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Surface waves on the spin-1/2 quantum magnetoplasma half-space

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We present a theoretical investigation on the propagation of surface waves on the magnetized degenerate electron plasma half-space with spin effects. Using magnetohydrodynamic model with quantum effects due to the Bohm potential, Fermi degenerate pressure and electron spin, the dispersion relations of surface plasmon polaritons (SPPs) are derived. The dispersion relation of electrostatic surface waves is also obtained by taking electrostatic limit.

1. Introduction

The investigation on surface waves began with the pioneering theoretical works by Trivelpiece and Gould (1959). They restricted their attentions to electrostatic surface waves on cold cylindrical plasma columns. The effect of a finite temperature on the dispersion relation of electrostatic surface waves was studied by Ritchie (1963), who applied a simple hydrodynamical model, and Guernsey (1969), who adopted the kinetic theory. Vedenov provided a general dispersion relation without limiting to electrostatic approximation, which showed that in cold plasma half-space, surface plasma polaritons existed with frequencies ranging from $\omega_{spps} \rightarrow \omega_P/\sqrt{2}$ down to $\omega_{spps} = 0$ (Vedenov 1965).

Recently, the quantum plasmas has been a rapidly growing field of research due to its promising applications in quantum well (Manfredi and Hervieux 2007), plasmonics (Atwater 2007), spintronics (Wolf et al. 2001), astrophysics (Palmer et al. 2005) and ultra-cold plasmas (Robinson et al. 2000). Since the surface plasmon is the collective electrostatic excitation of free electrons near a plasma dielectric surface, the quantum effects will consequentially affect the propagation properties of the surface plasmon. The surface plasmon (SP)-polariton (SPP) on the vacuum-quantum plasma or quantum plasma-quantum plasma interface have aroused wide attention. Relevant researches include surface Langmuir oscillations in semi-bounded quantum plasmas (Chang and Jung 2008), electrostatic/electromagnetic modes on the quantum plasma half-space (Lazar et al. 2007), propagation of the transverse electric surface modes on semi-bounded quantum plasma in the presence of external magnetic field (Mohamed 2010), self-excited surface plasmon polaritons at the interface of counterstreaming plasmas (Lazar et al. 2009), and the surface waves on the relativistic quantum plasma half-space (Zhu et al. 2013). Meanwhile, some works on wave phenomena in bounded plasmas are investigated by the Trivelpiece-Gould (TG) modes in cylindrical geometry, such as rotating field confinement of pure electron plasmas using TG Modes (Anderegg et al. 1998), dispersion properties of Trivelpiece-Gould waves in periodic

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plasma waveguides (Zaginaylov 2001), and collisional damping of plasma waves on a pure electron plasma column (Anderson and O'Neil 2007). The corresponding quantum case was discussed by H. Tercas (Tercas et al. 2008). They derived the dispersion relation for the electrostatic waves below the electron plasma frequency in a dense quantum plasma, and showed that in the classical case the dispersion relation reduces to the expression obtained for the well-known TG modes.

The above researches only consider the nonspin quantum effects (e.g., the Bohm potential, the Fermi degeneracy pressure, and certain quantum electrodynamical effects). It was shown that quantum features due to intrinsic magnetic moment of electrons become noticeable and spin effects in plasmas are found to be somewhat different from those of nonspin quantum effects in plasmas. In this paper, we will present a theoretical investigation on the propagation of surface waves on the magnetized degenerate electron plasma half-space, with a special focus on the properties caused by the electron spin.

This paper is organized as follows. In Sec. 2, starting from the quantum magnetohydrodynamic model, the dielectric function of spin quantum plasmas is derived. In Sec. 3, the dispersion relation of SPPs and surface waves are obtained. In Sec. 4, the discussion and conclusion for quantum effects are presented.

2. Dielectric function

The spin quantum plasmas is described by the coupled equations for the density n, the fluid velocity $\mathbf{v}$ and the spin vector $\mathbf{s}$. The density n and the fluid velocity $\mathbf{v}$ satisfy the continuity equation (Brodin and Marklund 2007; Marklund and Brodin 2007)

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0, \tag{1}$$

and the momentum equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v} = -\frac{e}{m}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right) - \frac{2T_{Fe}}{mn_0}\nabla n + \frac{\hbar^2 n}{2m^2 n_0}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{2\mu_e}{m\hbar}\nabla(\mathbf{S}\cdot\mathbf{B}), \tag{2}$$

where $T_{Fe} = mv_{Fe}^2/2$ is the Fermi temperature of electrons, $v_{Fe} = (3\pi^2 n_0)^{1/3}\hbar/m$ is the Fermi velocity of electrons. e is the magnitude the electron charge, m is the electron mass, and $\hbar$ is Planck's constant divided by 2π. For an electron, the magnetic moment is given by $\mu_e = -\mu_B$, where $\mu_B = e\hbar/2mc$ is the Bohr magneton.

The spin vector satisfy the evolution equation (Brodin and Marklund 2007; Marklund and Brodin 2007)

$$\frac{d\mathbf{S}}{dt} = \frac{2}{\hbar}\mu_B\mathbf{B}\times\mathbf{S}, \tag{3}$$

where the spin-thermal coupling terms and the nonlinear spin fluid contribution are neglected. Neglecting the spin inertia, the spin vector is determined from $\mathbf{B}\times\mathbf{S} = 0$, which has a solution (Brodin and Marklund 2007)

$$\mathbf{S} = -\frac{\hbar}{2}\eta\left(\frac{\mu B}{k_B T_{Fe}}\right)\hat{\mathbf{B}}, \tag{4}$$

where B denotes the magnitude of the magnetic field, and $\hat{\mathbf{B}}$ is a unit vector in the direction of the magnetic field. $\eta(\alpha) = \tanh\alpha$ is the Brillouin function, where $\alpha = \mu_B B_0/(k_B T_F)$.

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After linearization, the first order of the continuity equation and the momentum equation are obtained as

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 = 0, \tag{5}$$

and

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{e}{m}\left(\mathbf{E}_1 + \frac{\mathbf{v}_1}{c} \times \mathbf{B}_0\right) - \frac{2T_{Fe}}{mn_0}\nabla n_1 + \frac{\hbar^2}{4m^2 n_0}\nabla\nabla^2 n_1 - \frac{2\mu_e}{m\hbar}\nabla(\mathbf{S} \cdot \mathbf{B}_1), \tag{6}$$

where $n_1 \ll n_0$ is a small electron density perturbation in the equilibrium number density n_0.

The electric field $\mathbf{E}_1$ and magnetic field $\mathbf{B}_1$ are governed by the Maxwell equations

$$\nabla \times \mathbf{E}_1 = -\frac{1}{c}\frac{\partial \mathbf{B}_1}{\partial t}, \tag{7}$$

$$\nabla \times \mathbf{B}_1 = \frac{1}{c}\frac{\partial \mathbf{E}_1}{\partial t} + \frac{4\pi}{c}\mathbf{J}, \tag{8}$$

where $\mathbf{J} = -en_0\mathbf{v}_1 + c\mathbf{J}_M$ is the total current density, and $\mathbf{J}_M = -\nabla \times (2n_0\mu_B\mathbf{S}/\hbar)$ is the magnetization spin current.

We suppose the external magnetic field as $\mathbf{B}_0 = B_0\hat{z}$ which respect to the propagation direction determined by the wavenumber $\mathbf{k} = k\hat{y}$ of the wave. Consider the propagation of a linearly polarized electromagnetic wave in the plasma system. The polarized electric field is $\mathbf{E}_1 = E_1\hat{x}$ and polarized magnetic field is $\mathbf{B}_1 = B_1\hat{z} = -kcE_1/\omega\hat{z}$.

Under the above assumptions, the linearized continuity (5) and motion (6) can be written as

$$n_1 = \frac{n_0}{\omega}\mathbf{k} \cdot \mathbf{v}_1, \tag{9}$$

and

$$-im\omega\mathbf{v}_1 = -en_0\left(\mathbf{E}_1 + \frac{\mathbf{v}_1}{c} \times \mathbf{B}_0\right) - \frac{i}{3}mv_{Fe}^2 n_1\mathbf{k} - \frac{i\hbar^2 k^2}{4m}n_1\mathbf{k} + i\eta(\alpha)\mu_B n_0 B_1\mathbf{k}. \tag{10}$$

The magnitude of spin magnetization current can be written as

$$J_M = i\mu_B\eta(\alpha)n_0k^2v_y/\omega. \tag{11}$$

By solving the above (9)–(11) along with Maxwell (7) and (8), we can obtain the following dispersion relation

$$\omega^2 = k^2c^2 + \tilde{\omega}_p^2, \tag{12}$$

where $\tilde{\omega}_p^2 = \omega_p^2[1+\frac{2\omega_c\eta(\alpha)\mu_B ck}{e\omega^2(1-A_e)}+\frac{\eta^2(\alpha)\mu_B^2c^2k^4}{e^2\omega^2(1-A_e)}][\frac{\omega^2(1-A_e)}{\omega^2(1-A_e)-\omega_c^2}]$ is introduced as the magnetization plasma frequency. $\omega_c = eB_0/(mc)$ is the electron cyclotron frequency, and

$$A_e = \left(v_{Fe}^2 + \frac{\hbar^2k^2}{4m^2}\right)\frac{k^2}{\omega^2}. \tag{13}$$

The dielectric function ε for spin-1/2 quantum plasmas is obtained as

$$\varepsilon = 1 - \frac{\tilde{\omega}_p^2}{\omega^2}. \tag{14}$$

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3. Dispersion relation

The surface plasmon (SP)-polariton (SPP) is the collective electrostatic/electromagnetic excitation of electrons on the vacuum-plasma or plasma-plasma interface. The SPP propagates along the plasma interface and is evanescent in plasmas. At the interface of a vacuum and plasmas, the surface modes are described by the dispersion relation (Fourkal et al. 2006; Lazar et al. 2009; Moslem et al. 2009; Yu and Zhelyazkov 1978):

$$\varepsilon(k^2c^2-\omega^2)^{1/2}=-(k^2c^2-\omega^2\varepsilon)^{1/2}, \tag{15}$$

where $\varepsilon(\omega,k)$ is the dielectric function of plasmas. For cold plasmas, $\varepsilon(\omega,k)=1-\omega_p^2/\omega^2$, but for spin quantum plasmas, the dielectric function is more complex and has been obtained in the last section.

Now, inserting (14) into (15), we get the dispersion relation of SPPs on the spin quantum plasma half-space

$$\frac{\omega^2}{k^2c^2}=1+\frac{\omega^2}{\omega^2-\tilde{\omega}_p^2}. \tag{16}$$

In (16), there involves three types of quantum effects, i.e., the statistical effects, Bohm potential, and spin magnetization. Without external magnetic field, i.e., $B_0\to 0$, (16) is degenerated to the well known dispersion relation of surface waves on the cold plasma half-space

$$\omega^2=\frac{1}{2}\omega_p^2+k^2c^2-\frac{1}{2}\sqrt{\omega_p^4+4k^4c^4}. \tag{17}$$

The electrostatic limit of equation (16) is taken by $c\to\infty$, as a result, we have the dispersion relation of electrostatic surface waves on the spin quantum plasma half-space

$$\omega^2=\frac{1}{2}\tilde{\omega}_p^2. \tag{18}$$

When $B_0\to 0$, (18) is degenerated to the well known dispersion relation for the electrostatic surface waves on the cold plasma half-space. Equation (18) indicates that surface Langmuir oscillations can propagate on the cold plasma half-space due to quantum effects.

4. Discussion and conclusion

In this section, we adopt the parameters of metal for quantitative calculation. The parameters chosen are $n_0=10^{24}\mathrm{cm}^{-3}$, $B_0\sim 10^9\mathrm{Gs}$, $T\sim 10^4\mathrm{K}$, $\omega_p=5.64\times 10^{16}\mathrm{s}^{-1}$, and $v_F=3.57\times 10^8$ cm/s. In quantum plasmas, since the de Broglie wavelength λ_B of electrons becomes comparable to, or even larger than the average interparticle distance of electrons (viz., $\lambda_B^3 n_0\geqslant 1$), quantum effects are expected to play a crucial role in plasma dynamic. From the expression $\lambda_B^3 n_0\geqslant 1$, we have

$$\frac{n_0}{T^{3/2}}\geqslant\left(\frac{\sqrt{mk_B}}{\hbar}\right)^3\sim 10^{16}\mathrm{cm}^{-3}/\mathrm{K}^{3/2}. \tag{19}$$

Obviously, the parameters of metals satisfy the quantum condition $\lambda_B^3 n_0\geqslant 1$.

Figure 1 describe the curves of the electromagnetic waves in classical plasmas and spin quantum plasmas. Figure 2 depicts the curves of the dispersion relation (16) and (17), and it shows that the frequency of SPPs has an obvious blue-shift produced by quantum effects, and the quantum correction can reach 10^{-1}, which is significant and

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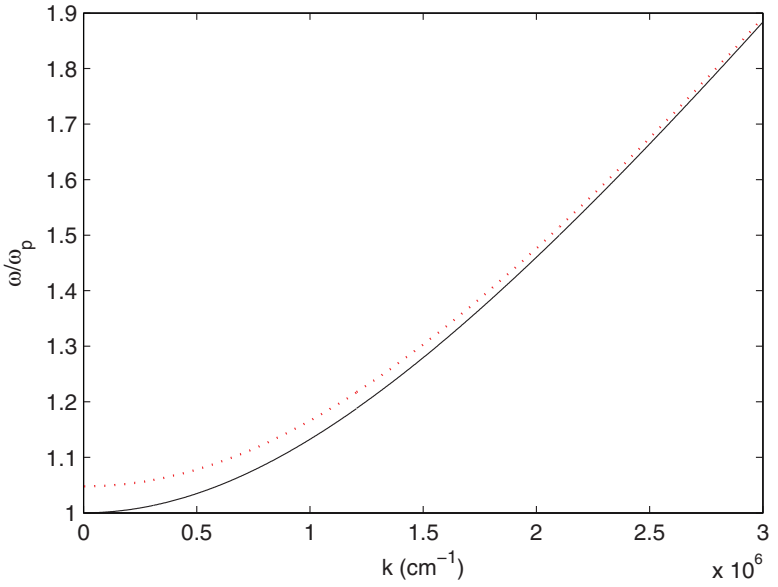


FIGURE 1. Solid line: Electromagnetic waves in cold plasmas. Dashed line: Electromagnetic waves in spin quantum plasmas. $n_0 = 10^{24}\text{cm}^{-3}$, $\omega_p = 5.64 \times 10^{16}\text{s}^{-1}$.

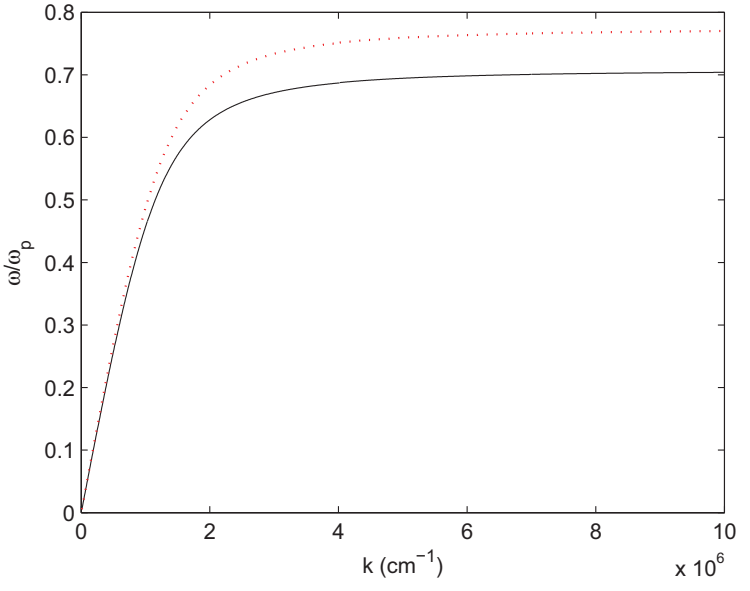


FIGURE 2. Solid line: Surface plasmon polaritons on a cold plasma half-space. Dashed line: Surface plasmon polaritons on spin quantum plasma half-space. $n_0 = 10^{24}\text{cm}^{-3}$, $\omega_p = 5.64 \times 10^{16}\text{s}^{-1}$, and $v_F = 3.57 \times 10^8\,\text{cm/s}$.

observable in experiments. It is well known that electron Landau damping occurs due to the energy exchange between an wave with phase velocity v_{ph} and electrons in the plasma with velocity approximately equal to v_{ph}. In quantum plasmas, electrons are degenerate and obey the Fermi-Dirac distribution. Since electrons are bounded under the Fermi level, the velocities of electrons are less than the Fermi velocity

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v_{Fe}. Therefore, when $v_{Fe} < v_{ph}$, there is no resonant electron and electron Landau damping will not occur. However, when $v_{Fe} > v_{ph}$, there exist resonant electrons and electron Landau damping will occur. After simple calculation we can found that when wavevector $k < 1.12 \times 10^8 \mathrm{cm}^{-1}$, there is no electron Landau damping. When $k > 1.12 \times 10^8 \mathrm{cm}^{-1}$, the electron Landau damping will occur and prevent wave propagation. In the Fig. 2, wavevector changes from 0 to $10^7 \mathrm{cm}^{-1}$, therefore, electron Landau damping does not exist in this range.

To summarize, we present a theoretical investigation on the propagation of surface waves on the magnetized degenerate electron plasma half-space with spin effects. Using magnetohydrodynamic model with quantum effects due to the Bohm potential, Fermi degenerate pressure and electron spin, the dispersion relations of SPPs are derived. The dispersion relation of electrostatic surface waves is also obtained by taking electrostatic limit.

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