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Task planning for serial redundant manipulators

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SUMMARY

The first stage in developing a task planning language for redundant manipulators is that of constructing an algorithm to produce a schedule of joint displacements for the specified task. This algorithm may be based upon the use of an inverse infinitesimal kinematic analysis for the majority of the path with a finite inverse analysis used to provide an initial joint configuration and periodically to correct for any joint errors that may have accumulated. Here, it is assumed that there is complete correlation and coherence of solutions between these analyses, otherwise problems, such as a lack of repeatability may occur. This may be accomplished by modelling the manipulator as an elastic system which permits the development of a task planning method by using analyses which are all mutually coherent. In this paper, the method is developed and demonstrated using a redundant manipulator that simultaneously avoids obstables, singularities and joint motion limitations. The task planning method and associated analyses are applicable to the general serial redundant manipulator with an arbitrary degree of redundancy.

KEYWORDS: Task planning; Redundant manipulators; Elastic system.

1. INTRODUCTION

The purpose of this paper is to provide the basis for planning the tasks undertaken by serial redundant manipulators. There are essentially three methods for task planning; teach and replay procedures, simulation and inverse analytic approaches which are all applicable to non-redundant manipulators. Teach and replay procedures are less widely applicable to redundant manipulators and hence, simulation is the most popular approach because there is currently no satisfactory inverse analytic approach; consequently, this paper addresses this area.

The primary task of a manipulator is the task that the end effector is required to perform which, in geometric terms, normally defines the required, successive positions and orientations of the end effector. There may be additional requirements which the manipulator should fulfil such as, for example, the avoidance of obstacles, and these are called secondary tasks. For non-redundant manipulators, secondary tasks are not normally realisable since there is only a sufficient number of independently controllable joints to fulfil the primary task. The addition of extra joints permits the secondary tasks to be performed since the general dexterity of the manipulator is increased. The classes of manipulators possessing these extra joints, that provide for the performance of secondary tasks, are known as redundant manipulators. A manipulator with n independently controlled inputs which is performing primary tasks described by m independent co-ordinates is redundant if $n > m$, otherwise it is non-redundant.

In the present context, task planning is the process of developing a schedule of the independent inputs required to satisfy the specified tasks. The scheduling of non-redundant manipulators for given primary tasks is well-established, so this paper is concerned with the development of the additional procedures required for the simultaneous fulfilment of primary and secondary tasks by serial redundant manipulators.

2. KINEMATICS OF SERIAL REDUNDANT MANIPULATORS

Since manipulator tasks are often specified in terms of the end effector position and orientation, the successful execution of primary and secondary tasks requires a position solution to the inverse kinematics of the manipulator. If D represents an m-dimensional vector that uniquely describes the position and orientation of the end effector, and q represents an n-dimensional vector of joint positions, or simply the joint configuration, then the forward position kinematics may be expressed as

$$D = D(q) \tag{1}$$

The inverse solution of equation (1) is a non-trivial problem, since the analysis is under-constrained by $(n - m)$ equations. This is worsened by the fact that equation (1) is non-linear and consists of transcendental functions meaning that the determination of the solution is complex and computational slow. It is for this reason that most of the existing formulations are at the infinitesimal or rate level, as the resulting equations are linear. The forward rate kinematics may be written as

$$\delta D = J\delta q \tag{2}$$

where J is the $(m \times n)$ manipulator Jacobian that transforms from the infinitesimal joint space to the infinitesimal task space. However, there are a number of disadvantages of the infinitesimal methods. Firstly, the resulting set remains deficient by $(n - m)$ equations, but the linearity of these rate equations allows the use of well-established methods of solutions, such as the

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generalised inverse. Secondly, the use of an infinitesimal solution to predict changes in position provides only an approximation of the finite inverse position solution and after successive implementations, the accumulative errors become significant. Thirdly, solutions are dependent upon initial conditions or the initial joint configuration.

There are further problems if the inverse rate solution for redundant manipulators is not formulated properly. Previous research work[1,2] has shown that many existing methods are dimensionally improper when applied to manipulators of mixed joint types, leading to improper and non-interpretable solutions. This is apparent since many solutions involve the term (JJ^T), which is not applicable to manipulators of mixed joint types. Furthermore, a formulation of the inverse rate solution which does not allow for the instantaneous changes in the Jacobian results in a loss of control of the manipulator's configuration. This has especially serious consequences when the manipulator is required to perform cyclical tasks.

Typically, primary tasks are specified in terms of a large number of end effector positions, for example during path following. Attempting to use the inverse position analysis at each of these positions would result in a process which is far too alow to be of any practical use. However, the use of the inverse rate method on its own is equally unacceptable because of the reasons enumerated above. Thus, a procedure which combines the best features of both should be used to form the basis for a task planning method for redundant manipulators. Such a combined method would firstly use the inverse position analysis to determine the initial joint configuration, and subsequent steps would use an infinitesimal method. At certain intervals during the execution of the task, the finite analysis may be used to correct any configuration errors that may have accumulated as a result of approximating the steps with infinitesimal displacements. As a consequence, the computational time is significantly reduced, since for the majority of the task the computationally attractive infinitesimal analysis is used and the heavy computation of a finite inverse position analysis is kept to a minimum.

In order for the combined method to be effective, the methods must be properly formulated, as discussed above, and there must be total correlation and consistency of solutions between the two levels. If these properties are inherent, then, the infinitesimal analysis would compensate for geometric changes of the manipulator and would be applicable to repeatable tasks.

3. THE ELASTIC MODEL

The development of a combined method that overcomes the problems discussed in Section 2 requires the introduction of a suitable manipulator model. One such model is the conceptual elastic model as proposed by Mussa-Ivaldi and Hogan,[3] Kerr et al.[4] and Chung et al.[5] These researchers formulated the model only at the infinitesimal level, but it is possible to formulate the model at the finite displacement level,[6,7] giving rise to the inverse position kinematic solution. This model represents each joint as an elastic spring system such that

$$Q + s(q - q_0) = 0_{n\times 1} \tag{3}$$

where Q, q and q_0 are n-dimensional vectors of generalised joint forces, configuration and unloaded configuration respectively, and s is an $(n \times n)$ diagonal matrix of joint stiffnesses. The static equilibrium relationship between an m-dimensional externally applied generalised wrench vector, w, at the end effector and the generalised joint forces, Q, may be represented by

$$Q + J^T w = 0_{n\times 1} \tag{4}$$

Equating equations (3) and (4) yields

$$s(q - q_0) - J^T w = 0_{n\times 1} \tag{5}$$

Equation (5) represents the static equilibrium between the externally applied wrench at the end effector and the generalised joint forces.

3.1 The Finite Analysis of the Elastic Model

As discussed in Section 2, $(n - m)$ additional equations are required for a finite inverse position kinematic analysis. Hence, the number of equations in equation (5) must be reduced, yet maintain the full representation of the manipulator's static equilibrium requirement. Since the interest is in the joint variables and not the end effector variables, the wrench may be eliminated. By partitioning the n static equations into m and $(n - m)$ equations,[6] the wrench may be eliminated between the equation sets and this process provides the required $(n - m)$ static equations as

$$[A_{12}^T A_{11}^{-T} \vdots -I_{(n-m)}]q = 0_{(n-m)\times n} \tag{6}$$

where $A = s^{-1}J^T (\in \mathbb{R}^{n\times m})$.

Here, subscript 11 represents a partitioned set of m equations and subscript 12, the remaining $(n - m)$ equations. Augmenting the $(n - m)$ static equations with the m kinematic equations from equation (1) provides a soluble set of n equations in n unknowns yielding a solution that satisfies the end effector requirement and is in a state of static equilibrium. This solution is the finite inverse position kinematic analysis.

A direct solution for the augmented set of equations requires a numerical search in an n-dimensional space, and for manipulators with an appreciable number of joints and degree of redundancy, this may prove to be extremely time-consuming. However, it should be noted that by treating the equation sets separately and utilising the manipulator geometry, it is possible to reduce the search space from n to only $(n - m)$. Recalling that the computational time is approximately proportional to the power of the dimension of the search space, it is clear that the reduction has significant savings. Powell's method[8] for multi-dimensional non-linear functions is employed to search in this $(n - m)$ dimensional space.[6]

One of the inherent parameters in the model is the

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stiffness matrix and, in previous work,[7] the selection of an appropriate stiffness matrix in order to change the joint configuration in a predictable manner, that would realise secondary tasks, has been shown to be difficult. The selection of appropriate joint stiffnesses from a joint configuration is termed a finite inverse stiffness analysis. Such an analysis may be developed[9] from equation (5) and may be represented by the homogeneous equation

$$[J_{12}^T J_{11}^{-T} \Delta q_{11} - \Delta q_{22}] s_v = 0_{(n-m)\times n} \tag{7}$$

where s_v is an n-dimensional vector comprised of the diagonal elements of s. The $(n \times n)$ matrix, Δq, is a diagonal matrix comprised of the elements of the n-dimensional vector, $(q - q_0)$. The n stiffnesses of s_v are appropriately selected such that equation (7) is satisfied.

3.2 The Infinitesimal Analysis of the Elastic Model

Previously published work on the infinitesimal method[3–5] provides a relationship between the infinitesimal changes in the joint configuration as a result of infinitesimal changes in the end effector position and orientation. The capabilities of the infinitesimal method may be extended[10] to include the infinitesimal changes in joint configuration as a result of infinitesimal changes in elastic parameters, such as the joint stiffnesses and unloaded joint configuration. Due to the linear properties at the infinitesimal level, these changes may be included by the use of superposition. Such an infinitesimal method is referred to as a general infinitesimal method. The existence of such a method is of fundamental importance in applying the elastic model to a task planning method, since smooth progression from one desirable joint configuration and elastic state to another is realisable. Such an analysis may be developed by formulating the solution to equation (2) in the form of a general solution in terms of particular and homogeneous components. Secondly, equation (5) is differentiated, which provides n infinitesimal manipulator static equilibrium equations

$$\delta s(q - q_0) + s(\delta q - \delta q_0) - (\delta J^T w + J^T \delta w) = 0_{n\times 1} \tag{8}$$

As mentioned in Section 2, the term representing the instantaneous changes in the manipulator Jacobian, $(\delta J^T w)$, had previously been ignored resulting in the non-integrability of equation (8), which subsequently gives rise to the joint space drift or the repeatability problem. In order to avoid this problem and to ensure a complete correlation and consistency of solutions between the finite and infinitesimal solutions, in this analysis, the term is not ignored. It may be shown,[3,5,10] that

$$\delta J^T w = \Gamma \delta q \tag{9}$$

where Γ is an $(n \times n)$ matrix of second order terms that may be formulated using

$$\Gamma_{i,k} = \left[\sum_{j=1}^{m} \frac{\partial^2 D_{j,k}}{\partial q_i \, \partial q_k} w_j\right] = \left[\sum_{j=1}^{m} \frac{\partial J_{j,k}}{\partial q_i} w_j\right] \begin{Bmatrix} i = 1, \ldots, n \\ j = 1, \ldots, m \\ k = 1, \ldots, n \end{Bmatrix} \tag{10}$$

Combining equations (8) and (9) with the general solution of equation (2), and simplifying, the general infinitesimal equation may be expressed as

$$\begin{aligned} \delta q = &\begin{bmatrix} J_{11}^{-1} + J_{11}^{-1} J_{12} G_7^{-1} G_1 J_{11}^{-1} \\ -G_7^{-1} G_3 J_{11}^{-1} \end{bmatrix} [\delta D] \\ &+ \begin{bmatrix} -J_{11}^{-1} J_{12} G_7^{-1} G_3 & -J_{11}^{-1} J_{12} G_7^{-1} G_4 \\ G_7^{-1} G_3 & G_7^{-1} G_4 \end{bmatrix} [\delta s_v] \\ &+ \begin{bmatrix} -J_{11}^{-1} J_{12} G_7^{-1} G_5 & -J_{11}^{-1} J_{12} G_7^{-1} G_6 \\ G_7^{-1} G_5 & G_7^{-1} G_6 \end{bmatrix} [\delta q_0] \end{aligned} \tag{11}$$

where

$$G_1 = [J_{12}^T J_{11}^{-T} (\Gamma_{11} - s_{11}) - \Gamma_{21}] \in \mathbb{R}^{(n-m)\times m}$$

$$G_2 = [J_{12}^T J_{11}^{-T} \Gamma_{12} - (\Gamma_{22} - s_{22})] \in \mathbb{R}^{(n-m)\times(n-m)}$$

$$G_3 = [J_{12}^T J_{11}^{-T} \Delta q_{11}] \in \mathbb{R}^{(n-m)\times m}$$

$$G_4 = [-\Delta q_{22}] \in \mathbb{R}^{(n-m)\times(n-m)}$$

$$G_5 = [-J_{12}^T J_{11}^{-T} s_{11}] \in \mathbb{R}^{(n-m)\times m}$$

$$G_6 = [s_{22}] \in \mathbb{R}^{(n-m)\times(n-m)}$$

$$G_7 = [G_2 - G_1 J_{11}^{-1} J_{12}] \in \mathbb{R}^{(n-m)\times(n-m)}$$

The first term of equation (11) permits the smooth progression from one kinematic state to another for a constant elastic state. The remaining two terms permit the smooth progression from one elastic state to another for a given end effector state. All three terms together permit the smooth progression from one kinematic and elastic state to another.

4. TASK PLANNING

As a consequence of the consistency and correlation of solutions between the three analyses, they may be used together to realise secondary tasks such as the avoidance of obstacles within the workspace, kinematic singular configurations and the restrictions of joint motion ranges. Unlike many existing methods within the literature, the simultaneous execution of multiple tasks may be achieved without the need for any re-formulation of the analyses.

4.1. The Task Planning Method

Suppose that the primary task is for the end effector to move smoothly between two points within its workspace and a secondary task is also specified. At each end point of the path, a configuration is required. By generating the self-motion manifold of the manipulator (the infinite set of possible joint configurations for a specified end effector position and orientation) at each end point, such a pair of joint configurations may be selected from their respective manifolds. The actual configuration selected depends upon the nature of the secondary task. For example, if a workspace obstacle is to be avoided, then the configurations that avoid the obstacle are suitable selected. Having selected the joint configurations, the kinematic states at each end are fully defined.

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In order to determine the elastic state for each of the selected joint configurations at each end point, an inverse stiffness analysis is applied to the two configurations in order to provide two sets of joint stiffnesses. Hence, the elastic states at each end are fully defined.

The general infinitesimal analysis is now applied between the two kinematic and elastic states. Here, both the end effector position and joint stiffnesses are linearly interpolated between the two end states and the corresponding changes in joint configuration are calculated. The result is a smooth progression from one kinematic and elastic state to another. If required, a finite inverse analysis may be periodically executed in order to correct for any errors that may have accumulated from the inherent approximation of infinitesimal work.

This task planning method is a general procedure and hence, is applicable to both planar and spatial serial redundant manipulators of an arbitrary degree of redundancy. As a simple illustration of the concurrent use of the three analyses, the avoidance of obstacles by a planar manipulator is presented in Section 4.2.

4.2. Avoiding Workspace Obstacles

For the purposes of simplicity, a 3R planar manipulator is used for this and the subsequent example. If the orientation of the end effector is not specified, then the manipulator has a degree of redundancy of one ($m = 2, n = 3$). The manipulator is required to follow a specified straight-line path between two circular obstacles, labelled A and B in Figure 1, and avoid them. The demonstration is very simple, but the method may be applied to more complex tasks as will be seen in a later example.

The self-motion of the manipulator at the initial end effector position is generated and an initial configuration is selected from the self-montion that avoids obstacle A. Using this procedure, the final configuration is also specified. These initial and final states are specified by subscript A and subscript B respectively corresponding to obstacles A and B. For the pair of configurations, an inverse stiffness analysis is applied which provides two equations from equation (7). The pair of equations may be solved separately in terms of a general solution to provide two distinct coresponding joint stiffnesses. The numerical example follows.

The 3R planar redundant manipulator is defined with link lengths $a_i = 30$ cm and an unloaded joint configuration of $q_{oi} = 0°$ $(i = 1, \ldots, 3)$. The end effector at the initial and final positions are specified as

$$D_A = \begin{bmatrix} -20.000 \\ 70.000 \end{bmatrix} \text{cm and } D_B = \begin{bmatrix} 70.000 \\ 40.000 \end{bmatrix} \text{cm respectively.}$$

From the self-motion, at D_A and D_B, the initial and final configurations are selected to avoid the obstacles. There are techniques for selecting a suitable joint configuration from the manifold that avoids these obstacles, such as the well-known potential field method. This method exhibits problems when the manipulator is required, for example, to perform tasks in the vicinity of a corner, since the potential fields of the two faces (obstacles) that meet at the corner interfere with each other. One method of overcoming this, which is favoured by the authors, is to develop a distance relationship between the closest point on the manipulator and the boundaries of the obstacles using simple vector geometry. Any configuration that is greater than a pre-defined distance from the obstacles or threshold is regarded as acceptable. This obstacle avoidance selection procedure is applicable to the generalised serial manipulator and the formulation only requires the homogeneous transformation equations. A similar procedure may be extended to other performance criteria. Adopting this procedure, the configurations are selected appropriately as

$$q_A = [75.000° \ 83.444° \ -68.705°]^T$$

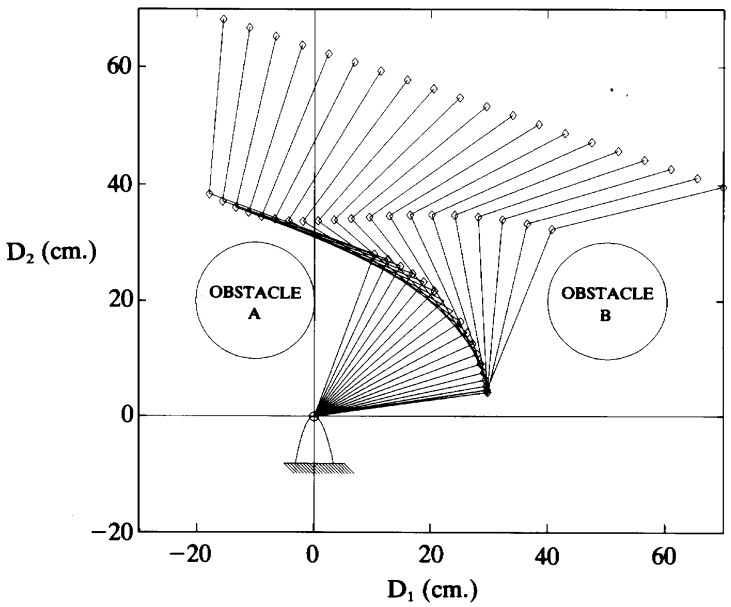


Fig. 1. Avoiding Workspace Obstacles.

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and

$$q_B = [30.000^\circ\ 32.059^\circ\ -64.930]^T$$

Applying the inverse stiffness analysis at each configuration provides two equations (from equation (7))

$$[0.815\ -0.659\ 1.000]s_{v(A)} = 0 \tag{12}$$

$$[-0.585\ 1.000\ 0.017]s_{v(B)} = 0 \tag{13}$$

The corresponding general solutions to equation (12) and (13) are

$$s_{v(A)} = \begin{bmatrix} 0.809 \\ 1.000 \\ 0.000 \end{bmatrix} s_{2(A)} + \begin{bmatrix} -1.227 \\ 0.000 \\ 1.000 \end{bmatrix} s_{3(A)} \tag{14}$$

$$s_{v(B)} = \begin{bmatrix} 1.709 \\ 1.000 \\ 0.000 \end{bmatrix} s_{2(B)} + \begin{bmatrix} 0.029 \\ 0.000 \\ 1.000 \end{bmatrix} s_{3(B)} \tag{15}$$

To ensure positive stiffnesses in equation (14), select $s_{2(A)} = 3$ N cm and $s_{3(A)} = 1$ N cm, and for equation (15) select $s_{2(B)} = s_{3(B)} = 1$ N cm for convenience. This provides normalised joint stiffnesses of

$$s_{v(A)} = \begin{bmatrix} 0.400 \\ 1.000 \\ 0.333 \end{bmatrix} \text{N cm and } s_{v(B)} = \begin{bmatrix} 1.000 \\ 0.576 \\ 0.576 \end{bmatrix} \text{N cm}$$

It is interesting to note that the presence of these free stiffness variables permits the potential for satisfying further requirements, or for optimisation. The end effector path and the joint stiffnesses are linearly interpolated between their respective values, such that the general infinitesimal method may be applied between the two kinematic and elastic states. During the path, inverse position analyses may be applied periodically to correct for any errors that may have accumulated. Additionally, the distance between the closest point on the manipulator and the obstacle boundary may be continually monitored in case of collision. Should a predicted collision occur, corrective action is necessary, such as re-selecting the configurations from the manifold and re-interpolating. As would be expected, the final joint configuration from such a method provides the specified final joint configuration. The path is successfully followed while avoiding both the obstacles, as seen in Figure 1.

4.3. Performing Multiple Secondary Tasks

Again, the same 3R planar redundant manipulator is used from the previous example in Section 4.2. Each joint of the manipulator is constructed such that the joint angle range extends from $-\pi$ to $+\pi$. The manipulator is required to follow a defined outline of a square, with two defined obstacles within the workspace, as shown in Figure 2. The corners of the square are labelled A, B, C and D and the co-ordinates of each corner are

$$A(40, -10)\text{ cm} \quad B(65, -10)\text{ cm}$$

$$D(40, -35)\text{ cm} \quad C(65, -35)\text{ cm}$$

The two obstacles are defined in the form of vertical barriers. The first obstacle is 110 cm long and is placed from the top end of the work area at co-ordinates (20, 90) cm to (20, −20) cm The second obstacle is 45 cm. long and is placed from the bottom end of the workspace at co-ordinates (20, −90) cm to (20, −45) cm. With both obstacles in place, this leaves a vertical 25 cm opening between them. The home end effector position, H is

$$D_H = [-10\ -20]^T \text{ cm}$$

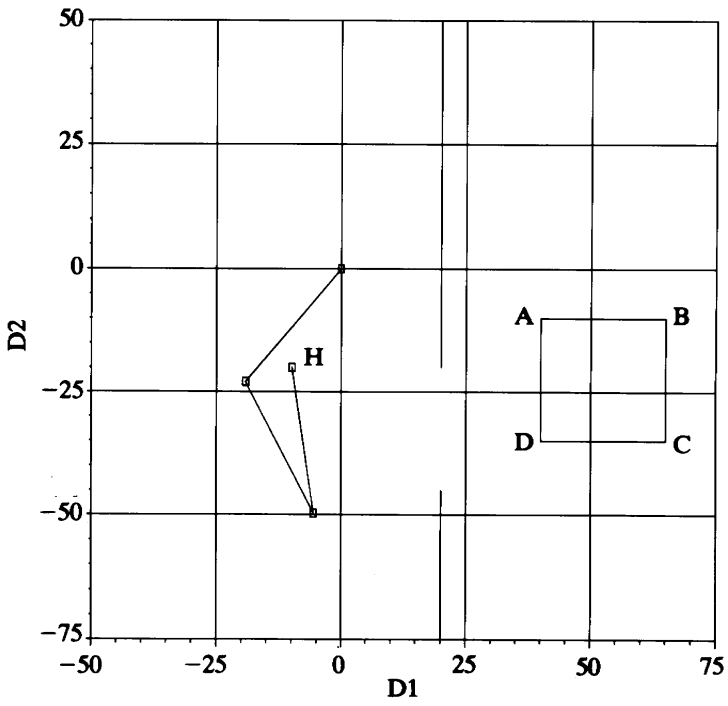


Fig. 2. The Workspace with the Obstacles, Square and Home Configuration.

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and the home joint configuration, q_H, is chosen, fairly arbitrarily from the self-motion set as

$$q_H = [-130.000^\circ \; 67.156^\circ \; 161.295^\circ]^T$$

In this example, during the approach to the square, the end effector must move from the home position, D_H, through the opening and then to corner A of the square. In order to accomplish this, it is beneficial to select an end effector position at the opening that maximises the distance from both barriers. For this purpose, the mid-point of the opening is selected; this position is labelled E and is at $(20, -32.5)$ cm. A suitable joint configuration must now be selected at E that would avoid collision with the obstacles. This configuration is selected from the manifold using the obstacle avoidance selection procedure as described in the previous example in Section 4.2. The inverse stiffness analysis is now applied at both configurations in order to select a set of suitable joint stiffness. The general infinitesimal method is then applied between the joint configuration at H, q_H, and the end effector position at E, D_E, using the computed joint stiffnesses. Using the same procedure at E, the configuration is selected at A and the general infinitesimal method is similarly executed between E and A. Hence, applying an inverse stiffness analysis to the home configuration, H, this gives, from equation (7)

$$[1.000 \quad -0.689 \quad -0.670]s_{v(H)} = 0 \tag{16}$$

where $s_{v(H)}$ is an n-dimensional vector of joint stiffnesses in joint configuration H. If the joint stiffnesses are chosen to satisfy equation (16), then the configuration, q_H, is in a state of static equilibrium. The next stage is to link the home configuration with the position at the mid-point of the obstacles, position E, and in order to do this, a configuration for E must be selected from the self-motion of the manipulator at E which is illustrated in Figure 3.

It is clear from Figure 3 that a range of this self-motion subset results in collision with the upper obstacle. A suitable configuration for position E is chosen from those which are far from collision as

$$q_E = [-100.000^\circ \; 28.339^\circ \; 129.947^\circ]^T$$

where q_E represents the configuration at E. Applying the inverse stiffness analysis to this configuration gives

$$[-0.698 \; -0.293 \; 1.000]s_{v(E)} = 0 \tag{17}$$

Solutions to equations (16) and (17) are

$$s_{v(H)} = \begin{bmatrix} 0.689 \\ 1.000 \\ 0 \end{bmatrix} s_{2(H)} + \begin{bmatrix} 0.670 \\ 0 \\ 1.000 \end{bmatrix} s_{3(H)} \tag{18}$$

$$s_{v(E)} = \begin{bmatrix} -0.420 \\ 1.000 \\ 0 \end{bmatrix} s_{2(E)} + \begin{bmatrix} 1.432 \\ 0 \\ 1.000 \end{bmatrix} s_{3(E)} \tag{19}$$

For simplicity, select values of $s_{2(H)} = s_{3(H)} = 1$ N cm which gives, when normalised,

$$s_{v(H)} = \begin{bmatrix} 1.000 \\ 0.736 \\ 0.736 \end{bmatrix} \text{N cm}$$

For equation (19), provided that the ratio of

$$\frac{s_{2(E)}}{s_{3(E)}} \leq 3.408$$

then $s_{v(E)}$ has all elements positive. For simplicity, select

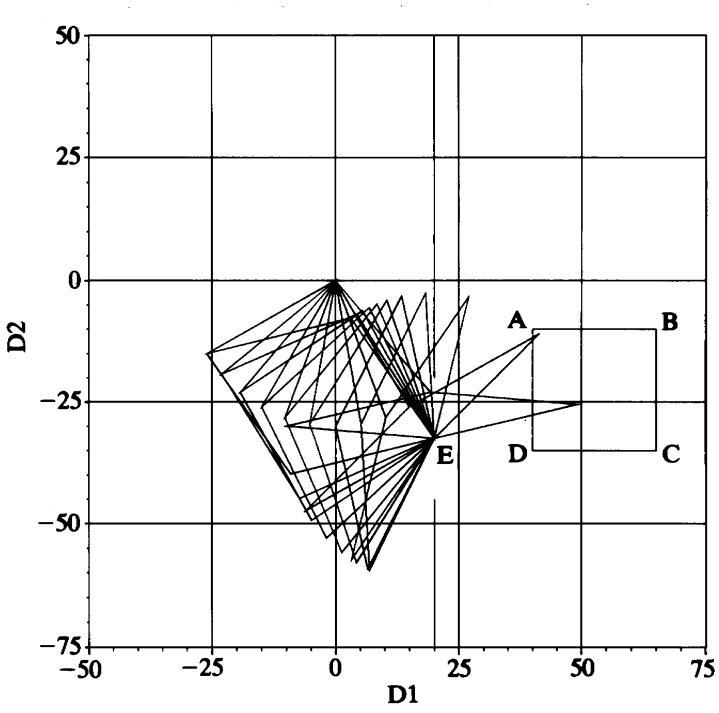


Fig. 3. A Subset of the Self-motion at Position E.

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the ratio equal to one, such that, $s_{2(E)} = s_{3(E)} = 1$ N cm which gives, when normalised,

$$s_{v(E)} = \begin{bmatrix} 1.000 \\ 0.989 \\ 0.989 \end{bmatrix} \text{N cm}$$

Applying the general infinitesimal method between the home configuration, q_H, and the end effector position at E, D_E, using the above joint stiffnesses results in the solution given by the selected joint configuration at E. This path is illustrated in Figure 4.

Four configurations are selected at A, B, C and D using the same procedure. Between E and A and along each side of the square there exists a pair of configurations (one for each corner), and the inverse stiffness analysis is applied to each configuration in order to select a suitable set of joint stiffnesses. The general infinitesimal analysis is then applied between the configurations. After applying this procedure five times the manipulator has returned to corner A. For the return from the square at corner A, back to the home position, H, the same path and configurations may be used in reverse, provided that the joint stiffnesses are the same at the start and finish of the cycle, which would be the result in the absence of joint space drift. Joint configurations for the four corners A, B, C and D are all selected using the obstacle avoidance selection procedure; these are

$$q_A = [-100.000° \quad 88.553° \quad 69.653°]^T$$

$$q_B = [-65.000° \quad 59.805° \quad 46.764°]^T,$$

$$q_C = [-72.000° \quad 44.617° \quad 41.524°]^T$$

and

$$q_D = [-125.000° \quad 100.405° \quad 28.534°]^T.$$

The inverse stiffness analysis applied in turn to these selected configurations, together with choices of the 'free' stiffness variables to ensure positive stiffnesses throughout, gives the joint stiffnesses at the corners as

$$s_{v(A)} = \begin{bmatrix} 0.400 \\ 0.500 \\ 1.000 \end{bmatrix} \text{N cm}, \quad s_{v(B)} = \begin{bmatrix} 0.735 \\ 0.500 \\ 1.000 \end{bmatrix} \text{N cm},$$

$$s_{v(C)} = \begin{bmatrix} 0.703 \\ 0.500 \\ 1.000 \end{bmatrix} \text{Nc m}, \quad \text{and} \quad s_{v(D)} = \begin{bmatrix} 0.207 \\ 0.500 \\ 1.000 \end{bmatrix} \text{N cm}$$

It is now possible to use the general infinitesimal method from E to A and along each side of the square, and the complete path from the home position H to the following of the square and back to H is shown in Figure 5.

5. DISCUSSION

It is clear that the three analyses—the inverse position analysis, the inverse stiffness analysis and the general infinitesimal method may be used collectively according to a structured procedure in order to accomplish primary and secondary tasks successfully.

The secondary tasks that were selected for demonstration were the simultaneous avoidance of obstacles within the workspace, kinematic singular configurations and the restriction of joint motion ranges. Such secondary tasks are generally regarded as probably the most common tasks for redundant manipulators and the combined usage of the three methods as the basis of a powerful task planning method is clear. However, the application of such a task planning method to other secondary tasks is currently under investigation.

A pair of selected configurations that fulfil the relevant local requirements will not guarantee a smooth and collision-free path, since prior to executing the general

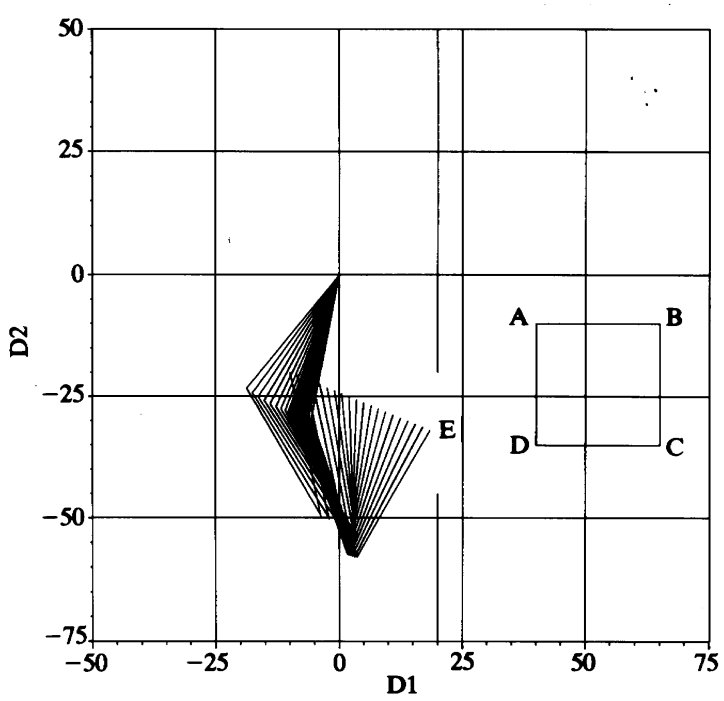


Fig. 4. The Path from the Home Configuration, H to the Mid-point, E.

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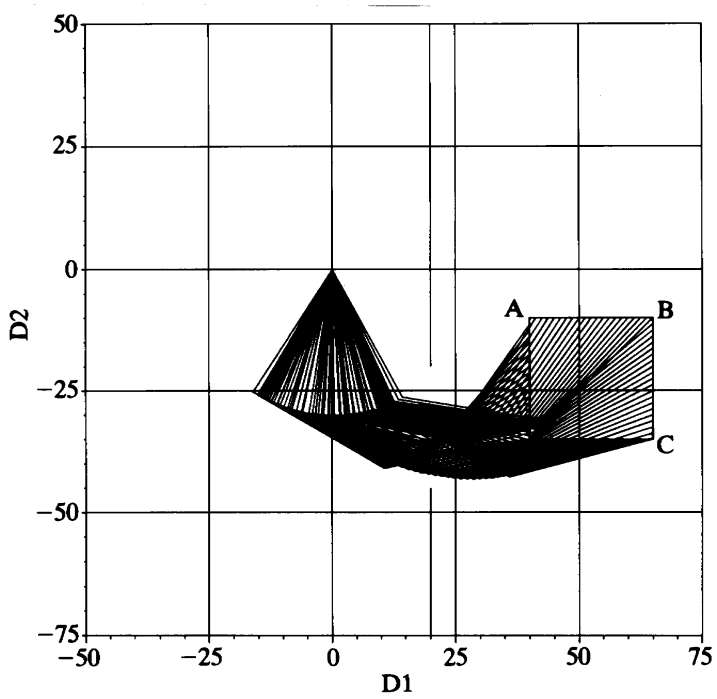


Fig. 5. The Complete Path that Avoids all Obstacles.

infinitesimal method between the configurations, the operator has no method of predicting the nature of intermediate configurations. The final joint configuration is the only parameter that may be predicted as the operator selected the input parameters to achieve this. However, further sub-division of the task will alleviate this problem. Corrective action may be introduced by re-selecting a joint configuration from the manifold using the obstacle avoidance selection procedure as discussed in Section 4.2. Spatial simulations have proved that this procedure is essential, since a selection based upon pure visual data is difficult.

The specified tasks described in this paper are relatively simple and, hence the possibility of applying this method to highly complex tasks remains to be seen. For instance, if there are many obstacles within the workspace, an obvious problem would be the difficulty of selecting a suitable joint configuration from the self-motion manifold of the manipulator. This would be especially difficult if, for example, the only suitable configuration is required to move through a kinematic singularity. However, the use of way points, like that of the mid-point E in the example from Section 4.3, would certainly modularise the task into easier, manageable parts.

The task planning method has tremendous scope for utilising the infinite possible joint configurations from the self-motion manifold. In the example demonstrated in Section 4, the variation of the elastic states was achieved only by varying the joint stiffnesses. The unloaded joint configuration remained constant and by varying this elastic parameter there is more potential utility of the manifold. More potential is also achievable by recognising that there are m free variable joint stiffnesses in equation (7).

It should be noted that the method provides a schecule of required joint positions. As presented, time is not featured explicitly, but this may be addressed in the same manner as it is with non-redundant manipulators.

6. CONCLUSIONS

An established elastic model for a redundant manipulator has been used as the basis for a task planning method. The model permits consistent inverse position and rate analyses to be developed. There is complete consistency and correlation of results between the finite and infinitesimal levels. The combined method uses a finite and infinitesimal analysis which exploits the advantages of both methods, yet overcomes the disadvantages of each. The analyses and its associated task planning method provide the first stage in developing a task planning language for redundant manipulators since it is the basis for an algorithm to produce a schedule of joint displacements for the specified task. Furthermore, the task planning method and its associated analyses are applicable to the general redundant manipulator of an arbitrary degree of redundancy.

7. ACKNOWLEDGEMENT

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