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PARTIAL REDUNDANCY OF MOMENT CONDITIONS

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In this paper, we first transform a set of moment conditions into a set of transformed moment conditions, based on which the efficient partial generalized method of moments estimation for part of a parameter vector is defined. Given the set of transformed moment conditions, we then show that the conditions for partial redundancy of an additional set of moment conditions given an original set of moment conditions simply become the conditions for full redundancy of the second subset of transformed moment conditions given the first subset of transformed moment conditions. Thus the transformed moment conditions proposed in this paper unify partial redundancy of moment conditions with full redundancy of moment conditions. Using transformed moment conditions, we then straightforwardly derive necessary and sufficient conditions for partial redundancy of one or two subset(s) of moment conditions given the other when the large set of moment conditions consists of three subsets of moment conditions. The paper also provides several easily checkable sufficient conditions for partial redundancy of one set of moment conditions given other sets of moment conditions.

1. INTRODUCTION

It is well known in the generalized method of moments (GMM) estimation literature that adding additional (valid) moment conditions to existing ones will not decrease the asymptotic efficiency of a GMM estimator. However, sometimes using more moment conditions may not increase the asymptotic efficiency of a GMM estimator. In this case, we say that the additional moment conditions are (fully) redundant (for the estimation of all parameters) given the original set of moment conditions. Similarly, adding extra moment conditions sometimes may not improve the asymptotic efficiency of the GMM estimation of some of the parameters. In this case, the extra moment conditions are said to

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be partially redundant (for the estimation of those parameters) given the original set of moment conditions. More precisely, let $\hat{\theta}$ be the optimal GMM estimator of θ_0 based on a set of moment conditions $E[g_1(y,\theta_0)] = 0$. Now add an extra set of moment conditions $E[g_2(y,\theta_0)] = 0$ and let $\tilde{\theta}$ be the optimal GMM estimator of θ_0 based on the larger set of moment conditions $E[g(y,\theta_0)] \equiv E[g_1(y,\theta_0)', g_2(y,\theta_0)']' = 0$. Then the moment conditions $E[g_2(y,\theta_0)] = 0$ are said to be (fully) redundant (for the estimation of θ_0) given the moment conditions $E[g_1(y,\theta_0)] = 0$, if the asymptotic variances of $\hat{\theta}$ and $\tilde{\theta}$ are the same. Similarly, if we partition θ_0 as $(\theta_{10}', \theta_{20}')'$, the moment conditions $E[g_2(y,\theta_0)] = 0$ are said to be partially redundant (for the estimation of θ_{10}) given the moment conditions $E[g_1(y,\theta_0)] = 0$, if the asymptotic variances of $\hat{\theta}_1$ and $\tilde{\theta}_1$ are the same.

Breusch, Qian, Schmidt, and Wyhowski (1999) derive necessary and sufficient conditions for full redundancy of moment conditions when the largest set of moment conditions consists of *two or three* subsets of moment conditions, and they also obtain necessary and sufficient conditions for partial redundancy of moment conditions when the largest set of moment conditions consists of only *two* subsets of moment conditions. Breusch et al. derive their partial redundancy conditions only algebraically and do not provide any intuitive explanation of the partial redundancy conditions, whereas they obtain the full redundancy conditions through sequential linear projection of moment functions so that the full redundancy conditions have nice intuitive interpretation.

This paper aims to remedy the Breusch et al. asymmetric treatment of full and partial redundancy of moment conditions. Specifically, we will make the following three new contributions. First, we will define efficient partial GMM estimation for a subset (θ_{10}) of a parameter vector θ_0, based on a set of transformed moment conditions. Second, using a set of transformed moment conditions and the efficient partial GMM estimation idea, we will show that partial redundancy conditions for θ_{10} can be derived as the full redundancy conditions of a set of transformed moment conditions given another set of transformed moment conditions. Thus, the set of transformed moment conditions (based on which the efficient partial GMM estimation of θ_{10} alone is defined) unifies the full redundancy with the partial redundancy of moment conditions. Last, using transformed moment conditions, we will straightforwardly derive necessary and sufficient conditions for partial redundancy of various subsets of moment conditions given the others when the large set of moment conditions consists of three subsets of moment conditions.

The notion of partial redundancy of moment conditions is also related to the problem of partial identification.[1] (See, e.g., Phillips, 1989.) Suppose that we have available a set of moment conditions $E[g_1(y,\theta_0)] = 0$ with $G_{11} \equiv E[\nabla_{\theta_1}' g_1(y,\theta_0)] \equiv E[\partial g_1(y,\theta_0)/\partial \theta_1']$ having full column rank and $G_{12} \equiv E[\nabla_{\theta_2}' g_1(y,\theta_0)] = 0$,[2] where $\theta_0 = (\theta_{10}', \theta_{20}')'$. Then, the moment conditions $E[g_1(y,\theta_0)] = 0$ partially identify θ_{10} given θ_{20}. Now, suppose that we have available an extra set of moment conditions $E[g_2(y,\theta_0)] = 0$ with $G_{22} \equiv E[\nabla_{\theta_2}' g_2(y,\theta_0)]$ having full column rank. Then, the enlarged set of moment

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conditions $E[g_1(y,\theta_0)', g_2(y,\theta_0)']' = 0$ identifies the entire parameter vector $\theta_0 = (\theta_{10}', \theta_{20}')'$. Thus, an interesting question is under what conditions the GMM estimator of θ_{10} alone, based on the sample moment conditions $N^{-1}\sum_{i=1}^{N} g_1(y_i,\theta) \xrightarrow{p} 0$,[3] is as efficient as the corresponding subvector of the (joint) GMM estimator of θ_0, based on the enlarged set of moment conditions $E[g_1(y,\theta_0)', g_2(y,\theta_0)']' = 0$, where N is the sample size and $\theta \equiv (\theta_{10}', \bar{\theta}_2')'$ with $\bar{\theta}_2$ being a consistent estimate of θ_{20}. It turns out that the GMM estimator of θ_{10} based on the sample moment conditions $N^{-1}\sum_{i=1}^{N} g_1(y_i,\theta) \xrightarrow{p} 0$ is as efficient as the GMM estimator of θ_{10} based on the enlarged set of moment conditions $E[g_1(y,\theta_0)', g_2(y,\theta_0)']' = 0$, if $G_{21} - \Omega_{21}\Omega_{11}^{-1}G_{11} = G_{22}A$ for some matrix A, where $G_{21} \equiv E[\nabla_{\theta_1}' g_2(y,\theta_0)]$ and $\Omega_{st} \equiv E[g_s(y,\theta_0)g_t(y,\theta_0)']$ for $s, t = 1,2$. (See, e.g., Breusch et al., 1999, Theorem 8.) One special case for this condition to be true is when $E[g_2(y,\theta_0)] = 0$ just-identifies θ_{20} given θ_{10}; that is, G_{22} is nonsingular.

The rest of the paper is structured as follows. In the next section, we first define efficient partial GMM estimation and then show that the partial redundancy of an additional set of moment conditions given an original set of moment conditions can be derived as the full redundancy of a set of transformed moment conditions given another set of transformed moment conditions. We complete the section with discussions on partial redundancy of various subsets of moment conditions given the others when the large set of moment conditions consists of three subsets of moment conditions. Section 3 concludes the paper.

2. PARTIAL REDUNDANCY OF MOMENT CONDITIONS

Suppose that we have a set of moment conditions:

$$E[g(y_i,\theta_0)] = 0, \qquad i = 1,\ldots,N, \tag{1}$$

where $g(y,\theta)$ is a $k \times 1$ vector of moment functions and $\theta_0 = (\theta_{10}', \theta_{20}')'$ is a $p \times 1$ vector of (true) parameters to be estimated, with θ_{10} $p_1 \times 1$, θ_{20} $p_2 \times 1$, and $p = p_1 + p_2$. We assume that $\Omega \equiv \mathrm{Var}[g(y_i,\theta_0)]$ is positive definite and that $G \equiv E[\nabla_\theta' g(y_i,\theta_0)]$ has full column rank such that the moment conditions in (1) successfully identify the unknown parameter vector θ_0. For simplicity of exposition, we assume that the data $y_1, y_2, \ldots, y_N$ are independent and identically distributed (i.i.d.). (The i.i.d. assumption could be relaxed, so long as $N^{-1/2}\sum_{i=1}^{N} g(y_i,\theta_0)$ satisfies a central limit theorem.)

A GMM estimator is said to be optimal when the weighting matrix used equals the inverse of the covariance matrix of the moment functions (evaluated at the true parameter values). In this paper, we only consider optimal GMM estimators.

One of the main purposes of this paper is to find the necessary and sufficient conditions for partial redundancy of one set of moment conditions given the others. However, before we can succinctly present the main results of the pa-

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per, we first need to introduce the concept of efficient partial GMM estimation for part of a parameter vector, as summarized in Theorem 1.

THEOREM 1. *The* GMM *estimator of* θ_{10} *from the optimal (joint)* GMM *estimation of* $\theta_0 = (\theta_{10}', \theta_{20}')'$, *based on moment conditions* (1), *is asymptotically equivalent to the optimal partial* GMM *estimator of* θ_{10} *alone (by treating* θ_{20} *as known), based on the following set of transformed moment conditions:*

$$E[g^*(y_i, \theta_0)] \equiv E[P_{G_2}(\Omega) g(y_i, \theta_0)] = 0, \tag{2}$$

where $G_2 \equiv E[\nabla'_{\theta_2} g(y_i, \theta_0)]$ *and the asymmetric projection matrix* $P_{G_2}(\Omega) \equiv I_k - G_2(G_2'\Omega^{-1}G_2)^{-1}G_2'\Omega^{-1}$.[4]

Using the standard result for the asymptotic distribution of a GMM estimator and the partitioned-matrix inverse rule, we can easily prove this theorem. So we omit its proof here. Also, to save space, we will omit the derivations for expressions (8) and (15), which follow. Detailed derivations of these results are available upon request.

Remark 1. In Theorem 1, we assumed that θ_{20} is known, which is usually not true in practice. However, it is straightforward to show that replacing θ_{20} by a root-N consistent estimate (say, $\bar{\theta}_2$) will not affect the first-order asymptotics of the optimal partial GMM estimator of θ_{10}, based on the sample moment conditions $N^{-1}\sum_{i=1}^{N} g^*(y_i, \theta_{10}, \bar{\theta}_2) \xrightarrow{p} 0$ (see, e.g., Newey and McFadden, 1994, Theorem 6.1). Also, by standard GMM arguments, consistent estimates of the covariance matrix Ω and the expected derivative matrix G will not affect the asymptotic distribution of a GMM estimator. Thus, in the rest of the paper, we will assume that θ_{20}, Ω, and G are known.

A word on the notation is now in order: from now on we will use $g^*(y, \theta)$ and $g^*(y, \theta_1, \theta_2)$ interchangeably and similarly for other functions with θ as an argument. Also, to simplify expressions, we will suppress arguments of moment functions whenever the context is clear.

We now use Theorem 1 to derive the partial redundancy condition of one set of moment conditions given another. We first consider the case when the set of moment conditions in (1) consists of two subsets:

$$E[g(y_i, \theta_0)] \equiv E[g_1(y_i, \theta_0)', g_2(y_i, \theta_0)']' = 0, \tag{3}$$

where the dimensions of g_1 and g_2 are $k_1 \times 1$ and $k_2 \times 1$, respectively, with $k_1 + k_2 = k$.

Let $G_{nj} = E[\nabla'_{\theta_j} g_n(y_i, \theta_0)]$ (for $n, j = 1,2$) and $\Omega_{st} = E[g_s(y_i, \theta_0) g_t(y_i, \theta_0)']$ (for $s, t = 1,2$). We assume that $\Omega \equiv \mathrm{Var}[g(y_i, \theta_0)]$ is positive definite and that (G_{11}, G_{12}) has full column rank so that θ_0 is identified by $E[g_1(y_i, \theta_0)] = 0$ alone.

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We now seek to find the optimal moment conditions for the efficient partial GMM estimation of θ_{10}, which are implied by the moment conditions in (3). To this end, we define

$$r_2(y_i, \theta_0) = g_2 - \Omega_{21}\Omega_{11}^{-1} g_1. \tag{4}$$

(So r_2 is the linear projection error of g_2 onto g_1; r_2 summarizes all the independent new information when g_2 is added to g_1.) Then, by Theorem 1, the GMM estimator of θ_{10} from the optimal joint GMM estimation of $\theta_0 = (\theta_{10}', \theta_{20}')'$, based on the following set of orthogonalized moment conditions:

$$E[g^{\perp}(y_i, \theta_0)] \equiv E[g_1(y_i, \theta_0)', r_2(y_i, \theta_0)']' = 0, \tag{5}$$

is asymptotically equivalent to the optimal partial GMM estimator of θ_{10} (by treating θ_{20} as known), based on the following set of transformed moment conditions:

$$E[g^{\perp *}(y_i, \theta_0)] \equiv E[P_{G_2^{\perp}}(\Sigma) g^{\perp}(y_i, \theta_0)] = 0, \tag{6}$$

where $G_2^{\perp} \equiv E[\nabla_{\theta_2}' g^{\perp}(y_i, \theta_0)] = (G_{12}', R_{22}')'$ and $\Sigma \equiv \mathrm{Var}[g^{\perp}(y_i, \theta_0)] = \mathrm{diag}(\Omega_{11}, (\Omega^{22})^{-1})$, with $R_{22} \equiv E[\nabla_{\theta_2}' r_2(y_i, \theta_0)] = G_{22} - \Omega_{21}\Omega_{11}^{-1} G_{12}$ and $\Omega^{22} \equiv (\Omega_{22} - \Omega_{21}\Omega_{11}^{-1}\Omega_{12})^{-1}$.

We now proceed to simplify the set of transformed moment conditions (6). Thus, we define

$$g_1^* = P_{G_{12}}(\Omega_{11}) g_1,$$

$$r_2^* = r_2 - R_{22}(G_{12}'\Omega_{11}^{-1} G_{12})^{-1} G_{12}'\Omega_{11}^{-1} g_1. \tag{7}$$

(So, by Theorem 1, $E(g_1^*) = 0$ is the set of optimal moment conditions, which is implied by $E(g_1) = 0$, for the efficient partial GMM estimation of θ_{10}.) Then, it is now not difficult (though tedious) to show that

$$g^{\perp *} = \begin{bmatrix} I_{k_1} & -G_{12}(G_{12}'\Omega_{11}^{-1} G_{12})^{-1} R_{22}' \Psi^{-1} \\ 0 & (\Omega^{22})^{-1}\Psi^{-1} \end{bmatrix} \begin{bmatrix} g_1^* \\ r_2^* \end{bmatrix} \tag{8}$$

with $\Psi \equiv (\Omega^{22})^{-1} + R_{22}(G_{12}'\Omega_{11}^{-1} G_{12})^{-1} R_{22}'$.[5] These moment functions are obviously a nonsingular transformation of $(g_1^{*\prime}, r_2^{*\prime})'$. Thus, the optimal partial GMM estimator of θ_{10}, based on (6), is asymptotically equivalent to the optimal partial GMM estimator of θ_{10}, based on $E(g_1^{*\prime}, r_2^{*\prime})' = 0$. Therefore, combining this result with Theorem 1 and noticing that the set of orthogonalized moment conditions in (5) is also a nonsingular transformation of the set of moment conditions in (3), we obtain the following important result.

Result. The GMM estimator of θ_{10} from the optimal joint GMM estimation of $\theta_0 = (\theta_{10}', \theta_{20}')'$, based on moment conditions (3), is asymptotically equiva-

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lent to the optimal partial GMM estimator of θ_{10} (by treating θ_{20} as known), based on the following set of transformed moment conditions:

$$E[g_1^*(y_i,\theta_{10})', r_2^*(y_i,\theta_{10})']' = 0. \tag{9}$$

Thus, combining this result with Theorem 1 and noticing that g_1^* and r_2^* are uncorrelated with each other, we can now easily prove Theorem 2, as follows.

THEOREM 2. *g_2 is partially redundant (for the estimation of θ_{10}) given g_1 if and only if*

(a) $E[\nabla'_{\theta_1} r_2^*(y_i,\theta_{10},\theta_{20})] = 0$, *or*
(b) $R_{21} = R_{22}(G_{12}'\Omega_{11}^{-1}G_{12})^{-1}G_{12}'\Omega_{11}^{-1}G_{11}$ *with* $R_{21} \equiv E[\nabla'_{\theta_1} r_2(y_i,\theta_0)] = G_{21} - \Omega_{21}\Omega_{11}^{-1}G_{11}$.

Condition (b) of Theorem 2 is the same as the partial redundancy condition derived algebraically by Breusch et al. (1999). However, condition (a) is new. One way to intuitively understand this new result is as follows. The result stated immediately before Theorem 2 implies that g_2 is partially redundant given g_1 if and only if r_2^* is fully redundant given g_1^*, as the GMM estimation problem based on moment conditions (9) is defined for θ_{10} only (by treating θ_{20} as known). Therefore, using the uncorrelatedness of g_1^* and r_2^* and the full redundancy condition of Theorem 1 of Breusch et al., we know that r_2^* is redundant given g_1^* if and only if $E(\nabla'_{\theta_1} r_2^*) = 0$. Thus, r_2^* summarizes all of the potentially nonredundant new information for the estimation of θ_{10} when an extra set of moment conditions g_2 is added to the original set of moment conditions g_1. Then, the problem of whether g_2 is informative (for the estimation of θ_{10}) given g_1 becomes simply whether the additional information r_2^* is informative by checking whether $E(\nabla'_{\theta_1} r_2^*) = 0$. Therefore, the set of transformed moment conditions (9) unifies partial redundancy of moment conditions with full redundancy of moment conditions.

Our discussion to this point has been for moment conditions consisting of no more than two subsets. We now turn to deriving necessary and sufficient conditions for partial redundancy of one or two subsets of moment conditions given the other subsets when the largest set of moment conditions consists of three subsets:

$$E[g(y_i,\theta_0)] \equiv E[g_1(y_i,\theta_0)', g_2(y_i,\theta_0)', g_3(y_i,\theta_0)']' = 0, \tag{10}$$

where g_1, g_2, and g_3 are $k_1 \times 1$, $k_2 \times 1$, and $k_3 \times 1$, respectively, with $k_1 + k_2 + k_3 = k$.

We continue to assume that (G_{11}, G_{12}) has full column rank (so that the first subset of moment conditions $E[g_1(y_i,\theta_0)] = 0$ alone can identify the entire parameter vector θ_0) and that $\Omega \equiv \mathrm{Var}[g(y_i,\theta_0)]$ is positive definite.

Applying Theorem 2 to moment conditions (10), we can easily show that $(g_2', g_3')'$ is partially redundant given g_1, if and only if g_2 is partially redundant given g_1 and g_3 is also partially redundant given g_1.

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In the rest of this section, we will consider the partial redundancy of g_3 given $(g_1', g_2')'$. It turns out that the partial redundancy condition for g_3 given $(g_1', g_2')'$ is a little bit more complicated than the one for $(g_2', g_3')'$ given g_1 as a result of the potential interaction between g_1 and g_2.

Similar to the definition of r_2^* in the second expression of (7) we define

$$r_3^* = r_3 - R_{32}(G_{12}'\Omega_{11}^{-1}G_{12})^{-1}G_{12}'\Omega_{11}^{-1}g_1, \tag{11}$$

where $r_3 \equiv g_3 - \Omega_{31}\Omega_{11}^{-1}g_1$ and $R_{32} \equiv E[\nabla_{\theta_2}' r_3(y_i, \theta_0)] = G_{32} - \Omega_{31}\Omega_{11}^{-1}G_{12}$, with $\Omega_{31} \equiv E[g_3(y_i, \theta_0)g_1(y_i, \theta_0)']$ and $G_{32} \equiv E[\nabla_{\theta_2}' g_3(y_i, \theta_0)]$.

We further define e_3 as the linear projection error of g_3 onto g_1 and g_2. Then, using the fact that $(g_1', r_2')'$ is a nonsingular transformation of $(g_1', g_2')'$, we can also express e_3 as

$$e_3 = g_3 - \Omega_{31}\Omega_{11}^{-1}g_1 - \tilde{\Omega}_{32}\Omega^{22}r_2 = r_3 - \tilde{\Omega}_{32}\Omega^{22}r_2 \tag{12}$$

using the definition of $r_3 = g_3 - \Omega_{31}\Omega_{11}^{-1}g_1$, where $\tilde{\Omega}_{32} \equiv E(g_3 r_2') = \Omega_{32} - \Omega_{31}\Omega_{11}^{-1}\Omega_{12}$.

Thus, by Theorem 2 with g_1 replaced by $(g_1', r_2')'$, g_3 is partially redundant given $(g_1', g_2')'$ (or equivalently, $(g_1', r_2')'$), if and only if

$$E[\nabla_{\theta_1}' e_3^*(y_i, \theta_{10}, \theta_{20})] = 0, \tag{13}$$

where

$$\begin{aligned} e_3^* &\equiv e_3 - E_{32}(G_2^{\perp\prime}\Sigma^{-1}G_2^{\perp})^{-1}G_2^{\perp\prime}\Sigma^{-1}(g_1', r_2')' \\ &= e_3 - E_{32}(G_{12}'\Omega_{11}^{-1}G_{12} + R_{22}'\Omega^{22}R_{22})^{-1}(G_{12}'\Omega_{11}^{-1}g_1 + R_{22}'\Omega^{22}r_2) \end{aligned} \tag{14}$$

using $G_2^{\perp} = (G_{12}', R_{22}')'$ and $\Sigma = \text{diag}(\Omega_{11}, (\Omega^{22})^{-1})$ and where $E_{32} \equiv E[\nabla_{\theta_2}' e_3(y_i, \theta_{10}, \theta_{20})]$. (So, similar to the interpretation of r_2^* earlier, e_3^* summarizes all of the potentially nonredundant new information [for the estimation of θ_{10}] when g_3 is added to g_1 and g_2.)

The partial redundancy condition $E[\nabla_{\theta_1}' e_3^*(y_i, \theta_{10}, \theta_{20})] = 0$, as given in (13), is not intuitive and is hard to check in applications. Thus, to obtain easily checkable conditions, we now wish to find the relationship between the partial redundancy of g_3 given $(g_1', g_2')'$ and the partial redundancy of g_2 given g_1 and the partial redundancy of g_3 given g_1; that is, we now seek to decompose e_3^* into a linear combination of r_2^* (which contains all of the potentially nonredundant new information for the estimation of θ_{10} when g_2 is added to g_1) and r_3^* (which contains all of the potentially nonredundant new information for the estimation of θ_{10} when g_3 is added to g_1). In fact, by using the definitions of r_2^* in the second expression in equation (7) and r_3^* in (11), it is straightforward to show that

$$e_3^* = r_3^* - [\tilde{\Omega}_{32} + E_{32}(G_{12}'\Omega_{11}^{-1}G_{12} + R_{22}'\Omega^{22}R_{22})^{-1}R_{22}']\Omega^{22}r_2^*. \tag{15}$$

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Thus, substituting it into (13), we see that g_3 is partially redundant given $(g_1', g_2')'$ if and only if

$$E(\nabla'_{\theta_1} e_3^*) = E(\nabla'_{\theta_1} r_3^*) - [\tilde{\Omega}_{32} + E_{32}(G'_{12}\Omega_{11}^{-1}G_{12} + R'_{22}\Omega^{22}R_{22})^{-1}R'_{22}] \times \Omega^{22}E(\nabla'_{\theta_1} r_2^*) = 0. \quad \textbf{(16)}$$

Then, combining it with Theorem 2(a), we obtain the following corollary.

COROLLARY.

(a) *g_3 partially redundant given g_1 does not imply that g_3 is partially redundant given $(g_1', g_2')'$. Furthermore, g_3 partially redundant given g_1 and g_3 partially redundant given g_2 do not imply that g_3 is partially redundant given $(g_1', g_2')'$.*

(b) *If g_3 is partially redundant given g_1 and g_2 is partially redundant given g_1, then g_3 is partially redundant given $(g_1', g_2')'$.*

(c) *If g_3 is partially redundant given $(g_1', g_2')'$ and g_2 is partially redundant given g_1, then g_3 is partially redundant given g_1.*

This corollary is important. Result (b) indicates that when an extra set of moment conditions (g_3) is also partially redundant given g_1, then there is no need to reconsider any of the previously partially redundant moment conditions (g_2). However, result (a) indicates that g_2 partially redundant given g_1 does not necessarily imply that g_2 continues to be partially redundant given g_1 and g_3. Thus, every time we add an extra set of partially nonredundant moment conditions (g_3) to an existing set of moment conditions (g_1), we have to reconsider all the previously partially redundant moment conditions (g_2).[6] Therefore, deducing partial redundancy through this kind of pairwise comparison involves ambiguity. On the contrary, result (c) offers a practical solution to this uncertainty. More specifically, result (c) indicates that if we find that the newly added moment conditions (g_3) are partially redundant given the original set of moment conditions (g_1) and the previously partially redundant moment conditions (g_2), then the extra set of moment conditions (g_3) must be also partially redundant given g_1. Thus, we can ignore both g_2 and g_3 and proceed to efficiently estimate the parameters of interest (θ_{10}) based on $E(g_1) = 0$ only. Therefore, there is no ambiguity involved if we evaluate the partial redundancy of an additional set of moment conditions through this kind of sequential conditioning.

3. CONCLUSIONS

In this paper, we first transformed a set of moment conditions into a set of transformed moment conditions, based on which the efficient partial GMM estimation for part of a parameter vector is defined. Given the set of transformed moment conditions, we then showed that the conditions for partial redundancy of an extra set of moment conditions given an original set of moment condi-

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tions simply become the conditions for full redundancy of the second subset of transformed moment conditions given the first subset of transformed moment conditions. Thus, the transformed moment conditions unify partial redundancy of moment conditions with full redundancy of moment conditions, which as a consequence also remedies the Breusch et al. (1999) asymmetric treatments of full and partial redundancy of moment conditions. Using transformed moment conditions, we also straightforwardly derived necessary and sufficient conditions for partial redundancy of one or two subsets of moment conditions given the other(s) when the large set of moment conditions consists of three subsets. Finally, we provided several easily checkable sufficient conditions for partial redundancy of one set of moment conditions given others. The method employed in this paper can be straightforwardly extended to the case when a set of moment conditions consists of more than three subsets, though the algebra required is more involved.

NOTES

1. I am grateful to Professor Peter Phillips for pointing out this connection.

2. The assumption that $G_{12} \equiv E[\nabla'_{\theta_2} g_1(y, \theta_0)] = 0$ is without loss of generality as we can always use reparameterization to transform a problem with G_{11} having full column rank and G_{12} not having full column rank into a problem of G_{11} having full column rank and $G_{12} = 0$.

3. Under the condition $G_{12} = 0$, replacing θ_{20} with any root-N consistent estimate of it will not change the first-order asymptotic distribution of the optimal GMM estimator of θ_{10} based on moment conditions $E[g_1(y, \theta_0)] = 0$. See, for example, Theorem 6.1 of Newey and McFadden (1994).

4. The optimal weighting matrix used for the partial GMM estimation of θ_{10} is the generalized inverse of $P_{G_2}(\Omega)\Omega P_{G_2}(\Omega)'$, which equals $\Omega^{-1}P_{G_2}(\Omega)$ or $P_{G_2}(\Omega)'\Omega^{-1}$.

5. The proof of (8) uses the following matrix equality:

$$(G'_{12}\Omega_{11}^{-1}G_{12} + R'_{22}\Omega^{22}R_{22})^{-1} = (G'_{12}\Omega_{11}^{-1}G_{12})^{-1} - (G'_{12}\Omega_{11}^{-1}G_{12})^{-1}R'_{22}\Psi^{-1}R_{22}(G'_{12}\Omega_{11}^{-1}G_{12})^{-1}.$$

See, for example, Rao (1973, Complements and Problems, 2.9, p. 33).

6. One of the referees pointed out that this phenomenon is somewhat similar to that of Granger non-causality (GNC); that is, a GNC property in a multivariate time series may alter with the addition (or subtraction) of time series.

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