J. Plasma Physics (2006), *vol.* 72, *part* 6, *pp.* 1117–1121. © 2006 Cambridge University Press
doi:10.1017/S0022377806005769 Printed in the United Kingdom

Multi-scale approach to the solution of nonlinear MHD evolution of heliotron plasma

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(Received 12 August 2005 and accepted 25 November 2005)

Abstract. A multi-scale nonlinear magnetohydrodynamic (MHD) evolution scheme is developed for the numerical analysis of a heliotron plasma as beta increases. In this scheme, the fast time scale dynamics is given by the nonlinear MHD code based on the reduced MHD equations and the slow dynamics is carried out with three-dimensional static equilibrium code. The time evolution is calculated iteratively with a linear interpolation technique for the equilibrium quantities. This scheme is applied to the analysis of the Large Helical Device plasma. Self-organization of the pressure profile induced by the interchange mode is obtained.

1. Introduction

In the Large Helical Device (LHD) experiments, good confinement of the plasma has been observed even in region of parameters predicted to be linearly unstable to ideal interchange modes. In order to investigate the potential mechanism for the plasma stabilization, we have developed a nonlinear magnetohydrodynamic (MHD) code, NORM, based on the reduced MHD (RMHD) equations [1]. Our initial results showed a synergistic effect between the resistive and ideal interchange instabilities. For a series of the beta value [1, 2], we found that the nonlinear deformation of the pressure profile caused by the resistive instabilities at low beta can stabilize the ideal modes at higher beta. In this analysis, however, the calculations were separately carried out for each beta value.

To consistently understand the plasma dynamics, we need to examine a continuous evolution of the plasma as the beta increases. In the RMHD formulation, the position of the plasma boundary and the averaged field line curvature are fixed in the time evolution of the nonlinear analysis once an equilibrium is given. On the other hand, the interchange mode is sensitive to these quantities. Therefore, we have to evolve not only the perturbation but also the equilibrium to include the beta increase effect. However, equilibrium evolution takes place over times of the order of 10 ms and the basic time scale linked to the instabilities that induces the energy transfer is 0.5 μs. Therefore, the evolutions involve a change of time scales of the order of 10^5–10^6. Thus, we have developed a numerical scheme to treat this multi-scale problem. We consider an iterative scheme with the VMEC and the NORM codes including a linear interpolation of the equilibrium quantities. Here the VMEC code is a three-dimensional (3D) static free-boundary equilibrium code [3]. We apply the scheme to the LHD plasma to see how the plasma evolves in the increase of beta.

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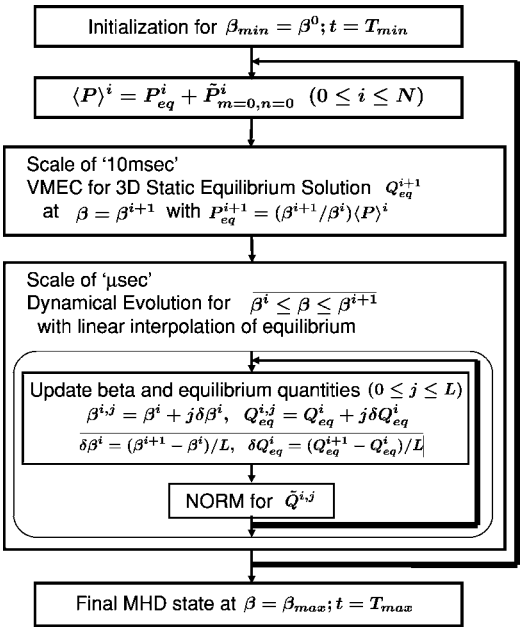


Figure 1. Flow chart of the numerical scheme.

2. Numerical scheme for multi-scale analysis

We consider the numerical scheme for the multi-scale problem in the range of $\beta_{\min} \leqslant \beta \leqslant \beta_{\max}$. Here β denotes the volume average beta value. Figure 1 shows the flow chart of the scheme.

At first, we divide the whole range of β into N intervals. We focus on the interval of $\beta^i \leqslant \beta \leqslant \beta^{i+1}$ and consider the equilibrium evolution from β^i to β^{i+1} $(0 \leqslant i \leqslant N$, $\beta^0 = \beta_{\min}, \beta^N = \beta_{\max})$. At the end of the nonlinear calculation for $\beta^{i-1} \leqslant \beta \leqslant \beta^i$, both the equilibrium and the perturbed parts of the pressure at β^i are already obtained as shown later, which are denoted by $P^i_{\rm eq}$ and $\tilde{P}^i$, respectively. Then, the average pressure profile at β^i can be calculated, which is defined by $\langle P \rangle^i = P^i_{\rm eq} + \tilde{P}^i_{0,0}$. Here $\tilde{P}^i_{0,0}$ is the $(m,n) = (0,0)$ component of $\tilde{P}^i$, where m and n are the poloidal and the toroidal mode numbers, respectively. We assume that the equilibrium pressure at β^{i+1} is given by $P^{i+1}_{\rm eq} = (\beta^{i+1}/\beta^i)\langle P \rangle^i$. By employing this pressure, we calculate the equilibrium at β^{i+1} with the VMEC code. Then we obtain the equilibrium quantity for β^{i+1}, which is denoted by $Q^{i+1}_{\rm eq}$ generically. This part of the scheme gives the evolution in the time scale of ~ 10 ms.

Next, we consider the calculation of the nonlinear dynamics with the NORM code for the interval of $\beta^i \leqslant \beta \leqslant \beta^{i+1}$. We can calculate the nonlinear dynamics only independently for different beta values. Thus, in order to keep a smooth continuity of the perturbation, we divide the nonlinear calculation of this interval into $L+1$ steps. The beta value and the equilibrium quantity are updated every step by utilizing a linear interpolation. The beta value of the jth step $\beta^{i,j}$ is given by $\beta^{i,j} = \beta^i + j\delta\beta^i$ for $0 \leqslant j \leqslant L$, where $\delta\beta^i = (\beta^{i+1} - \beta^i)/L$. The equilibrium quantity of the jth step $Q^{i,j}_{\rm eq}$ is given by $Q^{i,j}_{\rm eq} = Q^i_{\rm eq} + j\delta Q^i_{\rm eq}$, where $\delta Q^i_{\rm eq} = (Q^{i+1}_{\rm eq} - Q^i_{\rm eq})/L$.

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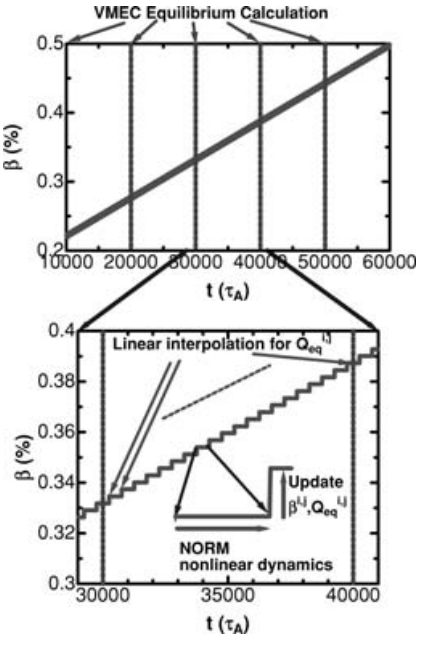


Figure 2. Time evolution of the beta value.

It is noted that $\beta^{i,L} = \beta^{i+1,0} = \beta^{i+1}$ and $Q_{\rm eq}^{i,L} = Q_{\rm eq}^{i+1,0} = Q_{\rm eq}^{i+1}$. Then, the nonlinear dynamics of the perturbation $\tilde{Q}^{i,j}$ is calculated for $\beta^{i,j}$ and $Q_{\rm eq}^{i,j}$ during the time range of $\delta t_j(\tau_{\rm A})$ at the jth step. Here $\tau_{\rm A}$ denotes the poloidal Alfvén time, and $1\tau_{\rm A}$ corresponds to $0.5\,\mu$s in the LHD plasma. Therefore, we can examine the nonlinear evolution in the time scale of microseconds in this part.

In the end of the interval of $\beta^i \leqslant \beta \leqslant \beta^{i+1}$, we set $\tilde{P}_{0,0}^{i+1} = \tilde{P}_{0,0}^{i,L}$. This quantity is used in the VMEC calculation for the next interval. By employing appropriate initial value of $\langle P \rangle^0$ at $\beta^0 = \beta_{\rm min}$ and $t = T_{\rm min}$ and continuing this iteration scheme until β reaches $\beta_{\rm max}$, we can obtain the solution for the whole beta region.

3. Nonlinear evolution of LHD plasma

We apply the multi-scale scheme to the LHD plasma with the vacuum magnetic axis located at $R_{\rm ax} = 3.6$ m. We choose the parameters of $\beta_{\rm min} = 0.221\%$, $\beta_{\rm max} = 0.498\%$, $N = 5$, $L = 20$ and $\delta t_j = 500\tau_{\rm A}$. The beta value is increased stepwise as shown in Fig. 2. The VMEC calculation is carried out every $10\,000\tau_{\rm A}$ under the no net-current constraint and free boundary condition. We assume a resistivity of $S = 10^6$, where S denotes the magnetic Reynolds number.

To give the initial state, we calculate an equilibrium at $\beta_{\rm min}$ with the pressure profile $P_{\rm eq}^0 = P_0(1-\rho^2)(1-\rho^8)$, where ρ denotes the square root of the normalized toroidal magnetic flux. Since this equilibrium is linearly unstable for the interchange mode, we follow the nonlinear evolution and obtain a saturation of the mode at $t = 10\,000\tau_{\rm A}$. We employ the saturated state as the initial state and set $t = 10\,000\tau_{\rm A}$ to $T_{\rm min}$. Then, we increase the beta value following the multi-scale scheme. Figure 3 shows the time evolution of the kinetic energy. The total energy varies smoothly in the evolution, which indicates the present multi-scale approach works well. Figure 4

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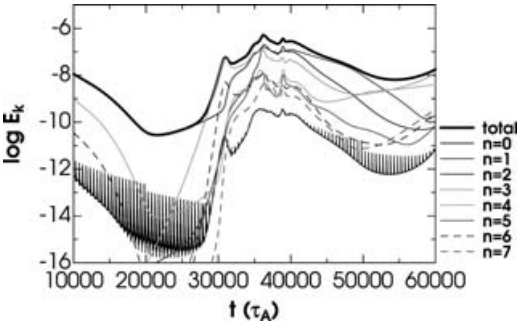


Figure 3. Time evolution of the kinetic energy.

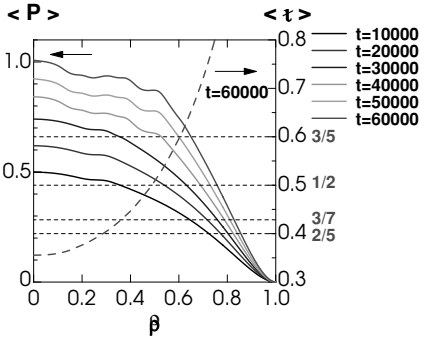


Figure 4. Profiles of average pressure and rotational transform.

shows the profiles of the average pressure. The pressure increases stably in the time evolution without any disruptive phenomena, while a disruptive pressure degradation was obtained for the equilibrium pressure profile fixed to $P_{\rm eq}^0$ (see [1]). Figure 4 also shows the profile of the rotational transform at $t = 60\,000\tau_{\rm A}$. The profile is hardly changed during the whole time range in the present analysis.

Figure 5 shows the time evolution of the total pressure profile. The profile is deformed continuously as the beta value increases. The deformed structure depends on the interchange mode excited at each beta value. In the present case, the free energy of the mode is not accumulated so much as the disruptive phenomenon is caused. Therefore, the driving force of the mode is weak and the saturated mode width is narrow. Hence, the pressure deformation due to each mode is localized around the corresponding resonant surface through the whole time sequence. This result indicates that the pressure in the LHD can increase with releasing the free energy gradually so as to suppress disruptive phenomena. It can be considered as a self-organization of the pressure.

4. Conclusions

We have developed a multi-scale scheme for the nonlinear MHD analysis, which is based on the iterative calculations of the static equilibrium and the nonlinear dynamics of resistive interchange. The former gives the plasma parameters over the long time scale while the latter gives the evolution of the pressure profile. This

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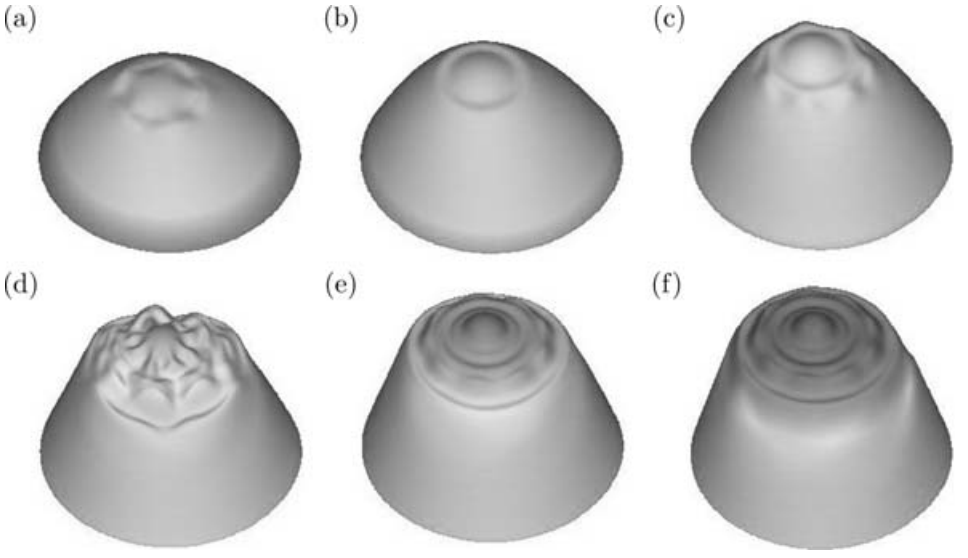


Figure 5. Bird's eye view of the total pressure profile in the region of $\rho \leqslant 0.8$ at (a) $t = 10\,000\tau_A$, (b) $t = 20\,000\tau_A$, (c) $t = 30\,000\tau_A$, (d) $t = 40\,000\tau_A$, (e) $t = 50\,000\tau_A$ and (f) $t = 60\,000\tau_A$.

scheme allows us to follow the time evolution of the plasma while beta increases in a continuous manner. The numerical results confirm the synergy between resistive and ideal instabilities. The resistive interchange instability modifies conveniently the plasma pressure profile causing the stabilization of the ideal modes. This dynamical evolution leads to a self-organization of the pressure profile that opens a stable path to high beta for the LHD plasmas.

Acknowledgements

This work is performed with the support of the NIFS Collaborative Research Program NIFS05ULDD101 and is also partly supported by the Grant-in-Aid for Scientific Research (C) 17560736 of the Japan Society for the Promotion of Science.

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