J. Plasma Physics (2010), *vol.* 76, *parts* 3&4, *pp.* 409–418. © Cambridge University Press 2010
doi:10.1017/S0022377809990614

Solitary waves in a dusty adiabatic electronegative plasma

A. A. MAMUN[1], K. S. ASHRAFI[1] and M. G. M. ANOWAR[1,2]

[1] Department of Physics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh (mamun_phys@yahoo.co.uk)

[2] Department of Physics, National University, Gazipur 1704, Bangladesh

(Received 25 November 2009 and accepted 3 December 2009, first published online 15 January 2010)

Abstract. The dust ion-acoustic solitary waves (SWs) in an unmagnetized dusty adiabatic electronegative plasma containing inertialess adiabatic electrons, inertial single charged adiabatic positive and negative ions, and stationary arbitrarily (positively and negatively) charged dust have been theoretically studied. The reductive perturbation method has been employed to derive the Korteweg-de Vries equation which admits an SW solution. The combined effects of the adiabaticity of plasma particles, inertia of positive or negative ions, and presence of positively or negatively charged dust, which are found to significantly modify the basic features of small but finite-amplitude dust-ion-acoustic SWs, are explicitly examined. The implications of our results in space and laboratory dusty electronegative plasmas are briefly discussed.

1. Introduction

About 18 years ago, Shukla and Silin (1992) have first theoretically shown the existence of dust ion-acoustic (DIA) waves in a dusty plasma containing mobile ions, Boltzmann electrons and negatively charged stationary dust. They (Shukla and Silin 1992) have pointed out that the low-frequency DIA waves with a phase speed much smaller (larger) than electron (ion) thermal speed naturally exist in such a dusty plasma due to the conservation of equilibrium charge density $n_{e0} + n_{d0} z_d = n_{i0}$, and the strong inequality $n_{e0} \ll n_{i0}$, where n_{i0}, n_{e0}, and n_{d0} are the ion, electron, and dust number density, respectively, z_d is the number of electrons residing onto the dust grain surface, and e is the magnitude of an electron charge. The linear dispersion relation of the DIA waves is (Shukla and Silin 1992) $\omega^2 = (n_{i0}/n_{e0})k^2 C_i^2/(1+k^2\lambda_{De}^2)$, where $C_i = (k_B T_{e0}/m_i)^{1/2}$ is the ion-acoustic speed (with T_{e0} being the electron temperature at equilibrium and m_i being the ion mass, k_B being the Boltzmann constant) and $\lambda_{De} = (k_B T_{e0}/4\pi n_{e0} e^2)^{1/2}$ is the electron Debye radius. When a long wavelength limit (viz. $k^2\lambda_{De}^2 \ll 1$) is considered, the dispersion relation for the DIA waves becomes $\omega = (n_{i0}/n_{e0})^{1/2} k C_i$. This form of spectrum is similar to the usual ion-acoustic wave spectrum for a plasma with $n_{i0} = n_{e0}$ and $T_{i0} \ll T_{e0}$ (where T_{i0} is the ion temperature at equilibrium). However, in dusty plasmas we usually have $n_{i0} \gg n_{e0}$ and $T_{i0} \simeq T_{e0}$. Therefore, a dusty plasma cannot support the usual ion-acoustic waves, but can do the DIA waves of Shukla and Silin (1992). The existence of these DIA waves have been confirmed by a

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

laboratory experiment of Barkan et al. (1996). The linear properties of the DIA waves in such a dusty plasmas are now well understood from both theoretical and experimental points of view (Shukla and Silin 1992; Mendis and Rosenberg 1994; Barkan et al. 1996; Shukla and Rosenberg 1999; Shukla 2001; Shukla and Mamun 2002, 2003; Rosenberg and Shukla 2002; Shukla 2004; Merlino and Goree 2004). The nonlinear propagation of the DIA waves in a such dusty plasma (containing electrons, ions, and negatively charged dust) have also been studied by many authors during the last few years (Bharuthram and Shukla 1992; Nakamura and Sharma 2001; Mamun and Shukla 2002a, b, 2005; Shukla and Eliasson 2004; Shukla 2007; Mamun 2008; Shukla and Eliasson 2009).

Recently, the nonlinear propagation of the DIA waves in a dusty electronegative plasma (DENP) (dusty plasma (Geortz 1989; Shukla and Mamun 2002; Fortov et al. 2005; Ishihara 2007; Morfill and Ivlev 2009; Shukla and Eliasson 2009) with a significant amount of negative ions (Vender et al. 1995; Kimura et al. 1998; Berezhnoj et al. 2000; Franklin 2002; Chabert et al. 2007; Coates 2007; Meige et al. 2007; Plihon et al. 2007; Chung 2009) whose contribution cannot be neglected in any way) have received a great deal of interest in understanding the electrostatic disturbances in both space environments and laboratory devices (Mamun and Shukla 2003; Annaratone et al. 2004; D'Angelo 2004; Annaratone and Allen 2005; Kim and Merlino 2006; Merlino and Kim 2006; Rosenberg and Merlino 2007, 2009). A number of attempts on nonlinear DIA waves in such a DENP (Sayed et al. 2008; Mamun et al. 2009a, b, c) have already been made. Sayed et al. (2008) have investigated the DIA solitary waves (SWs) by considering cold negative and positive ions, inertialess isothermal (Boltzmann) electrons, and stationary negatively charged dust. Mamun et al. (2009a, b) have considered inertial cold negative ions, inertialess isothermal (Boltzmann) positive ions and electrons, and stationary negatively charged dust, and have studied small-amplitude (Mamun et al. 2009a) as well as large-amplitude (Mamun et al. 2009b) DIA shock waves. On the other hand, motivated by a recent laboratory experiment of Ghim and Hershkowitz (2009), Mamun et al. (2009c) have considered inertial cold positive ions, inertialess isothermal (Boltzmann) negative ions and electrons, and stationary negatively charged dust, and have examined the possibility for the formation of the DIA SWs and double layers (DLs). However, (i) model of Sayed et al. (2008) is valid for cold positive and negative ion species, (ii) model of Mamun et al. (2009a, b) is valid for cold negative ion species, but not for isothermal (Boltzmann) positive ion species, (iii) model of Mamun et al. (2009c) is valid for cold positive ion species, but not for isothermal (Boltzmann) negative ion species. We note that in all of these investigations (Sayed et al. 2008; Mamun et al. 2009a, b, c) isothermal (Boltzmann) electrons are considered. Therefore, in our present work we attempt to generalize these investigations (Sayed et al. 2008; Mamun et al. 2009a, b, c) by considering an adiabatic DENP containing inertialess adiabatic electrons, inertial adiabatic positive and negative ions, and stationary arbitrarily (positively or negatively) charged dust, and theoretically examine the basic features of the DIA SWs in such a DENP.

The paper is organized as follows. The basic equations governing the DENP system (under consideration) are given in Sec. 2. The Korteweg de-Vries (KdV) equation is derived by employing the reductive perturbation method in Sec. 3. The stationary SW solution of KdV equation and its basic features are studied in Sec. 4. A brief discussion is finally presented in Sec. 5.

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

2. Governing equations

We consider a one-dimensional, collisionless, unmagnetized DENP consisting of inertialess adiabatic electrons, inertial single-charged adiabatic positive ions, inertial single charged adiabatic negative ions, and arbitrarily (positively or negatively) charged stationary dust. Thus, at equilibrium we have $n_{p0} = n_{e0} + n_{n0} - jz_d n_{d0}$, where n_{p0}, n_{e0}, n_{n0}, and n_{d0} are, respectively, positive ion, electron, negative ion, and dust number densities at equilibrium, z_d is the number of electrons residing on the dust grain surface, and $j = +1$ ($j = -1$) for positively (negatively) charged dust. We are interested in the nonlinear propagation of the low phase speed (in comparison with the electron thermal speed) perturbation mode on the time scale of the DIA waves. The time scale of the DIA waves is much faster than the dust plasma period so that dust can be assumed to be stationary. The nonlinear dynamics of these low-frequency (purely electrostatic) DIA waves in such a DENP is described by the normalized equations of the form

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{2.1}$$

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\alpha}{n_p}\frac{\partial p_p}{\partial x}, \tag{2.2}$$

$$\frac{\partial u_n}{\partial t} + u_n \frac{\partial u_n}{\partial x} = \sigma\frac{\partial \phi}{\partial x} - \frac{\sigma\alpha_n}{n_n}\frac{\partial p_n}{\partial x}, \tag{2.3}$$

$$\frac{\partial \phi}{\partial x} = \frac{1}{n_e}\frac{\partial p_e}{\partial x}, \tag{2.4}$$

$$\frac{\partial p_s}{\partial t} + u_s \frac{\partial p_s}{\partial x} + \gamma_s p_s \frac{\partial u_s}{\partial x} = 0, \tag{2.5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu_e n_e - n_p + \mu_n n_n - j\frac{z_d n_{d0}}{n_{p0}}, \tag{2.6}$$

where $s = p\ (n)\ e$ for the positive ion (negative ion) electron species, n_p (n_n) n_e is the positive ion (negative ion) electron number density normalized by n_{p0} (n_{n0}) n_{e0}, u_p (u_n) u_e is the positive ion (negative ion) electron fluid speed normalized by the positive ion-acoustic speed $C_p = (k_B T_{e0}/m_p)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_{e0}/e$, $\alpha = T_{p0}/T_{e0}$, $\alpha_n = T_{n0}/T_{e0}$, $\mu_e = n_{e0}/n_{p0}$, $\mu_n = n_{n0}/n_{p0}$, $\sigma = m_p/m_n$, m_p (m_n) is the positive (negative) ion mass, k_B is the Boltzmann constant, $\gamma_s = C_p^s/C_v^s$, where C_p^s (C_v^s) is the specific heat of the species at constant pressure (volume), p_s is the pressure normalized by $n_{s0}k_B T_{s0}$, and T_{s0} is the temperature at equilibrium. The time variable t is normalized by the positive ion plasma period $\omega_{pp}^{-1} = (m_p/4\pi e^2 n_{p0})^{1/2}$, and the space variable x is normalized by the Debye radius $\lambda_D = (k_B T_{e0}/4\pi e^2 n_{p0})^{1/2}$.

3. Derivation of KdV equation

To study the electrostatic DIA SWs in the DENP model under consideration, we construct a weakly nonlinear theory (Washimi and Taniuti 1966) of small but finite-amplitude DIA waves, which leads to a scaling of the independent variables through

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

the stretched coordinates (Washimi and Taniuti 1966):

$$\zeta = \epsilon^{1/2}(x - V_p t), \tag{3.1}$$

$$\tau = \epsilon^{3/2} t, \tag{3.2}$$

where ϵ is a small parameter measuring the weakness of the dispersion, V_p is the unknown phase speed (to be determined later) normalized by C_p. It may be noted here that ζ is normalized by the Debye radius (λ_D) and τ is normalized by the positive ion plasma period (ω_{pp}^{-1}).

We can expand the perturbed quantities n_s, u_s, p_s, and ϕ as (Washimi and Taniuti 1966)

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots, \tag{3.3}$$

$$u_s = u_0 + \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \cdots, \tag{3.4}$$

$$p_s = 1 + \epsilon p_s^{(1)} + \epsilon^2 p_s^{(2)} + \cdots, \tag{3.5}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{3.6}$$

where u_0 is the drift speed of the plasma species normalized by C_p.

Now, using (3.1)–(3.6) into (2.1)–(2.6) one can obtain the first-order continuity equations, momentum equations, pressure equations, and Poisson's equation which, after simplification, yield

$$n_p^{(1)} = \frac{1}{(V_p - u_0)^2 - \alpha\gamma_p}\phi^{(1)}, \tag{3.7}$$

$$n_n^{(1)} = -\frac{\sigma}{(V_p - u_0)^2 - \sigma\alpha_n\gamma_n}\phi^{(1)}, \tag{3.8}$$

$$n_e^{(1)} = \frac{1}{\gamma_e}\phi^{(1)}, \tag{3.9}$$

$$u_p^{(1)} = \frac{V_p - u_0}{(V_p - u_0)^2 - \alpha\gamma_p}\phi^{(1)}, \tag{3.10}$$

$$u_n^{(1)} = -\frac{\sigma(V_p - u_0)}{(V_p - u_0)^2 - \sigma\alpha_n\gamma_n}\phi^{(1)}, \tag{3.11}$$

$$u_e^{(1)} = \frac{V_p - u_0}{\gamma_e}\phi^{(1)}, \tag{3.12}$$

$$p_p^{(1)} = \frac{\gamma_p}{(V_p - u_0)^2 - \alpha\gamma_p}\phi^{(1)}, \tag{3.13}$$

$$p_n^{(1)} = -\frac{\sigma\gamma_n}{(V_p - u_0)^2 - \sigma\alpha_n\gamma_n}\phi^{(1)}, \tag{3.14}$$

$$p_n^{(1)} = \phi^{(1)}, \tag{3.15}$$

$$V_p = u_0 + \left(\frac{b + \sqrt{b^2 - 4\mu_e c}}{2\mu_e}\right)^{1/2}, \tag{3.16}$$

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

where $b = \alpha\mu_e\gamma_p + \sigma\alpha_n\mu_e\gamma_n + \sigma\mu_n\gamma_e + \gamma_e$ and $c = \sigma\alpha\alpha_n\mu_e\gamma_p\gamma_n + \sigma\alpha_n\gamma_e\gamma_n + \sigma\alpha\mu_n\gamma_e\gamma_p$. Equation (3.16) is the linear dispersion relation for the DIA waves propagating in the DENP, under consideration. Now, following the same procedure one can obtain the next higher order continuity equations, momentum equations, pressure equations, and Poisson's equation. Using these new higher order equations, along with (3.7)–(3.16), one can eliminate $n_s^{(2)}$, $p_s^{(2)}$, $u_s^{(2)}$, and $\phi^{(2)}$, and can finally obtain

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\zeta} + B\frac{\partial^3\phi^{(1)}}{\partial\zeta^3} = 0, \tag{3.17}$$

where

$$A = \frac{a^2d^2(g_1 - g_2 - g_3)}{2(V_p - u_0)(d^2 + \sigma\mu_n a^2)}, \tag{3.18}$$

$$B = \frac{a^2d^2}{2(V_p - u_0)(d^2 + \sigma\mu_n a^2)}, \tag{3.19}$$

in which

$$a = (V_p - u_0)^2 - \alpha\gamma_p,$$

$$d = (V_p - u_0)^2 - \sigma\alpha_n\gamma_n,$$

$$g_1 = \frac{3(V_p - u_0)^2 - \alpha\gamma_p(2 - \gamma_p)}{a^3},$$

$$g_2 = \frac{3\sigma^2\mu_n(V_p - u_0)^2 - \sigma^3\alpha_n\gamma_n\mu_n(2 - \gamma_n)}{d^3},$$

$$g_3 = \frac{\mu_e(2 - \gamma_e)}{\gamma_e^2}.$$

Equation (3.17) is the KdV equation describing the nonlinear propagation of the DIA waves in the DENP.

4. SW solution of the KdV equation

The stationary SW solution of the KdV (3.17) is obtained by transforming the independent variables ζ and τ to $\xi = \zeta - U_0\tau$ and $\tau = \tau$, where U_0 is a constant speed normalized by C_p, and imposing the appropriate boundary conditions, viz. $\phi^{(1)} \to 0$, $d\phi^{(1)}/d\xi \to 0$, $d^2\phi^{(1)}/d\xi^2 \to 0$ at $\xi \to \pm\infty$. Thus, one can express the stationary SW solution of the KdV (3.17) as

$$\phi^{(1)} = \phi_m \operatorname{sech}^2(\xi/\Delta), \tag{4.1}$$

where the amplitude ϕ_m (normalized by k_BT_{e0}/e), and the width Δ (normalized by λ_D) are given by

$$\phi_m = 3U_0/A, \tag{4.2}$$

$$\Delta = \sqrt{4B/U_0}. \tag{4.3}$$

It is clear from (4.2), (3.18), and (3.16) that the SWs will be associated with either positive ($\phi_m > 0$) or negative ($\phi_m < 0$) potential depending on the sign of A

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

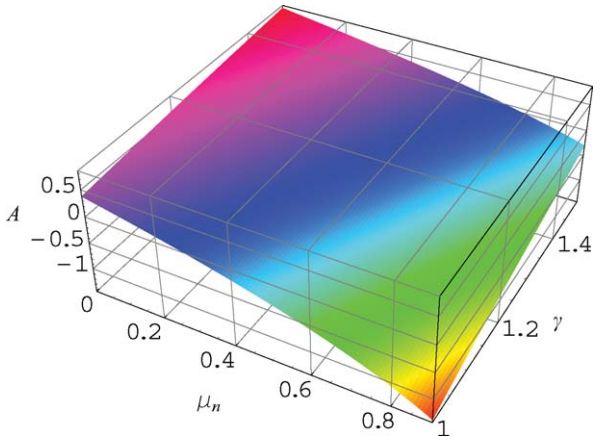


Figure 1. (Color online) The variation of A with μ_n and $\gamma(=\gamma_e=\gamma_p=\gamma_n)$ for $j=1$ (positive dust), $\sigma=0.1$, $\alpha=1$, $\alpha_n=0.1$, and $\mu_d=0.3$.

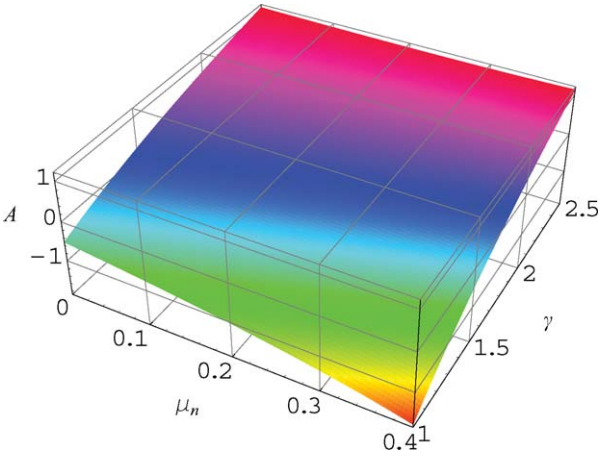


Figure 2. (Color online) The variation of A with μ_n and γ for $j=-1$ (negative dust), $\sigma=0.1$, $\alpha=1$, $\alpha_n=0.1$, and $\mu_d=0.3$.

(since $U_0>0$). Figures 1–4 show how the basic features (polarity, amplitude, and width) of the DIA SWs in DENP are modified by the effects of the adiabaticity of electrons and ions (γ), presence of negative ions (μ_n) and dust (μ_d), and polarity of the charged dust (j). These figures clearly indicate the parametric regimes for the existence of the DIA SWs associated with either positive or negative potential. On the other hand, Figs 5–8 indicate the variation of the amplitude and width of such positive and negative DIA solitary structures. Figures 5 and 7 imply that the amplitude (width) of the positive SWs decreases (increases) with γ, whereas Figs 6 and 8 indicate that the amplitude (width) of the negative SWs increases (decreases) with γ.

5. Discussion

We have considered a fully ionized collisionless, unmagnetized DENP consisting of inertialess adiabatic electrons, inertial single-charged adiabatic positive ions,

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

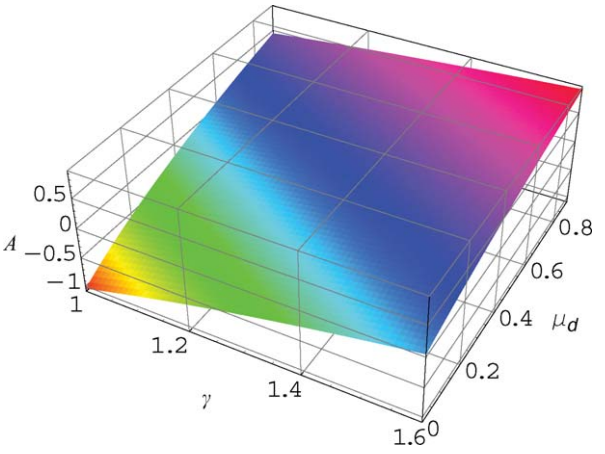


Figure 3. (Color online) The variation of A with γ and μ_d for $j = 1$ (positive dust), $\mu_n = 0.5$, $\sigma = 0.1$, $\alpha = 1$, and $\alpha_n = 0.1$.

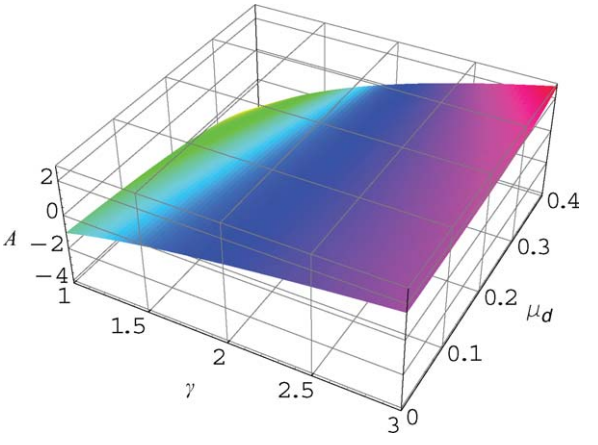


Figure 4. (Color online) The variation of A with γ and μ_d for $j = -1$ (negative dust), $\mu_n = 0.5$, $\sigma = 0.1$, $\alpha = 1$, and $\alpha_n = 0.1$.

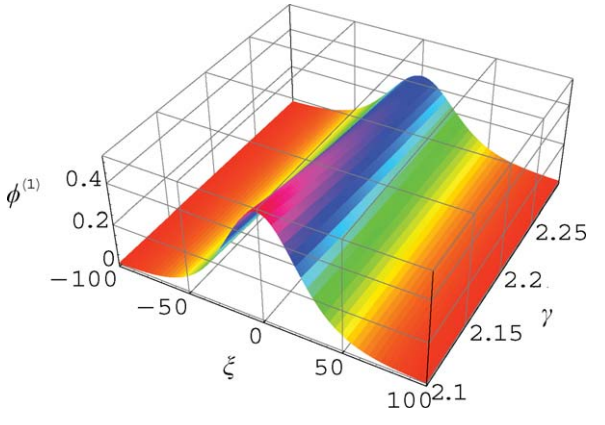


Figure 5. (Color online) The solitary potential profile (i.e., variation of $\phi^{(1)}$ with ξ and with a upper range γ (i.e., from 2.1 to 2.3)) for $j = -1$ (negative dust), $U_0 = 0.1$, $\mu_d = 0.3$, $\sigma = 0.1$, $\alpha = 1$, $\mu_n = 0.4$, and $\alpha_n = 0.1$.

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

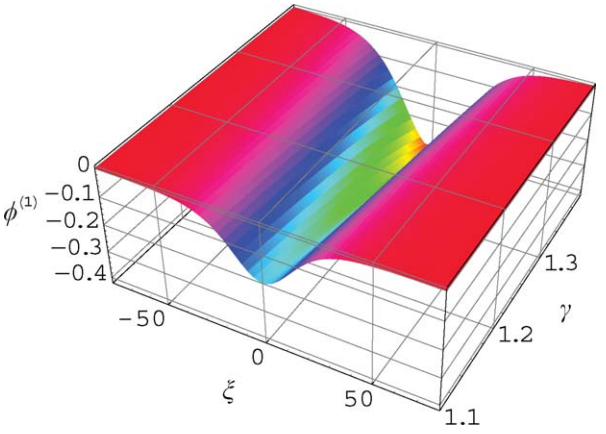


Figure 6. (Color online) The solitary potential profile (i.e., variation of $\phi^{(1)}$ with ξ and with a lower range of γ (i.e., from 1.1 to 1.4)) for $j = -1$ (negative dust), $U_0 = 0.1$, $\mu_d = 0.3$, $\sigma = 0.1$, $\alpha = 1$, $\mu_n = 0.4$, and $\alpha_n = 0.1$.

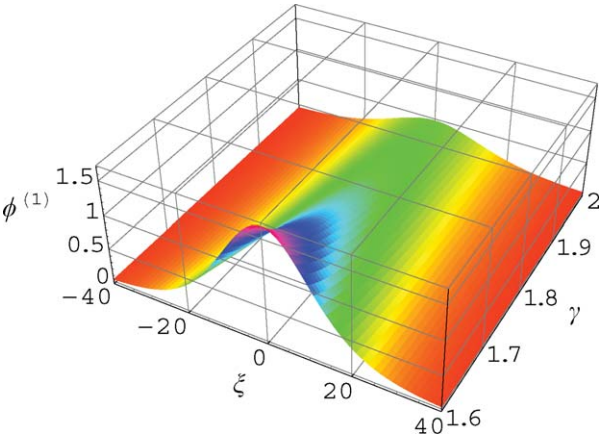


Figure 7. (Color online) The solitary potential profile (i.e., variation of $\phi^{(1)}$ with ξ and with a upper range of γ (i.e., from 1.6 to 2)) for $j = 1$ (positive dust), $U_0 = 0.1$, $\mu_d = 0.1$, $\sigma = 0.1$, $\alpha = 1$, $\mu_n = 0.5$, and $\alpha_n = 0.1$.

inertial single-charged adiabatic negative ions, and arbitrarily (positively and negatively) charged stationary dust, and have studied the DIA SWs associated with both positive and negative potential by deriving the KdV equation. It has been found that the SWs with positive or negative potential can formed depending on the parametric regimes shown in Figs 1–4. It has been shown here that the adiabaticity of electrons, positive and negative ions, and the polarity of the stationary dust significantly modify the basic features (polarity, amplitude, and width) of the DIA SWs in DENP. It has also been found that the amplitude (width) of the positive SWs decreases (increases) with γ, whereas the amplitude (width) of the negative SWs increases (decreases) with γ. We note that the effects of the external magnetic field and the dust charge fluctuation on the DIA solitary structures in DENP are also problems of great interest, but are not within the scope of our present investigation.

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

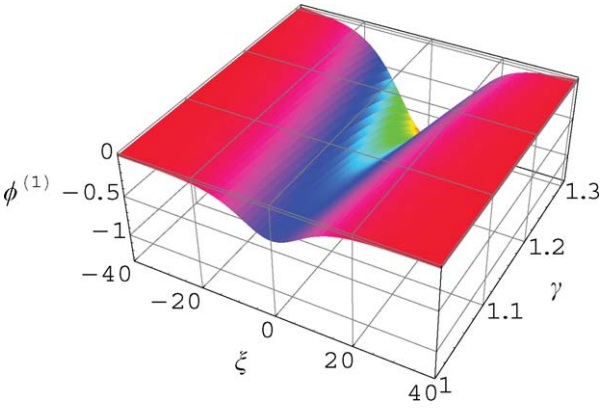


Figure 8. (Color online) The solitary potential profile (i.e., variation of $\phi^{(1)}$ with ξ and with a lower range of γ (i.e., from 1 to 1.3)) for $j = 1$ (positive dust), $U_0 = 0.1$, $\mu_d = 0.1$, $\sigma = 0.1$, $\alpha = 1$, $\mu_n = 0.5$, and $\alpha_n = 0.1$.

However, we hope that our present investigation should be useful in understanding the basic features (polarity, amplitude, and width) of the nonlinear electrostatic structures in space and laboratory DENPs (Mamun and Shukla 2003; Annaratone et al. 2004; D'Angelo 2004; Annaratone and Allen 2005; Kim and Merlino 2006; Merlino and Kim 2006; Rosenberg and Merlino 2007, 2009). We, finally, propose to perform a laboratory experiment (Kim and Merlino 2006; Merlino and Kim 2006) which may able to identify the features of the DIA solitary structures predicted in our present investigation.

Acknowledgements

This paper is dedicated to Prof. Dr. Padma Shukla, one of the world's leading scientists, and a guide of young talented scientists working in both developing and developed countries, on the occasion of his 60th birthday. We all wish his evergreen life with cherished goals.

References

Annaratone, B. M. and Allen, J. E. 2005 *J. Phys.* D **38**, 26.
Annaratone, B. M. et al. 2004 *Phys. Rev. Lett.* **93**, 185001.
Barkan, A. et al. 1996 *Planet. Space Sci.* **44**, 239.
Berezhnoj, S. V. et al. 2000 *Appl. Phys. Lett.* **77**, 800.
Bharuthram, R. and Shukla, P. K. 1992 *Planet. Space Sci.* **40**, 973.
Chabert, P. et al. 2007 *Phys. Plasmas* **14**, 093502.
Chung, T. H. 2009 *Phys. Plasmas* **16**, 063503.
Coates, A. J. et al. 2007 *Geophys. Res. Lett.* **34**, L22103.
D'Angelo, N. 2004 *J. Phys.* D **37**, 860.
Fortov, V. E. et al. 2005 *Phys. Rep.* **421**, 1.
Franklin, R. N. 2002 *Plasma Sources Sci. Technol.* **11**, A31.
Geortz, C. K. 1989 *Rev. Geophys.* **27**, 271.
Ghim, Y. and Hershkowitz, N. 2009 *Appl. Phys. Lett.* **94**, 151503.
Ishihara, O. 2007 *J. Phys.* D **40**, R121.

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press

Kim, S. H. and Merlino, R. L. 2006 *Phys. Plasmas* **13**, 052118.
Kimura, T. et al. 1998 *J. Phys. D: Appl. Phys.* **31**, 2295.
Mamun, A. A. 2008 *Phys. Lett.* A **372**, 1490.
Mamun, A. A. and Shukla, P. K. 2002a *IEEE Trans. Plasma Sci.* **30**, 720.
Mamun, A. A. and Shukla, P. K. 2002b *Phys. Plasmas* **9**, 1468.
Mamun, A. A. and Shukla, P. K. 2003 *Phys. Plasmas* **65**, 1518.
Mamun, A. A. and Shukla, P. K. 2005 *Plasma Phys. Contr. Fusion* **47**, A1.
Mamun, A. A. et al. 2009a *Phys. Lett.* A **373**, 2355.
Mamun, A. A. et al. 2009b *Phys. Rev.* E **80**, 046406.
Mamun, A. A. et al. 2009c *Phys. Plasmas* **16**, 114503.
Meige, A. et al. 2007 *Phys. Plasmas* **14**, 053508.
Mendis, D. A. and Rosenberg, M. 1994 *Annu. Rev. Astron. Astrophys.* **32**, 419.
Merlino, R. L. and Goree, J. 2004 *Phys. Today* **57**, 32.
Merlino, R. L. and Kim, S. H. 2006 *Appl. Phys. Lett.* **89**, 091501.
Morfill, G. E. and Ivlev, A. V. 2009 *Rev. Mod. Phys.* **81**, 1353.
Nakamura, Y. and Sharma, A. 2001 *Phys. Plasmas* **8**, 3921.
Plihon, N. et al. 2007 *Phys. Plasmas* **14**, 013506.
Rosenberg, M. and Merlino, R. L. 2007 *Planet. Space Sci.* **55**, 1464.
Rosenberg, M. and Merlino, R. L. 2009 *J. Plasma Phys.* **75**, 495 (2009).
Rosenberg, M. and Shukla, P. K. 2002 *J. Geophys. Res.* **107**, SIA 21–1.
Sayed, F. et al. 2008 *Phys. Plasmas* **15**, 063701.
Shukla, P. K. 2001 *Phys. Plasmas* **8**, 1791.
Shukla, P. K. 2007 *Plasma Phys. Contr. Fusion* **49**, A 221.
Shukla, P. K. and Eliasson, B. 2004 *Phys. Plasmas* **11**, 584.
Shukla, P. K. and Eliasson, B. 2009 *Rev. Mod. Phys.* **81**, 23.
Shukla, P. K. and Mamun, A. A. 2002 *Introduction to Dusty Plasma Physics.* Bristol, UK: IoP Publishing Ltd, p. 195.
Shukla, P. K. and Mamun, A. A. 2003 *New J. Phys.* **5**, 17.1.
Shukla, P. K. and Rosenberg, M. 1999 *Phys. Plasmas* **6**, 1038.
Shukla, P. K. and Silin, V. P. 1992 *Phys. Scri.* **45**, 508.
Vender, D. et al. 1995 *Phys. Rev.* E **51**, 2436.
Washimi, H. and Taniuti, T. 1966 *Phys. Rev. Lett.* **17**, 996.

https://doi.org/10.1017/S0022377809990614 Published online by Cambridge University Press