Robotica: (2021) volume 39, pp. 452–467. © The Author(s), 2020. Published by Cambridge University Press.
doi:10.1017/S0263574720000508

Adaptive Robust Tracker Design for Nonlinear Sandwich Systems Subject to Saturation Nonlinearities

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(Accepted May 28, 2020. First published online: June 22, 2020)

SUMMARY
This paper addresses the tracking problem for uncertain nonlinear sandwich systems that consist of two nonlinear subsystems and saturation nonlinearity, which is sandwiched between the subsystems. The considered sandwich system is also subject to a nonsymmetric input saturation constraint. Due to the nonsmooth characteristics of sandwiched saturation nonlinearity and also the input saturation function, the design procedure deals with hard challenges. To overcome these difficulties, a recursive approach is suggested that consists of two phases. For the implementation of the proposed approach, a tracking problem is solved in each phase. In the first phase, the second subsystem with sandwiched saturation nonlinearity is considered and the output tracking problem of the desired time-varying reference signal is solved using backstepping method. The outcome of the first phase is the desired reference signal that should be tracked by the first subsystem in the next phase. In the second phase, the robust control input is designed for the first subsystem by employing adaptive sliding mode technique such that, despite the nonsymmetric input saturation constraint, model uncertainty and external disturbances, the output of the first subsystem follows the desired signal that is obtained in the previous phase. The simulation results for a mechanical sandwich system are illustrated to verify the effectiveness of the proposed control method.

KEYWORDS: Saturation nonlinearity; Nonlinear sandwich systems; Back-stepping approach; Adaptive sliding mode; Lyapunov stability; Robotic system.

1. Introduction

Many practical systems can be described as a connection of several distinct linear or nonlinear subsystems in the cascade structure in which static or dynamic nonsmooth nonlinear functions are located between the subsystems. This class of systems is called sandwich systems because the nonsmooth nonlinear functions are sandwiched between the subsystems.[1,2] The most important static nonlinearities are saturation and dead-zone, and the dynamic nonlinearities are hysteresis and backlash. These types of hard nonlinearities exist in a wide range of practical systems such as mechanical systems,[3] hydraulic valves,[4] mechatronic systems,[5] DC servo motors,[6] mechanical transmission systems,[7] etc. In industrial fields, many systems can be described as sandwich systems such as hydraulic actuator of aircraft elevator,[8] X–Y moving positioning stage system[9] and hydraulic actuator with a pilot valve.[10] The mentioned static and dynamic nonlinearities, which occur in system modeling, are due to the imperfect performance of system components, physical constraints of subsystems or inherent characteristics of the systems. The existence of these nonsmooth nonlinearities limits the functionality of the system, causes oscillation and an undesirable error, or even may lead to instability in closed-loop systems.[11–16] Therefore, the effects of such factors must be considered in the procedure of controller design.

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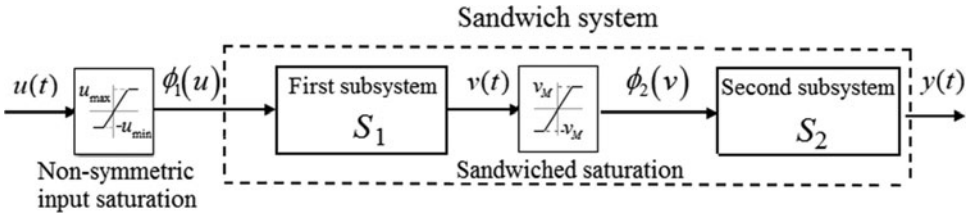


Fig. 1. Sandwich system with sandwiched saturation nonlinearity and non-symmetric input saturation constraint.

Saturation is one of the common static nonlinearities that exist in many practical systems. It often occurs due to the limited capacity of actuators, the physical limitation of system components and limited range of physical quantities such as voltage, torque, speed and so on;[17–19] therefore, saturation is a ubiquitous problem in practical systems.

A common architecture of sandwich systems with saturation nonlinearity, which is also subject to input saturation constraint, is shown in Fig. 1.

Due to the existence of nonsmooth nonlinear functions between the subsystems in the structure of sandwich systems, the analysis and control of such systems are more challenging. According to the above discussion, there are few works of literature devoted to sandwich systems. In Ref. [20], a generalized low-gain design methodology has been proposed for semi-global and global stabilization of a class of sandwich systems consisting of linear subsystems. This approach was extended in Ref. [21] to sandwich systems with discrete-time linear subsystems. The low and high-gain feedback design methodology has been introduced in Ref. [22] for the stabilization of sandwich systems. In Ref. [23], a dynamic output feedback controller has been presented for semi-global stabilization of a sandwich system with saturation nonlinearity. In Ref. [24], an approach has been proposed to analyze the controllability and observability of sandwich systems with dead-zone nonlinearity. A hybrid control scheme, consisting of inner and outer feedback loops, has been developed for sandwich systems with dead-zone nonlinearity in Ref. [25]. Authors of Ref. [26] solved the tracking problem of sandwich-like systems with dead-zone nonlinearity via the backstepping approach.

Motivated by the above observations, most of the papers have focused on sandwich systems that consist of linear subsystems; meanwhile, most of the practical systems consist of strictly nonlinear subsystems.

To the best of the authors' knowledge, an output tracking problem for sandwich systems with strictly nonlinear subsystems of any order, in the structure of Fig. 1, has not been studied until now, while this scheme is one of the common structures in practical systems.[27]

The objective of this paper is to design a robust control input $u(t)$ for the first subsystem (see Fig. 1) such that the output of the sandwiched system $y(t)$ can track the time-varying reference signal in the presence of an input saturation constraint, model uncertainties, external disturbances and also sandwiched saturation nonlinearity. According to the existence of saturation nonlinearities, the controller design deals with hard difficulties. To overcome these problems, the design procedure is divided into two phases. In the first phase, the sandwiched saturation nonlinearity and the second subsystem are considered and the tracking problem is solved for them by utilizing the backstepping approach along with the Nussbaum function such that the output $y(t)$ tracks the desired time-varying reference signal. The result of this phase is the design of a desired signal that should be tracked by the first subsystem in the next phase. In the second phase, the desired signal $v(t)$, which is extracted in the first phase, is considered as the desired output signal of the S_1 subsystem, and the task is to design the control input $u(t)$ for the S_1 subsystem such that the tracking problem is fulfilled for the first subsystem even if input saturation takes place. For the realization of control purpose in the second phase, an adaptive sliding mode technique is employed and appropriate adaptive laws have been extracted. Theoretical results and stability analysis are based on the Lyapunov approach. Simulation results of a practical sandwich system, consisting of a nonlinear mass spring system that handles a one-link manipulator, are provided to illustrate the effectiveness of the proposed method.

The remainder of this paper is as follows. In Section 2, some necessary preliminaries are introduced. The problem statement is explained in Section 3. Section 4 contains the main achievements of this paper with theoretical details. Simulation results are provided in Section 5; finally, Section 6 reports the conclusions.

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2. Preliminaries

In this section, some necessary lemma and definitions are given.

Definition 1.[28,29] *The function $\chi(\kappa)$ is a Nussbaum function if it satisfies the following properties:*

$$\lim_{s\to\infty} \sup \int_0^s \chi(\kappa)d\kappa = +\infty \tag{1}$$

$$\lim_{s\to\infty} \inf \int_0^s \chi(\kappa)d\kappa = -\infty \tag{2}$$

The usual types of Nussbaum functions are $\chi(\kappa)=\kappa^2\sin(\kappa)$, $\chi(\kappa)=\kappa^2\cos(\kappa)$ *and* $\chi(\kappa)=\exp(\kappa^2)\cos((\pi/2)\kappa)$.

Definition 2. *The Gaussian error function erf(x) is a continuous and differentiable function that is defined as:*

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{3}$$

Lemma 1.[30,31] *Suppose that $V(.)$ and $\lambda(.)$ are continuous functions on the time interval $[0,\ \infty)$, where $V(t)\geq 0,\ \forall t\in[0,\infty)$, and $N(\lambda)$ is an even smooth Nussbaum-type function. If the following inequality is realized,*

$$V(t)\leq c_0+e^{-\omega t}\int_0^t(\upsilon(\tau)\chi(\lambda)+1)\,\dot{\lambda}\,e^{\omega\tau}d\tau \qquad \forall\, t\in[0,\infty) \tag{4}$$

where c_0 represents a suitable constant, ω is a positive constant and $\upsilon(.)$ is an unknown time-varying parameter in the unknown closed set $l\in[l_-,\ l_+]$. Then, $V(t)$, $\lambda(.)$ and $\int_0^t(\upsilon(\tau)\chi(\lambda)+1)\,\dot{\lambda}\,e^{\omega\tau}d\tau$ are bounded for every $t\in[0,\infty)$.

3. Problem Statement

Considering a nonlinear sandwich system with the following nonlinear subsystems in the structure of Fig. 1:

$$S_1:\begin{cases}\dot{\eta}_i(t)=\eta_{i+1}(t) & i=1,2,...,m-1\\ \dot{\eta}_m(t)=f_1(\eta,t)+\Delta f_1(\eta,t)+g_1\phi_1(u)+d_1(t)\\ v(t)=\eta_1(t)\end{cases} \tag{5}$$

$$S_2:\begin{cases}\dot{\xi}_i(t)=\xi_{i+1}(t) & i=1,2,...,n-1\\ \dot{\xi}_n(t)=f_2(\xi,t)+g_2(\xi,t)\phi_2(v)+d_2(t)\\ y(t)=\xi_1(t)\end{cases} \tag{6}$$

where $\eta(t)\in R^m$ and $\xi(t)\in R^n$ are the state vectors of first and second subsystems, respectively. $f_1(\eta,t)$ and $f_2(\xi,t)$ are known continuous nonlinear functions; $\Delta f_1(\eta,t)$ represents the model uncertainties of the first subsystem; g_1 is a known constant; and $g_2(\xi,t)$ is an unknown time-varying function. Moreover, $d_1(t)$ and $d_2(t)$ are external disturbances, $\phi_1(u)\in R$ is the input of the S_1 subsystem and $\phi_2(v)\in R$ is the input of the S_2 subsystem where $\phi_1(.)$ and $\phi_2(.)$ are nonsmooth nonlinear functions due to input saturation constraints of each subsystem. Moreover, $v(t)\in R$ and $y(t)\in R$ are the outputs of the first and second subsystems, respectively.

Assumption 1. *The external disturbances $d_1(t)$ and $d_2(t)$ are unknown, time-varying and satisfy $|d_1(t)|<d_1^*$, $|d_2(t)|<d_2^*$, where d_1^*, d_2^* are known positive constants.*

Assumption 2. *The unknown function $\Delta f_1(\eta,t)$ is norm-bounded and satisfies $|\Delta f_1(\eta,t)|<\mu$, where μ is an unknown positive constant.*

Assumption 3. *The time-varying reference signal $y_r(t)$ is a smooth function.*

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The objective is to design a robust control law $u(t)$, such that the output of the second subsystem $y(t)$ tracks the time-varying reference signal $y_r(t)$, in the presence of nonsmooth nonlinear functions $\phi_1(.)$ and $\phi_2(.)$, unknown functions $\Delta f_1(\eta, t)$ and $g_2(\xi, t)$ and external disturbances $d_1(t)$ and $d_2(t)$.

4. Main Results

The main contribution of the paper is explained in this section. The existence of saturation functions $\phi_1(.)$ and $\phi_2(.)$ in the introduced structure causes hard challenges in the procedure of the controller design. To achieve a time-varying output tracking for sandwich systems, a novel technique is introduced. At first, the sandwiched saturation function and S_2 subsystem are augmented, and the desired signal $v(t)$ is designed based on the backstepping approach such that the output of the second subsystem (i.e. $y(t)$) tracks the time-varying reference signal $y_r(t)$. After that, the obtained signal $v(t)$ is considered as the desired output for the S_1 subsystem and called $v_r(t)$. Since the input saturation constraint is nonsymmetric, and also the model uncertainties and external disturbances exist, an adaptive sliding mode technique is proposed in this phase. A robust control law $u(t)$ is designed such that the output of the S_2 subsystem follows the desired signal $v_r(t)$ in the presence of an input saturation constraint. In the following, the details of each phase are elaborated.

4.1. Phase 1

In this phase, a backstepping-like approach is used to solve the tracking problem of the second subsystem.

The sandwiched saturation function between the two subsystems is defined as follows:

$$\phi_2(v(t)) = \begin{cases} v_M sign(v(t)) & |v(t)| \geq v_M \\ v(t) & |v(t)| \leq v_M \end{cases} \tag{7}$$

where $v_M > 0$ is an upper admissible value of the sandwiched saturation function. This function is approximated with a continuous Gauss error function, which is introduced in Definition 2:

$$\phi_2(v(t)) \approx v_M \times erf(av(t)) \tag{8}$$

where $a = \sqrt{\pi} \big/ (2v_M)$. For achieving the input signal of the saturation function, the following function is defined.

$$\Delta u = v_M erf(av(t)) - bv(t) \tag{9}$$

where $b > 0$ is a constant parameter. To facilitate the conditions for applying the backstepping approach, the following relation is considered:

$$\phi_2(v(t)) = bv(t) + \Delta u \tag{10}$$

It is assumed that there exists known constants $\underline{b}$, $\bar{b}$, and $\Delta^* > 0$ satisfies $b \in \left[\underline{b}, \bar{b}\right]$ and $|\Delta u| < \Delta^*$. By substituting Eq. (10) into Eq. (6), the equations of the S_2 subsystem can be rewritten as:

$$S_2 : \begin{cases} \dot{\xi}_i(t) = \xi_{i+1}(t) & i = 1, 2, ..., n-1 \\ \dot{\xi}_n(t) = f_2(\xi) + g_2(\xi, t)\,(bv(t) + \Delta u) + d_2(t) \\ y(t) = \xi_1(t) \end{cases} \tag{11}$$

To facilitate the controller design, error variables are defined as follows:

$$\begin{cases} e_1 = \xi_1(t) - y_r(t) \\ e_i(t) = \xi_i(t) - \beta_{i-1}(t) & i = 2, 3, ..., n \end{cases} \tag{12}$$

where $e(t) = [e_1(t), e_2(t), ..., e_n(t)]^T \in R^n$ is the error vector and β_is are the virtual control laws that should be designed in each step.

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In what follows, the procedure of the first, *i*th and the last steps are explained.

Step 1: Let's start with the first element of the error vector (i.e. $e_1 = \xi_1 - y_r(t) = y(t) - y_r(t)$) According to Eq. (11) and Eq. (12), $\dot{e}_1$ is obtained as:

$$\dot{e}_1 = \dot{\xi}_1 - \dot{y}_r(t) = e_2 + \beta_1 - \dot{y}_r(t) \tag{13}$$

Considering the positive-definite Lyapunov function candidate as $V(t) = \frac{1}{2}e_1^2$, the time derivative of $V(t)$ along Eq. (13) is:

$$\dot{V}_1(t) = e_1\dot{e}_1 = e_1\,(e_2 + \beta_1 - \dot{y}_r(t)) \tag{14}$$

$$\beta_1 = -c_1 e_1 + \dot{y}_r(t) \tag{15}$$

where $c_1 > 0$ is a design parameter. Substituting Eq. (15) into Eq. (14) yields:

$$\dot{V}_1(t) = -c_1 e_1^2 + e_1 e_2 \tag{16}$$

Continuing this procedure leads to the following results for the *i*th step.

Step i (for $i = 2, \ldots, n-1$): The *i*th element of the error vector is considered as $e_i(t) = \xi_i(t) - \beta_{i-1}$. Considering Eq. (11) and Eq. (12), its time derivative is calculated as:

$$\dot{e}_i(t) = \dot{\xi}_i(t) - \dot{\beta}_{i-1} = e_{i+1} + \beta_i - \dot{\beta}_{i-1} \tag{17}$$

In this step, the Lyapunov function is chosen as:

$$V_i(t) = V_{i-1}(t) + \frac{1}{2}e_i^2 \tag{18}$$

where $V_{i-1}(t) = \sum_{j=1}^{i-1} \frac{1}{2}e_j^2$. According to Eq. (17), the time derivative of Eq. (18) is computed as:

$$\begin{aligned}\dot{V}_i(t) &= \dot{V}_{i-1}(t) + e_i\dot{e}_i \\ &= \dot{V}_{i-1}(t) + e_i\left(e_{i+1} + \beta_i - \dot{\beta}_{i-1}\right)\end{aligned} \tag{19}$$

where $\dot{V}_{i-1}(t) = -\sum_{j=1}^{i-1} c_j e_j^2 + e_{i-1}e_i$. The virtual control law for the *i*th step is suggested as:

$$\beta_i = -e_{i-1} - c_i e_i + \dot{\beta}_{i-1} \tag{20}$$

where c_i s for $(i = 2, \ldots, n-1)$ are positive constants that should be chosen appropriately. Putting Eq. (20) into Eq. (19) leads to:

$$\dot{V}_i(t) = -\sum_{j=1}^{i} c_j e_j^2 + e_i e_{i+1} \tag{21}$$

Step *n*: In this step, the desired signal (i.e. $v_r(t)$) will be designed for achieving the output tracking of the time-varying signal $y_r(t)$. Consider the last element of the error vector as $e_n = \xi_n - \beta_{n-1}$. Then, one has:

$$\dot{e}_n = \dot{\xi}_n - \dot{\beta}_{n-1} = f_2(\xi, t) + g_2(\xi, t)\,(bv(t) + \Delta u) + d_2(t) - \dot{\beta}_{n-1} \tag{22}$$

The Lyapunov function of the last step ($V(t) = V_n(t)$) is selected as:

$$V(t) = V_{n-1}(t) + \frac{1}{2}e_n^2 = \sum_{i=1}^{n} \frac{1}{2}e_i^2 \tag{23}$$

Differentiating Eq. (23) yields:

$$\begin{aligned}\dot{V}(t) &= \dot{V}_{n-1}(t) + e_n\dot{e}_n \\ &= \dot{V}_{n-1}(t) + e_n\left(f_2(\xi, t) + g_2(\xi, t)bv(t) + d(\xi, t) - \dot{\beta}_{n-1}\right)\end{aligned} \tag{24}$$

where $\dot{V}_{n-1}(t) = -\sum_{i=1}^{n-1} c_i e_i^2 + e_{n-1}e_n$. Also, $d(\xi, t) = g_2(\xi, t)\Delta u + d_2(t)$ is considered a disturbance-like term that is bounded and satisfies $|d(\xi, t)| < \bar{d}$, where $\bar{d}$ is a known positive constant. By applying Young's inequality,[32] we have $e_n d(\xi, t) \leq e_n^2 + \frac{1}{4}\bar{d}^2$. Then Eq. (24) can be rewritten as:

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$$\dot{V}(t) \leq -\sum_{i=1}^{n-1} c_i e_i^2 + e_{n-1}e_n + e_n\left(f_2(\xi, t) + g_2(\xi, t)bv(t) - \dot{\beta}_{n-1}\right) + e_n^2 + \frac{1}{4}\bar{d}^2 \tag{25}$$

The main difficulty in the design of the desired signal $v(t)$, in phase 1, emerges in Eq. (25). It refers to the unknown time-varying control coefficient $g_2(\xi, t)b$. According to the unknown control coefficient, the traditional backstepping approach, which is used in the design of virtual control laws until the $(n-1)$th step, cannot be applied in the last step. For solving this problem, the following Nussbaum-type function, which is introduced in Definition 1, is employed:

$$\chi(\lambda) = \lambda^2 \cos(\lambda), \qquad \dot{\lambda} = \gamma e_n \beta_n \tag{26}$$

where γ is a positive constant parameter that should be chosen appropriately by the designer. The desired control signal $v(t)$ is chosen as:

$$v(t) = \chi(\lambda)\beta_n(t) \tag{27}$$

where

$$\beta_n(t) = (c_n + 1)\, e_n + e_{n-1} + f_2(\xi, t) - \dot{\beta}_{n-1} \tag{28}$$

and c_n is a positive constant. Now, adding the neutral term $(e_n\beta_n(t) - e_n\beta_n(t))$ and substituting Eq. (27) into Eq. (25) results in

$$\begin{aligned}\dot{V}(t) \leq -\sum_{i=1}^{n-1} c_i e_i^2 + e_{n-1}e_n + e_n\left(f_2(\xi, t) + g_2(\xi, t)b\chi(\lambda)\beta_n(t) + \beta_n(t) - \beta_n(t) - \dot{\beta}_{n-1}\right)\\ + e_n^2 + \frac{1}{4}\bar{d}^2\end{aligned} \tag{29}$$

Then, Eq. (29) can be rewritten as:

$$\begin{aligned}\dot{V}(t) \leq -\sum_{i=1}^{n-1} c_i e_i^2 + e_{n-1}e_n + e_n\left(g_2(\xi, t)b\chi(\lambda) + 1\right)\beta_n(t) - e_n\beta_n(t)\\ + e_n\left(f_2(\xi, t) - \dot{\beta}_{n-1}\right) + e_n^2 + \frac{1}{4}\bar{d}^2\end{aligned} \tag{30}$$

Replacing $\dot{\beta}_{n-1}$ from Eq. (28) into Eq. (30) leads to:

$$\dot{V}(t) \leq -\sum_{i=1}^{n} c_i e_i^2 + e_n\beta_n(t)\left(g_2(\xi, t)b\chi(\lambda) + 1\right) + \frac{1}{4}\bar{d}^2 \tag{31}$$

By replacing $e_n\beta_n(t) = \frac{\dot{\lambda}}{\gamma}$ from Eq. (26) into Eq. (31), the following result is achieved:

$$\dot{V}(t) \leq -\sum_{i=1}^{n} c_i e_i^2 + \frac{1}{\gamma}\left(g_2(\xi, t)b\chi(\lambda) + 1\right)\dot{\lambda} + \frac{1}{4}\bar{d}^2 \tag{32}$$

Considering Eq. (23), Eq. (32) can be rewritten as:

$$\begin{aligned}\dot{V}(t) &\leq -\sum_{i=1}^{n} c_i e_i^2 + \frac{1}{\gamma}\left(g_2(\xi, t)b\chi(\lambda) + 1\right)\dot{\lambda} + \frac{1}{4}\bar{d}^2\\ &\leq -c\sum_{i=1}^{n} e_i^2 + \frac{1}{\gamma}\left(g_2(\xi, t)b\chi(\lambda) + 1\right)\dot{\lambda} + \frac{1}{4}\bar{d}^2\\ &= -2cV(t) + \frac{1}{\gamma}\left(g_2(\xi, t)b\chi(\lambda) + 1\right)\dot{\lambda} + \frac{1}{4}\bar{d}^2\end{aligned} \tag{33}$$

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where $c = \min\{c_i\}$, for $i = 1, 2, ..., n$. Multiplying both sides of Eq. (33) by e^{2ct} yields:

$$\dot{V}(t)e^{2ct} \leq -2cV(t)e^{2ct} + \frac{e^{2ct}}{\gamma}\left(g_2(\xi, t)b\chi(\lambda) + 1\right)\dot{\lambda} + \frac{1}{4}\bar{d}^2 e^{2ct} \tag{34}$$

Then, integrating both sides of Eq. (34) over $[0,\ t]$ gives:

$$V(t)e^{2ct} - V(0) \leq \frac{1}{\gamma}\int_0^t \left(g_2(\xi, \tau)\, b\chi(\lambda) + 1\right) e^{2c\tau}\dot{\lambda}\, d\tau + \frac{1}{8c}\bar{d}^2\left(e^{2ct} - 1\right) \tag{35}$$

Now, both sides of Eq. (35) are multiplied with e^{-2ct} and one has:

$$V(t) \leq V(0)\, e^{-2ct} + \frac{e^{-2ct}}{\gamma}\int_0^t \left(g_2(\xi, \tau)\, b\chi(\lambda) + 1\right) e^{2c\tau}\dot{\lambda}\, d\tau + \frac{1}{8c}\bar{d}^2\left(1 - e^{-2ct}\right) \tag{36}$$

Considering $\left(V(0)\, e^{-2ct} + \frac{1}{8c}\bar{d}^2\left(1 - e^{-2ct}\right) \leq V(0) + \frac{1}{8c}\bar{d}^2 = c_0\right)$, then Eq. (36) can be written as:

$$V(t) \leq c_0 + \frac{e^{-2ct}}{\gamma}\int_0^t \left(g_2(\xi, \tau)\, b\chi(\lambda) + 1\right) e^{2c\tau}\dot{\lambda}\, d\tau \tag{37}$$

By applying Lemma 1, $\int_0^t \left(g_2(\xi, \tau)\, b\chi(\lambda) + 1\right) e^{2c\tau}\dot{\lambda}\, d\tau$ is bounded on the time interval $[0, \infty)$. Since $e^{-2ct} \to 0$, $V(t)$ is ultimately bounded, and considering the relation in Eq. (23), e_is for $i = 1, 2, ..., n$ are also ultimately bounded. By proper adjustment of c_is and γ, the tracking error $e_1 = y(t) - y_r(t)$ will converge to zero with a small ultimate bound.

The first phase leads to solving the tracking problem for the augmented system in Eq. (11). In the next phase, the designed signal in Eq. (27) is called $v_r(t)$ and will be considered the desired output signal for the S_1 subsystem. Now, the task is to design the control input $u(t)$ such that the output of the S_1 subsystem tracks the desired signal $v_r(t)$ in the presence of a nonsymmetric input saturation constraint $\phi_1(.)$ (i.e. $v(t) \to v_r(t)$).

4.2. Phase 2

In this subsection, the aim is to design a robust adaptive input control law $u(t)$, such that the output of the S_1 subsystem tracks the desired signal $v_r(t)$. The S_1 subsystem is subjected to a nonsymmetric input saturation constraint, which causes difficulties in the design of the control law. To deal with this problem, a new definition is employed to describe the saturation function, and based on the tracking purpose, the error vector is defined. Then, by using the sliding mode method, a robust adaptive controller is proposed that leads to the stabilization of the error dynamics.

The nonsymmetric saturation function in the input of the S_1 subsystem is defined as:

$$\phi_1(u(t)) = \begin{cases} u_{\max} & u(t) > u_{\max} \\ u(t) & -u_{\min} \leq u(t) \leq u_{\max} \\ -u_{\min} & u(t) < -u_{\min} \end{cases} \tag{38}$$

where $-u_{\min}$, $u_{\max}$ are the lower and upper limits of the saturation function, respectively. The nonsymmetric saturation function in Eq. (38) can be defined as:[33,34]

$$\phi_1(u(t)) = \sigma(u(t))\, u(t) \tag{39}$$

where

$$\sigma(u(t)) = \begin{cases} \dfrac{u_{\max}}{u(t)} & u(t) > u_{\max} \\ 1 & -u_{\min} \leq u(t) \leq u_{\max} \\ \dfrac{-u_{\min}}{u(t)} & u(t) < -u_{\min} \end{cases} \tag{40}$$

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Since $0 < \sigma\ (u(t)) \leq 1$, it exists that a constant α satisfies the following relation:[35]

$$0 < \alpha < \sigma\ (u(t)) \leq 1\ \forall u(t),\ t \in [0, \infty) \tag{41}$$

To achieve the tracking purpose, the error vector is defined as $z(t) = [z_1(t),\ z_2(t),\ ...,\ z_m(t)]^T$, where

$$\begin{cases} z_1(t) = \eta_1(t) - v_r(t) \\ z_i(t) = \eta_i(t) - v_r^{(i-1)}(t) \quad i = 2, 3, ..., m \end{cases} \tag{42}$$

It is assumed that the signal $v_r(t)$ and its derivatives up to the system's relative degree are known, bounded and continuous. The time derivative of this signal may be generated by appropriate filters given in literature.[36–38]
The dynamic equations of the error vector are computed as:

$$\begin{cases} \dot{z}_i(t) = z_{i+1}(t) \qquad i = 1, 2, ..., m-1 \\ \dot{z}_m(t) = f_1(\eta, t) + \Delta f_1(\eta, t) + g_1\phi_1\ (u(t)) + d_1(t) - v_r^{(m)}(t) \end{cases} \tag{43}$$

Now, the task is to design a robust adaptive controller such that the error vector $z(t)$ converges to the origin. For satisfying this purpose, an approach based on the adaptive sliding mode method is proposed. To implement the sliding mode method, two steps should be taken. The first step is to choose an appropriate sliding surface for satisfying the control objective, and the second step is to design a discontinuous control law that forces the error trajectories to reach the sliding surface.[39–43]
Based on the mentioned points, the sliding surface is suggested as:

$$s(t) = z_m(t) + \sum_{i=1}^{m-1} k_i z_i(t) = kz \tag{44}$$

where the vector $k = \left[k_1,\ k_2,\ ...,\ k_{m-1},\ 1\right] \in R^m$ should be designed. To stabilize the error dynamics, motion equations on the sliding surface (i.e. $s(t) = 0$) should be asymptotically stable. Since $s(t) = 0$ results in $z_m(t) = -\sum_{i=1}^{m-1} k_i z_i(t)$, then by considering Eq. (43), the dynamic equations of motion equations on the sliding surface (called the reduced-order model) are obtained as:

$$\begin{aligned} &\dot{z}_r(t) = Az_r(t) \\ &A = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ & ... & & & \\ -k_1 & -k_2 & -k_3 & ... & -k_{m-1} \end{bmatrix} \end{aligned} \tag{45}$$

where $z_r(t) = \left[z_1(t),\ z_2(t),\ ...,\ z_{m-1}(t)\right]^T$ is the state vector of the system in Eq. (45). For the asymptotic stability of the system in Eq. (45), k_is should be chosen such that the matrix A is a Hurwitz matrix. Realization of this fact yields $z_i(t) \to 0$ for $(i = 1,\ ...,\ m)$, and consequently, the tracking error converges to the origin, and the tracking objective for the output of the S_1 subsystem is satisfied. For reaching the surface, a discontinuous controller is needed, which is given in the next theorem.

Theorem 1. *A robust control law, which is described in Eq.* (46) *with adaptive laws in Eqs.* (48) *and* (49)*, guarantees the tracking of the desired signal $v_r(t)$ by the output of the S_1 subsystem in the presence of nonsymmetric input saturation constraint, model uncertainty and external disturbance.*

$$u(t) = \frac{-1}{g_1}\hat{\alpha}(t)\psi(t)sgn\ (s(t)) \tag{46}$$

where $\hat{\alpha}(t)$ and $\psi(t)$ are positive functions and

$$\psi(t) = |f_1(\eta, t)| + d_1^* + \varepsilon\hat{\mu}(t) + \left|v_r^{(m)}(t)\right| + \left|\sum_{i=1}^{m-1} k_i z_{i+1}(t)\right| \tag{47}$$

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The adaptive laws are obtained as:

$$\dot{\hat{\alpha}}(t) = \psi(t)\hat{\alpha}^3(t)\,|s(t)| \tag{48}$$

$$\dot{\hat{\mu}}(t) = |s(t)| \tag{49}$$

where $\hat{\alpha}\,(0)$ *and* $\hat{\mu}\,(0)$ *have small positive values,* $d_1^* > 0$ *is a known constant, and* $\varepsilon > 1$ *is the design parameter.*

Proof. Define $\tilde{\alpha}(t) = \hat{\alpha}^{-1}(t) - \alpha$ and $\tilde{\mu}(t) = \hat{\mu}(t) - \mu$. Let us consider the following Lyapunov function:

$$W(t) = \frac{1}{2}s^2(t) + \frac{1}{2}\tilde{\mu}^2(t) + \frac{1}{2}\tilde{\alpha}^2(t) \tag{50}$$

Then, the time derivative of Eq. (50) is calculated as:

$$\dot{W}(t) = s(t)\dot{s}(t) + \tilde{\mu}(t)\dot{\tilde{\mu}}(t) + \tilde{\alpha}(t)\dot{\tilde{\alpha}}(t) \tag{51}$$

where the time derivative of the sliding surface $s(t)$ is:

$$\begin{aligned}\dot{s}(t) &= \dot{z}_m(t) + \sum_{i=1}^{m-1} k_i \dot{z}_i(t) \\ &= f_1(\eta, t) + \Delta f_1(\eta, t) + g_1\phi_1\,(u(t)) + d_1(t) - v_r^{(m)}(t) + \sum_{i=1}^{m-1} k_i z_{i+1}(t)\end{aligned} \tag{52}$$

Inserting Eq. (52) into Eq. (51) yields:

$$\begin{aligned}\dot{W}(t) = s(t) &\left(f_1(\eta, t) + \Delta f_1(\eta, t) + g_1\phi_1\,(u(t)) + d_1(t) - v_r^{(m)}(t) + \sum_{i=1}^{m-1} k_i z_{i+1}(t)\right) \\ &+ \tilde{\mu}(t)\dot{\tilde{\mu}}(t) + \tilde{\alpha}(t)\dot{\tilde{\alpha}}(t)\end{aligned} \tag{53}$$

By considering Eq. (39), one has

$$\begin{aligned}\dot{W}(t) \le |s(t)| &\left(|f_1(\eta, t)| + |\Delta f_1(\eta, t)| + |d_1(t)| + \left|v_r^{(m)}(t)\right| + \left|\sum_{i=1}^{m-1} k_i z_{i+1}(t)\right|\right) \\ &+ g_1\sigma\,(u(t))\,u(t)s(t) + \tilde{\mu}(t)\dot{\tilde{\mu}}(t) + \tilde{\alpha}(t)\dot{\tilde{\alpha}}(t)\end{aligned} \tag{54}$$

Substituting $u(t)$ from Eq. (46) and using $s(t)sgn\,(s(t)) = |s(t)|$, Eq. (54) can be rewritten as:

$$\begin{aligned}\dot{W}(t) \le |s(t)| &\left(|f_1(\eta, t)| + |\Delta f_1(\eta, t)| + |d_1(t)| + \left|v_r^{(m)}(t)\right| + \left|\sum_{i=1}^{m-1} k_i z_{i+1}(t)\right|\right) \\ &- \sigma\,(u(t))\,\hat{\alpha}(t)\psi(t)\,|s(t)| + \tilde{\mu}(t)\dot{\tilde{\mu}}(t) + \tilde{\alpha}(t)\dot{\tilde{\alpha}}(t)\end{aligned} \tag{55}$$

where considering $\tilde{\alpha}(t) = \hat{\alpha}^{-1}(t) - \alpha$ and $\tilde{\mu}(t) = \hat{\mu}(t) - \mu$, then

$$\dot{\tilde{\alpha}}(t) = -\hat{\alpha}^{-2}(t)\dot{\hat{\alpha}}(t) \tag{56}$$

$$\dot{\tilde{\mu}}(t) = \dot{\hat{\mu}}(t) \tag{57}$$

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Now, using Eqs. (41), (56) and (57), one has:

$$\begin{aligned}\dot{W}(t) \le |s(t)| \left(|f_1(\eta, t)| + |\Delta f_1(\eta, t)| + |d_1(t)| + \left|v_r^{(m)}(t)\right| + \left|\sum_{i=1}^{m-1} k_i z_{i+1}(t)\right| \right) \\ - \alpha\hat{\alpha}(t)\psi(t)\,|s(t)| \\ + \tilde{\mu}(t)\dot{\hat{\mu}}(t) - \tilde{\alpha}(t)\hat{\alpha}^{-2}(t)\dot{\hat{\alpha}}(t)\end{aligned} \tag{58}$$

Furthermore, substituting the relations in Eqs. (47), (48) and (49) into Eq. (58) and considering Assumptions 1 and 2 yields:

$$\begin{aligned}\dot{W}(t) &\le |s(t)| \left(|f_1(\eta, t)| + \mu + d_1^* + \left|v_r^{(m)}(t)\right| + \left|\sum_{i=1}^{m-1} k_i z_{i+1}(t)\right| \right) - \alpha\hat{\alpha}(t)\psi(t)\,|s(t)| \\ &\quad + \tilde{\mu}(t)\,|s(t)| \\ &\quad - \tilde{\alpha}(t)\hat{\alpha}^{-2}(t)\psi(t)\hat{\alpha}^3\,|s(t)| \\ &= |s(t)| \left(|f_1(\eta, t)| + \mu + d_1^* + \left|\sum_{i=1}^{m-1} k_i z_{i+1}(t)\right| + \left|v_r^{(m)}(t)\right| + \varepsilon\hat{\mu}(t) - \varepsilon\hat{\mu}(t) \right) \\ &\quad - \alpha\hat{\alpha}(t)\psi(t)\,|s(t)| \\ &\quad + \tilde{\mu}(t)\,|s(t)| - \tilde{\alpha}(t)\hat{\alpha}(t)\psi(t)\,|s(t)|\end{aligned} \tag{59}$$

Referring to Eq. (47), one has:

$$\dot{W}(t) \le |s(t)| \left(\psi(t) + \mu - \varepsilon\hat{\mu}(t) - \alpha\hat{\alpha}(t)\psi(t)\right) + \tilde{\mu}(t)\,|s(t)| - \tilde{\alpha}(t)\hat{\alpha}(t)\psi(t)\,|s(t)| \tag{60}$$

Since $\tilde{\alpha}(t) = \hat{\alpha}^{-1}(t) - \alpha$ and $\tilde{\mu}(t) = \hat{\mu}(t) - \mu$, Eq. (60) can be rewritten as:

$$\begin{aligned}\dot{W}(t) &\le |s(t)| \left(\psi(t) + \mu - \varepsilon\hat{\mu}(t) - \left(\hat{\alpha}^{-1}(t) - \tilde{\alpha}(t)\right)\hat{\alpha}(t)\psi(t)\right) + (\hat{\mu}(t) - \mu)\,|s(t)| \\ &\quad - \tilde{\alpha}(t)\hat{\alpha}(t)\psi(t)\,|s(t)| \\ &= -(\varepsilon - 1)\,\hat{\mu}(t)\,|s(t)| \\ &= -U(t) \le 0\end{aligned} \tag{61}$$

where $U(t) = (\varepsilon - 1)\,\hat{\mu}(t)\,|s(t)|$ is a positive function ($\varepsilon > 1$). Integrating $\dot{W}(t) \le -U(t)$ over $[0,\ t]$ gives:

$$W(t) \le W(0) - \int_0^t U(\tau)d\tau \tag{62}$$

Since $\dot{W}(t) \le -U(t)$, $W(t)$ is a decreasing function, and according to Barbalat's lemma,[44] $\lim_{t\to\infty} \int_0^t U(\tau)d\tau$ is bounded and consequently:

$$\lim_{t\to\infty} U(t) = \lim_{t\to\infty} (\varepsilon - 1)\,\hat{\mu}(t)\,|s(t)| = 0 \tag{63}$$

Since $(\varepsilon - 1)$ and $\hat{\mu}(t)$ have positive values, $\lim_{t\to\infty} s(t) = \lim_{t\to\infty} k\,z(t) = 0$ and $z(t) \to 0$. Consequently, the robust asymptotic stability of the tracking error in the presence of mentioned constraints is proved.

Remark 1. The proposed control law in Eq. (46) has a discontinuous term, which leads to the chattering problem. To overcome this problem, the control law in Eq. (46) is approximated by the following function:

$$u(t) = \frac{-1}{g_1}\hat{\alpha}(t)\psi(t)\tanh\left(\frac{s(t)}{\tau_0}\right) \tag{64}$$

where $\tau_0 > 0$ is a small constant, and the continuous function $\tanh\left(s(t)/\tau_0\right)$ is an approximation of the discontinuous function $sgn\,(s(t))$. In this situation, $|s(t)|$ will decrease until it reaches the

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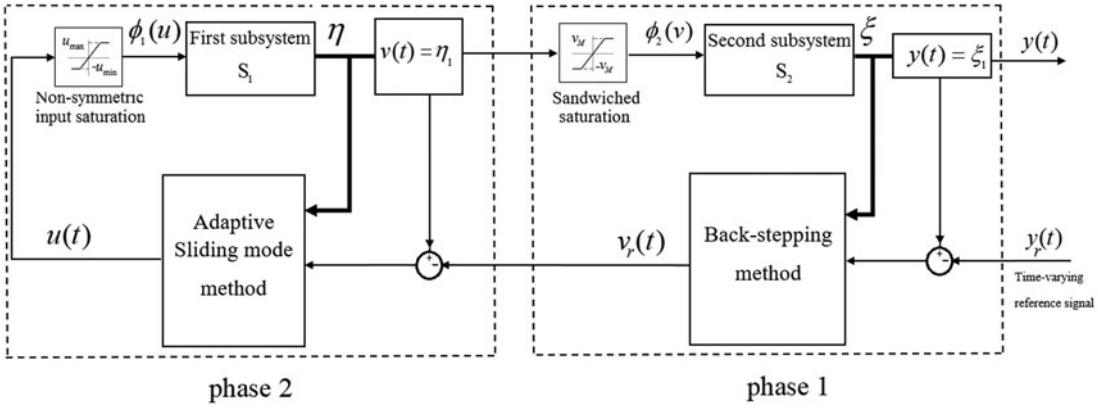


Fig. 2. Block diagram of the closed-loop system.

boundary layer $|s(t)| \leq \rho$ and remains there after that. Furthermore, by choosing a sufficiently small value for the parameter τ_0, the ultimate bound ρ can be reduced.[45] In this situation, the adaptive laws in Eqs. (48) and (49) are used outside the boundary layer.

The block diagram of a closed-loop system after the implementation of phases 1 and 2 is demonstrated in Fig. 2. As seen, in phase 1, considering an S_2 subsystem, sandwiched saturation function and time-varying reference signal $y_r(t)$, the signal $v_r(t)$ is achieved via the backstepping method. In phase 2, considering a nonsymmetric input saturation constraint, the dynamical equations of an S_1 subsystem and the output of phase 1 (which is $v_r(t)$), the control input $u(t)$ is obtained based on the adaptive sliding mode method.

5. Simulation Results

To validate the effectiveness of the proposed approach, a mechanical sandwich system consisting of two nonlinear systems is considered in the structure of Fig. 1. The first subsystem is replaced with a one-link manipulator system, and the second subsystem is substituted with a mass spring system.

The dynamical equations of the manipulator are considered as follows:[46]

$$S_1 : \begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = -\frac{A_0}{C}\eta_2 - \frac{L}{C}\sin(\eta_1) + \Delta f_1(\eta, t) + g_1\phi_1(u(t)) + d_1(t) \\ v(t) = \eta_1 \end{cases} \tag{65}$$

where η_1 is the link angular position and η_2 is the velocity of the manipulator. The dynamical equations of the mass spring–damper system are:[47]

$$S_2 : \begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \frac{1}{M}\left(-k_0\xi_1 - 0.5\xi_1^3 - b_0\xi_2 - 0.5\xi_2^2\right) + g_2(\xi, t)\phi_2(v(t)) + d_2(t) \\ y(t) = \xi_1 \end{cases} \tag{66}$$

where ξ_1 and ξ_2 are displacement and velocity of the mass, respectively. The parameters of the subsystems are $A_0 = 1,\ L = 2.2,\ C = 0.5,\ B = 5, M = 1,\ b_0 = 2,\ k_0 = 2$.
External disturbances, model uncertainties and control coefficients are considered as $d_1(t) = 0.5\sin(2t)$, $d_2(t) = 0.5\sin(t)$, $\Delta f_1(\eta, t) = 0.2\eta_2 + 4\eta_2\sin(\eta_1)$, $g_1 = \frac{B}{C}$ and $g_2(\xi, t) = \frac{1}{M + 0.1\sin(t)}$.

For simulation, the design parameters are chosen as $c_1 = 5,\ c_2 = 20,\ k_1 = 200$. The goal is to track the reference signal $y_r(t) = \sin(t)$. The lower and upper bounds of the nonsymmetric input saturation constraint are considered as $-u_{\min} = -2,\ u_{\max} = 3$, respectively. Moreover, the upper admissible value of the sandwiched saturation function is assumed as $v_M = 3$. The simulation results are presented in Figs. 3–7.

Figure 3 shows the output tracking performance of an S_2 subsystem. It is clear that the output $y(t)$ robustly tracks the reference signal. The time response of the saturated input signal of the S_2 subsystem is illustrated in Fig. 4.

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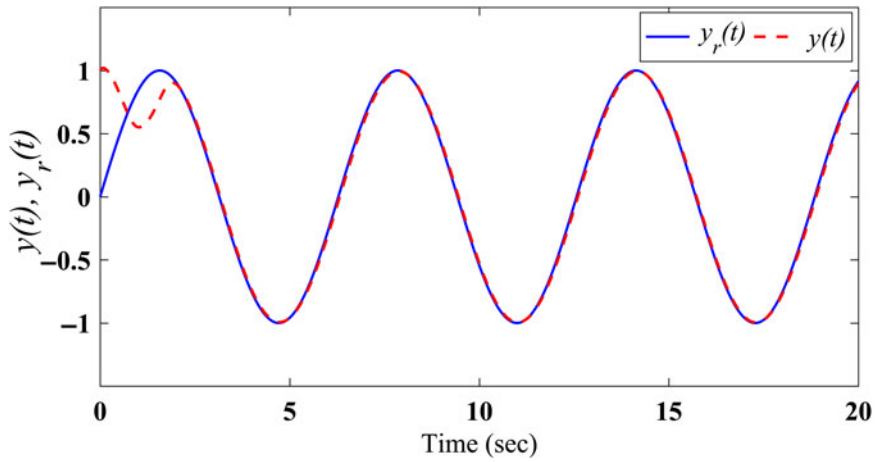


Fig. 3. Time responses of the output $y(t)$ and the desired output $y_r(t)$ of the S_2 subsystem.

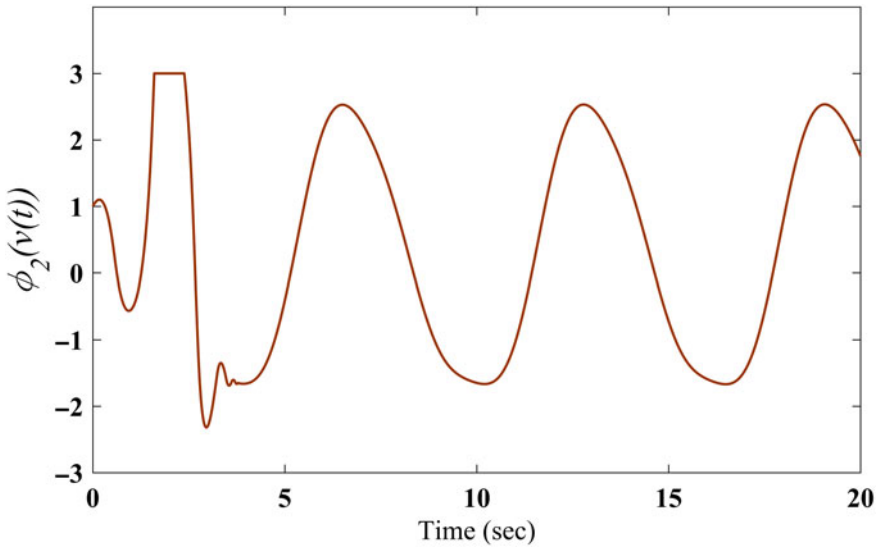


Fig. 4. Time response of $\phi_2(v(t))$ (saturation level is $v_M = 3$).

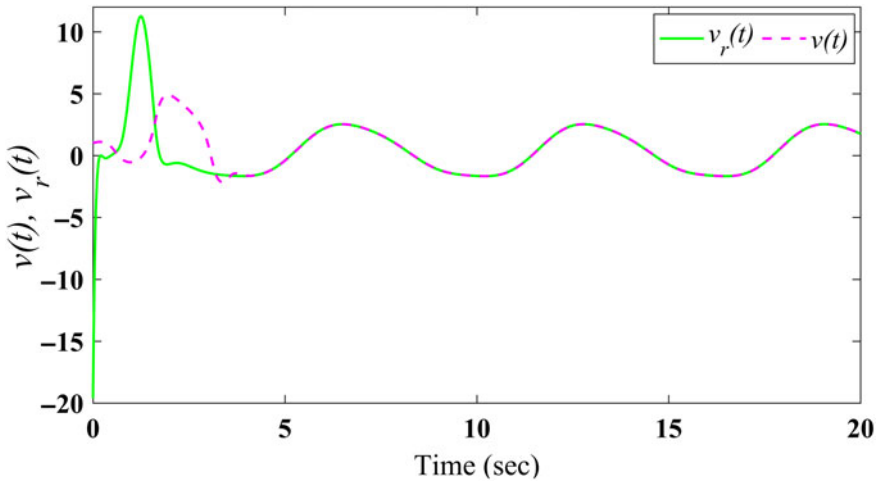


Fig. 5. Time response of the actual output $v(t)$ and desired output $v_r(t)$ of the S_1 subsystem.

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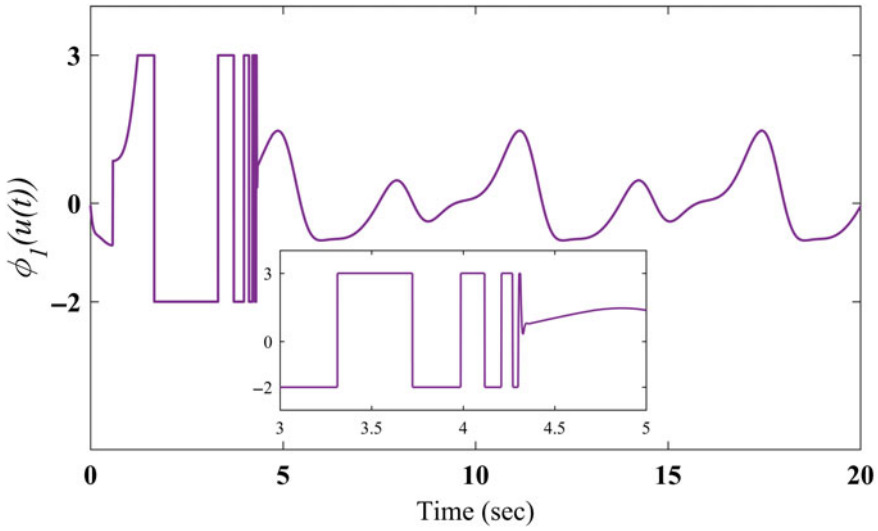


Fig. 6. Time response of the saturated control input ϕ_1 $(u(t))$ of the S_1 subsystem for $u_{\max} = 3,\ u_{\min} = -2$.

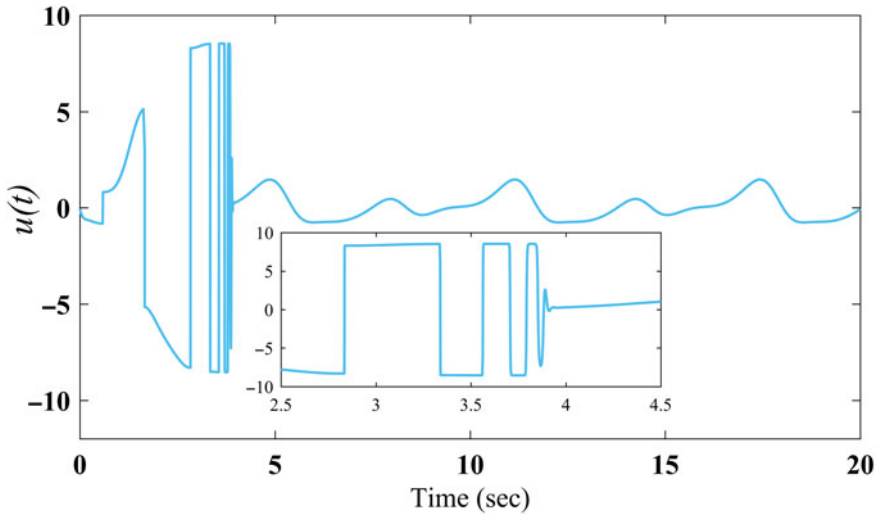


Fig. 7. Time response of the control input $u(t)$.

The output of phase 1 (i.e., $v_r(t)$) and the output of S_1 subsystem (i.e., $v(t)$) are illustrated in Fig. 5. This figure verifies the ability of the proposed approach, in the second phase, to output tracking despite nonsymmetric input saturation constraint and model uncertainties. Figure 6 demonstrates the saturated control input of the S_1 subsystem. The time response of the control input for achieving the tracking goal is also illustrated in Fig. 7.

From the simulation results, it is obvious that, despite the occurrence of saturation in the input of the first subsystem and also between the two subsystems, the tracking of the time-varying reference signal is realized for the sandwich system with an effective performance. It illustrates the capability of the proposed controller to achieve the tracking purpose for sandwich systems in the presence of mentioned limitations.

To show the efficiency of the proposed approach, output tracking in the second phase is compared to the controller given in Ref. [48] for a similar signal $v_r(t)$. As shown in Fig. 8, the given controller in Ref. [45] has no acceptable performance in output tracking in the face of a nonsymmetric input saturation constraint.

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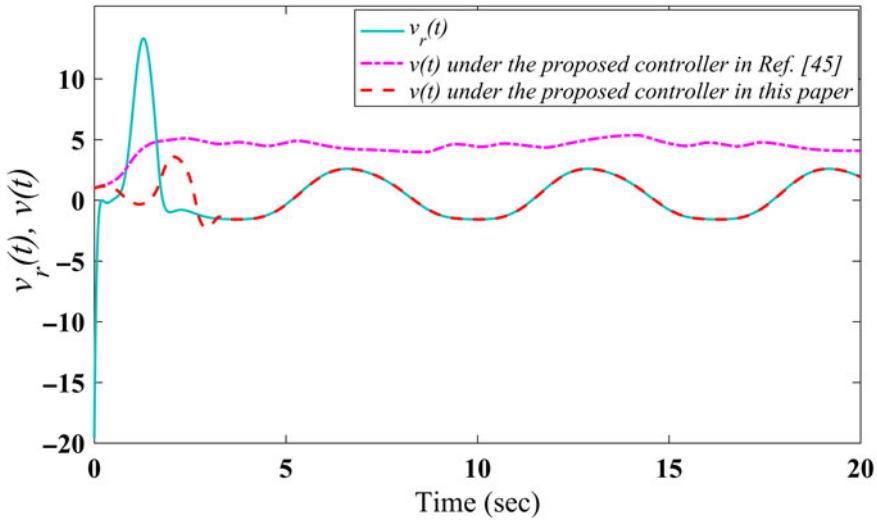


Fig. 8. Time response of the actual output $v(t)$ and desired output $v_r(t)$ of the S_1 subsystem under the proposed controller and the controller given in Ref. [45].

6. Conclusions

In this paper, a robust controller is presented for the output tracking problem of sandwich systems with saturation nonlinearities. The designing procedure is divided into two phases. In the first phase, the tracking problem is solved for the second subsystem by combining the backstepping approach and Nussbaum function. In the second phase, the robust control law is designed via the adaptive sliding mode technique for tracking the desired signal that is obtained in the first phase. To verify the efficiency of the given method, the proposed controller is applied to a practical sandwich system. The simulation results illustrate the applicability and performance of the proposed method.

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