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ALMOST ERDŐS SETS

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Abstract A set $S \subseteq \mathbb{R}$ is *almost Erdős* if, for every $\varepsilon > 0$, there exists a set $E \subseteq \mathbb{R}$ of positive Lebesgue measure such that $\{x \in S : ax + b \notin E\}$ is nonempty for all $|a| > \varepsilon$ and $b \in \mathbb{R}$. In this note, we show that any decreasing null sequence (x_n) with decay rate greater than $1/2$ is an almost Erdős set.

Keywords: almost Erdős sets; decreasing null sequences; Lebesgue measure; Hausdorff dimension; additive combinatorics

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1. Introduction

The lower decay rate ρ of a decreasing null sequence (x_n) is given by

$$\rho = \liminf_{n\to\infty} \frac{x_{n+1}}{x_n}. \tag{1}$$

Evidently, $\rho \in [0, 1]$. When $\rho \in (0, 1]$, we say that (x_n) has a positive lower decay rate. It was proved independently by Eigen and Falconer [2, 4] that if $\rho = 1$, then there exists a set E of positive Lebesgue measure such that $\{n \in \mathbb{N} : ax_n + b \notin E\}$ is nonempty for any $a \neq 0$ and $b \in \mathbb{R}$. This relates to the well-known question posed by Erdős [3]:

> Given an infinite set S of real numbers, is there a set $E \subseteq \mathbb{R}$ of positive Lebesgue measure such that $\{x \in S : ax + b \notin E\}$ is nonempty for all $a, b \in \mathbb{R}$?

This is known as the Erdős similarity problem. Erdős was motivated by a classical result of Steinhaus [10] for finite sets, to which the answer to the above question is negative. In [11], the term *Erdős set* was coined for any set S for which the answer to the above question is positive. A set that is not Erdős is known as a *universal set*; see, e.g., [6, 9]. Steinhaus' result implies that finite sets are not Erdős. Any unbounded or dense set S is Erdős, since one may take E to be a Cantor-type set of positive Lebesgue measure. It is still unknown whether there exists an infinite set that is not Erdős (equivalently,

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universal). An analogue of the problem in topological spaces is considered by [5], while a more recent result involving the construction of special Cantor-type sets can be found in [7].

In the spirit of this problem, we introduce the following notion: a set $S \subseteq \mathbb{R}$ is *almost Erdős* if, for every $\varepsilon > 0$, there exists a set $E \subseteq \mathbb{R}$ of positive Lebesgue measure such that $\{x \in S : ax + b \notin E\}$ is nonempty for all $|a| > \varepsilon$ and $b \in \mathbb{R}$. If S is almost Erdős, then so is any S' containing S; the closure of an almost Erdős set is still almost Erdős, and any affine translation of an almost Erdős set is again almost Erdős. Consequently, it suffices to restrict attention to decreasing null sequences. Building on Falconer's work [4], we show that whenever $\rho > 1/2$, the sequence (x_n) is an almost Erdős set (Theorem 1). The argument may also be extended to sequences (x_n) with positive decay rate $\rho \leq 1/2$; however, in this case, we are only able to construct compact sets E_+ and E_- of positive measure with the property that E_+ misses any copy of (x_n) whenever $b, |a| > \varepsilon$, while E_- misses any copy of (x_n) whenever $-b, |a| > \varepsilon$ (Theorem 2).

Moreover, we show that if $(x_n)_{n\in\mathbb{N}}$ is a decreasing null sequence with $\rho > 1/2$, then there exists a set E of positive Hausdorff dimension such that $\{n \in \mathbb{N} : ax_n + b \notin E\}$ is infinite for all $a \neq 0$ and $b \in \mathbb{R}$ (Theorem 3). We conclude with some corollaries concerning translates of sequences (x_n) by a real number b.

2. Almost Erdös sequences

Theorem 1 *Let $(x_n)_{n\in\mathbb{N}}$ be a decreasing null sequence with $\rho > 1/2$. Then, for every $\varepsilon > 0$, there exists a set $E \subseteq \mathbb{R}$ of positive measure such that $\{n \in \mathbb{N} : ax_n + b \notin E\}$ is nonempty for all $|a| > \varepsilon$ and $b \in \mathbb{R}$. Moreover, the measure of E is uniformly bounded away from zero.*

Proof. Let $\rho > 1/2$. Then there exists $N \in \mathbb{N}$, $N \geq 3$, such that

$$\rho > \frac{1}{2} + \frac{1}{N}.$$

Let (r_k) be a decreasing null sequence such that $r_1 < 1/2$ is sufficiently small. Then, for every $k \in \mathbb{N}$ and $b \in [0, r_k]$, we have

$$\frac{r_k/N + b}{r_k + b} \leq \frac{r_k/N + r_k}{r_k + r_k} = \frac{1}{2N} + \frac{1}{2} < \frac{1}{N} + \frac{1}{2} < \rho - r_k < \frac{x_{n+1}}{x_n} \quad \text{for all } n \geq n_k. \tag{2}$$

The inequalities in (2) imply that, for $b \in [0, r_k]$, the intervals

$$\frac{1}{x_n}\,(r_k/N + b,\, r_k + b) \quad \text{and} \quad \frac{1}{x_{n+1}}\,(r_k/N + b,\, r_k + b)$$

overlap for all $n \geq n_k$. Consequently, the interval $(r_k/N + b, \infty)/x_{n_k}$ is entirely contained in the complement of the set

$$\bigcap_{n\geq n_k} \frac{1}{x_n}\,(E_k + b), \quad \text{where } E_k = \bigcup_{j\in\mathbb{Z}} [jr_k,\, jr_k + r_k/N] \quad \text{for all } k \in \mathbb{N}, \tag{3}$$

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and therefore

$$\bigcap_{n\geq n_k} \frac{1}{x_n}\,(E_k + b) \subseteq (-\infty,\, r_k/N + b]/x_{n_k}. \tag{4}$$

On the other hand, for $b \in [0, r_k]$ we have

$$\begin{aligned}\frac{r_k - b}{2r_k - r_k/N - b} &\leq \frac{r_k}{2r_k - r_k/N}\\ &= \frac{1}{2 - 1/N} < \frac{1}{2} + \frac{1}{2N} < \rho - r_k < \frac{x_{n+1}}{x_n} \quad \text{for all } n \geq n_k,\end{aligned}$$

implying that the intervals

$$\frac{1}{x_n}\,(-2r_k + r_k/N + b,\, -r_k + b) \quad \text{and} \quad \frac{1}{x_{n+1}}\,(-2r_k + r_k/N + b,\, -r_k + b)$$

overlap for all $n \geq n_k$. Therefore, the interval $(-\infty,\, -r_k + b)/x_{n_k}$ is entirely contained in the complement of the set (3), and so

$$\bigcap_{n\geq n_k} \frac{1}{x_n}\,(E_k + b) \subseteq [-r_k + b,\, \infty)/x_{n_k}. \tag{5}$$

Together with (4), we obtain

$$\bigcap_{n\geq n_k} \frac{1}{x_n}\,(E_k + b) \subseteq [-r_k,\, 2r_k]/x_{n_k} \quad \text{for all } k \in \mathbb{N}. \tag{6}$$

By periodicity of E_k, (6) holds for all $b \in \mathbb{R}$.

Now let $\varepsilon > 0$. We may choose (r_k) so that (r_k/x_{n_k}) is itself a null sequence. Then there exists $k_\varepsilon \in \mathbb{N}$ such that $r_k < \varepsilon x_{n_k}/2$ for every $k \geq k_\varepsilon$. Consequently,

$$\bigcap_{n\geq n_k} \frac{1}{x_n}\,(E_k + b) \subseteq [-\varepsilon,\, \varepsilon] \quad \text{for all } k \geq k_\varepsilon. \tag{7}$$

This implies that, for any $|a| > \varepsilon$, there exists $n \geq n_k$ such that $ax_n - b \notin E_k$ whenever $k \geq k_\varepsilon$. Taking $k = k_\varepsilon$ and letting $E = E_{k_\varepsilon}$ yields the candidate set. Finally, we estimate the measure of E. Let λ denote the Lebesgue measure on $\mathbb{R}$. Then

$$\lambda(E) = \lambda(E_{k_\varepsilon}) \geq \lambda(E_{k_\varepsilon} \cap [0,1]) \geq \frac{r_{k_\varepsilon}}{N}\left\lfloor \frac{1}{r_{k_\varepsilon}} \right\rfloor > \frac{r_{k_\varepsilon}}{N}\left(\frac{1}{r_{k_\varepsilon}} - 1\right) \geq \frac{1}{N} - \frac{r_1}{N} > \frac{1}{2N},$$

since $r_1 < 1/2$. Thus, the set E has a measure uniformly bounded away from zero. □

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Theorem 2 *Let $(x_n)_{n\in\mathbb{N}}$ be a decreasing null sequence with $\rho > 0$. If $\rho \le 1/2$, then, for every $\varepsilon > 0$, there exist sets $E_1, E_2 \subseteq \mathbb{R}$ of positive measure such that*

$$\{n \in \mathbb{N} : ax_n + b \notin E_1\} \textit{is nonempty for all } b > \varepsilon,\ |a| > \varepsilon,$$

and

$$\{n \in \mathbb{N} : ax_n + b \notin E_2\} \textit{is nonempty for all } b < -\varepsilon,\ |a| > \varepsilon.$$

Moreover, the measures of the sets E_1 and E_2 are uniformly bounded away from zero.

Proof. Let $\rho \le 1/2$. Then there exists $N \ge 3$, $N \in \mathbb{N}$, such that

$$\frac{1}{N} < \rho \le \frac{1}{N-1}.$$

W.l.o.g., take (r_k) such that $r_k < N\rho - 1$ for all $k \in \mathbb{N}$. For every $k \in \mathbb{N}$ and $b \in [0, r_k]$, we have

$$\frac{(N\rho - 1 - r_k)r_k + b}{Nr_k + b} \le \frac{N\rho - r_k}{N+1} < \rho - \frac{r_k}{N+1} < \frac{x_{n+1}}{x_n}, \quad \text{for all } n \ge n_k.$$

Consequently, for $b \in [0, r_k]$, the intervals

$$\frac{1}{x_n}((N\rho - 1 - r_k)r_k + b,\ Nr_k + b) \quad \text{and} \quad \frac{1}{x_{n+1}}((N\rho - 1 - r_k)r_k + b,\ Nr_k + b)$$

overlap for all $n \ge n_k$. Thus, the interval $((N\rho - 1 - r_k)r_k + b, \infty)/x_{n_k}$ is contained in the complement of the set

$$\bigcap_{n\ge n_k} \frac{1}{x_n}(E_k^1 + b),$$

where

$$E_k^1 = [0, (N\rho - 1 - r_k)r_k] \cup \bigcup_{j\ge N} [jr_k,\ jr_k + (N\rho - 1 - r_k)r_k], \quad \text{for all } k \in \mathbb{N}. \tag{8}$$

Because $\inf E_k^1 = 0$ for all $k \in \mathbb{N}$, together with the inclusion

$$\bigcap_{n\ge n_k} \frac{1}{x_n}(E_k^1 + b) \subseteq (-\infty, (N\rho - 1 - r_k)r_k + b]/x_{n_k},$$

it follows that

$$\bigcap_{n\ge n_k} \frac{1}{x_n}(E_k^1 + b) \subseteq [0, (N\rho - 1 - r_k)r_k + b]/x_{n_k} \subseteq [0, (N\rho - r_k)r_k]/x_{n_k}, \quad \text{for all } k \in \mathbb{N} \tag{9}$$

for all $b \in [0, r_k]$.

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Now let $b \geq Nr_k$. Then, from (8), it follows that $E_k^1 + b \subseteq E_k^1 + \widehat{b}$, where $b \equiv \widehat{b} \mod r_k$. Consequently, in view of (9), we have

$$\bigcap_{n \geq n_k} \frac{1}{x_n} (E_k^1 + b) \subseteq [0, (N\rho - r_k)r_k]/x_{n_k}, \quad \text{for all } k \in \mathbb{N}, \tag{10}$$

for all $b \in [0, r_k] \cup [Nr_k, \infty)$.

Now let $\varepsilon > 0$. Again, we may choose (r_k) so that (r_k/x_{n_k}) is itself a null sequence. Then, there exists $k_\varepsilon \in \mathbb{N}$ such that $Nr_k < \varepsilon x_{n_k}$ for every $k \geq k_\varepsilon$. Hence, for all $b > \varepsilon > \varepsilon x_{n_k} > Nr_k$, we have*

$$\bigcap_{n \geq n_k} \frac{1}{x_n} (E_k^1 + b) \subseteq [0, \varepsilon], \quad \text{for all } k \geq k_\varepsilon.$$

This implies that, for any $|a| > \varepsilon$, there exists $n \geq n_k$ such that $ax_n - b \notin E_k$ whenever $k \geq k_\varepsilon$. Denote $E_1 = E_{k_\varepsilon}^1$. Then, similar calculations show that

$$\begin{aligned} \lambda(E_1) = \lambda(E_{k_\varepsilon}^1) &\geq \lambda(E_{k_\varepsilon}^1 \cap [0,1]) \\ &\geq (N\rho - 1 - r_{k_\varepsilon})r_{k_\varepsilon} \left\lfloor \frac{1 - Nr_{k_\varepsilon}}{r_{k_\varepsilon}} \right\rfloor \\ &> (N\rho - 1 - r_1)(1 - r_1(N+1)) > \frac{1}{3}(N\rho - 1), \end{aligned}$$

by letting $r_1 < (N\rho - 1)/(N+1)$.

Now we show the existence of E_2. Note that, for all $b \in [0, r_k]$, the inequalities

$$\frac{-r_k + b}{-2Nr_k + (N\rho - 1 - r_k)r_k + b} \leq \frac{-r_k}{-2Nr_k + (N\rho - 1 - r_k)r_k} \leq \frac{1}{N} - \frac{r_k}{N} < \rho - \frac{r_k}{N} < \frac{x_{n+1}}{x_n}$$

hold for all $n \geq n_k$.

Therefore, for $b \in [0, r_k]$, the intervals

$$\frac{1}{x_n}(-2Nr_k + (N\rho - 1 - r_k)r_k + b,\ -r_k + b) \quad \text{and} \quad \frac{1}{x_{n+1}}(-2Nr_k + (N\rho - 1 - r_k)r_k + b,\ -r_k + b)$$

overlap for all $n \geq n_k$. Consequently, the interval $(-\infty, -r_k + b)/x_{n_k}$ is contained in the complement of the set

$$\bigcap_{n \geq n_k} \frac{1}{x_n} (E_k^2 + b),$$

where

$$E_k^2 = [-r_k, -r_k + (N\rho - 1 - r_k)r_k] \cup \bigcup_{j \leq -2N} [jr_k,\ jr_k + (N\rho - 1 - r_k)r_k], \quad \text{for all } k \in \mathbb{N}. \tag{11}$$

* Since (x_n) is a decreasing null sequence, we may normalize it so that $x_n < 1$ for all $n \in \mathbb{N}$.

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Because $\sup E_k^2 \le 0$ for all $k \in \mathbb{N}$, together with the inclusion

$$\bigcap_{n \ge n_k} \frac{1}{x_n}\,(E_k^2 + b) \subseteq [-r_k + b, \infty)/x_{n_k},$$

it follows that

$$\bigcap_{n \ge n_k} \frac{1}{x_n}\,(E_k^2 + b) \subseteq [-r_k + b, 0]/x_{n_k} \subseteq [-r_k, 0]/x_{n_k}, \quad \text{for all } k \in \mathbb{N}, \tag{12}$$

for all $b \in [0, r_k]$.

Now, let $b \le -2Nr_k$. Then, from (11), it follows that $E_k^2 + b \subseteq E_k^2 + \widehat{b}$, where $b \equiv \widehat{b}$ mod $(-r_k)$. In view of (12), we obtain

$$\bigcap_{n \ge n_k} \frac{1}{x_n}\,(E_k^2 + b) \subseteq [-r_k, 0]/x_{n_k}, \quad \text{for all } k \in \mathbb{N}, \tag{13}$$

for all $b \in [0, r_k] \cup (-\infty, -2Nr_k]$.

Avoiding repetitions, for any $\varepsilon > 0$, there exists $k_\varepsilon \in \mathbb{N}$ such that, for $b < -\varepsilon$, we have

$$\bigcap_{n \ge n_k} \frac{1}{x_n}\,(E_k^2 + b) \subseteq [-\varepsilon, 0], \quad \text{for all } k \ge k_\varepsilon.$$

Thus, for any $|a| > \varepsilon$, there exists $n \ge n_k$ such that $ax_n - b \notin E_k$ whenever $k \ge k_\varepsilon$. Denote $E_2 = E_{k_\varepsilon}^2$. Then, similarly as for E_1, we have

$$\begin{aligned}
\lambda(E_2) = \lambda(E_{k_\varepsilon}^2) &\ge \lambda(E_{k_\varepsilon}^2 \cap [-1, 0]) \\
&\ge (N\rho - 1 - r_{k_\varepsilon}) r_{k_\varepsilon} \left\lfloor \frac{1 - 2Nr_{k_\varepsilon}}{r_{k_\varepsilon}} \right\rfloor \\
&\ge (N\rho - 1 - r_1)(1 - r_1(2N+1)) > \frac{1}{3}(N\rho - 1),
\end{aligned}$$

by letting $r_1 < (N\rho - 1)/(2N + 1)$.

In particular, the sets E_1 and E_2 have measures uniformly bounded away from zero. □

3. A set of positive Hausdorff dimension avoiding copies of a null sequence

Theorem 3 *Let $(x_n)_{n\in\mathbb{N}}$ be a decreasing null sequence with $\rho > 1/2$. Then, there exists a set $E \subseteq \mathbb{R}$ of positive Hausdorff dimension such that*

$$\{n \in \mathbb{N} : ax_n + b \notin E\} \textit{is infinite for all } a \ne 0 \textit{and } b \in \mathbb{R}.$$

Proof. The inequalities in (2) still hold for all $n \ge n_m$, for any fixed m, that is, by replacing r_k with r_m in $\rho - r_k$. This gives us the freedom to choose any null sequence

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(r_k), independent of whether (r_k/x_{n_k}) is a null sequence or not. Then, inclusion (6) still holds with x_{n_k} replaced by x_{n_m}.

Considering E_k restricted to the interval $[0,1]$ and taking the intersection with respect to $k \geq 1$ yields

$$\bigcap_{n \geq n_m} \frac{1}{x_n}(E+b) \subseteq \{0\}, \quad \text{where } E = \bigcap_{k \geq 1} E_k.$$

Consequently, for every $a \neq 0$ and $b \in \mathbb{R}$, there exists $n \geq n_m$ such that

$$ax_n - b \notin E.$$

By moving along the subsequence $\{n_m\}$, we obtain that

$$ax_n - b \notin E \quad \text{infinitely often.}$$

For $n \geq 1$, let

$$D_n = \bigcap_{k \leq n} E_k.$$

Then $E \subseteq D_n$ for every $n \geq 1$, so any cover (open or closed) for D_n is also a cover for E.

Now, choose the null sequence $r_k = N/(N+1)^k$ for $k \in \mathbb{N}$ with $N \in \mathbb{N}$, $N \geq 3$. The set D_n consists of disjoint closed intervals of length $1/(N+1)^n$, and there are

$$\left(\left\lfloor \frac{r_{n-1}}{Nr_n} \right\rfloor + 1\right)\left(\left\lfloor \frac{r_{n-2}}{Nr_{n-1}} \right\rfloor + 1\right) \cdots \left(\left\lfloor \frac{r_1}{Nr_2} \right\rfloor + 1\right)\left(\left\lfloor \frac{1}{r_1} \right\rfloor + 1\right) = 2^n$$

such intervals.

Given a set $U \subseteq \mathbb{R}$, let $|U| = \sup\{|x-y| : x, y \in U\}$. For $s, t > 0$ and $V \subseteq \mathbb{R}$, denote

$$H^s_t(V) = \inf\Big\{ \sum_{n \geq 1} |U_n|^s : |U_n| \leq t,\ V \subseteq \bigcup_{n \geq 1} U_n \Big\}.$$

Then,

$$H^s_{r_n}(E) \leq H^s_{r_n}(D_n) \leq 2^n \Big(\frac{r_n}{N}\Big)^s = 2^n \Big(\frac{1}{N+1}\Big)^{sn}.$$

For the choice $s = \log 2/\log(N+1)$, we obtain

$$H^s(E) := \lim_{n \to \infty} H^s_{r_n}(E) \leq 1.$$

Next, we show that $H^s(E) > 0$. It suffices to show that, for some $\alpha > 0$, it holds

$$\sum_{n \geq 1} |U_n|^s \geq \alpha, \quad \text{for any cover } \{U_n\} \text{of } E.$$

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W.l.o.g., we may assume that each U_n is a closed interval. Because E is compact, we may assume the cover $\{U_n\}$ is finite. For each U_n, let $k \in \mathbb{N}$ be such that

$$\frac{1}{(N+1)^{k+1}} = \frac{r_{k+1}}{N} \leq |U_n| < \frac{r_k}{N} = \frac{1}{(N+1)^k}.$$

Then U_n intersects D_k in at most one of its subintervals, since the intervals are of width r_k/N and are separated by a distance of $r_k(1-1/N)$. Next, U_n intersects D_{k+1} in at most two intervals, since each subinterval of D_k contains at most two subintervals of D_{k+1}. Inductively, for every $j \geq k$, the interval U_n intersects D_j in at most 2^{j-k} subintervals. Note that

$$2^{j-k} = 2^j(N+1)^{-sk} \leq 2^j(N+1)^s|U_n|^s.$$

We take j large enough so that $(N+1)^{-(j+1)} \leq |U_n|$ for all U_n. Any interval I of D_j contains a nested sequence of subintervals

$$I \supseteq I_1 \supseteq I_2 \supseteq \ldots,$$

where I_m is a subinterval of D_{j+m} inside I. Since $r_k \to 0$, the lengths shrink to zero, so $\bigcap_m I_m$ is nonempty and contained in E. Therefore, any cover $\{U_n\}$ must intersect all 2^j intervals of D_j; otherwise, it would miss a point in E.

We write $j = j_n + k_n$ for each U_n, then

$$2^j \leq \sum_n 2^{j_n} = \sum_n 2^{j-k_n} \leq \sum_n 2^j(N+1)^s|U_n|^s,$$

implying

$$\sum_n |U_n|^s \geq \frac{1}{(N+1)^s} = \frac{1}{2}.$$

□

Remark 4. The set E in Theorem 3 has positive Hausdorff dimension

$$s = \frac{\log 2}{\log(N+1)}$$

and Hausdorff measure at least $1/2$.

4. Some corollaries

Corollary 5. *Let $(x_n)_{n\in\mathbb{N}}$ be a decreasing null sequence with $\rho > 1/2$. Then there exists a set $E \subset \mathbb{R}$ of positive measure such that, for all $b \in \mathbb{R}$, the set $x_n + b \notin E$ is nonempty.*

Proof. This follows from Theorem 1 by taking $\varepsilon < 1$ and $a = 1$. □

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An analogous corollary holds as well for decreasing null sequences with $0 < \rho \leq 1/2$.

Corollary 6. *Let $(x_n)_{n\in\mathbb{N}}$ be a decreasing null sequence with $0 < \rho \leq 1/2$. Then, for every $\varepsilon \in (0,1)$, there exist sets $E_1, E_2 \subset \mathbb{R}$ of positive measure such that*

$$x_n + b \notin E_1 \text{is nonempty for all } b > \varepsilon,$$

and

$$x_n + b \notin E_2 \text{is nonempty for all } b < -\varepsilon.$$

Proof. This follows from Theorem 2 by taking $\varepsilon < 1$ and $a = 1$. □

Corollary 5 relates to [1, Theorem 1(ii), Borwein–Ditor]. Borwein and Ditor showed that there exists a set E of positive measure and a null sequence (x_n) such that, for each $b \in \mathbb{R}$, $x_n + b \notin E$ infinitely often. Corollary 5 also relates to [8, Theorem 3, P. Komjáth], where it is proved that for any null sequence (x_n) and $\varepsilon > 0$, there exists a set $E \subset [0,1]$ of measure $\lambda(E) > 1 - \varepsilon$ such that, if $b \in [0,1]$, then $x_n + b \notin E$ infinitely often.

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