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PLURAL QUANTIFICATION LOGIC: A CRITICAL APPRAISAL

HANOCH BEN-YAMI
Philosophy Department, Central European University

Abstract. I first show that most authors who developed Plural Quantification Logic (PQL) argued it could capture various features of natural language better than can other logic systems. I then show that it fails to do so: it radically departs from natural language in two of its essential features; namely, in distinguishing plural from singular quantification and in its use of an 'is-one-of' relation. Next, I sketch a different approach that is more adequate than PQL for capturing plural aspects of natural language semantics and logic. I conclude with a criticism of the claim that PQL should replace natural language for specific philosophical or scientific purposes.

The idea of *plural reference* has by now attained wide currency in philosophy. It already exists in Strawson and Geach's works from the fifties and sixties (Strawson, 1950, 1952; Geach, 1962), but it acquired more influence on mainstream philosophy with Black's (1971) paper. It was picked up by Armstrong (1978), Boolos (1984, 1985), van Inwagen (1990), Lewis (1991), and others, and in the last decade or so has been pivotal to many works in philosophy of language and logic. It has been explained and defended by the mentioned authors and later ones, who have also argued effectively against the singularist presupposition of so much of modern logic, which 'thinks of reference, first and foremost, as relating names and other *singular* terms to their objects.' (Quine, 1992, p. 27; italics added). I shall therefore assume in this paper without argument that various kinds of word and phrase are occasionally used to refer to more than a single particular.[1]

More specifically, I shall assume, together with much of the literature, that plural pronouns ('we', 'they'), plural demonstrative phrases ('these books'), plural definite descriptions ('the students', 'my children'), and conjunctions of referring expressions ('I and John', 'Mary and the children') are sometimes used to refer to several individuals. Two examples of sentences in which such expressions would normally be used as plural referring expressions are:

1. These books are new.
2. The students have arrived.

When I say that the noun phrases in these sentences are used to refer to several individuals, I do *not* mean that they refer to a single individual which is somehow composed of several ones, for example, a set or a mereological sum: they refer to the plurality as a plurality. As I have said, this analysis has been sufficiently developed and defended in the literature, so I shall assume it here without argument.

Having acknowledged the semantic category of plural referring expressions, the question arises, what should we do with them in logic? (I deliberatelyphrase the question somewhat

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[1] For recent developments, clarification, and defense of the idea of plural reference, see, for instance, Ben-Yami (2004, Part I), Yi (2005–2006, Part I), and McKay (2006, Chapters 1–2).

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vaguely, for I discuss below two different ways of giving it more precise meaning.) The most influential logic system that incorporates plural referring expressions was developed by Boolos (1984). Boolos interpreted the predicates of monadic second-order logic as plural referring expressions, and consequently distinguished singular from plural quantification. For instance, in Boolos's notation, while singular quantification is exemplified by the first-order formula $\forall x(Px)$, translating 'Everthing is P', plural quantification is exemplified by the second-order formula $\exists X(Xa)$, which translates 'There are some things one of which is a'. Several authors have recently developed Boolos's logic in great detail.[2]

All these logic systems start from some version of the Predicate Calculus and enrich it in various ways. Most authors, Boolos included, use as their basis Frege's classical Predicate Logic, with unary quantifiers; McKay (2006) uses mainly binary restricted quantifiers. They all enrich their calculus with plural variables and with a constant relation, said to translate natural language's 'is one of' or similar phrases (more details below). And most importantly, all distinguish singular quantification from plural quantification. For this last reason, and because of the close affinity of all these logic systems, I shall refer to all of them collectively as *Plural Quantification Logic*, or PQL.

These formal languages may be constructed for two distinct purposes, which need not be mutually exclusive. On the one hand, they may be intended as a substitute for natural language or some other formal language—the first-order Predicate Calculus, say—to be used as a tool for developing a certain theory, such as one laying the foundations of arithmetic. On the other hand, they may be developed in order to supply a more adequate analysis of the semantics of natural language—more adequate than the one supplied by the first-order Predicate Calculus, for instance. As we shall see, both purposes motivate the literature.

I will have little to say on the adequacy of these logic systems for the first purpose. The little I do have to say is said in the last, fourth section of this paper. My main aim here is to criticize the adequacy of these systems for analyzing the semantics and logic of natural language. I shall also argue for the superiority for that end of a different approach. I proceed as follows. In the first section I show that many of the authors who have developed PQL do think they can analyze by means of it various constructions of natural language better than can be done by means of the standard version of the Predicate Calculus. In the second section I criticize the adequacy of these systems for that end. In the third I introduce a different approach to quantification, also involving plural *reference*, but one that does not distinguish singular from plural *quantification*; I then show how this alternative approach fares better than do the plural quantification systems with respect to the difficulties mentioned in the second section, and I also mention some additional advantages it has. The fourth section concludes the article with some critical comments on the alleged possible applications of Plural Quantification Logic.

§1. Boolos opens his (1984) paper with a question:

> Are quantification and cross-reference in *English* well represented by the devices of standard logic. . . ?

[2] Primarily Yi (2005–2006), McKay (2006), and Oliver & Smiley (2006); but see also Rayo (2002) and Linnebo (2003). For surveys see Linnebo (2004) and Rayo (2007).

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He replies that a 'fairly widely held' view concludes that

> the variety of inferences that cannot be dealt with by first-order logic (with identity) is by no means as great or as interesting as the variety that can be handled by the predicate calculus, even without identity… (p. 55)

And he continues with his own skeptical position:

> It is the conclusion of this view that I want to take exception to.… It seems to me that we really do not know whether there is much or little in the province of logic that the first-order predicate calculus with identity cannot treat. In the first part of this paper I shall present and discuss some data which suggest that there may be rather more than might be supposed, that there may be an interesting variety both of quantificational and referential *constructions in natural language* that cannot be represented in standard logical notation and of valid inferences for whose validity these constructions are responsible. Whether quantification and cross-reference in *English* are well represented by standard logic seems to me to be an open question, at present.

Similarly, after having given several examples of sentences and inferences in English that are, as he puts it, nonfirstorderizable, Boolos concludes:

> It is because of these examples that I think that the question whether the first-order predicate calculus with identity adequately represents quantification, generalization, and cross-reference in *natural language* ought to be regarded as a question that hasn't yet been settled. (p. 62; italics added, here and above)

Boolos is explicitly interested in representing, in a language of logic, quantification and cross-reference in English or in natural language more generally. He claims that some data suggest that 'the first-order predicate calculus with identity', or 'standard logical notation', cannot handle an interesting variety of such constructions. The analysis of the semantics and logic of *natural language* by means of a richer system of logic is accordingly Boolos's purpose in the first part of his paper (pp. 54–64).

Boolos would even like his logic language to have a greater semantic proximity to natural language than the one supplied by logical equivalence (mutual entailment). He therefore also looks 'at a number of sentences whose most natural representations are given by second-order formulas, but second-order formulas that turn out to be equivalent to first-order formulas.' (p. 62). One of his examples is

3. There are some monuments in Italy of which no one tourist has seen all.

Its 'correct symbolization' 'might appear to require a second-order formula', such as

4. $\exists X(\exists x Xx \wedge \forall x[Xx \rightarrow Mx] \wedge \neg\exists y[Ty \wedge \forall x(Xx \rightarrow Syx)])$.

However, since (3) can be paraphrased as 'No tourist has seen all the monuments in Italy', symbolized as

5. $\exists x Mx \wedge \neg\exists y[Ty \wedge \forall x(Mx \rightarrow Syx)]$,

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(3) is logically equivalent to the first-order (5). Still, (4) 'captures more of the quantificational structure' of (3) than does (5), 'just as $\neg\neg p$ can sometimes be a better symbolization than p of "It's not the case that John didn't go"' (pp. 62–63).

So one of the main reasons Boolos developed his plural quantification interpretation of monadic second-order logic is to capture quite closely the semantics and logic of natural language sentences. Boolos also had other reasons for interpreting monadic second-order logic this way, primarily logic's ontological commitments (see Boolos, 1985), but the semantics and logic of natural language is clearly one of his main targets.

Other authors have also presented their versions of Boolos's Plural Quantification Logic as intended to capture the semantics and logic of natural language (although this need not be their only purpose either). Here are some excerpts from Yi (2005–2006). His first abstract's first sentence, with added italics, is: 'Contemporary accounts of logic and language cannot give *proper treatments of plural constructions of natural languages*.' After having noted some inferential relations between sentences of *English* containing plurals, for example, that the first of the following two sentences entails the second one:

6. Venus and Serena are tennis players, and they won a U.S. Open doubles title;
7. There are some tennis players who won a U.S. Open doubles title;

Yi observes (p. 460):

> To explain the logic of plurals, we need a system of logic, a post-Fregean system, that has wider scope than the Fregean systems.

And this is indeed what his plural quantification system is supposed to achieve (pp. 461–462):

> Plural languages contain natural paraphrases of basic plural constructions.... So we can explain the logic of the plural constructions using a system of logic that characterizes the logical relations among plural language sentences. To do so, I formulate a system of logic that I call (*first-order*) *plural logic*.

And similar claims appear elsewhere in his paper (cf. Yi, 1999, for instance Section III). Like Boolos, a main objective of Yi in developing Plural Quantification Logic is capturing the semantics and logic of natural language sentences.

Similarly, McKay claims that 'the standard formalization of [first-order logic] does not provide adequate resources for properly representing many ordinary things that we say.' (2006, p. 1; see also p. 5). He sees an advantage of his approach over Yi's in that his work (McKay's) 'develops a language with restricted quantification, and, accordingly, a larger range of quantifiers and *a more immediate link with natural language*.' (p. 6, footnote 4; italics added). His language 'enables us to do a better job of modeling the features of English sentences...' (p. 7). And so on. As we see, McKay too intends his Plural Quantification Logic to provide resources for properly representing natural language sentences, and he would like the link to natural language to be as immediate as possible.

Linnebo's purpose in developing his Plural Quantification Logic is also evidently akin to Boolos, Yi, and McKay's. He opens his (2003) paper by claiming that

> English contains two sorts of object quantifiers. In addition to ordinary *singular* quantifiers, as in the sentence 'There is a Cheerio in the bowl', there are *plural* quantifiers, as in 'There are some Cheerios in the bowl'.

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He then tries to 'regiment the plural quantification of English in a formal language' (p. 73; see also Linnebo, 2004, Introduction and Sections 1.0 and 1.1). Although Linnebo does not clarify the meaning of regimentation in his paper, he has in mind, like Rayo (see below), Quine's (1960, § 33) notion, which presupposes no exact synonymy (private communication). Accordingly, Linnebo is partly interested in capturing constructions of natural language that cannot be captured by singular quantification alone, and partly interested in developing a substitute language for some specific aims.

Not all authors consider themselves as trying to achieve in their language of logic a representation of quantification, cross-reference, or any other such feature of natural language. Rayo (2002) writes on his 'regimentation' of natural language that it has 'no presupposition of synonymy, or of sameness of "logical form"'. All he requires is that, 'to our satisfaction, whatever we hoped to achieve by way of our original sentences can be achieved closely enough by way of their paraphrase.' (p. 437). Of course, this would require some analysis of what the paraphrased natural language sentences 'achieved' or said; so if it turns out that what they said is quite far from what their plural logic paraphrases say, we might not be that satisfied. But since Rayo (2002) is interested that his language of logic should reduce 'unclarities and ambiguities' that might interfere with the goals of scientists and philosophers, he is less committed to the representation in his system of natural language constructions than Boolos or the other authors I discussed (this is even more marked in Rayo, 2007). I will say a few things on Rayo's goals in the last section, but my main criticisms, in Section II, are not directed at them.

As we have seen, many authors who develop versions of Plural Quantification Logic, from Boolos on, aim to capture in their languages, at least to a high degree of accuracy, the semantics and logic of natural language sentences containing plural constructions. It is therefore legitimate to try and assess whether their logic systems succeed in doing that. This is the subject of our next section.

§2.

2.1. The language of PQL. I start with a short exposition of the formal language of Plural Quantification Logic. Different authors develop it in slightly different ways, but these differences are irrelevant to the criticisms I would like to raise. Moreover, a minimalist version of PQL, containing what is common to all or most versions, would suffice for my purpose. I shall therefore ignore in this section some of the specific elaborations of specific authors, although a fully developed PQL system may require some or even all of them. For instance, unlike Oliver & Smiley (2006), the system presented below does not contain functors or a description operator. Unlike Yi (2005–2006), I do not distinguish predicates with plural argument places from predicates with singular argument places (or from predicates with some plural, some singular argument places). Unlike McKay (2006), and following most of the literature, I use unary and not binary quantifiers; also unlike him, I have no compound terms. In addition, since symbolism is not uniform in the literature, I have to make a choice: like Rayo and Linnebo, I use *xx*, *yy*, and so forth for plural variables, a symbolism probably first introduced by Burgess & Rosen (1997, p. 154); I use $\preccurlyeq$ for the 'is one of' relation. My exposition follows most closely Linnebo (2004).

The language of Plural Quantification Logic, L, is constructed as follows:

1. L has the following *terms*:
    - singular variables x, y, etc.
    - plural variables *xx*, *yy*, etc.

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- singular constants a, b, etc.
- plural constants aa, bb, etc.

2. L has the following *predicates*:

   - two dyadic logical predicates: = (identity) and $\preccurlyeq$ (the 'is one of' relation)
   - non-logical predicates R_i^n (for every adicity n and every natural number i)

3. L has the following *formulas*:

   - $R_i^n(t_1, \ldots, t_n)$ is a formula when R_i^n is an n-adic predicate and t_j are terms
   - $t_1 = t_2$ is a formula when t_1 and t_2 are singular terms
   - $t \preccurlyeq T$ is a formula when t is a singular term and T a plural term
   - the usual rules for sentential connectives
   - $\exists v(\phi)$, $\forall v(\phi)$, $\exists vv(\phi)$, and $\forall vv(\phi)$ are formulas when ϕ is a formula, v a singular variable and vv a plural one.

To get some feeling of the system's relation to natural language, I translate a few sentences into it. For the sake of clarity, I occasionally write English predicates and terms instead of L ones.

8. They lifted the piano.
9. lifted(aa, b)
10. Some children lifted the piano.
11. $\exists xx(\forall y(y \preccurlyeq xx \rightarrow child(y)) \wedge lifted(xx, the\ piano))$
12. There are some horses that are all faster than Zev and also faster than the sire of any horse that is slower than all of them. (Boolos's example (1984, p. 58); his formalization is adapted below to L.)
13. $\exists xx(\forall y(y \preccurlyeq xx \rightarrow faster(y, Zev)) \wedge \forall z[\forall y(y \preccurlyeq xx \rightarrow faster(y, z)) \rightarrow \forall y \forall t((y \preccurlyeq xx \wedge sire(t, z)) \rightarrow faster(y, t))])$

We can now proceed to a criticism of PQL as supplying an adequate analysis of plural constructions of natural language. My criticism focuses on the two main additions to PQL: first, the distinction between singular and plural quantification; secondly, the relation 'is one of'.

2.2. There is, there are. All versions of Plural Quantification Logic distinguish the natural language construction 'There is an x' from 'There are xx', as expressing two different kinds of quantificational constructions, singular and plural. I quote again the opening sentences of Linnebo (2003):

> English contains two sorts of object quantifiers. In addition to ordinary *singular* quantifiers, as in the sentence 'There is a Cheerio in the bowl', there are *plural* quantifiers, as in 'There are some Cheerios in the bowl'.

Accordingly, sentences containing these different constructions are translated differently into PQL. For instance,

14. There is a horse in the attic.
15. There are horses in the attic.

are translated by, correspondingly,

16. $\exists x$(horse $x \wedge$ in-the-attic x)
17. $\exists xx \forall y(y \preccurlyeq xx \rightarrow$ (horse $y \wedge$ in-the-attic y)).

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But do these different English constructions indeed express different semantic constructions, 'two sorts of quantifiers'? We all know that different grammar is not always an indication of different semantic structure. And we do think that the different copulas in 'I *am* hungry' and 'He *is* hungry' do not indicate different semantic structures (both sentences are translated by 'hungry(a)'). So the fact that (14) and (15) have different syntax is insufficient evidence for the existence of two semantically distinct existential structures in English.

This suspicion is strengthened by the fact that some other languages—for example, German, French, Spanish, and, in the present tense, Hebrew—do not have this kind of divergence between the sentences that translate sentences (14) and (15). Their respective translations into German, for instance, are

18. Es gibt ein Pferd im Dachboden.
19. Es gibt Pferde im Dachboden.

The existential construction is expressed by '*es gibt*' in German, with no distinction in form between plural and singular. So if uniformity and difference in grammar were reliable criteria of respective semantic uniformity and difference, we should conclude that there is no difference between plural and singular existential constructions in German. Consequently, at least one of (18) and (19) would not be an adequate translation of the corresponding English sentence. But the corresponding sentences do say the same thing in English and German. Accordingly, in this case grammatical uniformity or divergence is insufficient indication of semantic uniformity or divergence in any language, whether or not it has different singular and plural forms for the existential construction.

One might perhaps try to argue that the 'two sorts of quantifiers' exist also in German, French, and the other languages mentioned, but that their presence there is indicated by the difference in the number of the noun alone—singular versus plural. It seems clear, however, that the plausibility of the idea of 'two sorts of quantifiers' derives from the traditionally claimed correspondence between formal logic's existential quantifier '$\exists x$' and natural language's existential construction 'there is', and its apparent parallel in the correspondence between '$\exists xx$' and 'there are'. This plausibility is lost when other languages are checked, and therefore additional work should be done to motivate the introduction of the distinction.

Moreover, the English existential construction 'there is/are' is probably related to the demonstrative use of 'there' in the following sentences:

20. A horse is *there*, in the attic.
21. Horses are *there*, in the attic.[3]

The difference in copula between these sentences is a result of agreement in number, and, like agreement in gender in some languages, it does not indicate any semantic distinction between the sentences. Accordingly, the different verb forms ('is', 'are') in the existential constructions may also result from number agreement, and indicate no additional semantic distinction. They may express no more of a semantic distinction than do the copulas 'am' and 'is' in 'I am tired' versus 'She is tired'.

[3] The difference between the constructions 'There are *q Ss*' and '*q Ss* are there', where *q* is a quantifier, seems to be in the fact that a sentence of the latter form, as any sentence of the form '*q Ss* are *P*', presupposes reference to *Ss*, while the sentence 'There are *q Ss*' does not. This is why it makes no sense to say 'There are all/most horses', while it does make sense to say 'All/most horses are there': 'all', 'most', and other proportional quantifiers make sense only with a logically prior reference to several *Ss*.

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I am of course not contesting the claim that there *is* a genuine semantic distinction between singular and plural referring expressions. Neither am I contesting the consequent claim, that this distinction should make us develop a logic system for natural language, which, unlike the standard version of the Predicate Calculus, applies to both kinds of referring expression. What I am questioning is whether that logic would be justified in distinguishing two sorts of quantifiers in natural language, singular and plural. Since philosophers developing PQL rely on allegedly semantically different existential constructions in natural language to justify this distinction, I am trying here to show that these grammatically different constructions in English do not justify such a semantic distinction. That should make us modify logic in a different manner than does PQL (with its apparatus of singular and plural variables and quantification, and so forth) if we wish to accommodate the plural idiom within logic.

Next, the supposed distinction between plural and singular quantification cannot be reproduced in the same way with quantifiers other than the existential one. Consider, for instance, the universal quantifier, 'all' in English. What would be the different constructions in natural language that express singular versus plural quantification involving 'all'? Is the quantification in 'All *Ss* are *P*' singular or plural? There is no distinction in form here. But if the existential plural is different from the singular one, then there should be distinct plural and singular universal quantifiers, since the universal quantifier is definable by means of the existential one. And a universal plural quantifier is indeed straightforwardly definable in PQL. Similarly, although distinct singular and plural numerical quantifiers can be introduced into PQL—$\exists_3 x$ versus $\exists_3 xx$, say—nothing similar exists in natural language: 'Three *S* are *P*' does not distinguish singular from plural quantification. So the lack of a singular–plural distinction for universal quantification in natural language, as well as for quantification with other quantifiers, supports the claim that there is no such distinction in the existential case either, English grammar notwithstanding.

One should not confuse different *quantificational* constructions with different modes of *predication*. Consider the following two sentences:

22. All the children formed a circle.
23. All the children sang.

In the first sentence the predication is collective, while in the second it is distributive. Namely, while from (23) it follows that each and every child sang, it does not follow from (22) that each and every child formed a circle. So do we also have different sorts of quantification in the two sentences, albeit grammatically unmarked? In that case perhaps there would be some justification for counting the quantification in (22) plural in some sense.

That this is not the case can be seen from the following, 'mixed' sentence:

24. All the children formed a circle and sang.

While 'formed a circle' is predicated collectively, 'sang' is predicated distributively. So if the collective–distributive distinction necessitated different sorts of quantification (say plural versus singular), the quantification in (24) should be different from at least one of those in (22) and (23). Yet 'all' does not strike us as ambiguous in any sense between (24) and any of the preceding sentences. Accordingly, the collective–distributive distinction for predication does not justify a singular–plural distinction for the universal quantifier, or for any other quantifier.

In the case of universal quantification, English, like many other languages, contains two different quantifier words, 'all' and 'every', where the latter, but not the former, forces

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predication to be read distributively and not collectively (the same applies to 'each'). In 'Every *S* is *P*', '*P*' *has* to apply to each and every *S* on its own (for the distinctions between these quantifiers, see Vendler, 1962). But we have just seen that these different modes of predication, collective versus distributive, do not necessitate different modes of quantification, plural versus singular. So again, an additional argument is needed in order to establish that 'all' and 'every' indicate any distinction in quantification, beyond the distinction in possible predications they involve. Moreover, such different quantifier words do not exist for numerical quantifiers (as well as most other quantifiers), although a distinction between plural and singular quantification in their case as well is straightforward in PQL. So the all–every distinction offers but lame support to the claim that there is a plural–singular quantification distinction in natural language.

It is even doubtful whether a semantic distinction between singular and plural forms exists in English for the *particular* quantifier, 'some'. We have, indeed, the following different grammatical constructions:

25. Some *animals* escaped from the zoo yesterday.
26. Some *animal* escaped from the zoo yesterday.

But as we teach our students, sentence (26) at most implicates, in Grice's (1967, Chapter 2) sense, singularity. It is true, although possibly misleading, if used when more than a single animal escaped from the zoo. If a person working in the zoo utters it after noticing a hole in the fence when he starts his morning shift, he will not be found mistaken if it turns out that several animals ran away. Moreover, perhaps even sentence (25) can be true when a single animal escaped: if our guard uttered it instead of (26), we would not think he had made a mistake in case we later found that only the elephant had escaped. Sentence (25) seems to be different from sentence (26) in the *evidence* on the ground of which the speaker should assert it (its assertibility conditions), and not in its truth conditions.

(Although the predication in sentences (25) and (26) is distributive, our considerations in the previous paragraph apply to collective predication as well. This can be easily seen if we replace the distributive 'escaped from the zoo' by the collective 'killed the guard's cat', say.)

Yet even if a distinction in truth conditions between sentences (25) and (26) is justified, this does not yet justify claiming that they involve two sorts of *quantification*. The sentences

27. Exactly one animal escaped from the zoo yesterday

27a Exactly two animals escaped from the zoo yesterday

have different truth conditions, since they contain different *quantifiers*. But this does not lead us to claim that they involve different sorts of *quantification*; namely, singular and plural, or something of this kind. We can translate both sentences into the classical Predicate Calculus using only singular quantification. So even if sentence (26) were true just in case exactly one animal escaped from the zoo, and consequently sentences (25) and (26) had different truth conditions, this would not yet justify the claim that there exists in English a distinction between singular and plural quantification for the particular quantifier. One could still reasonably claim that the *quantifier* 'some' is ambiguous between its use followed by a noun in the singular (exactly one) and by a noun in the plural (at least one). This ambiguity claim is supported by the fact that when sentences (25) and (26) are translated into other languages, *different* quantifiers are often used in the translations (e.g., *einige* and *ein* in German).

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Let me clarify the distinction drawn in the previous paragraph between different quantifiers and different sorts of quantification. Already in the classical version of the Predicate Calculus we distinguish different *quantifiers*—primarily ∃ and ∀, of course, and others can also be introduced. All these quantifiers are substitutable in formulas without affecting their grammaticality, or join the same kind of variables and can operate on the same formulas. By contrast, PQL recognizes different kinds of variable, substitutable by different kinds of term; and the different quantifier-cum-variables turn different open formulas into closed ones when operating on them. The semantic principles for interpreting these different constructions also differ. In this consists the distinction in the formal language between different sorts of *quantification*, a distinction the plural quantification logician seeks in natural language.[4]

Turning back to the natural language, the existential construction in it is usually taken to be logically equivalent to particular quantification. We translate both 'Some *Ss* are *P*' and 'There are *Ss* that are *P*' into the standard version of the Predicate Calculus by '$\exists x(Sx \wedge Px)$'. Accordingly, if there is no semantic distinction between singular and plural particular quantification, it is doubtful whether there is any between singular and plural existential constructions.

There is some irony in the fact that the existential construction, 'there is an *S*' or 'there are *Ss*', is currently taken as the paradigmatic form of quantification.[5] Aristotelian logic did not consider it in its theory of inference. In fact, as far as I know, the existential construction had never been part of standard Aristotelian or Scholastic logic. Aristotelian logic discussed and systematized inferential relations involving the universal and *particular* quantifiers, 'all' and 'some' in English. However, when Frege developed his logic, then, following the peculiarities of his system, he identified particular quantification and existential construction (see Frege, 1879, Section 12 and footnote). This is despite the significant (and cross-linguistic) grammatical differences between the existential construction and quantifiers. For instance, while 'some', 'all', 'seven', 'many', 'most', and other quantifiers are unary determiners, this is not the case with the existential construction.[6] It is possible that the identification of the existential construction as a quantificational device is an artificial by-product of Frege's system. In order not to prejudge the matter, I use the phrase 'existential *construction*', and not 'existential *quantification*', when discussing natural language.

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[4] But can't we continue (25) but not (26) with 'and *they* must be hungry by now', while the latter but not the former can be continued with 'and *it* must…'? And surely these additional clauses have different truth conditions: only the former must be verified by several animals. So doesn't this change of pronoun number and truth conditions indicate that different variables are involved in (25) and (26), and accordingly different quantification?
I think these pronouns are Evans's E-type, and as he argued against Geach, the additional clause presupposes the truth of its antecedent (Evans, 1977). Since (25) and (26) have different assertibility conditions, expressed by the different number of their nouns, and since we need to assume not only their truth but also their assertibility if we wish to refer back to them with anaphoric pronouns, the need for different E-type pronouns arises. So we still don't need to assume that they involve different sorts of variables and quantification.

[5] McKay emphasized in correspondence that this observation does not apply to him. Except where considering others' examples, he always relates quantification to the particular quantifier, as one would expect with restricted quantifiers.

[6] See Ben-Yami (2004, Section 6.5) for a more detailed discussion of distinctions between quantification and existential constructions.

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Nevertheless, since the existential quantifier *is* a paradigmatic quantifier in our constructed logic languages—the Predicate Calculus on all its versions—we are used to see quantification in natural language too as expressed by the existential construction. Consequently, Plural Quantification Logic starts with an apparent distinction between singular and plural existential constructions in natural language, and interprets it as a distinction between different modes of *quantification*. But even if such semantic distinctions do exist for the existential construction—a claim I am criticizing in this subsection—it still does not follow that there are any parallel distinctions for natural language quantification. Whatever is our judgment of the forms of the former, a separate argument would be needed to establish the existence of parallel forms of the latter.

Turning back to the existential construction, is there indeed a semantic distinction between its grammatically singular and plural forms? Consider again sentences (14) and (15):

14. There is a horse in the attic.
15. There are horses in the attic.

I believe all agree that (14) is true even if there are several horses in the attic. On the other hand, suppose that while sitting in your living room, you suddenly hear sounds of neighing and stamping coming from the attic. You exclaim: 'There are again horses in our attic!': you would be considered right, I believe, if this time it is only a single one. So it seems there is no difference in truth conditions between (14) and (15). As with sentences (25) and (26), the differences are in assertibility conditions: you should assert (14) and not (15) just in case you have evidence leading to a single horse. (These assertibility conditions probably need some elaboration and refinement, but these are irrelevant to our purposes here.)

And again, as we argued above for sentences (25) and (26), even if sentences (14) and (15) do have different truth conditions, this does not necessarily show that they involve two different modes of *quantification*; they can still involve two different *quantifiers*. Similarly, the sentences 'There are exactly two horses in the attic' and 'There are exactly three horses in the attic' have different truth conditions, but this is because they contain different quantifiers, and not because they involve different modes of quantification, the way PQL does.

I therefore conclude that the evidence is against an existence of a semantic distinction between singular and plural existential constructions in natural language, and even more so against a semantic distinction between singular and plural quantification. Accordingly, Plural Quantification Logic, which assumes such a distinction, does not capture in this respect the semantics of plural constructions in natural language.

Notice that I am not claiming that singular quantification in natural language is *reducible* to plural quantification, or vice versa. We can reduce the sentential connective 'or' to the sentential connectives 'not' and 'and', yet language clearly contains these three semantically distinct connectives, 'not', 'and', and 'or'. I am claiming that natural language does not contain semantically distinct plural and singular quantificational constructions. PQL advocates can accept the reducibility claim. McKay in fact shows (pp. 120–121) how singular variables can be eliminated in his system, by reducing singular quantification to plural one (cf. Rayo, 2002, pp. 452–453). But I cannot see how PQL advocates could accept the latter, no-distinction claim, if they are to claim that their formal systems represent the semantics of natural language. As we saw in the previous subsection, all versions of PQL do distinguish plural from singular quantification. Indeed, McKay's reduction of

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singular quantification clearly shows that it is *not* identical with plural quantification. Using restricted quantifiers, the singularly quantified formula '[Qx: Fx] Gx' is equivalent, in his PQL system, to the following plural formula:

$$[Qyy : [\forall xx : xxAyy \wedge Ixx]Fxx][\forall xx : xxAyy \wedge Ixx]Gxx$$

('Ixx' means that xx is an individual, and 'A' is the *among* relation, McKay's variation on *is one of*. I am using the plural variables I introduced above, and not McKay's.) The difference between singular and plural quantification is striking. So if there is no semantic distinction in natural language between singular and plural quantification, PQL, which distinguishes the two, does not capture the way quantification functions in natural language.

Responding, in correspondence, to my argument in the previous paragraph, McKay claimed that the complexity of his reductive formula comes from the fact that he is also representing the distributivity of the predicates 'Fxx' and 'Gxx'. But, he observed, if we allow singular quantification with nondistributive predicates, this analysis does not even work. He is inclined to think, however, that the minimal formula '[Qxx: Fxx] Gxx' *always* represents what is said, while the more complex formula represents what follows in case the predicates are distributive, which is often a Gricean implicature.

If the formula '[Qxx: Fxx] Gxx' were always the correct representation of a sentence of the form 'Q F are G' of English, the question would still remain, why there is no distinction between singular and plural quantification in natural language, although it is so natural in the language of PQL? Consider the quantifier 'seven', say: '7x' and '7xx' are distinct in PQL, but this distinction does not exist in natural language. This discrepancy between the languages strongly suggests that their basic approach to quantification is different.

But even ignoring this difficulty, the formula '[Qxx: Fxx] Gxx' is of course the basic formula of plural quantification as well, according to McKay's version of PQL. In this way we end with a single form of quantification in natural language, the distinction between singular and plural variables becomes redundant, and our quantification is consequently neither singular nor plural. This uniform analysis of quantification is quite unlike what is found in the Plural Quantification Logic literature, from Boolos on. It is, in fact, close in various respects to the analysis I shall sketch in the third section of this paper. I still think it has some faults compared with the analysis I shall present there, but these are unrelated to the specific claims of PQL.[7] I therefore leave it here as a possible significant modification of PQL, with a much better claim than the original for supplying an analysis of natural language quantification.

2.3. Is one of. All versions of PQL use a dyadic logical relation-sign, $\preccurlyeq$ in our PQL language above, which is supposed to translate natural language's 'is one of' relation-phrase or some similar construction. The 'is one of' phrase was prominent already in Boolos's examples. Linnebo (2004, Section 1.1) writes that the dyadic logical predicate $\preccurlyeq$ should 'be thought of as … the relation *is one of*'. Oliver and Smiley use $\preccurlyeq$ as 'a primitive two-place logical predicate' 'to express the relation of inclusion', which they see as generalizing the identity relation. They write (2006, p. 327):

> When b is a plural term the $\preccurlyeq$ in $a \preccurlyeq b$ will naturally be read as *is/are among*, or equivalently *is one of/are a number of*. When b stands for a

[7] For some criticisms, see my comments on the generalized quantifiers analysis of natural language quantification in Chapters 6 and 7 of Ben-Yami (2004). I am currently working on a more detailed paper criticizing this analysis.

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> single thing, however, $a \preccurlyeq b$ can only be understood as an identity. The best English reading of $\preccurlyeq$ is therefore disjunctive: *is/are* or *is/are among*, as the case may be.

Yi (2005–2006, Section 3.2.3, p. 485) writes that in addition to identity, '[p]lural languages have another logical predicate: "is one of", a two-place plural predicate whose second argument place is neutral while its first argument place is singular. It is the refinement[8] of the English predicate "*to* be one of" (or its singular form "is one of").' McKay's (2006, p. 59) designated logical relation is a translation of natural language's *among*, which, unlike *is one of*, can take both singular and *plural* referring expressions in its first argument place. But *is one of* and his *among* are interdefinable (135):

> Most discussions of the logic or semantics of plurals use the fundamental relation *x is one of Y* rather than our relation *X are among Y*. I have been understanding *x is one of Y* in a way that makes these interdefinable.

Our discussion of *is one of* will therefore apply to McKay's *among* as well.

I prefer 'is one of' over 'among' for our purposes here, as the latter has undesired aspects in its meaning. Its use in all the following examples is not the use we are interested in, although together they form a family of related meanings: 'a house among the trees', 'You are among friends', 'This attitude is common among the under-25s', 'Discuss it among yourselves first', and 'They divided the money up among the children' (all examples taken from *Oxford Advanced Learner's Dictionary*). If we want to specify in what sense we use 'among' in PQL, we do it by saying that it is synonymous with 'is one of' or 'are some of'. We therefore better discuss the latter directly.

Is *is one of* a relation at all? I suspect it is not. I shall argue that these copula plus quantifier plus preposition do not even form a syntactic or semantic unit.

The fact that 'is' and 'one of' do not form together a syntactic or semantic unit is illustrated by the fact that we use 'one of' in both the following ways:

28. John is one of the children.
29. One of the children is missing.

I suppose no one would claim that 'one of' has different grammar or meaning in (28) compared to (29). But in (29) it is a quantifier plus preposition used to form a noun phrase together with the definite noun 'the children'. It is not used as part of an expression indicating a relation between a single particular and several particulars. The quantifier 'one' functions in (29) like the quantifiers in the following sentences:

30. All/Some/Many/None of the children are sleeping.

None of these expresses, or is part of a complex phrase that expresses a relation between some particulars (one or more) and some other particulars. Analogously, 'one' does not express such a relation in (29), nor is it part of a complex phrase that expresses such a relation. And since 'one' and 'of' do not change their syntactical role or semantic contribution

[8] It is not clear to me in what sense predicates and structures of our constructed languages are, according to Yi, *refinements* of their natural language sources. We indeed leave out various characteristics of these sources while constructing artificial languages; but this would make the artificial language's predicates and structures simplifications or partial representations of their sources, and not their refinements.

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between (28) and (29), they are not part of a phrase expressing a relation between one and several particulars in (28) either.

The fact that 'is' and 'one of' do not form a syntactic or semantic unit in sentences like (28) is also evident from sentences such as the following:

31. Joe is one of Australia's prominent philosophers and a very nice person as well.
32. Joe is a very nice person and one of Australia's prominent philosophers.

'Is' and 'one of' are separable. 'Is', in (31) and (32), is related to 'a very nice person' in the same way in which it is related to 'one of Australia's prominent philosophers'. I believe no one would claim 'is a' expresses any relation; so neither does 'is one of'. 'Is' indicates the mode of predication, and does not combine with the determiner that may appear in the noun phrase following it to express any relation.

Accordingly, we may say that the philosophers who developed PQL analyze the phrase 'is one of the *Ps*' in '*a* is one of the *Ps*' as being of the structure

(is one of) (the *Ps*),

while it is more justified to analyze it as being of the structure

(is) (one of the *Ps*).

'One of' is part of the noun phrase, and not part of a relation term preceding the noun phrase. From this analysis it is obvious that 'is one of' forms no syntactic unit, and that consequently—PQL notwithstanding—it expresses no relation.

But we can, and should, carry this deconstructive analysis even further. Consider the following two sentences:

33. Plato is one philosopher.
34. Plato is one of the philosophers.

We have already noted that 'is' is a separate grammatical unit, the copula or verb. Now in both sentences, the grammatical predicate contains a quantifier and a general term. But while in (33) the general term is indefinite, 'philosopher', in (34) it is definite, 'the philosophers'. Accordingly, in order to form a noun phrase from the combination of quantifier plus general term, we have to insert 'of' between them in (34), but not in (33). But it is doubtful whether, apart from this syntactic role, 'of' has here any semantic role as well.

More generally, let '*q*' be a quantifier and '*a*' a referring expression, either singular or plural. We should then distinguish the two sentence forms: '*a* is/are *q Ps*' and '*a* is/are *q* of the *Ps*'. The preposition 'of' is inserted after the quantifier only if it is followed by a definite noun phrase: 'the students' but not 'students', 'my children' but not 'children'. However, in sentences of both forms, the contribution of the quantifier *q* to meaning is the same, whether or not it is followed by 'of'. 'Of' does not combine with the quantifier to form a semantic unit. In fact, in some circumstances, change of form seems not to bring about any change in meaning: is there any difference in meaning between '*He* and *she* are two English pronouns' and '*He* and *she* are two of the English pronouns'? If indeed there is none, then 'of' may have a merely syntactic role, and no semantic one, in these sentences. Moreover, some languages that do distinguish definite from indefinite noun phrases do not recognize, for some quantifiers, two distinct sentence forms that depend on which of these nouns follows the quantifier. In Hebrew, for instance, no such distinction exists for the quantifier 'most': '*a* are most *Ps*' and '*a* are most of the *Ps*' are translated into Hebrew by

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the same sentence. This lack of distinction corroborates the claim that 'of' does not make in such cases any contribution to meaning.[9]

It thus seems that the preposition 'of' does not form a syntactic or semantic unit together with the quantifier. Turning back to 'is one of', the correct grammatical analysis of the phrase 'is one of the *Ps*' in '*a* is one of the *Ps*' seems to be

(is) ((one) (of) (the *Ps*));

Or perhaps,

(is) ((one) (of the *Ps*)).

'Is one of' is neither a syntactic nor a semantic unit. The contribution of 'one' to the meaning of such sentences is according to the same rules as the contribution of any quantifier '*q*' in a sentence of the form '*a* is/are *q Ps*', where the quantifier is followed by an indefinite noun and 'of' is not needed. Thus, in '*a* is one of the *Ps*', 'is' indicates affirmative predication; 'one' contributes to meaning according to the rules that govern quantifiers' semantic contribution; and 'of' has a syntactic role, gluing quantifier to definite noun. 'Is one of' expresses no relation. It is not even a unit that expresses anything. Plural Quantification Logic departs radically from natural language in this respect.

Notice that I am not arguing that 'is one of' is not semantically *simple*. This in itself should not have constituted a problem for PQL. What I am arguing is that it is not even a semantic or syntactic unit, and that therefore it is not a phrase that can express a relation. In

The man wearing a hat turned slowly,

the phrase 'the man wearing a hat' is semantically complex, containing the definite article, a common noun, and a defining clause; nevertheless, from both syntactic and semantic points of view it is a unit, a noun phrase designating the subject of that sentence. Similarly, neither is the phrase 'turned slowly' semantically simple, for it contains a verb modified by an adverb, yet it serves as the predicate of that sentence. By contrast, the string of words 'a hat turned', which is also contained in that sentence, is neither a syntactic nor a semantic unit. 'Is one of', I claim, is more like the latter than like any of the former.

Since we dived quite deep into the sea of grammar in this subsection, we should check whether our claims here agree with the analyses linguists have supplied.[10] First, practically all linguists agree that, say, in (28), namely,

John is one of the children

[9] Commenting on the argument in the last few paragraphs, an anonymous referee wrote that 'there are languages with prepositions with the effect of 'not among' (the German 'ausser' may be an example). Switching from 'of' to such a preposition would have a major semantic effect.' I am not sure which German sentences the referee had in mind, but granting him his claim, it does not follow that 'of' has a comparable semantic role in the sentences under discussion. 'Of' might be filling in a space where a *different* phrase might be inserted, which would then make a significant semantic contribution; but it does not follow that 'of' makes such a contribution when it is in that slot. Similarly, from the fact that a gene might be inserted somewhere along a chromosome, and it would then have a major phenotypic effect, it does not follow that what is currently at that place is not junk DNA, with a merely structural role.

[10] I am indebted to Anna Babarczy and Anna Szabolcsi, who helped me find my way in contemporary linguistics literature. I have also profited from Sauerland & Yatsushiro (2004), although it is not mentioned in the body of my paper.

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'is', the verb, is to be separated from 'one of the children', the noun phrase. The basic structure of 'is one of the children' is

(is) (one of the children).

In this respect it does not differ from, for instance, 'met two of the boys' in 'John met two of the boys', whose basic structure is

(met) (two of the boys).

No one would claim, I believe, that 'met two of' expresses any relation. The same applies, according to my analysis and common linguistic wisdom, to 'is one of'.

Secondly, what of 'one of' in (28)? There is apparently no clearly agreed standard analysis of this partitive construction. The traditional analysis, however, supports mine, as do most analyses I could find. For instance, while discussing the related noun phrase 'three of the books', when appearing as direct object, Selkirk (1977), Jackendoff (1977), Ladusaw (1982), Barker (1998), and Matthewson (2001) all analyze it as being of the form

(three) (of the books),

in which 'three of' is not a constituent. Analogously, 'one of' in 'John is one of the children' would be no constituent either, the structure of the noun phrase being

(one) (of the children).

Similarly, Szabolcsi (2001) analyzes the structure of 'any of the books' as (any)(of the books), again in support of my analysis and not PQL's. The only exception I could find is Keenan & Stavi (1986), who analyze the phrase 'three of the books' as having the structure (three of the) (books)—closer to the position of PQL, although not identical with it.

It thus seems that also according to the current opinion in linguistics, 'is one of' is no syntactic unit. And according to the position of most linguists, 'one of' should not be accounted as a syntactic unit either. My analysis is compatible with what one founds in contemporary linguistics, and it is even strongly supported by the latter.

As a last resort, could a Plural Quantification Logician maintain that the preposition 'of' is the part of speech that expresses the relation that $\preccurlyeq$ expresses in PQL? This seems indeed to be Schein's (2006, passim) position. But I suspect this is too heavy a burden for such small a word. As I said above, it seems that 'of' has only a syntactic function in the sentences we consider. Moreover, even its use between quantifier and noun (e.g., in 'most of the children') does not translate uniformly. For instance, while 'of' can be used after 'all' and 'most' in English ('all/most of these books'), it has no parallel in the Hebrew translation of these phrases. This supports the claim that it does not express any one relation in all its uses, even in its uses between quantifier and noun. Lastly, in face of the diverse uses of 'of', one need not be of a particular Wittgensteinian disposition in order to reject the claim that 'There *must* be something common' to these uses. Just consider its diverse uses in phrases like 'a friend of mine', 'the role of the teacher', 'the paintings of Monet', 'the lid of the box', 'a member of the team', 'the result of the debate', 'the people of Wales', 'a story of passion', and so on. 'Of' has a structural, not a semantic function.

Let me emphasize that I am *not* arguing that $\preccurlyeq$ expresses no relation in the language of PQL. My point is that PQL does not capture a semantic feature of natural language by means of it. The syntax and semantics of natural language and PQL are significantly different in this respect.

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2.4. Conclusion of this section. We saw in Section I that many authors who develop PQL aim to capture with it the semantics and logic of natural language sentences containing plural constructions. We saw in this section that two of PQL's key additions to the Predicate Calculus are a distinction between singular and plural quantification, allegedly demonstrated by the English forms 'there is' and 'there are'; and the alleged logical relation expressed by natural language's 'is one of' or similar constructions. We then argued that there is no distinction in natural language between singular and plural quantification of the kind PQL contains; and that 'is one of' does not express any relation in natural language, that it is not even a unit expressing anything. Assuming that our arguments are sound, we must conclude that Plural Quantification Logic has failed in its attempt to represent the logic and semantics of plural constructions of natural language.

It is of course still possible that PQL should have some other application (we return to this claim in the last section). However, one main reason for developing it, as we saw in Section I, was to capture the semantics and logic of natural language. And this, I have argued, it cannot do.

§3. Still, some treatment of the semantics and logic of plurals in natural language is called for. There are, for instance, some inferences in natural language that involve plural reference and that cannot therefore be translated into valid inferences of the standard version of the Predicate Calculus. To borrow, with some modifications, an example from Yi (2005–2006, p. 460), which was mentioned above, the following inference is obviously valid:

35. Venus and Serena won a U.S. Open doubles title.
36. Venus and Serena are tennis players.
37. Some tennis players won a U.S. Open doubles title.

Sentence (37) follows from (35) and (36). However, predication in (35) is collective (it is not true, for instance, that Serena won a U.S. Open doubles title). We cannot therefore represent that validity in the Predicate Calculus by breaking our sentences into sentences involving only singular reference and translating the latter into the calculus. Some other semantic and deductive system is necessary for that purpose.

In the rest of this section I introduce such a system and show how it can deal with such inferences. My presentation of this system, however, will be rudimentary, for several reasons. First, we already have well over 8000 words behind us, and this paper should not be too long. Secondly, my primary purpose in this paper is critical, showing inadequacies of PQL; my purpose in presenting the alternative system is just to give an idea of how it can do better than PQL what the latter attempted. Thirdly, this system has been developed in other works.[11] On the other hand, none of these works attempted to apply the deductive system to sentences of the form '*aa* are *P*', where '*aa*' is a plural definite noun phrase, such as 'Venus and Serena', 'they', 'these books', or 'the students'. It is therefore appropriate that I show here how this can be done.

As many readers would not be familiar with my system, I start with a concise presentation of its main ideas. I then proceed to apply it to inferences of the kind just mentioned.

[11] See Ben-Yami (2004), Lanzet & Ben-Yami (2004), and Lanzet (2006), where the deductive system is developed in detail (with some variations between the different versions). I develop the semantics of plurals in detail in Ben-Yami (2004).

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Let us start by considering again sentences (1) and (2):

1. These books are new.
2. The students have arrived.

As noted above, many authors claim that 'these books' and 'the students' are used in these sentences as plural referring expressions. They are logical subject terms or arguments, and not predicates. This has been powerfully argued in several works, referred to above. It is also my position.

But now let us slightly change these sentences:

38. Some of these books are new.
39. Most of the students have arrived.

I find it very plausible to maintain that the noun phrases 'these books' and 'the students' do not change their semantic role between the former and latter pair of sentences. That is, if we claim that 'these books' functions as a plural referring expression in (1), we should claim that it does so in (38) as well, and similarly for 'the students'.

But this claim has far-reaching consequences. The Fregean analysis of, for instance, the sentence 'Some *S* are *P*' is $\exists x(Sx \wedge Px)$; that is, the general term following the quantifier 'some' is analyzed as semantically predicative, contributing to sentence meaning in the same way as does the grammatical predicate *P*. This construal is preserved in all later versions and descendents of Frege's calculus. 'Some *S* are *P*' is translated into a system using restricted quantifiers, for instance, by '$[\exists x\colon Sx]\ Px$', where *S* is again taken to be semantically a predicate.

By contrast, if we are right and 'these books' does not change its way of contributing to sentence meaning between (1) and (38), then at least in some sentences of the form '*q S* are *P*' (or '*q* of *S* are *P*'), the general term '*S*' following the quantifier '*q*' (or '*q* of') is logically *not* a predicate but a referring expression or a logical subject term; semantically, it is closer to 'Socrates' than to 'philosopher' in 'Socrates is a philosopher'.

Thus, according to this analysis, if a predicate has as one of its grammatical arguments a noun phrase of the form '*q S*', then *S* functions semantically not as a predicate but as a referring expression or logical subject term. This would also be the case with, for example, 'the children' in 'Paul invited some of the children', where 'some of the children' is the grammatical direct object. This analysis has first been developed by Geach (1962), in the last chapter of his *Reference and Generality*.

In the previous section I argued that the preposition 'of' makes just a syntactic contribution to noun phrases like, for instance, (38)'s 'some of these books'. Accordingly, on my analysis, in that noun phrase the quantifier is 'some', the referring expression—a *plural* referring expression—is 'these books', and 'of' is used as syntactic glue, binding them together into a noun phrase. (One should distinguish between the referential use of definite and indefinite nouns, for example, 'these books' versus 'books'. For reasons given above I shall not discuss this here, but see Ben-Yami (2004, Chapter 5.))

My concise presentation here is obviously not intended to bear the weight of overturning the Fregean tradition, but just as a preliminary to the application of my system to inferences used in the PQL literature to show that Frege's calculus is insufficient. The justification of the general approach is found elsewhere. In Ben-Yami (2004) I show various advantages of this analysis of general terms in quantified noun phrases in argument position over their Fregean analysis as semantically predicative. I show how this analysis explains away an alleged ambiguity of the copulative structure; how it explains semantic analogies between

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empty singular terms and empty general terms; the classes of quantifiers of natural language; the semantic need for active–passive voice distinction, converse relation-terms or similar reordering devices; and more. It is also very difficult to find any arguments in the literature against the Aristotelian logical-subject analysis and for the Fregean predicative analysis. The only philosopher who tried to argue in any detail against the logical-subject analysis or for the predicative one is, unsurprisingly, Frege himself. I criticize his arguments in Ben-Yami (2006).

In Ben-Yami (2004) I also develop a deductive system on the basis of this analysis, which I prove to be sound. In Lanzet & Ben-Yami (2004) and Lanzet (2006) formal systems are constructed, on the basis of the semantics and deductive system of Ben-Yami (2004); Lanzet proves there that these formal systems are also complete, and in Lanzet (2006) he proves that the formal system developed there contains, in a sense specified in that work, the first-order Predicate Calculus. So the deductive system built on the basis of the semantic analysis of the mentioned general terms as logical subjects is obviously at least as powerful as the first-order Predicate Calculus. We should now see how it applies to arguments containing plural unquantified noun phrases, such as the argument (35) to (37) above.

The full presentation of this deductive system involves a discussion of bound anaphora, which would distract us from the main focus of this paper. I shall therefore discuss here only inferences with sentences that do not contain bound anaphors. For the same reason I shall mostly ignore issues of multiple quantification. We should also introduce the notion of a quantified noun phrase *governing* a sentence, which is the analogue of a quantifier having wide scope. An intuitive understanding of this notion would suffice for our needs here. The noun phrase 'some students', with 'some' expressing the particular quantifier, governs the first two but not the last three of the following five sentences:

40. Some students were late.
41. Some students studied with every professor.
42. It's not the case that some students were late.
43. Every professor taught some students.
44. If some students were late, then John was upset.

It will be noticed that in accordance with my earlier distinction between quantification and existential construction, I discuss here the particular quantifier—'some' in English—and not the existential construction. I shall not discuss the existential construction below.

Before turning to the deductive system, a few words on how this system handles collective and distributive predication, as well as on matters of scope. I present the system here with substitutional semantics for quantification, but an objectual account can also be supplied (objectual quantification is used in Lanzet & Ben-Yami, 2004; Lanzet, 2006).[12]

[12] Substitutional semantics for quantification faces the problem of undesignated objects. For instance, although the sentence 'Some raindrop will never be designated by any singular referring expression' is obviously true, it seems to come out false on substitutional semantics. But objectual semantics faces an analogous difficulty, that of objects that are not the value of any interpretation of any constant or variable. *I* find the formula '$\exists x$(x is not the value of any interpretation of any constant or variable)' intuitively true, but it comes out false on objectual quantification; while if one thinks it is indeed false, then one is committed to strong Platonism about interpretation functions, a Platonism which, *mutates mutandis*, may save substitutional semantics as well. For lack of space, I shall not develop these cryptic remarks any further here; for a more detailed discussion of these issues, see Section 8.6 of the revised version of my book. But I hope one would grant me the use of substitutional semantics at least for heuristic purposes.

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Suppose the quantified noun phrase '*q A*' governs sentence *S*. Then *S* is true iff a definite noun phrase, designating *q* of the particulars '*A*' designates, can be substituted for '*q A*', generating a true sentence. For instance,

45. Two boys lifted the piano

is true iff some definite noun phrase designating two boys, say 'Peter and Paul', can be substituted for 'two boys', generating a true sentence. If the predication is meant collectively, we cannot give any further specification of truth conditions. Suppose, however, that the predication is meant distributively, and let us substitute the noun phrase '*a* and *b*' for 'two boys'. We get the sentence:

46. *a* and *b* lifted the piano,

which is logically equivalent to:

47. *a* lifted the piano and *b* lifted the piano.

Thus, we can say that a sentence *S*, governed by a noun phrase '*q A*', is true, when it involves distributive predication relative to the argument place of '*q A*', just in case we can substitute for '*q A*' *q* singular referring expressions, referring to *q* different particulars to which '*A*' refers, all of these substitutions generating true sentences.

Our truth conditions rule for quantified sentences also yields an explanation of matters of scope. Consider the sentence, 'Two students failed three exams', and suppose the predication is meant distributively. This sentence is governed by the noun phrase 'two students'. It is therefore true just in case we can substitute two names of two different students for 'two students' and get true sentences. Suppose we get true sentences this way just in case we substitute 'Paul' and 'Jane' for 'two students'. Now 'Paul failed three exams' is true, according to our rule, just in case we can substitute the names of three different exams for 'three exams' and get a true sentence; similarly for 'Jane failed three exams', although these names need not be names of the same exams. We see that in this way we get the correct truth conditions. It is also easy to see that we would get, according to this rule, the correct, and different, truth conditions, if we analyzed the sentence 'Three exams were failed by two students'.

I acknowledge that this account is both condensed and does not address several important issues, for instance ambiguity. A detailed discussion is found in Ben-Yami (2004, Chapter 7). I brought it here mainly to give a rough idea of how this alternative approach deals with issues that supply much of the motivation for PQL.

We shall now consider some derivation rules of my system. I bring them here in the form in which I formulated them in Ben-Yami (2004), with some minor stylistic alterations. Here is the Universal Elimination derivation rule:

> *Universal Elimination*. Suppose the noun phrase '*q S*' governs sentence (i), where '*q*' is the universal quantifier. Suppose further that sentence (j) is '*a* is an *S*', where '*a*' is a singular definite noun phrase. Then, in any line (k), one can write the sentence identical to sentence (i) apart from the fact that in it '*a*' has been substituted for 'every *S*'. Line (k) relies on the lines on which lines (i) and (j) rely. Its justification is written 'UE, i, j'.

The following inference contains an application of UE:

1	(1)	Every man is mortal.	Premise
2	(2)	Socrates is a man.	Premise
1, 2	(3)	Socrates is mortal.	UE, 1, 2

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The numbers in brackets are line numbers. The lines written to the left of the line number are the numbers of the lines on which that line relies (a premise relies only on itself).

The following is the system's Particular Introduction rule:

> *Particular Introduction*. Suppose sentence (i) contains the singular definite noun phrase '*a*', and that if we substitute '*q S*' for '*a*', then this appearance of '*q S*' will govern sentence (i). Suppose further that sentence (j) is '*a* is an *S*'. Then in any following line (k) one can write the sentence identical to (i) apart from the fact that in it '*q S*', where '*q*' is the particular quantifier, has been substituted for '*a*'. Line (k) relies on the lines on which lines (i) and (j) rely. Its justification is written 'PI, i, j'.

An example:

1	(1)	Socrates is s philosopher.	Premise
2	(2)	Socrates is Athenian.	Premise
1, 2	(3)	Some Athenians are philosophers.	PI, 1,2

Both rules specify that '*a*' has to be a *singular* definite noun phrase. However, there is nothing which *should* restrict '*a*' to being singular. It may as well be a *plural* definite noun phrase. My original formulation of the rules was restricted to singular noun phrases because I was trying to show how this deductive system is applicable to inferences to which the Predicate Calculus is also applicable. But since in this paper we are trying to go beyond the bounds of the standard version of the calculus, we can remove this restriction. Accordingly, if we allow '*a*' in the Particular Introduction rule to stand for both singular and *plural* definite noun phrases, the following inference is valid:

1	(1)	Venus and Serena won a U.S. Open doubles title.	Premise
2	(2)	Venus and Serena are tennis players.	Premise
1, 2	(3)	Some tennis players won a U.S. Open doubles title.	PI, 1, 2

This inference, similar to the one Yi brought as an example of an inference that shows the insufficiency of Frege's Predicate Calculus, does not require any addition to the system considered here. On the contrary, if we *remove* an unnecessary constraint on that system, the system is then applicable to this inference as well. This is an obvious advantage of this system over Frege's.

Its advantages over Plural Quantification Logic are also clear. First, I argued above that, by contrast to PQL, natural language recognizes no distinction between singular and plural quantification. We see that neither does this system need any such distinction in order to apply to natural language sentences involving plural referring expressions. Secondly, unlike PQL, this system makes no use of an alleged logical relation 'is one of', which I argued is not even a syntactic or semantic unit in natural language. So it is possible that this system gives a better analysis of the semantics and logic of natural language than does PQL. Other advantages of this system over Fregean logic more generally, discussed in Ben-Yam (2004), also support this conclusion.

I shall bring here just one more derivation rule of this alternative system, in order to demonstrate how it generalizes to plural definite noun phrases and how it enables us in this way to show the validity of various forms of inference. The derivation rule I bring is Leibniz Law, that if two expressions denote the same thing, they are substitutable *salva veritate*. First, the singular version of this law:

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> *Leibniz Law*. Suppose that sentence (i) is '*a is b*', where '*a*' and '*b*' are singular definite noun phrases, and that '*a*' appears in sentence (j) too. Then in any line (k) one can write the sentence identical to sentence (j) apart from the fact that in it '*a*' has been substituted by '*b*' in some or all of its appearances. Line (k) relies on the lines on which lines (i) and (j) rely. Its justification is written 'LL, i, j'.

Again, the restriction in Leibniz Law, that '*a*' and '*b*' be *singular* definite noun phrases, is unnecessary. Let us therefore remove it. The following inference is then valid:

1	(1)	Grice and Strawson are the authors of 'In Defense of a Dogma'.	Premise
2	(2)	Grice and Strawson studied in Oxford.	Premise
1, 2	(3)	The authors of 'In Defense of a Dogma' studied in Oxford.	LL, 1, 2

Similar inferences are brought by Oliver & Smiley (2001, p. 289) and Yi (2005–2006, p. 460) in order to demonstrate the insufficiency of the Predicate Calculus and the need for a logic of plurals. We once again see how the alternative system presented here can be applied to these inferences by merely removing an unnecessary initial constraint on its rules.

We of course have to show that these derivation rules remain sound when the singularity constraint is eliminated. Although this can easily be done, I shall not do it here: proofs on the system would obviously take much space and thus distract us from the main line of argument of this paper. The reader is referred to the works mentioned in footnote 11, where such proofs are given for the singular case; their generalization for the plural case is straightforward.

I supplied in this section a description, in outline, of alternative semantics and logic for natural language. We saw how the deductive system of this approach is straightforwardly generalizable to constructions involving plural definite noun phrases. This straightforward generalization is a result of the fact that the system contained plural reference as an essential part: it treats quantification as involving the use of general terms as plural referring expressions. It also does not distinguish between singular and plural forms of quantification. In this way the alternative system satisfies the need, correctly identified by the writers mentioned above, of developing a logic of plurals. And it does that without the difficulties we saw confront Plural Quantification Logic. Plural Quantification Logic is neither adequate nor necessary for understanding the logic of plurals in natural language.

§4. However, as was said earlier, an artificial language may be developed not for analyzing natural language, but rather for *replacing* it in certain uses. The formula language of algebra is an example of a language that is much more efficient than natural language for many mathematical needs. Another example is the symbolic language of chemistry, which represents much more compactly than natural language the structure and reactions of molecules and other particles. Yet another, quite different example of an artificial language that replaced natural language in specific uses is the language of musical notation: without it, most masterpieces of Western music would not have been possible.

From its inception, modern logic was also meant to serve such replacement purposes. Frege (1879, Preface) intended his formula language primarily for laying the foundations of arithmetic, and also as a tool for geometry, mechanics, and physics. Similar claims about the aim of an artificial logic language are found in the works of many of the philosophers who made major contributions to its development (cf. Russell, 1957, pp. 387–388; Quine,

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1953, pp. 150–151). The Predicate Calculus was of course used, from Frege onward, for analyzing natural language as well. But whether or not it is adequate for the latter purpose, it is a different question whether it is an appropriate substitute for natural language in some uses.

This replacement view is also found with respect of Plural Quantification Logic among the authors who developed it. The one most explicit about this purpose of PQL is probably Rayo. As we saw above, Rayo claims that the unclarities and ambiguities that, according to him, plague ordinary discourse might interfere with the goals of scientists and philosophers. We then develop an artificial language, with no presupposition of synonymy, so that 'the old discourse is surrendered in favor of the new' (2002, p. 437). The subjects Rayo thinks may be served by PQL are ontology or metaphysics, the foundations of arithmetic and set theory, and the meta-theory of logic (see Rayo, 2007).

Although PQL may find some such use, I think Rayo failed to justify the construction of a substitute language. The artificial languages I mentioned above, as examples of languages that replaced natural language for certain purposes, did not replace it because of any ambiguities or unclarities which, according to Rayo, plague it. They replaced it because they can express much more concisely and perspicuously than it the required contents. In fact, they are explained by means of natural language; so it seems they should have inherited any unavoidable ambiguity or unclarity their natural language explanations possessed. The ability of natural language to be accurate—although perhaps not as concise or perspicuous—in expressing the contents they were constructed to express, is therefore confirmed by theirs.

Accordingly, *if* we are to learn from any existing and successful substitute artificial language, then if PQL is to replace natural language in some use, it should express more concisely and perspicuously than the latter the required contents. But a survey of the translations offered in the literature would reveal, I think, that the contrary is the case: the translated sentences are always, or almost always clearer and more compact than their translations. I therefore doubt whether PQL can be justified as a substitute language in *this* way.

Advocates of PQL as a substitute language might try a different justification. Perhaps, they might claim—as did Frege of his calculus—it would be easier to carry with PQL long and formal chains of inferences—easier than with natural language. And since PQL has greater semantic resources than Frege's original calculus, admitting plural reference as well, it could be more powerful than Frege's calculus in various mathematical areas.

We often find in the sciences, as well as in nonscientific areas, long, complex, and valid chains of inference, without any recourse to formal logic. So long and complex reasoning seems not to require a formal logic language. And there is no reason for thinking that long and complex reasoning in metaphysics or any other area of philosophy should require one. It is even doubtful whether any area of mathematics, apart from mathematical logic itself, would gain from recasting its theorems in the language of formal logic. More than a century after its development, mathematical logic has not made much significant inroad, to the best of my knowledge, into other areas of mathematics.[13]

[13] The Predicate Calculus is indeed occasionally used to demonstrate some logical distinctions; primarily, perhaps, the significance of quantifier order, for example, the distinction between $\forall x \exists y$ and $\exists y \forall x$. But even if it did play some historical role in making us conscious of this distinction, it is certainly unnecessary for capturing it: we all understand the difference between 'Every boy loves some girl' and 'Some girl is loved by every boy'.

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And again, if Plural Quantification Logic departs from natural language in its treatment of quantification, as I claimed above, one should justify the need for such a departure when laying the foundations of arithmetic or engaging in any related project. If natural language indeed does not distinguish singular from plural quantification, why should we introduce such a distinction when pursuing those projects? I am not familiar with arguments supporting the desirability of such departure.

Perhaps such arguments can be supplied; we should not commit the fallacy of the argument from ignorance. But they have not yet been supplied. So we currently have no reason for thinking that PQL should *replace* natural language for some investigations in science, philosophy, or mathematics. And I have also argued that, contrary to the intentions of most of its developers, PQL cannot serve us for *analyzing* the semantics and logic of natural language. My conclusion is, therefore, that we have no reason for thinking that Plural Quantification Logic can have any applications.

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PHILOSOPHY DEPARTMENT,
CENTRAL EUROPEAN UNIVERSITY
1051 BUDAPEST, HUNGARY
E-mail: benyamih@ceu.hu

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