SENSOR CONFIGURATION OPTIMIZING IN MODAL IDENTIFICATION BY SIEGE ANT COLONY ALGORITHM

S. Feng* **J.-Q. Jia** **J.-C. Zhang**

State Key Laboratory of Coastal and Offshore Engineering
Dalian University of Technology
Dalian, China

ABSTRACT

Proper monitor planning is a vital component of structural health monitoring (SHM) project. An extremely important part of the monitor planning is the placement of sensors, usually in the form of acceleration sensors. For the placement of three-dimensional acceleration sensors, the state of practice is to select the sensor configuration by previous experiences. However, this results in a waste of many sensors. A novel method called siege ant colony algorithm (SAC) is proposed in this paper. This method is built on the previous ant colony optimization (ACO) in the direction of improving efficiency and accuracy when applied to optimal sensor placement (OSP) problems in large-scale structure monitoring. This method is applied and compared with standard approaches using the Hanjiang transmission tower.

Keywords: Structural health monitoring, Modal identification, Optimal sensor placement, Siege ant colony algorithm.

1. INTRODUCTION

Large-scale structures are wildly used all over the world. The environment loads (wind, earthquake, erosion etc.) have great influences on the safety of structures. structural health monitoring SHM is a subject focuses on how to make use of significant technological advances in sensing and communication technology to enhance safety of structures. In the project of SHM [1-8], the sensors (stress, acceleration, displacement etc.) are installed on the structure and report in real-time the safety condition. For the three-dimensional acceleration sensor monitoring of large-scale structures, the state of practice is to select the sensor configuration by previous experience. However, this results in a waste of many sensors. The sensor configuration should be selected that render the corresponding target mode shape partitions as linearly independent as possible and, at the same time, the final sensor location must be optimal in some sense such that testing resources are conserved. Additionally, the calculation of sensor placement should be automated in some fashion to facilitate engineers [6, 9].

There are many researches focused on optimal sensor placement (OSP). Salama *et al*. [10] proposed using modal kinetic energy as a means of ranking the importance of candidate sensor locations. Henshell and Ong [11] propose using the mass-to-stiffness ratio associated with each candidate sensor location to find the optimal sensor configuration. An effective independence method is proposed by Kammer [12] to sensor placement. And this method can be also extended to 3-D [13]. A sensor and actuators placement method is proposed by Batou, A. [14] basing on updating of computational models. The purpose of the method is to render the measured data are as sensitive as possible with respect to updating parameters. Bishop, A. N. [15] studied the method to optimize the sensor-target geometries. The aim of that is to identify those relative sensor-target geometries which result in a measure of the uncertainty ellipse being minimized. Guo, H. Y. *et al*. [16] use the genetic algorithms to search for the optimal locations of sensors. The optimal placement of piezoelectric sensors and actuators has been studied by Han, J. H. *et al*. [17]. And Genetic algorithm is used to find efficient locations of piezoelectric sensors and actuators of a smart composite place. Optimal sensor placement problems are considered for a thermo-elastic solid body by Herzog, R. *et al*. [18] The main idea of that is the real-time capable prediction of the displacement of the tool center point in a machine tool by temperature measurements. The Convex Optimization is used to solve the problem of choosing a set of sensor measurements by Joshi, S. *et al*. [19]. Krause, A. *et al*. [20] tackled the combinatorial optimization problem of maximizing the mutual information between the chosen locations and the locations which are not selected. Far too many sensor placement approaches exist to mention them all in this paper.

Most of the aforementioned approaches were applied

* Corresponding author (fengshuotougao@126.com)

DOI : 10.1017/jmech.2016.87
Copyright © 2016 The Society of Theoretical and Applied Mechanics
https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

to simple structure or one-dimensional sensor monitoring project. However, the calculation efficiency and accuracy still cannot meet the requirement of three-dimensional monitoring engineering in large-scale structures. Moreover, the existing methods performed with poor adaptability in different OSP cases. In this paper, a novel method called SAC is proposed. This method is built on the previous ant colony optimization [21] in the direction of improving efficiency and accuracy when applied to OSP problems in large-scale structure monitoring. Section 2 outlines the main features and detailed implementation on SAC. Section 3 presents the practical monitoring project of Hanjiang transmission tower. Section 4 is dedicated to the discussion of SAC, which is compared favorably with ACO and GA. At last, some conclusions and comments of further researches are given in section 5.

2. SIEGE ANT COLONY ALGORITHM (SAC)

OSP is a kind of very complex problem, especially for three-dimensional sensors placed in the vibration test. There are quite a lot of influence parameters in this kind of optimal problem. SAC, the algorithm presented in this paper, builds on the previous ACO in the direction of improving efficiency and accuracy when applied to OSP. The main idea is that of having a set of agents, called ant (soldier ants and worker ants), search in parallel for good sensor configurations, and cooperate through pheromone mediated indirect and global communication. Informally, each ant constructs a sensor configuration in an iterative way: it adds new cities to a partial solution by exploiting both information gained from past experience and a random judgment. Past experience takes the form of pheromone deposited by previous ants, while random judgment is simply a random choice determined by an ant. Three parts (1 ant colony assembling process. 2 ant colony siege process. 3 pheromone total update process) are included in SAC, and the final optimal sensor configuration will be found by ant colony in an iterative fashion.

2.1 Ant Colony Assembling Process

There is a clear division of labor in real ant colony. Different kinds of ants have different kinds of works in this organization, which is beneficial to ant colony search for food quickly and efficiently. Inspired by real ant colony, two kinds of ants (soldier ants and worker ants) are set in SAC. Soldier ant and worker ant are assigned with different kinds of tasks, which will be explained in detail at next process. The number of soldier ants and worker ants are expressed as M_s and M_w. The total number of cities is set to N. In this process, all kinds of ants are assigned to the colony uniformly and randomly, which can avoid ants crowded in a specific area.

Step 1. The ant colony is divided into small teams, which contain the same number of ants. The number of teams is equal to the number of cities (N). Then, the number of ants in a team can be calculated as $M_{team} = (M_s + M_w)/N$.

Step 2. Disturb the sequence orders of both soldier ants and worker ants. And then, select the soldier ants and worker ants randomly by proportion of M_s/M_w. Assign the selected ants to team 1.

Step 3. Repeat step 2 until all of teams are filled. And then, we prepare N city cards, which marked different number ($1 \sim N$). The number in the city card denotes the initial city of the team. Assign city cards to each team, and the initial cities of ants can be obtained. Take the ith team for example. Assuming the city card of the ith team is j, then all of the ants in the ith team will begin their searching from city j.

Note: Sometimes, the value of M_{team} in step1 may not be an integer. We can truncate M_{team} to the integer position and the remainder ants are assigned to the cities randomly. In this case, each team can only incorporate one remainder ant.

After the ant colony assembling process, the initial city positions of ants will be obtained, and the phenomenon of ants crowded in a specific area can be avoided.

2.2 Ant Colony Siege Process

In the real ant colony of word wild, the soldier ants always forage for food firstly. And then, a large amount of pheromone will be deposited by the soldier ants when soldier ants encountered prey. After that, a great many of worker ants will be attracted by the pheromone toward the location of of prey. Finally, all of the ants kill or carry the prey together. This hunting strategy is proved quite effective.

Inspired by the real ant colony, an ant colony siege process is proposed in this paper to optimize the sensor configuration. In SAC, a large amount of pheromone will be deposited by the soldier ant when the ant comes across a relative good solution. And then, the worker ants will be attracted by the soldier ants and gather to the area of the relative good solution. At this time, some better solutions around the soldier ant may discovered by the other ants. This method is more efficient than the original ACO [21] which has a low discrimination index in pheromone level. It should be noted that there are a few differences between ant colony in SAC and real ant colony. The biggest difference is that the soldier ants can only attract worker ants crawling in the next iteration in SAC.

In SAC, when ants arrive at a decision point in which they have to decide which city to select as their next city, they make their decisions by the level of pheromone. The more pheromone the road towards next city have, the bigger probability the ants would choice this road. The probability can be calculated by (1)

$$p_k \begin{cases} \dfrac{[\tau(r,s)]}{\sum_{u \in J_k(r)} \tau(r,u)} & if\ s \in J_k(r) \\ 0 & otherwise \end{cases} \tag{1}$$

Where $p_k(r, s)$ is the probability the ant k selects the road between city r and city s. $\tau(r, s)$ is the pheromone level between city r and city s. $J_k(r)$ is the candidate

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

city of ant k in the city r. Each ant will calculate the probabilities of the candidate roads according to the level of pheromone. And then, the decision will be made by each ant.

For an individual ant, while walking, ant deposits pheromone on the road passed by and follows, in probability, pheromone previously deposited by other ants. The level of pheromone deposited by an ant depends on the goodness of its whole trip (i.e. the goodness of sensor configuration). The better the whole trail, the higher pheromone level the ant would deposit on its roads with. The level of pheromone can be calculated by (2)

$$\Delta\tau_k(r,s)=\frac{Q}{mfx_k}\quad (k=1,2\ldots(M_s+M_w))$$
$$\begin{cases} r\in \text{the trail of ant } k \\ s\in \text{the trail of ant } k \end{cases} \tag{2}$$

In which $\Delta\tau_k(r, s)$ is the pheromone level deposited by ant k between city r and city s. mfx_k is the objective function value corresponding to the whole trip of ant k. In other words, mfx_k denotes the quality of sensor configuration corresponding to ant k. Q is a constant which is an adjuster between objective function value and pheromone. Ants in the colony can cooperate through pheromone-mediated communication by (1) and (2). The bad ants (bad sensor configuration) can learn from good ant (good sensor configuration) by this way.

However, the OSP is a kind of very complex problem. Original ACO cannot work well on it. A hunting process is proposed in this paper to improve the optimization ability. A large amount of pheromone will be deposited by soldier ant when the ant comes across the prey. And then, the worker ants will follow the guidance of pheromone and gather to the prey. The ant colony work together by this method and the best sensor configuration will be obtained in an iteration fashion. The detail of ant colony siege process is showed as follows:

Step 1. All of the ants select their roads by (1), at the same time, deposit pheromone by (2). And then, calculate the objective function value of each ant.

Step 2. Rank the ants by their objective function values. After that, select M_s ants who own the better objective value as the soldier ants. Record the trails of the soldier ants.

Step 3. Improve the pheromone amount level of the trips corresponding to soldier ants. The pheromone amount is calculated by

$$\Delta\tau_b(r,s)=\xi\cdot\frac{Q}{mfx_b}\quad (b=1,2\ldots M_s)$$
$$\begin{cases} r\in \text{the trail of ant } k \\ s\in \text{the trail of ant } k \end{cases} \tag{3}$$

In which $\Delta\tau_b(r, s)$ is the pheromone amount deposited by soldier ant b between city r and city s. ξ is the pheromone magnification factor.

2.3 Pheromone Total Update

The original ACO is always used in TSP problem. The ant deposited pheromone on the roads according to the sequences of the cities in its trip. However, the traditional pheromone process cannot be used in OSP problem. The reason for this can be explained by Fig. 1.

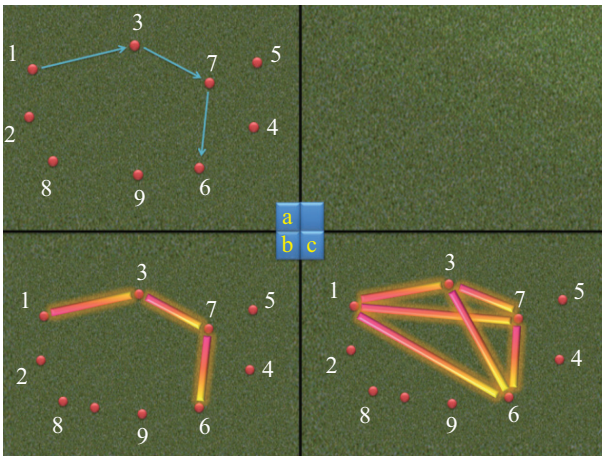


Fig. 1 Pheromone depositing strategy between different algorithms. (a) The trip of the ant. (b) Pheromone depositing strategy of ACO. (c) Pheromone depositing strategy of SAC.

Assuming the trip of an ant is 1-3-7-6 (Fig. 1(a)), and the pheromone configuration in original ACS is showed in Fig. 1(b) we can easily observe that the cities' sequence order of pheromone configuration is the same as that of ant's trip. However, this pheromone updating method is meaningless in OSP. This is because of the sequence order of cities in OSP is not important. In other words, the ants with trip of 1-3-7-6 and 3-7-1-6 denote the same sensor configuration. i.e. All of the selected sensor number of the two trips are No. 1 No. 3 No. 6 and No. 7. The ant with the trip of 1-3-7-6 should attract the ant with the trip of 3-7-1-6 (or 3-6-7-1 1-3-6-7 etc.) through pheromone-mediate communication in OSP. However, this goal cannot be achieved by traditional pheromone updating process in original ACO.

In order to solve this problem, a new method called pheromone total update process is proposed. The total roads between all of the cities visited by an ant are deposited by pheromone. The strategy is showed in Fig. 1(c). Assuming the trip of an ant is 1-3-7-6, firstly, the trip of 1-3-7-6 is deposited by pheromone. After that, the roads of 1-3,1-6,1-7,3-6,3-7,6-7 are also deposited by pheromone, which is showed in Fig. 1(c) the amount level of pheromone are still calculated by (2) and (3). Redundant iterations can be avoided by this process, and calculation efficiency can be improved by this pheromone updating method.

The ant colony searches for the best solution in an iterative fashion. After several times of iteration, the pheromone deposited on the roads became redundant so as to confuse the ants. Therefore, the pheromone, like the pheromone deposited by real ants in the word wild, should be evaporated. The pheromone evaporation process is executed after each time of iteration. And the remaining amount level of pheromone is calculated by (4):

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

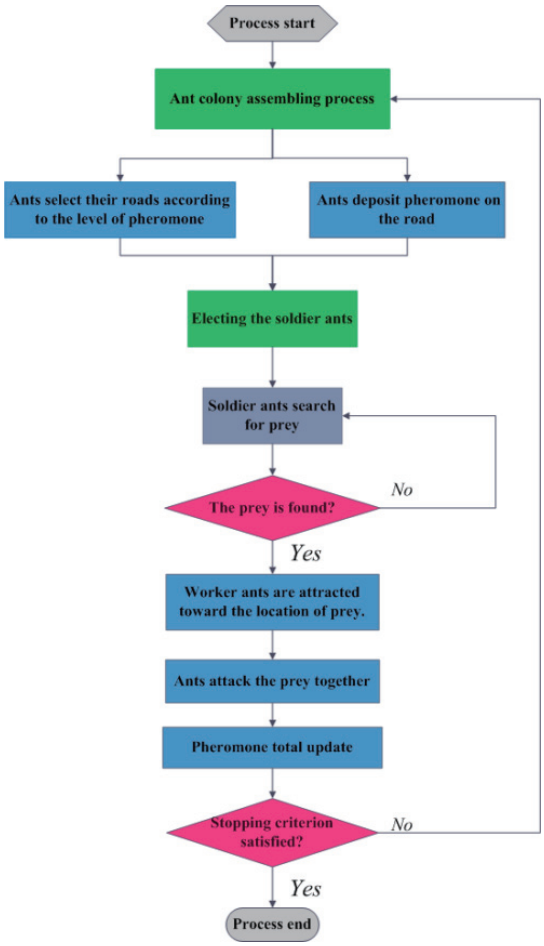


Fig. 2 Flowchart of the SAC for OSP.

$$\tau(r,s) = (1-\alpha)\cdot\tau(r,s) + \sum_{k=1}^{(M_s+M_w)} \Delta\tau_k(r,s) \tag{4}$$

In which $\tau(r, s)$ is the remaining amount level of pheromone between city r and city s. α is a parameter ($0 \le \alpha \le 1$) which denotes the pheromone evaporating speed.

The three processes are executed by an iterative fashion, and the flowchart of SAC is showed in Fig. 2. The ants in colony will cooperate through pheromone-mediated communication. The best sensor configuration will be obtained by several times of iterations, which can be fully implemented with the commercial software of MATLAB [22].

2.4 Objective Function

The objective function value is a standard to compare the goodness of sensor configurations. The objective function presented in this paper is derived from the famous modal assurance criterion (MAC), which is deduced by Carne and Dohmann [23]. The vibration behavior and monitoring technology are involved in MAC, which can evaluate the linear dependent degree of mode shape information collected by the sensor configuration in vibration test.

$$MAC_{ij} = \frac{(\phi_i^T \phi_j^T)^2}{(\phi_i^T \phi_i^T)(\phi_j^T \phi_j^T)} \tag{5}$$

Where ϕ_i^T and ϕ_j^T denote the ith and jth column vectors in matrix Φ. And Φ is the mode shape matrix collected by sensors. The superscript T is the transpose of vector ϕ.

MAC matrix is a diagonal matrix. The off-diagonal elements of MAC range between 0 and 1. $MAC_{ij} = 0$ denotes that the ith column vector and jth column vectors in matrix Φ are linearly independent. On the contrary, $MAC_{ij} = 1$ means that there is a high degree of linearly dependent between the ith column vector and jth column vector in matrix Φ. For an optimal sensor configuration, the off-diagonal elements values of MAC should be smaller compared with other sensor configurations. Therefore, the objective function can be expressed as follow:

$$mfx_x = \max\left\{MAC_{ij}^x\right\}_{i\neq j} \tag{6}$$

3. THE HANJIANG TRANSMISSION TOWER

The Hanjiang transmission tower is located in Han river of Hubei province in China. The tower is 122 meters high and 20.24 meters wide. The cross arm is 110 meters above ground level. And the end of cross arm is 10 meters from the middle point of tower. The main structure of the tower is made of high-strength 16mm thick steel, which is connected by flange. Q235 steel is used in diagonal rod, which is connected with main structure by solder. The picture of Hanjiang transmission tower and the FE model are showed in Fig. 3. The FE model is built by commercial finite

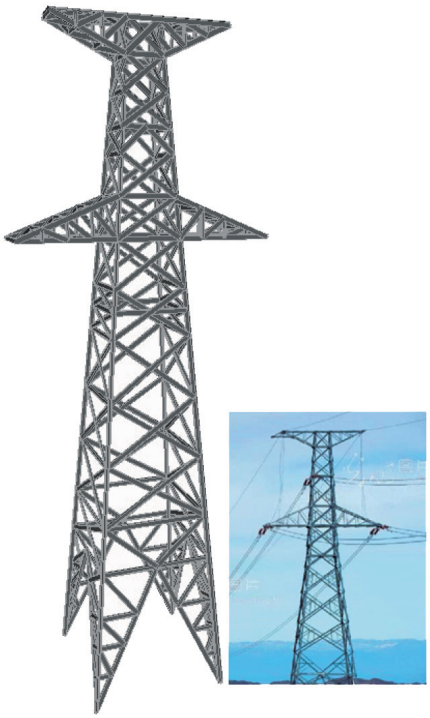


Fig. 3 FE model of Hangjiang transmission tower.

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

Table 1 The vibration characteristics of Hangjiang transmission tower

mode	1	2	3	4	5	6
Period (sec)	0.574076	0.542533	0.433003	1.340448	0.262971	0.221878
Frequency (cyc/sec)	1.7419	1.8432	2.3095	2.9373	3.8027	4.507
mode	7	8	9	10	11	12
Period (sec)	0.210534	0.18292	0.172076	0.169519	0.16438	0.145358
Frequency (cyc/sec)	4.7498	5.4669	5.8114	5.8991	6.0835	6.8795

element software of SAP2000 [24]. To facilitate the model analysis and other model-related studies, we reduce the nodes by equivalent reduced-order FEM. And only translational DOFs are considered for sensor installation. Finally, a total of 88 nodes are available for sensor installation. Dynamic analysis is executed to calculate the first 12 mode vectors, which is the input data in SAC. The vibration characteristics of Hangjiang transmission tower are showed in Table 1.

4. OPTIMIZATION RESULT AND DISCUSSION

The proportion of soldier ants and worker ants is an important parameter in SAC. If there are too many soldier ants, the attack information will be spread unchecked. So, the worker ants cannot determine which soldier ants to follow. This may lead to the scene of confusion. On the other hand, if the number of soldier ants is a small value, the ant colony will be leaded by a small group of soldier ants. As a result, the ant colony may crowd in to a specific place so quickly that a lot of optimal solutions are missed. On other words, the algorithm may fall into local optimum. The proportion of soldier ants and worker ants is calculated by $q_M = (M_s / M_s + M_w)$. In order to determine appropriate value of q_M, the value of q_M is set from 1% to 78%. And the objective function value of each q_M are recorded. An appropriate q_M should not only render the best objective function values obtained quickly, but also make the algorithm convergence stable.

In this part, 50 times of iterations are implemented in each value of q_M, and the mean objective function values of 25*th* to 50*th* iterations are recorded to evaluate the appropriate q_M. The curve is showed in Fig. 4.

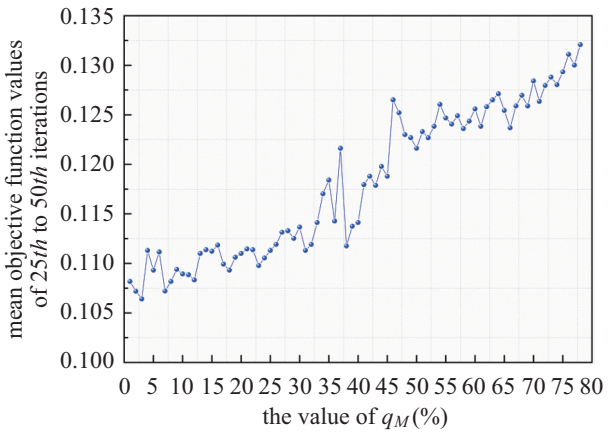


Fig. 4 The curve of mean objective function values of 25*th* to 50*th* iterations and q_M.

As respected, the curve in Fig. 4 increases with the increment of q_M. We can observe from the figure that the vertical axis value keeps a small value when q_M is 0.1% ~ 10%. We chose the lowest point (q_M = 3%) in the curve, and the numbers of soldier ant is set to 12.

The numbers of sensors should be determined before vibration tests. We always expect to collect more useful vibration information with less testing resources. Thus, we set the numbers of sensors N from 1 to 88, wich is showed in Fig. 5.

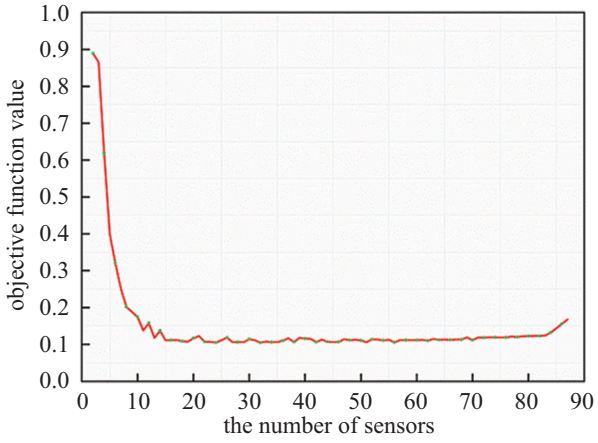


Fig. 5 The influence of sensor quantity on the objective function value.

It can be observed that the objective function value decreases with the number of sensors. That means the more sensors installed in the tower, the more vibration information will be obtained by the sensor configuration. It is expected that the objective function value will keep decreasing in a large slope. However, the objective function value begin to stabilize from N = 10 ~ 18. The same phenomenon is also happened in [25] and [26]. The reason for this is that when the numbers of sensors excess a value, redundant vibration information will be collected by the added sensors installed in the structure. Mathematically speaking, the mode vectors monitored by the added sensors have a large linear relationship with the mode vectors obtained by previous sensors. Thus, we select the point corresponding to a small slop value (N = 13) as the numbers of sensors.

What should be noted is that the SAC belongs to heuristic algorithms. In all kinds of heuristic algorithms, slight fluctuations will be happened if the parameters are not set properly. This is a characteristic of heuristic algorithms. The calculation results are influenced little by the slight fluctuations. In this part, the purpose of

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

our work is to find out the best number of sensors rather than obtain the final calculation result. We just need to know the tendency of the curve. Therefore, the parameters of SAC are set randomly to facilitate the engineers. So, the slight fluctuations is happened when the number of sensors is larger than 80. However, we can still obtain the relative stable point $N = 13$. Therefore, the slight fluctuations can be neglected to facilitate the engineers. The same circumstances can also be found in the research of [25], [26] etc.

The value of ξ (the pheromone magnification factor) will be calculated in this part. ξ is a parameter which expresses the impact area of the soldier ants. The value of ξ should be compatible with the ratio of soldier ants. And q_M is calculated by previous analysis. The value of ξ should be determined such that the objective function value is as small as possible. At the same time, the calculation result should be stable in some sense such that the algorithm is reliable. The mean and variance values of the second half objective function values are calculated in each ξ in this part. The values of ξ range from 0 to 2000 with evenly spaced increments (total 50 values) are test. Only 50 times of iteration rather than full calculation are performed in each ξ since we just want to obtain the best value of ξ. The mean and variance value of the second half objective function values in each ξ are showed in Fig. 6.

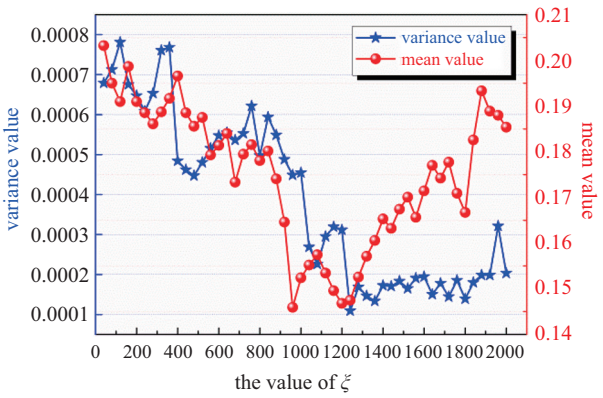


Fig. 6 The mean and variance value of of the second half objective function values in each ξ.

What should be noted is that Only 50 iterations are executed rather than full calculation in this part. This is because of that when the iteration times value is set to a value, which is greater than a certain value, the curves' tendencies of ξ in different iteration times are similar. The best value of ξ can be obtained just by the tendency. Therefore, it is not necessary to make sure the procedure is totally converged in this part. On the other hand, we know that the procedure is converged in 35*th* iteration from Fig. 7. And the algorithm has already converged in 50 iterations. Therefore, it is not necessary to be worried about if 50 iterations are enough.

The mean values of objective function values have a downward trend in the beginning as we can see from Fig. 6. And the variance values have the same tendency. This is because of that the contribution of soldier ant has a positive influence to the accuracy and stability of algorithm. However, the mean values rise sharply at last. This suggests that the iteration results are unsatisfactory when the weight of soldier ants influences is relative large. It can also be observed that the variance values close to 0, and mean values increase greatly when the value of ξ beyond a threshold. This means that the algorithm tends to fall into local optimum when ξ is a large value, and the best sensor configuration is very difficult to obtained in this case. In conclusion, the value of ξ is set to 900, which can balance the two aspects of considerations.

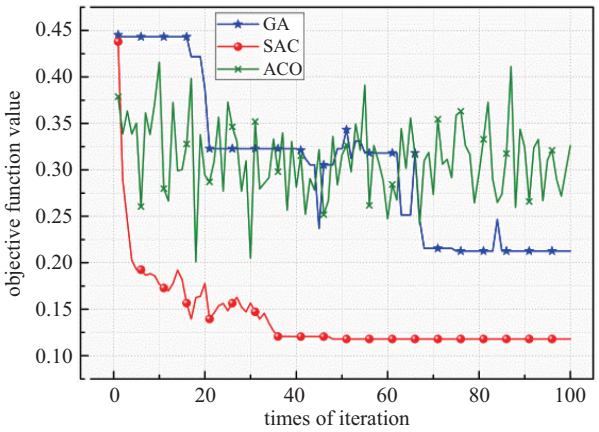


Fig. 7 The objective function value after each time of iteration calculated by ACO (Case 1) SAC (Case 2) and GA (Case 3).

Three cases are involved in this paper to solve the OSP problem of Hanjiang transmission tower. And the comparisons are made between the three different algorithms.

Case 1. ACO
Case 2. SAC
Case 3. GA

The population size in GA [26] is set the same value as the amount of ants in ACO and SAC in this paper. And the genes crossover probability and genes mutation probability are set to the best values by tentative calculation. The objective function value after each time of iteration in three cases is recorded. 100 times of iteration are performed in this three cases. And the results are showed in Fig. 7.

It can be easily observed in Fig. 7 that the curve of ACO is unstable during the whole process of iteration. The minimum objective function value is founded in the 19*th* iteration. However, the largest value is obtained in the 87*th* iteration. This means that the convergence solution cannot be obtained by ACO. From the curve of GA we can find that the dates decline rapidly before 20*th* iteration. And the curve becomes stable after 68*th* iteration. Finally the lowest point is emerged in 90*th* iteration in GA. Compared with the above two algorithms, the data in SAC converge very quickly. The results are completely stable in 56*th* iteration and the data after the 35*th* iteration is very close to the optimal value. That means the SAC algorithm has a good global convergence property. And the calculation efficiency of SAC is improved by 17% compared with GA.

The 3 different algorithms are performed 50 times re-

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

spectively. Whole process is executed in each case, and optimal results are recorded. The maximum values of off-diagonal elements in each mode partition of *MAC* matrix calculated by ACO, SAC, GA are showed in Fig. 8. The similar results can also be obtained in Fig. 7. We use the column chart in Fig. 8 in order to express it clearly. The values of SAC in each mode partition are much smaller than that of ACO and GA, which suggests the sensor configuration obtained by SAC is much better than ACO and GA. The final sensor configuration obtained by 3 different algorithms is illustrated in Fig. 9. The final optimal results of maximum value, minimum value, and mean value of each algorithm are showed in Table 2. The mean results of ACO, SAC, GA are 0.33, 0.11 and 0.21 respectively. That means the calculation accuracy of SAC is improved by 66% and 47% compared with ACO and GA.

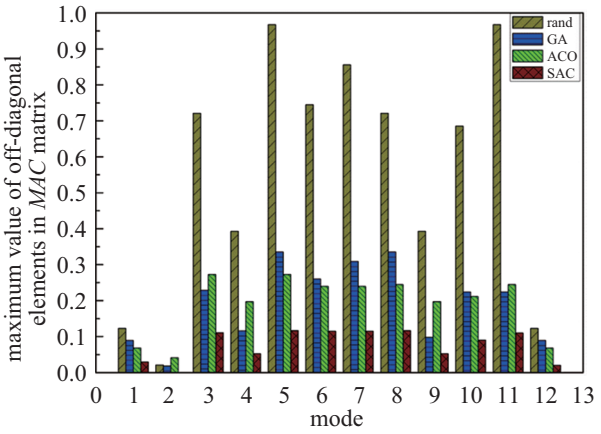


Fig. 8 The maximum values of off-diagonal elements in each mode partition of *MAC* matrix calculated by ACO, SAC and GA.

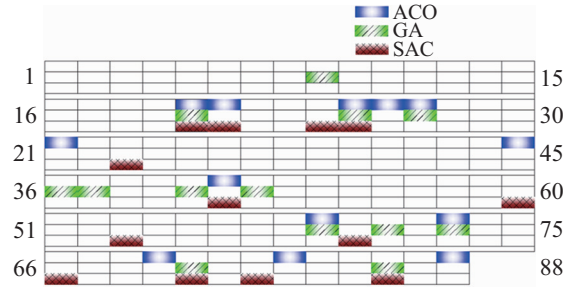


Fig. 9 The final sensor configuration obtained by ACO, SAC and GA.

Table 2 The optimal results of maximum value, minimum value, and mean value of ACO (Case 1) SAC (Case 2) and GA (Case 3)

	ACO	SAC	GA
maximum value	0.51	0.14	0.31
minimum value	0.31	0.09	0.18
mean value	0.33	0.11	0.21

5. CONCLUSIONS

OSP problem of large-scale structure is a kind of very complex problem, especially for the three-dimensional sensors. The existing methods are not effective enough in accuracy and stability for vibration tests. SAC, the algorithm presented in this paper, builds on the previous ACO in the direction of improving efficiency and accuracy. The main idea is that of having a set of agents, called ant (soldier ants and worker ants), search in parallel for good sensor configurations, and cooperate through pheromone mediated indirect and global communication. The final optimal sensor configuration will be found by ant colony in an iterative fashion. An engineering example is involved in this paper, and the algorithms of ACO, SAC and GA are applied on it. After that, some comparisons are made between the 3 algorithms and some conclusions are obtained.

1. The optimal result calculated by ACO is not stable enough in OSP problem of three-dimensional sensors. And the data is hard to converge. However, the suboptimal solution can still be obtained when the iteration times is sufficiently large.
2. The output data of GA converge quickly in the beginning. However, the curve begins to fluctuate after a threshold, which means a suboptimal solution may be obtained by this method.
3. The SAC performs much better than traditional OSP methods. The calculation accuracy have been improved by 66% and 47% compared with ACO and GA.
4. The SAC proposed in this paper is designed for the optimal problems of three-dimensional sensor placement in vibration tests of SHM. Whether other optimal problems can be solved by SAC still needs further researches.

ACKNOWLEDGEMENTS

The authors would like to thank the support by Natural Science Foundation of China under award number 50878037.

NOMENCLATURE

$p_k(r, s)$	The probability ant k selects the road between city r and city s;
$\tau(r, s)$	The pheromone level between city r and city s;
$J_k(r)$	The candidate city of ant k in the city r;
$\Delta\tau_k(r, s)$	The pheromone level deposited by ant k between city r and city s;
mfx_k	The objective function value corresponding to the whole trip of ant k;
Q	The adjuster between objective function value and pheromone;
$\Delta\tau_b(r, s)$	The pheromone amount deposited by soldier ant b between city r and city s;
ξ	The pheromone magnification factor;

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

$\tau(r, s)$ The remaining amount level of pheromone between city r and city s;
α The parameter which denotes the pheromone evaporating speed;
C heat capacity, J/(kg·K);
ϕ_i^T The ith column vector in matrix Φ;
Φ The mode shape matrix collected by sensors;
ϕ^T The transpose of vector ϕ;

REFERENCES

1. Giurgiutiu, V., "Tuned Lamb Wave Excitation and Detection with Piezoelectric Wafer Active Sensors for Structural Health Monitoring," *Journal of Intelligent Material Systems and Structures*, **16**, pp. 291-305 (2005).
2. Farrar, C. R. and Worden, K., "An Introduction to Structural Health Monitoring," *Philosophical Transactions of the Royal Society a-Mathematical Physical and Engineering Sciences*, **365**, pp. 303-315 (2007).
3. Chen, C. H., Shen, Y. L. and Shin, C. S., "Using Distributed Brillouin Fiber Sensor to Detect the Strain and Cracks of Steel Structures," *Journal of Mechanics*, **26**, pp. 547-551 (2010).
4. Lynch, J. P. and Loh, K. J., "A Summary Review of Wireless Sensors and Sensor Networks for Structural Health Monitoring," *Shock and Vibration Digest*, **38**, pp. 91-128 (2006).
5. Wan, Z., Li, J. D., Jia, M. and Li, J. L., "Structural Health Monitoring (Shm) of Three-Dimensional Braided Composite Material Using Carbon Nanotube Thread Sensors," *Journal of Mechanics*, **29**, pp. 617-621 (2013).
6. Raghavan, A. and Cesnik, C. E. S., "Review of Guided-Wave Structural Health Monitoring," *Shock and Vibration Digest*, **39**, pp. 91-114 (2007).
7. Harasawa, S. and Sato, M., "Helical Buckling of Slender Beam Structures Surrounded by an Elastic Medium," *Journal of Mechanics*, **31**, pp. 241-247 (2015).
8. Wirowski, A., Michalak, B. and Gajdzicki, M., "Dynamic Modelling of Annular Plates of Functionally Graded Structure Resting on Elastic Heterogeneous Foundation with Two Modules," *Journal of Mechanics*, **31**, pp. 493-504 (2015).
9. Yam, L. H., Yan, Y. J. and Jiang, J. S., "Vibration-Based Damage Detection for Composite Structures Using Wavelet Transform and Neural Network Identification," *Composite Structures*, **60**, pp. 403-412 (2003).
10. Salama, M., Rose, T. and Garba, J., "Optimal Placement of Excitations and Sensors for Verification of Large Dynamical Systems," *Proceedings of the 28th Structures, Structural Dynamics, and Materials Conference*, pp. 6-8 (1987).
11. Henshell, R. and Ong, J., "Automatic Masters for Eigenvalue Economization," *Earthquake Engineering & Structural Dynamics*, **3**, pp. 375-383 (1974).
12. Kammer, D. C., "Sensor Placement for on-Orbit Modal Identification and Correlation of Large Space Structures," J*ournal of Guidance, Control, and Dynamics*, **14**, pp. 251-259 (1991).
13. Kammer, D. C. and Tinker, M. L., "Optimal Placement of Triaxial Accelerometers for Modal Vibration Tests," *Mechanical Systems and Signal Processing*, **18**, pp. 29-41 (2004).
14. Batou, A., "Model Updating in Structural Dynamics—Uncertainties on the Position and Orientation of Sensors and Actuators," *Journal of Sound and Vibration*, **354**, pp. 47-64 (2015).
15. Bishop, A. N., Fidan, B., Anderson, B. D., Doğançay, K. and Pathirana, P. N., "Optimality Analysis of Sensor-Target Localization Geometries," *Automatica*, **46**, pp. 479-492 (2010).
16. Guo, H., Zhang, L., Zhang, L. and Zhou, J., "Optimal Placement of Sensors for Structural Health Monitoring Using Improved Genetic Algorithms," *Smart Materials and Structures*, **13**, pp. 528 (2004).
17. Han, J.-H. and Lee, I., "Optimal Placement of Piezoelectric Sensors and Actuators for Vibration Control of a Composite Plate Using Genetic Algorithms," *Smart Materials and Structures*, **8**, pp. 257 (1999).
18. Herzog, R. and Riedel, I., "Sequentially Optimal Sensor Placement in Thermoelastic Models for Real Time Applications," *Optimization and Engineering*, pp. 1-30 (2014).
19. Joshi, S. and Boyd, S., "Sensor Selection Via Convex Optimization," *IEEE Transactions on Signal Processing*, **57**, pp. 451-462 (2009).
20. Krause, A., Singh, A. and Guestrin, C., "Near-Optimal Sensor Placements in Gaussian Processes: Theory, Efficient Algorithms and Empirical Studies," *The Journal of Machine Learning Research*, **9**, pp. 235-284 (2008).
21. Dorigo, M., Maniezzo, V. and Colorni, A., "Ant System: Optimization by a Colony of Cooperating Agents," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, **26**, pp. 29-41 (1996).
22. *MATLAB 2007*, MathWorks, Natick, Massachusetts, U.S.
23. Carne, T. G. and Dohrmann, C., "A Modal Test Design Strategy for Model Correlation," *Proceedings-Spie The International Society For Optical Engineering*, pp. 927-927 (1995).
24. *SAP2000*, Computers and Structures, Inc., Berkeley, California, U.S.
25. Yi, T. H., Li, H. N. and Gu, M., "Optimal Sensor Placement for Structural Health Monitoring Based on Multiple Optimization Strategies," *The Structural Design of Tall and Special Buildings*, **20**, pp. 881-

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press

900 (2011).

26. Yi, T.-H., Li, H.-N. and Gu, M., "Optimal Sensor Placement for Health Monitoring of High-Rise Structure Based on Genetic Algorithm," *Mathematical Problems in Engineering*, **2011**, pp. 1-12 (2011).

(Manuscript received January 26, 2016, accepted for publication August 22, 2016.)

https://doi.org/10.1017/jmech.2016.87 Published online by Cambridge University Press