Interpreting Quantum Field Theory*

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The availability of unitarily inequivalent representations of the canonical commutation relations constituting a quantization of a classical field theory raises questions about how to formulate and pursue quantum field theory. In a minimally technical way, I explain how these questions arise and how advocates of the Hilbert space and of the algebraic approaches to quantum theory might answer them. Where these answers differ, I sketch considerations for and against each approach, as well as considerations which might temper their apparent rivalry.

1. Introduction. This essay aims to provide a partial but accessible introduction to the interpretation of quantum field theory (QFT), focusing on the simplest case of linear field theories.[1] It is organized as follows. Section 2 considers the project of interpreting physical theories, as well as constraints to which interpretation answers. Section 3 sketches QFT in a way that highlights how those constraints might apply to *its* interpretation. Sections 4–8 explain how several questions, which are recognizably species of Section 2's interpretative genus, arise in connection with QFT. The questions I catalog are not exhaustive. Nor are the answers I place in the mouths of two fictional and extremist interpreters of QFT, the Hilbert Space Conservative and the Algebraic Imperialist.[2] These characters are expository devices, intended to make interpretative projects vivid, by starkly illustrating their pursuit. From the clash of these extremists emerge

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1. Introductions which survey interactions as well include Teller (1995) and Huggett (2000).

2. The latter is so dubbed, I believe, by Arageorgis (1995).

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several moderating suggestions, which depend on such overtly philosophical questions as the nature of physical possibility. My primary aim here is neither to develop nor to defend these suggestions, but to convey a sense of the interpretative landscape on which they appear. Although my exposition is often only rough, I try to direct those desirous of more precision or detail to references which might satisfy them.[3]

2. Interpreting Physical Theories. This section presents one take on the project of interpreting physical theories. While I do not claim that the construal of interpretative projects offered here is the only, or even the best, construal of such projects, I do suggest that many archetypal interpretative questions are interpretative questions in this section's sense, and that many prominent and probing criticisms of interpretations call those interpretations to task for violating constraints this section announces.

One business of philosophers of physics is the interpretation of physical theories. The interpreter asks: "Under what conditions is this theory true? What does it say the world is like?" (van Fraassen 1991, 242). A standard sort of answer (see van Fraassen 1989, ch. 9 and references therein) proceeds in two stages. At the first, *structure-specifying,* stage, the interpreter characterizes the structures by which the theory would represent physical reality.[4] At the second, *semantic,* stage, the interpreter characterizes the physical worlds which model the theory so structured. Thus an interpretation of a theory identifies the physical worlds possible according to the theory.[5]

For that bulk of central physical theories admitting a Hamiltonian formulation, the first-stage specification of theory structure can be decomposed into three elements. The first is an identification of the theory's *state space.* In the mercifully simple case of the classical theory of a particle of mass m moving in one dimension, the state space is comprised of ordered pairs (q, p) of position and momentum values. The second element of structure specification is an identification of the set of physical magnitudes or *observables* the theory recognizes. In the same simple case, observables are functions from elements of the state space to the reals; e.g., the physical magnitude *kinetic energy* E is given by the function. $f_E(q, p) = p^2/2m$. The third element of structure specification is the identification of the theory's *dynamics,* its account of how the states of systems change with time. In

3. Wald (1994) and Emch (1972) are excellent general ones.

4. Note 6 attempts to appease those purists who maintain that such characterization is the prerogative of physics, but not of its interpretation.

5. For accounts of science that amend, complicate, or challenge the idea that theories represent by picking out sets of possible worlds as models of their fundamental mathematical structures, see, for example, the essays in Morgan and Morrison 1999.

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the simple case, dynamics takes the form of trajectories through state space determined by Hamilton's equations once a Hamiltonian function $H(q, p)$ is provided.

With these elements in hand, interpretation can proceed to its second, *semantic* stage of characterizing the physical models of the theory. A semantics can take the form of an account of which propositions attributing determinate values to observable magnitudes recognized by the theory are true of a system represented by a state of the theory. A natural semantics for the simple classical theory is: *"Observable O has value x in state (q, p)" is true just in case* $f_o(q, p) = x$. With a fix on a theory's state space, observables, and dynamics, the interpreter can use a semantics for the theory to characterize the worlds of which the theory is true.

Examples of familiar interpretative questions posed about each of these four elements are not hard to come by.[6] A question about a physical theory's state space: does *absolute position* parameterize the state space of Newtonian theory? A question about the observables appropriate to a physical theory: must the observables of general relativity be generally covariant, and if so, in what sense? A question about the dynamics of a physical theory: does the wave function of a quantum system collapse upon measurement or doesn't it? A question about that same theory's semantics: do all propositions attributing determinate values to observables pertaining to quantum systems have truth values, or just some of them? If the latter, which ones?

Interpretative questions of the sorts just canvassed can interact. The venerable debate between substantivalists and relationalists about space(time) can be profitably cast as the issue of how best to harmonize answers to the three structure-specifying questions. In the quantum mechanical measurement problem, structure-specifying questions interact with semantic questions: if one adopts the no-collapse answer to the question about dynamics—that is, if one takes quantum dynamics to be unitary dynamics—while adopting a semantics that limits the observables with determinate values to those whose values the theory predicts with certainty, one is committed to the unpalatable consequence that most measurements leave laboratory apparatus in states we cannot take to represent measurement outcomes.

The measurement problem demonstrates that interpretations of physical theories can be subject to stringent *intrinsic* criteria of adequacy. By

6. The availability of natural examples is why I'm inclined to count even questions about theory-structure as "interpretative." But this is only bookkeeping; those distressed by the nomenclature can gather the questions I discuss here under the heading "foundational," without damage to my central contention, that they are questions to which philosophers of physics should, and do, attend.

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intrinsic criteria, I mean criteria we can articulate without making reference to other physical theories, criteria that (so to speak) arise between us, the theory we're interpreting, and the world to which the theory would apply. Intrinsic criteria include logical coherence, consistency with empirical data, and adequacy to experimental practice. They may even include simplicity and mathematical elegance. A vast expanse of the literature on the interpretation of quantum mechanics engages the intrinsic concern of whether it is possible to give an interpretation of quantum mechanics consistent with the empirical predictions of the minimally interpreted quantum statistical algorithm. Another expanse addresses the measurement problem just sketched, which constrains interpretation to secure the fundamental presupposition of laboratory practice, that experiments have outcomes.

Interpretation needn't be a hermetic activity. We may also subject interpretations to *extrinsic* criteria of adequacy, criteria that make reference to theories beside the theory interpreted, criteria that (so to speak) arise between us, the theory we're interpreting, and other theories to which we're committed—perhaps even, depending on the level of our commitment to these theories, their interpretations. We may, for instance, ask: how well does the theory as interpreted fit with environing theories? Or: How promising a resource is the theory as interpreted for the construction of novel theories? Examinations of whether Bohmian interpretations of quantum mechanics violate the letter or the spirit of the special theory of relativity express the first sort of extrinsic concern. Working physicists—for instance, quantum gravity theorists taking a stake in the interpretation of spacetime theories—often express the second sort of extrinsic concern.

With these preliminary remarks in mind, I want to survey some interpretative questions that arise in connection with QFT. Cushing, for one, might seem to query the point of such an exercise: "My contention is that at the foundational level there is essentially nothing new in quantum field theory that is not already present in non-relativistic quantum mechanics" (Cushing 1987, 25).[7] My contention is that there is sufficient novelty, both in the interpretation of the theory proper and in the fit of its interpretations with the interpretations of environing physical theories, that the quantum field is fertile ground for the philosopher of physics.

3. Quantum Field Theory. This section characterizes QFT in two ways.

7. Cushing no doubt means to provoke! The body of his article finds precedents in non-relativistic quantum mechanics for candidate problems novel to QFT, then develops an account of why QFT—particularly, the art of renormalization—is *methodologically* interesting. The interpretative questions I raise here are different from the candidate novelties Cushing considers and deflates.

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First, extrinsically (and so with a view toward extrinsic constraints on its interpretation), in terms of how it relates to other theories, developed and imagined. Second, (semi-) intrinsically, in terms of the features constitutive of a quantization of a classical field theory. The reference to the classical field theory is what makes this characterization only semi-intrinsic.

3.1. Extrinsic Characterization. Physicists have a classical theory of gravity (Einstein's General Theory of Relativity) and quantum theories of just about everything else, but no quantum theory of gravity. Signposts on the road to quantum gravity include theories which treat the interaction between quantized matter and gravity, classically described. The standard spacetime setting for QFT is flat Minkowski spacetime; there QFT is a quantum theory uncomplicated by gravitation. But QFT on *curved* spacetime sets quantized matter fields in the classical gravitation field expressed by the spacetime curvature. And semiclassical quantum gravity calculates the back reaction on the spacetime metric wrought by quantized matter fields by replacing the stress energy tensor T_{ab} in Einstein's Field Equations with its quantum expectation value $\langle T_{ab} \rangle$ for those fields. Both approaches are potential signposts on the road to quantum gravity. Interpreters of QFT should be mindful of its place(s) in the (still under construction) nexus of physical theories.

3.2 (Semi-) Intrinsic Characterization. Traditionally, to quantize a classical theory is to cast the theory in Hamiltonian form and then to promote its canonical observables q_k, p_k to symmetric operators $\hat{q}_k$, $\hat{p}_k$ obeying commutation relations corresponding to the Poisson brackets of the classical theory. The familiar Heisenberg form of the canonical commutation relations (CCRs), constituting a quantization of our mercifully simply classical theory (q, p), is

(3.1)

The standard realization of these CCRs is the *Schrödinger Representation,* on the Hilbert space $L^2(\mathbb{R})$ of square-integrable complex-valued functions $\psi(x)$ of the reals, with

$$\hat{q}\psi(x) = x\psi(x) \text{ and } \hat{p}\psi(x) = -i\frac{h}{2\pi}\frac{d\psi(x)}{dx}.$$

The Heisenberg form (3.1) of the CCRs is, however, mathematically infelicitous (see Summers 2001). The unboundedness of the position and momentum operators is to blame: they are not jointly defined on all of

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$L^2(\mathbb{R})$, so that (3.1) fails to be an operator equation for that Hilbert space. While this can perhaps be addressed by painstaking attention to domains of definition for canonical operators, there is a more convenient fix. It is to exponentiate $\hat{q}$ and $\hat{p}$ to obtain unitary operators, bounded and everywhere defined, and to re-express (3.1) in terms of those unitaries. In particular, where $\hat{U}(a) = \exp(\frac{-i2\pi a\hat{q}}{h})$ and $\hat{V}(a) = \exp(\frac{-i2\pi a\hat{p}}{h})$, if (3.1) holds, then for all $a, b \in \mathbb{R}$,

$$\hat{U}(a)\hat{V}(b) = \exp\left(\frac{-i2\pi ab}{h}\right)\hat{V}(b)\hat{U}(a). \tag{3.2}$$

Call (3.2) the *Weyl form* of the CCRs, or *Weyl relations* for short. Unitary operators satisfying the Weyl relations express the quantization of the one-dimensional classical theory. According to Stone's theorem, provided the families $\{\hat{U}(a)\}$ and $\{\hat{V}(b)\}$ are (weakly) continuous with respect to the real parameters a and b, the canonical operators $\hat{q}$ and $\hat{p}$ can be recovered as the infinitesimal generators of those families—the Heisenberg CCRs (3.1) are essentially the infinitesimal form of the Weyl relations (3.2).[8]

The quantization of a classical field theory to obtain a QFT proceeds along analogous lines. Textbook approaches (e.g., Kaku 1993, Ch. 3.2) begin with a classical field equation, for instance, the Klein-Gordon equation on a spacetime (M, g_{ab}) (n.b. $x = (\vec{x}, t)$ is a point in the four manifold M):

$$(g^{ab}\nabla^a\nabla_a - m^2)\varphi(x) = 0.$$

This arises from a Hamiltonian H that is a function of the configuration variable $\varphi(x)$ and a canonically conjugate momentum density $\pi(x) \equiv \frac{\partial L}{\partial\dot{\varphi}(x)}$ where L is the theory's Lagrangian density. To quantize Klein-Gordon theory is to replace φ and π with Hilbert space operators $\hat{\varphi}$ and $\hat{\pi}$ obeying the canonical commutation relations (CCRs) answering to the Poisson brackets of the classical field theory:

$$[\hat{\varphi}(x),\hat{\varphi}(y)] = [\hat{\pi}(x),\hat{\pi}(y)] = 0,$$
$$[\hat{\varphi}((\vec{x},t)), \hat{\pi}((\vec{x}',t))] = -i\delta^3(\vec{x}-\vec{x}').$$

Once again, to avoid technical difficulties involving the domains of definition of the unbounded operators appearing in these CCRs, they are typically recast in exponential form as Weyl relations.[9]

8. The Weyl relations (3.2) are not equivalent to the Heisenberg form (3.1) of the CCRs, but arise from their formal manipulation. See Segal 1967, §1.

9. Where the symplectic vector space (S, Ω) gives the structure of a classical theory, a Weyl System over (S, Ω) is a pair $(\mathcal{H}, \hat{W})$, where $\mathcal{H}$ is a Hilbert space and $\hat{W}$ a map from S to unitary operators on $\mathcal{H}$ s.t. for all $y_1, y_2 \in S$

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Thus to quantize a classical field theory is to find a Hilbert space on which act operators satisfying the Weyl relations arising from that theory. Where $\mathcal{H}$ is a Hilbert space and $\{\hat{W}(y)\}$ are operators on $\mathcal{H}$ satisfying the Weyl relations, I will call $(\mathcal{H}, \hat{W}(y)))$ a *representation* of the Weyl relations for that theory. We can, without prejudice to interpretive questions we encounter down the road, use these Weyl relations to characterize what features of a system must be held constant when asking what states are possible for that system. These features are, minimally, that the Weyl relations involved in the quantization of the system hold of the set of observables pertaining to the system. So a state over a set of observables satisfying canonical *anti*-commutation relations characteristic of the free fermion field is not a state possible, in this sense, for the free boson field, which is a system characterized by appeal to canonical commutation relations.

In what follows, I will have occasion to refer to some features of a textbook heuristic for quantizing the Klein-Gordon field on Minkowski spacetime to obtain the free boson field. This frequency-splitting heuristic proceeds, very roughly, as follows (Kaku, 1993; see also Wald 1994, Ch. 4.3). By a plane wave expansion, solutions to the classical Klein-Gordon equation can be Fourier-decomposed into uncoupled normal modes with angular frequency ω_k, and so the classical field can be modeled as an infinite collection of independent oscillators. The heuristic quantization builds the single particle Hilbert space for the QFT from classical solutions which oscillate with positive frequency with respect to time. Exploiting the analogy with the simple harmonic oscillator, the textbook approach recasts the CCRs in terms of "creation" and "annihilation" operators $\hat{a}_k^\dagger$ and $\hat{a}_k$ for positive frequency field modes ω_k:

$$[\hat{a}_k, \hat{a}_{k'}] = 0 = \left[\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger\right], \left[\hat{a}_k, \hat{a}_{k'}^\dagger\right] = \delta_{kk'}\hat{I}. \tag{3.4}$$

Formal expressions for operators $\hat{\varphi}(x)$ and $\hat{\pi}(x)$ satisfying the CCRs can be constructed from these. The operators in (3.3) act on a symmetric Fock space (see Wald 1994, A.2) which is a direct sum of n-fold symmetrized tensor products of the single particle Hilbert space constructed from positive frequency solutions.

This Fock space features a state $|0\rangle$ such that $\hat{a}_k|0\rangle = 0$ for all k. $|0\rangle$ is the lowest energy eigenstate of the quantum Hamiltonian for the free bo-

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$$\hat{W}(y_1)\hat{W}(y_2) = \exp[i\Omega(y_1, y_2)/2]\hat{W}(y_1 + y_2)$$

$$\hat{W}^*(y) = \hat{W}(-y) \tag{3.3}$$

The map $\mathbb{R} \ni t \Rightarrow \hat{W}(ty)$ is required to be strongly continuous for any fixed element $y \in S$ (see Wald 1994, Ch. 2.2, or Baez, Segal, and Zhou 1992, Ch. 1.2.).

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son field (also handled by analogy with the simple harmonic oscillator). The state $(\hat{a}_k^\dagger)^n|0\rangle$ is an eigenstate of the Hamiltonian with the same energy a system of n particles each with energy $h\omega_k$ would have—provided the momenta and rest masses of these particles are given by standard relativistic expressions. Thus the quantization tempts a particle interpretation: the *vacuum* state $|0\rangle$ is the no particle state; the state $\hat{a}_k^\dagger|0\rangle$ obtained by acting on the vacuum once with the creation operator $\hat{a}_k^\dagger$ describes one particle of energy $h\omega_k$; the state $\hat{a}_k^\dagger\hat{a}_k^\dagger|0\rangle$ obtained by acting on the vacuum twice with the creation operator $\hat{a}_k^\dagger$ describes two particles, each of energy $h\omega_k$; the state obtained from *this* state by acting on it with an annihilation operator $\hat{a}_k$ describes one particle of energy $h\omega_k$ again, and so on. Every state in the representation can be approximated to arbitrary precision by acting on $|0\rangle$ with linear combinations of creation and annihilation operators. That is to say, $|0\rangle$ is *cyclic* with respect to the operators $\{\hat{a}_k, \hat{a}_k^\dagger\}$.

Of particular significance is the operator $\hat{N}_k = \hat{a}_k^\dagger\hat{a}_k$ – the *number operator* for particles of type k. Its expectation value in a state gives the mean number of particles of type k one would find in that state. Hence, $\hat{N} = \sum_j \hat{a}_j^\dagger\hat{a}_j$ is the *total number operator.* That for all k, $\langle 0|\hat{N}_k|0\rangle = \langle 0|\hat{N}|0\rangle = 0$ reinforces $|0\rangle$'s claim to be the vacuum or no-particle state.[10] The set of states available in a quantization with a Fock space structure can be understood in terms of excitations of an underlying ground state. Thus Fock space representations aid and abet the understanding of QFT as particle physics.

4. Unitary Inequivalence and the First Question of Interpretation. The topic of this section is the availability of unitarily inequivalent representations of the Weyl relations constituting the quantization of a classical field theory, an availability which drives the interpretative questions I consider in this essay. To describe what unitary inequivalence amounts to, I must review some features of the Hilbert space approach to quantum mechanics. On the Hilbert space approach, the space of states available for a quantum system is a space of normed vectors in (more generally, density operators on) a Hilbert space $\mathcal{H}$; quantum observables are bounded, self-adjoint operators on $\mathcal{H}$; once the Hamiltonian operator $\hat{H}$ is provided, Schrödinger's equation determines dynamical trajectories through state space. Thus we may associate a Hilbert space and a collection of operators with a quantum theory on the Hilbert space approach.

Now a definition. A Hilbert space $\mathcal{H}$, and a collection of operators $\{\hat{O}_i\}$ is *unitarily equivalent* to $(\mathcal{H}', \{\hat{O}_i'\})$ if and only if there exists a one-to-one, linear, norm-preserving transformation ("unitary map") U: $\mathcal{H} \to \mathcal{H}'$ such

10. Countering this temptation in the first instance are some distinctly unparticulate features of the theory so interpreted (see Teller 1995, Ch. 2).

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that $U^{-1}\hat{O}_i' U = \hat{O}_i$ for all values of i. The Born Rule invests a quantum theory with empirical content by assigning an observable $\hat{O}$ in a state $|\psi\rangle$ the expectation value $\langle\psi|O|\psi\rangle$. Unitary equivalence is a sufficient condition for every state in a theory ($\mathcal{H}$, $\{\hat{O}_i\}$) to have a *counterpart* in a theory ($\mathcal{H}'$, $\{\hat{O}_i'\}$), in this sense: the expectation values assigned observables $\hat{O}_i$ by a state $|\psi\rangle$ in the first theory are duplicated by those assigned $\hat{O}_i'$ by the counterpart state $U|\psi\rangle$ in the second. Because unitary maps are invertible, the counterpart relation is symmetric. In the case that two theories' observables are the sets of bounded, self-adjoint operators on their respective Hilbert spaces, the unitary equivalence of theories is not only sufficient but also necessary for every state in the first theory to have a counterpart in the second, and vice versa.[11]

The physical content of a state consists at least in its assignment of expectation values to the observables the theory recognizes. Thus the physical content of a theory consists at least in the set of such assignments it countenances. Hence two theories are physically equivalent only if their sets of expectation value assignments are suitably isomorphic. For quantum theories on the Hilbert space approach, according to the results of the previous paragraph, the suitable isomorphism in question is that furnished by unitary equivalence. Ergo it is prima facie reasonable to suppose that, on the Hilbert space approach, in order for two quantum theories to be physically equivalent, they must be unitarily equivalent. And so it is prima facie reasonable to regard unitarily *in*equivalent representations of the Weyl relations expressing the quantization of some classical theory as distinct physical theories.

I do not say that unitary equivalence is sufficient for physical equivalence on the Hilbert space approach, because I take physical equivalence to be a relation that obtains between *interpreted* theories, and I can readily imagine unitarily equivalent quantum theories which agree in the state sets they recognize but which under different interpretations assign different determinate properties to systems in their domain. Take a Bohmified quantum theory which makes *position* always determinate and one which makes *momentum* always determinate, for instance. Though unitarily equivalent, these interpreted theories aren't obviously physically equivalent. Hence my scruples about the sufficiency of unitary equivalence for physical equivalence. As far as I can tell, they don't affect the questions developed in this essay, which (put roughly) concern whether unitary equivalence is necessary for physical equivalence.

11. For a proof, see Clifton and Halvorson 2001, 431 and 461–462. From this they draw a different moral than I. They appeal to investigations of the inequivalent Minkowski and Rindler representations to urge that unitarily inequivalent representations should be regarded not as rival theories but as inequivalent, complementary descriptions within the same theory.

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If unitary equivalence is necessary for physical equivalence, and if to quantize a classical theory is to find a representation of the appropriate Weyl relations, quantization yields a unique physical theory only if all representations of the Weyl relations are unitarily equivalent. Provided that the state space of the classical theory is finite-dimensional, this unitary equivalence is secured by the Stone-Von Neumann theorem, which states, in essence, that all representations of the Weyl relations expressing the quantization of a finite dimensional classical theory are unitarily equivalent to the Schrödinger representation.[12] But QFTs are quantizations of—well, field theories, theories with infinitely many degrees of freedom. The Stone-von Neumann theorem does not cover their quantization. Indeed, the Weyl relations encapsulating the quantization of classical Klein-Gordon theory admit continuously many inequivalent representations. Given our working criterion of physical equivalence, this means that there are continuously many physically inequivalent quantum theories of the free boson field. This dramatic non-uniqueness prompts worries about the foundations of QFT, e.g., is there any such thing as *the* free boson field to begin with?!

Supporting evidence that unitarily inequivalent quantizations are physically inequivalent takes the following form. Suppose that $(\mathcal{H}, \hat{W}(y))$ and $(\mathcal{H}', \hat{W}'(y))$—henceforth, the *unprimed* and *primed* representations, respectively—are unitarily inequivalent quantizations of the Klein-Gordon field, each with Fock space structure. Then the vacuum state $|0\rangle$ of the unprimed representation cannot be expressed as a state in the primed representation, and mutatis mutandis. What's more, the unprimed total number operator $\hat{N}$ cannot appear as an operator in the primed representation, and mutatis mutandis. The primed and unprimed theorists cannot recognize one another's fundamental particle notions. Indeed, using one theorist's notions to plumb the particle content of the quantum field precludes using the other's. Any pair of states which one can distinguish by appeal to the probabilities they ascribe to propositions concerning umprimed particle contents are indistinguishable by appeal to the probabilities they as-

12. Summers 2001 gives an excellent review. Here is a statement of the theorem for the form of the Weyl relations given in note 9

Let (M, Ω) *be a finite dimensional symplectic vector space. Let* $(\mathcal{H}, \hat{W}(y))$ *and* $(\mathcal{H}', \hat{W}'(y))$ *be strongly continuous, irreducible, unitary representations of the Weyl Relations over that vector space. Then* $(\mathcal{H}, \hat{W}(y))$ *and* $(\mathcal{H}', \hat{W}'(y))$ *are unitarily equivalent.*

A representation $(\mathcal{H}, \hat{W}(y))$ is *irreducible* just in case the only subspaces of $\mathcal{H}$ that are invariant under $\hat{W}(y)$ are $\{0\}$ and $\mathcal{H}$. It should be emphasized that the Stone-von Neumann theorem does not apply to all finite dimensional theories, regardless of their Poisson algebra structure. For instance, the quantization of a classical particle moving in one dimension *on a circle* is not unique up to unitary equivalence! See Isham 1984, §4.6.1.

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cribe to propositions concerning primed particle content. And mutatis mutandis.[13] *Provided* that we're committed to articulating the physical content of a QFT in terms of particle notions—or even more cautiously, in terms of quanta of excitation of an underlying ground state—the differences between the physical contents of the primed and unprimed representations seem irreconcilable.

The questions of interpretation sketched here revolve around the availability of unitarily inequivalent representations of the Weyl relations for QFT. One question of interpretation this availability prompts is: what is the state space of a QFT? Is it the set of unit vectors (more generally, density matrices) residing in a single (unitary equivalence class of) representation(s) of the Weyl relations? Is every unit vector in every representation of the Weyl relations a possible state for the theory? Or does the state space of the theory lie somewhere between the stingy and the magnanimous options? Concerning the state space proper to QFT, these are interpretative questions in section 2's sense. Insofar as the Stone-von Neumann theorem banishes the specter of unitarily inequivalent representations from the quantization of classical theories whose configuration spaces are $\mathbb{R}^n$, they are interpretative questions that arise along with the attempt to quantize classical field theories.

5. Extremist Answers to the First Question. Here I develop two extremes in the continuum of possible answers to this first interpretative question. In what follows, as I raise additional interpretative questions born from QFT, I'll try to keep track of what these extremists have vested in their answers. These extremists are fictional devices, introduced to illustrate how foundational investigation of QFT might go, and how in its course interpretative questions intertwine. Against the backdrop of this rough illustration, the details of particular positions can be developed, and those positions accurately attributed to particular thinkers. But that task is subsequent to my present one, which is primarily to construct the backdrop.

According to my first extremist, whom I'll label the Hilbert Space Conservative (or just Conservative, for short), quantum states are what quantum states always were: normed elements of a Hilbert space. And the physical equivalence of quantum theories is what physical equivalence always was: unitary equivalence (or something for which unitary equivalence is necessary). The Conservative therefore maintains only one unitary equivalence class of representations of the Weyl relations to be *physical,* to bear the physical content of the theory. Enjoining us to pursue QFT in the ambit of this unitary equivalence class, she spurns representations of

13. Clifton and Halvorson (2001) make what can and can't be done with inequivalent particle notions much more precise.

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the Weyl relations outside of it as spurious, because they are physically inequivalent to the representations bearing the physical content of the theory.

Notice that the Hilbert spaces bearing the content of quantum theory, according to the Conservative, can be quite complicated. They can, like the Fock spaces of particle physics, be the direct sum of n-fold symmetrized tensor products of a single particle Hilbert space, or they can, like the Hilbert space for a single particle with both spin 1/2 and position degrees of freedom, be the Hilbert space obtained by taking the tensor product of a two-dimensional Hilbert space and an infinite-dimensional Hilbert space. The Conservative is not committed to the absurdity that the possible states of a system must all be representable as density matrices on some simple, non-composite Hilbert space. She is rather committed to the position that, however complicated the Hilbert space structure bearing a representation of the Weyl relations characteristic of a system, all states possible for the system can be represented on that Hilbert space structure.

Now, an immediate problem for the Conservative is the problem of privilege: by what criterion does she select *the* unitary equivalence class of *physical* representations from the infinitely many representations available? And, supposing she can articulate such a criterion, how is she to justify it?

Not much is made in QFT textbooks of the availability of unitarily inequivalent representations. That textbook QFT is almost invariably conducted in Minkowski spacetime explains this. Minkowski spacetime is highly symmetric and well structured. It is first of all *globally hyperbolic.* That is to say, it has a Cauchy surface, a spacelike hypersurface every inextendible causal curve crosses exactly once. A globally hyperbolic spacetime can be foliated by a one-parameter family of Cauchy surfaces, each of which has the same topology. Thus such a (four-dimensional) spacetime is topologically $\Sigma \times \mathbb{R}$, where Σ gives the topology of a three manifold. Minkowski spacetime is moreover *stationary.* That is to say, it is globally hyperbolic and it hosts a one-parameter family of isometries whose orbits are timelike. Thus a stationary spacetime admits a foliation by Cauchy surfaces such that the timelike transformations between these surfaces are symmetry transformations of the metric. Inertial time translations from one Cauchy slice to another in Minkowski spacetime are symmetries of the Minkowski metric (see Wald 1984, Ch. 8, for more on causal structure).

These symmetries furnish criteria for privilege. Recall the heuristic "frequency-splitting" quantization mentioned at the end of Section 3. This quantization builds the single particle Hilbert space of the QFT from classical solutions to the Klein-Gordon equation which are "positive frequency" with respect to time. A natural query to raise in this relativistic

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setting is: time according to whom? And the resoundingly satisfying answer available in Minkowski spacetime is: time according to some family of inertial observers, that is, time associated with some family of symmetries of the metric. It doesn't matter which; following the heuristic for different families of inertial observers yields unitarily equivalent quantizations. The frequency splitting prescription is unambiguous in Minkowski spacetimes because propagating a classical solution from one Cauchy surface to another, isometrically related to the first, does not alter its status as positive frequency.

To say that a heuristic for quantization may be unambiguously applied does not privilege the result of its application over other quantizations available. But here again the isometries of Minkowski spacetime serve the standard construction well. For in the presence of spacetime symmetries, a reasonable demand to place on a quantization with a Fock space structure is that its vacuum state be invariant under the full isometry group of the spacetime. Axiomatic approaches to QFT issue this demand in the form of an axiom. Its rough and ready justification is that the result of subjecting a spacetime with nothing in it to any of its symmetry transformations should be a spacetime with nothing in it. Suppose it is further demanded that this vacuum state be annihilated by all "negative frequency" operators (i.e., annihilation operators a_k associated with positive frequency modes ω_k).[14] The rough and ready justification of this demand is that the vacuum state be fully empty in the sense that it is not possible to obtain another state by removing something from the vacuum. The quantization meeting both demands is unique up to unitary equivalence. It is, in fact, the standard quantization obtained by the frequency-splitting heuristic. Thus the Hilbert space Conservative can solve her problem of privilege in Minkowski spacetime by appeal to that spacetime's symmetries: the standard quantization is the only quantization satisfying prima facie reasonable symmetry-presupposing and symmetry-driven demands.

Of course, the strategy of invoking spacetime symmetries to solve the problem of privilege fails when we consider the pursuit of QFT in more general spacetime settings. Recalling that QFT on curved spacetime might be one signpost on the road to quantum gravity, let us suppose that the Conservative is well-mannered enough to want her interpretation of QFT to extend to generic spacetime settings. Curved spacetimes won't in general share the symmetries of Minkowski spacetime—generic curved spacetimes will lack symmetries altogether. Interpreting QFT on curved spacetime,

14. This stability condition is connected to the requirement that the unitary operator implementing a one-parameter family of timelike isometries within a quantization have a positive generator. See Segal 1967, §VI, for a more precise statement. Notice that both versions of the requirement presuppose a global family of time-like isometries.

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the Conservative can't appeal to spacetime symmetries to implement the frequency-splitting heuristic or to resolve her problem of privilege. *The consideration of curved spacetimes shortcircuits the Conservative's privileging strategy, and forces those interested in the foundations of QFT to take the availability of unitarily inequivalent representations seriously.* Absent the consideration of curved spacetime settings, the interpretative questions I survey here are not gripping. Considering curved spacetime settings presents the Conservative with the problem of privilege. Leaving her with it for now, I turn next to my other extremist, the Algebraic Imperialist, or Imperialist, for short.

What spurs Algebraic Imperialism is the suggestive fact that each concrete Hilbert space representation of the Weyl relations gives rise to an abstract algebra, the *Weyl algebra,* which is representation-independent. To obtain this algebra, start with a concrete representation of the Weyl relations; construct finite, complex, linear combinations of the unitary operators bearing this representation to obtain a subalgebra of $\mathfrak{B}(\mathcal{H})$, the set of bounded linear operators on $\mathcal{H}$ and complete this subalgebra in the Hilbert space operator norm. The result is a C^* algebra,[15] the Weyl algebra. The key result for Imperialists is that Weyl algebras constructed from (possibly unitarily inequivalent) representations of the Weyl relations ($\mathcal{H}$, $\hat{W}(y)$) and ($\mathcal{H}'$, $\hat{W}'(y)$) are equivalent up to *isomorphism.

Abstract algebras can be realized in concrete Hilbert spaces. A map π: $\mathcal{A} \rightarrow \mathfrak{B}(\mathcal{H})$, from elements of an abstract algebra $\mathcal{A}$ to the set of bounded linear operators on a Hilbert space $\mathcal{H}$, furnishes a Hilbert space representation of the algebra.[16] Hilbert space representations of Weyl algebras for QFT have structure in excess of the algebraic structure. For instance, the map π: $\mathcal{A} \rightarrow \mathfrak{B}(\mathcal{H})$ has as its image only a proper subset of $\mathfrak{B}(\mathcal{H})$. That is, there are bounded, self-adjoint operators on concrete Hilbert space representations of Weyl algebras without direct correlates in those algebras. Concrete representations also make available operator topologies—for instance, the weak topology, which defines convergence for sequences of operators in terms of the Hilbert space inner product, or the strong topology, which defines convergence in terms of Hilbert space norm—

15. A C^*-*algebra* $\mathcal{A}$ is an algebra, over the field $\mathbb{C}$ of complex numbers, with an involution * satisfying: $(A^*)^* = A$, $(A + B)^* = A^* + B^*$, $(\lambda A)^* = \bar{\lambda} A^*$ and $(AB)^* = B^*A^*$ for all $A, B \in \mathcal{A}$ and all complex λ (where the overbar denotes the complex conjugate). In addition, a C^*-algebra is equipped with a norm, satisfying $\|A^*A\| = \|A\|^2$ and $\|AB\| \leq \|A\| \|B\|$ for all $A, B \in \mathcal{A}$, and is complete in the topology induced by that norm. I'll also assume that $\mathcal{A}$ contains a unit I such that $IA = AI$ for all $A \in \mathcal{A}$.

16. More precisely, a *representation* of a C^*-algebra $\mathcal{A}$ is a mapping $\pi : \mathcal{A} \Rightarrow \mathfrak{B}(\mathcal{H})$ from the abstract algebra into the algebra $\mathfrak{B}(\mathcal{H})$ of bounded linear operators on a Hilbert space $\mathcal{H}$ such that $\pi(\lambda A + \mu B) = \lambda\pi(A) + \mu\pi(B)$, $\pi(AB) = \pi(A)\pi(B)$, and $\pi(A^*) = \pi(A)^*$ for all $A, B \in \mathcal{A}$ and all complex numbers λ and μ.

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which can't be defined in terms of the abstract algebra, whose natural topology is furnished by the C^* norm $\|\cdot\|$. According to the Algebraic Imperialists, all this extra structure is extraneous-the Weyl algebra,[17] and not any of its concrete but disparate Hilbert space realizations, encapsulates the physical content of QFT: "[T]he important thing here is that the observables form some algebra, and not the representation Hilbert space on which they act" (Segal 1967, 128)).

Now, the algebraic approach to QFT identifies physical observables with self-adjoint elements of the Weyl algebra (i.e., elements A such that $A^* = A$). To assign these observables expectation values, the algebraic approach dispenses with the intermediaries of Hilbert space representations and Hilbert space states. Algebraic states assign observables expectation values directly. A *state* ω on an abstract algebra $\mathcal{A}$ is a linear functional $\omega : \mathcal{A} \rightarrow \mathbb{C}$ that is normed ($\omega(I) = 1$)) and positive ($\omega(A^* A) \geq 0$ for all $A \in \mathcal{A}$). The expectation value of an observable $A \in \mathcal{A}$ in state ω is simply ω (A). And the Algebraic Imperialist's answer to the first question of interpretation is that the state space of QFT is simply the space of algebraic states on the abstract Weyl algebra.

We can draw connections between the Conservative's and the Imperialist's notions of state. A state $|\psi\rangle$ in a Hilbert space $\mathcal{H}$ carrying a representation $\pi : \mathcal{A} \rightarrow \mathcal{B}(\mathcal{H})$ of an algebra $\mathcal{A}$ naturally gives rise to the algebraic state $\omega(A) = \langle\psi|\pi(A)|\psi\rangle$ for all $A \in \mathcal{A}$. Somewhat less obviously, an algebraic state ω over an algebra can be recast as state in a Hilbert space bearing a faithful representation of that algebra (a representation π of $\mathcal{A}$ is *faithful* iff $\pi(A) = 0 \Rightarrow A = 0$). Gel'fand, Naimark, and Segal showed how to construct this *GNS representation* (see Bratelli and Robinson 1987, § 3.2):

> **Theorem 1 (GNS).** *Given a C*-algebra $\mathcal{A}$ and a state ω: $\mathcal{A} \Rightarrow \mathbb{C}$ over that algebra, there exists a Hilbert space $\mathcal{H}_\omega$, a faithful representation π_ω $\mathcal{A} \Rightarrow \mathfrak{B}(\mathcal{H}_\omega)$, and a cyclic vector $|\Psi_\omega\rangle \in \mathcal{H}_\omega$ such that $\omega(A) = \langle\Psi_\omega|\pi_\omega(A)|\Psi_\omega\rangle$ for all $A \in \mathcal{A}$. Moreover the triple $(\mathcal{H}_\omega, \pi_\omega, |\Psi_\omega\rangle)$ is unique up to unitary equivalence.*

Thus we can translate every algebraic state to a Hilbert space state, and every Hilbert space state to an algebraic state. But it does not follow that the Conservative and the Imperialist notions of state are equivalent. Consider abstract algebraic states ω and ω' (for simplicity's sake assumed throughout to be pure states[18]) which give rise via the GNS construction

17. More properly, an axiom-governed association between abstract algebras and open bounded regions of spacetime (see Haag 1992).

18. As a consequence of this simplifying assumption, I do not discuss a notion of equivalence for representations, *quasi-equivalence,* that diverges from unitary equivalence

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to unitarily inequivalent concrete representations $(\mathcal{H}, \pi)$ and $(\mathcal{H}', \pi')$. (Two representations $(\mathcal{H}_1, \pi_1)$ and $(\mathcal{H}_2, \pi_2)$ of an abstract algebra $\mathcal{A}$ are *unitarily equivalent* just in case there is a unitary map U: $\mathcal{H}_1 \Rightarrow \mathcal{H}_2$ such that $U\pi_1(A)U^{-1} = \pi_2(A)$ for all $A \in \mathcal{A}$.) Both ω and ω' are, according to the Imperialist, possible states of the quantum field.

But not according to the Conservative. The set of states representable as density matrices on the GNS construction $(\mathcal{H}_\omega, \pi_\omega, |\Psi_\omega>)$ for ω is called the *folium* of ω. Giving rise to unitarily inequivalent GNS constructions, ω and ω' reside in different folia: one can't be represented as a density matrix on the GNS representation of the other. Now, the Conservative believes that all physically possible states lie in a single folium-this is just another way of saying that the state space of the theory is the space of density matrices on a Hilbert space. Because ω and ω' reside in different folia, the Conservative can't consider them both physically possible.

Let us pause to remark how stringent, and how bivalent, the Conservative's notion of physical possibility is.[19] Alerted to the availability of unitarily inequivalent representations of the Weyl relations, a more tolerant advocate of the Hilbert space approach might say: quantum-theoretic states reside in a single folium. But it's an empirical question which folium. She might even borrow algebraic resources to characterize the proper folium: where ω happens to be the algebraic state of the universe, it's ω's folium, that is, it's the set of states expressible as density matrices in the GNS construction arising from ω.[20] The tolerant Conservative regards these states as, so to speak, *more* possible than those residing in folia (e.g., ω''s) disjoint from ω's. But she thinks it excessive to relegate states outside ω's folium to outright physical impossibility. After all, had the state of the universe been ω' rather than ω, a folium of these outlying states would have comprised the quantum-theoretic state space. Intuitively, there are nearby, physically possible worlds the correct quantum theory of which is set in ω''s folium.

Considerations from the thermodynamic limit of quantum statistical mechanics suggest that this gentler way with physical possibility is also saner. Space prohibits developing these considerations here, but this is the general idea: equilibrium states at different temperatures can correspond

only when at least one of the representations in question arises from a mixed algebraic state. See Clifton and Halvorson 2001, 426–429.

19. Thanks to Gordon Belot for pursuing this, and other moderating maneuvers, with me.

20. I take there to be some similarity between the position impressionistically presented here and a claim Mathias Frisch (2000, 406–409) suggests, that the Maxwell equations don't on their own delimit what's physically possible according to classical electromagnetism, but only in conjunction with the evidently more "contingent" retardation condition.

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to unitarily inequivalent representations. Suppose that the state of the universe ω is an equilibrium state. Refusing to admit that physical possibility comes in degrees, the extreme Conservative would declare states outside ω's folium to be physically impossible. But in this setting, that amounts to declaring non-actual temperatures physically impossible, which seems modally draconian. Recognizing degrees of physical possibility, a gentler Conservative might begin to reconcile Algebraic and Hilbert space senses of the state space of QFT.

An extreme Conservative, on the other hand, riled by the physical significance with which the Imperialist would invest unitarily inequivalent representations, might challenge that Imperialist with a question of principle. What notion of physical equivalence supports and is supported by the Imperialist's identification of the state space of QFT? As a matter of historical fact, advocates of algebraic approaches did seem to regard their recognition of unitarily inequivalent representations as requiring some sort of apology, an apology some couched in terms of the notion of *weak equivalence.* The idea is that two representations are weakly equivalent exactly when no finite set of expectation values specified to finite accuracy can locate a state in one, rather than the other, representation. More precisely, $(\mathcal{H}, \pi)$ and $(\mathcal{H}', \pi')$ are weakly equivalent if and only if, given any Hilbert space state on the first representation and the algebraic state ω it yields, and given any finite set of elements of the abstract algebra of observables and any finite set of non-zero margins of experimental error, there exists a Hilbert space state on the second representation yielding an algebraic state ω' that reproduces within those margins of errors ω's assignment of expectation values to those observables.

Now the apologizing Imperialist "define[s] two representations to be *physically equivalent* if and only if they are weakly equivalent" (Robinson 1966, 488). A rough and ready verificationism suggests a justification of the equation of physical and weak equivalence. Run at the level of algebraic states, the justification goes like this. Any actual measurement performed on a system in the algebraic state ω will fix the expectation values of some finite collection of observables A_i, $i = 1, 2, \ldots, n$ within experimental error margins $\varepsilon_i > 0$. Accordingly, such a measurement cannot distinguish ω from any other state ω' satisfying $|\omega(A_i) - \omega'(A_i)| < \varepsilon_i$ for all *i.* Such states are, for all practical purposes, physically equivalent. And so, representations are for all practical purposes physically equivalent if they are weakly equivalent. What's more, all interesting concrete representations of the Weyl algebra are weakly equivalent (see Wald 1994, Ch. 4.5):

> **Theorem 2 (Fell).** *Let* $(\mathcal{H}_1, \pi_1)$ *and* $(\mathcal{H}_2, \pi_2)$ *be faithful but possibly unitarily inequivalent representations of the C*-algebra* $\mathcal{A}$. *Let* $A_1, \ldots A_n$

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> $\in \mathcal{A}$, *and let* $\varepsilon_1, \ldots, \varepsilon_n > 0$. *Suppose* ω_1 *is an algebraic state corresponding to a Hilbert space state on* $\mathcal{H}_1$. *Then there exists a state* ω_2 *corresponding to a Hilbert space state on* $\mathcal{H}_2$ *such that for all* $i = 1, \ldots, n$, *it is the case that* $|\omega_1(A_i) - \omega_2(A_i)| < \varepsilon_i$.

Weyl algebras are simple, and any non-zero representation of a simple algebra is faithful. Thus Fell's theorem encompasses all non-zero representations of Weyl algebras. Defining physical equivalence as weak equivalence, the apologetic Imperialist can conclude that, "All faithful representations of [the Weyl algebra] are physically equivalent" (Robinson 1966, 488)—and chastise the Conservative for elevating surplus structure to physical significance:

> It is in this new notion of equivalence in field theory that the algebraic approach has its greatest justification. All the physical content of the theory is contained in the algebra itself; nothing of fundamental significance is added to a theory by its expression in a particular representation. (Robinson 1966, 488)

Weak equivalence underwrites a satisfactory response to the Conservative's challenge only if weak equivalence is an adequate explication of physical equivalence. We just now imagined a verificationist justification of the explication. But such a justification fails, *even by operationalist lights.* For, as Summers (2001) has observed, even verificationists ought to take states to be predictive instrumentalities. They would therefore expect the equivalence of states to extend to their predictions concerning future measurements. But Fell's theorem offers us no assurance that a state on $\mathcal{H}$ for all practical purposes indistinguishable from a state on $\mathcal{H}'$ with respect to some set $\{A_i\}$ of observables will continue to mimic the first state's predictions with respect to an expanded or altered set of observables. The practical indistinguishability of states that secures the weak equivalence of the representations bearing them is only a backward-looking or static sort of indistinguishability. Before recognizing a state in one theory as a suitable counterpart to a state in another, an operationalist should demand not only backward-looking but forward-looking indistinguishability: the indistinguishability of putative counterpart states conceived as predictive instrumentalities. Thus the appeal to Fell's theorem should not convince even the operationalist that weak equivalence is an acceptable notion of physical equivalence.

Apologies for algebraic liberality invoking weak equivalence fall flat. I believe that, at this stage at least, the best way for an Imperialist to articulate his criterion of physical equivalence is the bold way. Just as his notion of algebraic state deposes the Hilbert space notion of state, so too a criterion of physical equivalence suited to the algebraic approach should

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depose the Hilbert space criterion of unitary equivalence. Placing the physical content of a quantum theory in an algebra of observables, the bold Imperialist would maintain that two quantum theories are physically equivalent only if their associated algebras are *-isomorphic. The bold Imperialist would not flinch in the face of the observation that he thereby elevates a continuum of states giving rise to unitarily inequivalent GNS representations to physical significance. For, he would maintain, he has no reason to take unitary equivalence seriously as a notion of physical equivalence to begin with.

As Clifton and Halvorson (2001) have aptly observed, we encounter here a hurdle liable to be faced by attempts to use accounts of physical equivalence, gleaned from the general philosophy of science literature, to adjudicate the claims of the Imperialist and the Conservative. Take Reichenbach's (1938) notion, that physical theories are equivalent if and only if they are confirmed equally by every possible empirical datum. Conservative and Imperialist alike can agree with Reichenbach-provided they disagree with each other about which empirical data are possible. The Conservative will characterize possible data in terms of observables associated with bounded operators on concrete Hilbert spaces; the Imperialist will characterize the same data in terms of elements of abstract Weyl algebras.

The boldness of championing *-isomorphism as a criterion, adapted to the algebraic approach, of physical equivalence, does not preclude a conciliatory gesture on the part of the Imperialist. Joining the gentle Conservative in recognizing degrees of physical possibility, he could concede that states in the folium of the actual state of the universe ω—states representable as density matrices on a single Hilbert space—are more physically significant than states in outlying folia. When it comes to answering the first interpretative question, invoking degrees of possibility may serve to reconcile algebraic and Hilbert space ways of thinking.

6. The Second Interpretative Question: Observables. I've sketched an interpretative question concerning QFT's state space. Closely related questions concern the observables the theory honors. Given that the Imperialist reckons as observables only self-adjoint elements of the abstract Weyl algebra, and given that the image of these elements under a representation π in a concrete Hilbert space forms only a proper subset of self-adjoint operators on that Hilbert space—and so only a proper subset of the observables we're accustomed, on the Hilbert space approach, to recognize—one might anticipate that algebraic liberality about states comes at the apparent cost of stinginess about observables. This section rehearses several versions of the charge that the Algebraic Imperialist recognizes fewer observables than he should.

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Applied to a particle moving in one dimension, unmitigated Imperialism can deny even the canonical position and momentum magnitudes observable status. Recall (cf. equations (3.1) and (3.2)) that position $\hat{q}$ and momentum $\hat{p}$ are recoverable from the families of unitaries $\{\hat{U}(a)\}$ and $\{\hat{V}(b)\}$ satisfying the Weyl relations only if those families are (weakly) continuous. Recall also that the Weyl algebra is based on the integral, not the infinitesimal, form of the CCRs—speaking loosely, the unitaries, but not their infinitesimal generators, are in the algebra. The fact underlying the first stinginess charge that there are algebraic states—called *irregular,* or *singular* states—in whose GNS representations the families of operators satisfying the Weyl relations aren't weakly continuous. This foils the recovery of position or momentum as the infinitesimal generators of those families of unitary operators. Put bluntly, there are algebraic states from whose GNS representations position or momentum observables are missing! These are precisely the states which make possible what the Hilbert space approach could not accomplish: the assignment of precise, punctal, and dispersion-free values of position or momentum to quantum systems (see Clifton and Halvorson 2001).[21] Still, the cost of withholding observable status from position or momentum and the inconvenience of having to deal with non-separable Hilbert spaces that represent singular states, may strike some as unattractive. A slightly moderated Imperialism follows Segal (1967, 127) in admitting only regular states as physically reasonable. The regularity requirement applies to the QFT context as well-there too continuity requirements may be imposed on representations of the Weyl algebra.

Let $\pi : \mathcal{A} \Rightarrow \mathcal{B}(\mathcal{H})$ be a representation of the Weyl algebra in a concrete Hilbert space. Some Hilbert space operators are in the image of the representation; others, like position and momentum (provided the representation is regular), are recoverable from these. But any concrete Hilbert space representation harbors a host of operators without straightforward correlates, direct or indirect, in the algebra. These *parochial* operators, wage-earning observables on the Hilbert space approach, are otiose accretions according to the Imperialist. Underlying further charges of stinginess against the Imperialist are the facts that number operators and projection operators are, in general, parochial operators. They appear only as (what the Imperialist reckons) surplus structure in concrete representations of the Weyl algebra, and have no analogs in the algebra itself. But number operators play a prominent roles in the in-practice understanding of QFT as particle physics, and projection operators are central

21. A distinct technical gadget developed to rationalize the attribution to quantum systems of punctal position or momentum values is the *rigged Hilbert Space.* See Gel'fand and Vilenkin 1964 for a presentation.

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to many foundational treatments of quantum theory. Surely, these stinginess charges run, number and projection operators should be counted as observables; surely the algebraic approach is deficient for failing to so count them.

One sort of Imperialist defense would be to demonstrate the harmlessness of the deficiency. Such a demonstration might take the form of a recovery project: we use number operators to interpret scattering experiments and projection operators to calculate probabilities. If algebraic resources alone are sufficient to this pair of purposes, the absence from those resources of literal, rather than functional, counterparts to number operators and projection operators shouldn't vex us.[22] This recovery project is obviously a daunting interpretative task. Impartial judges will count this as grounds for the development, not the dismissal, of algebraic approaches—and see how, or if, that development progresses.

Another sort of defense against these stinginess charges moderates extreme Imperialism. The moderating move is to attribute physical significance to observables not represented by self-adjoint elements of the abstract algebra. Where ω is the actual state of the universe, moderated Imperialism elevates to observable status self-adjoint operators on the Hilbert space of the GNS construction associated with ω. These will include a total number operator and a full set of projection operators, although, of course, which total number operator and which set of projection operators is not specifiable in advance of the state of the universe. Suggesting that which physical magnitudes there are (in addition to those represented by self-adjoint elements of the abstract algebra) depends on what the state of the universe is, Imperialism so moderated, is no doubt controversial. But the core idea of Imperialism so moderated—that some observables are automatically physically relevant, others contingently so—is a natural counterpart to the gentle Conservative's maneuver of reckoning some possibilities more physical than others.

A final stinginess charge against the Imperialist is grounded, not in expectations tutored by the Hilbert space approach, but in extrinsic constraints on the interpretation of QFT. Consider the stress-energy tensor, which the Einstein field equations relate to spacetime curvature. The Imperialist's algebra will not in general contain an element corresponding to the stress-energy tensor (see Wald 1994, Ch. 4.6). But to erect semiclassical quantum gravity as a signpost on the road to quantum gravity, we must be able to assign the stress-energy tensor an expectation value. Now, we can add the stress-energy tensor to QFT's catalog of observables by following a mathematical prescription for assigning it an expectation value in a given algebraic state. But the prescription succeeds only for

22. Wald (1994), Ch. 4.5, attempts an algebraic recovery of probability calculations.

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some algebraic states, viz., those possessed of a leading singularity structure of Hadamard form (see Wald 1994, ch. 4.6). Algebraic states in this way sustaining the stress-energy tensor as an observable are known as *Hadamard states.* If one's interpretative predilections are to insist that the stress energy tensor is an observable, then one might maintain (as do Kay and Wald (1991)) that for an algebraic state to be physical, it must be Hadamard. Call this *the Hadamard condition.*

Imposing this condition would have repercussions for the debate between the Hilbert Space Conservative and the Algebraic Imperialist. One strikes generally against the Imperialist: the case of the stress-energy tensor establishes that there are physically significant magnitudes without counterpart in the abstract algebra. Another strikes against the apologetic Imperialist's explication of physical equivalence as weak equivalence. Any two algebraic states give rise to weakly equivalent representations, but not all algebraic states are Hadamard. If states must be Hadamard to be physical, two representations can be weakly equivalent even if only one of them is physically possible. But a physically possible representation can't be physically equivalent to a physically impossible one! Thus weak equivalence is not physical equivalence. Similar considerations would depose the Imperialist's bold notion of physical equivalence as *-isomorphism.

The Hadamard condition actually reinforces Hilbert Space Conservatism. In spacetimes admitting a compact Cauchy surface—i.e. closed universes—all Hadamard vacuum states define unitarily equivalent representations. In such spacetimes, the Conservative can solve her problem of privilege simply by imposing the Hadamard condition. Recognizing only those representations of the Weyl relations whose vacua are Hadamard, she recognizes only a unitary equivalence class of representations. "There does not appear to be any necessity even to consider states lying outside this single Hilbert space" (Wald 1994, 97).

In spacetimes with noncompact Cauchy surfaces—open universes—things don't work out quite so nicely for the Conservative. There it is not the case that all Hadamard vacuum states give rise to unitarily equivalent representations. Still, a result due to Verch extends the Conservative a flimsier sort of equivalence between (possibly unitarily inequivalent) representations arising from Hadamard states ω and ω'. These states are *locally indistinguishable,* in the sense that (roughly) for any finite region of spacetime, and any Hilbert space state in the GNS representation ($\mathcal{H}$, π) arising from ω, there's a state in the GNS representation ($\mathcal{H}'$, π') arising from ω' reproducing *exactly* the first state's assignment of expectation values to all the observables pertaining to that region of spacetime. "Consequently, the states in unitarily inequivalent quantum field theory constructions arising from different Hadamard vacuum states can be distinguished only by making measurements of observables over unbounded

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regions of spacetime" (Wald 1994, 97). If she privileges a unitary equivalence class of representations arising from Hadamard states in an open universe, the Conservative can argue, the only otherwise (by the lights of the Hadamard condition) physically reasonable states she excludes from consideration are those indistinguishable from states she recognizes, even with respect to infinitely many observables measured infinitely accurately, so long as each of those observables pertains to a bounded spacetime region.

But now perhaps the Hilbert Space Conservative has said too much. She has suggested that it doesn't matter which unitary equivalence class of Hadamard vacua she privileges, because one choice is locally indistinguishable from any other. Thereby granting legitimacy to an operationalism according to which only locally distinguishable differences are genuine differences, she undermines principled resistance to operationalist equations of weak and physical equivalence. Still, she can oppose the equation on the grounds, cultivated above, that it falls short of requirements even operationalists would impose on equivalence.

Anyway, her reinforcement of Hilbert Space Conservatism is only as strong as the Hadamard condition to which it appeals. And the following might count as reasons to reject the Hadamard condition as a criterion of physical reasonableness. First, not all (by a variety of criteria) "physically interesting" states are Hadamard states. For example, neither the Unruh nor the Boulware vacua of extended Schwarzschild spacetime are Hadamard. Imposing the Hadamard condition disqualifies these states as physically significant. Second, there are spacetimes which harbor no Hadamard states—for example, Kerr Spacetime (see Kay and Wald 1991). Imposing the Hadamard condition would declare such *spacetimes* unphysical! Finally, as the next section explains, there might be a sense of "dynamical evolution" adapted to QFT for which it is not the case that the set of Hadamard states is closed under dynamical evolution. Assuming that it would be shocking for a "physical" state of affairs to be carried to an "unphysical" state of affairs by the dynamics of a physical theory, such breakdown of dynamical closure would be a strike against the Hadamard condition as a criterion of physical reasonableness.

7. A Third Interpretative Question: Dynamics? I've canvassed interpretative questions that arise in connection both with the state space and with the observable set of QFT. I turn next to an interpretative question concerning the theory's dynamics. I revert to my extremists to introduce the problem.

7.1 Unitary Implementability: The Question. For the Hilbert Space Conservative, QFT dynamics are what quantum dynamics always were: unitary evolution through Hilbert space. If $|\psi(0)\rangle$ is the state of the field at

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some time $t = 0$, there's a family of unitary operators $U(t)$ such that $U(t)|\psi(0)\rangle$ is the state of the field at any later time t. Having jettisoned Hilbert spaces as essential to QFT, the Algebraic Imperialist has jettisoned this account of the theory's dynamics as well. In its stead, he implements quantum field dynamics by means of automorphisms of the abstract algebra of observables (that is, maps from $\mathcal{A}$ to itself). Restricting attention to globally hyperbolic spacetimes,[23] let Σ and Σ' be Cauchy surfaces for such a spacetime. The algebraic approach associates with each Cauchy surface an abstract algebra. It follows from the Primitive Causality and Isotony axioms of axiomatic algebraic QFT (see Haag 1992) that $\mathcal{A}(\Sigma) = \mathcal{A}(\Sigma')$, and that both are identical to the global algebra $\mathcal{A}(M)$—henceforth, simply $\mathcal{A}$— associated with all of spacetime.[24] Given an identification map $i : \Sigma \Rightarrow \Sigma'$ between points of two Cauchy surfaces, the deterministic evolution of the *classical* field induces an automorphism $\alpha^{i}_{\Sigma\Sigma'}$ of this algebra.[25] This automorphism is "dynamical" in at least the sense that it is determined by the dynamics of the classical field. Whether it is dynamical in more full-blooded senses is among the interpretative questions this section discusses.

If one prefers one's "dynamics" in the form of state evolution, one can use $\alpha^{i}_{\Sigma\Sigma'}$ to construct, from an algebraic state ω, another algebraic state $\omega' := \omega'(x) = \omega(\alpha^{i}_{\Sigma\Sigma'}x)$ for all $x \in \mathcal{A}$. Notice that I did not say "a later algebraic state." Both ω and ω' are *global* states, that is states on the algebra of observables associated with the full spacetime. In addition to the familiar realization of dynamics as a map from the instantaneous state of a system at one time to its instantaneous state at another time, deterministic theories make available a roundabout notion of dynamics as a map from one possible history of a system to another possible history of a system. Crudely, and in our case, the roundabout notion arises as follows. For a deterministic theory, Cauchy data on a surface Σ determine a complete history h of the universe. Suppose we used an identification map

23. The spacetime at issue in the non-unitary pure-to-mixed state transition by which Hawking would describe black hole evaporation is not globally hyperbolic. Nor is QFT proper the theoretical setting of Hawking's result. So I do not discuss it here. Belot et al. (1999) survey reactions to Hawking's "information loss paradox."

24. Primitive Causality requires that if region O is a subset of region O''s domain of dependence (defined in Wald 1994, Ch. 8), then $\mathcal{A}(O) \subset \mathcal{A}(O')$. But the entire spacetime lies in the domain of dependence of a Cauchy surface. So $\mathcal{A}(M) \subset \mathcal{A}(\Sigma)$. The Isotony axiom requires that if O is a subset of O', then $\mathcal{A}(O) \subset \mathcal{A}(O')$. But a Cauchy surface is a subset of the entire spacetime. So $\mathcal{A}(\Sigma) \subset \mathcal{A}(M)$. Thus $\mathcal{A}(\Sigma) = \mathcal{A}(M)$. Likewise $\mathcal{A}(\Sigma') = \mathcal{A}(M)$.

25. Arageorgis et al. (2002) supply an account of how the evolution of the classical fields between Σ and Σ' fixes this automorphism, as well as a more detailed discussion of unitary implementability in general.

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between points of Σ and points of Σ' to reassign this Cauchy data to the surface Σ'. Then the reassigned data would determine a compete history, call it h', of the universe. Thus, given a pair of Cauchy surfaces and an identification map between them, a deterministic theory induces a map between histories of the universe. Dynamics of this sort at the level of a classical field theory lift to dynamical automorphisms $\alpha^{\iota}_{\Sigma\Sigma'}$ of the algebra expressing its quantization. States, such as ω and ω' above, related by these dynamical automorphisms, are nevertheless global states, states-for-all-time.

Now we have on the one hand the Hilbert space notion of dynamics as unitary evolution, and on the other, the dynamical automorphisms of the Weyl algebra. As I've indicated, one interpretative question is whether these automorphisms are properly understood as giving quantum dynamics. Section 7.3 addresses several miens of this hydra-like question. In the meantime, I will provisionally assume that the automorphisms are properly understood as dynamical in order to raise a further interpretative question: are the algebraic and the Hilbert space notions of dynamics equipollent? In particular, is it the case that every dynamical evolution implementable as an automorphism on the abstract algebra is also implementable as a unitary evolution in some fixed Hilbert space? I will say that an automorphism α on $\mathcal{A}$ is *unitarily implementable in a representation* $(\mathcal{H}, \pi)$ of $\mathcal{A}$ iff there exists a unitary $\hat{U}$ on $\mathcal{H}$ such that for all $x \in \mathcal{A}$, $\pi(\alpha x) = \hat{U}\pi(x)\hat{U}^{-1}$. In that case, just as α can be used to identify the evolute of an arbitrary element x of the algebra, so can $\hat{U}$ be used to identify the evolute of that element's representative $\pi(x)$ in the concrete Hilbert space. Where $(\mathcal{H}, \pi)$ provides the GNS representation of an algebraic state ω, I will say that an automorphism is *unitarily implementable with respect to* ω if and only if it satisfies the condition just stated.

7.2. Unitary Implementability: Some Results. Consider first the case of a static,[26] globally hyperbolic spacetime. Let t be the static time coordinate, and assume that the level surfaces $t = constant$ are Cauchy. The preferred identification map ι between isometrically related Cauchy surfaces Σ and Σ' is the one given by following the trajectories of the non-rotating Killing field $\partial/\partial t$. The requirement that the vacuum state be invariant under the automorphism $\alpha^{\iota}_{\Sigma\Sigma'}$, where ι is the preferred identification, guarantees that $\alpha^{\iota}_{\Sigma\Sigma'}$ is unitarily implementable with respect to the vacuum state. Under some technical assumptions, Kay (1978) establishes the much stronger

26. A static spacetime is a stationary spacetime whose time-like Killing field (the orbits of which are its timelike isometries) are hypersurface orthogonal. The metric of a stationary spacetime is independent of Killing time; the metric of a static spacetime is also time orthogonal (i.e., $g_{\alpha 4} = 0$ for $\alpha = 1, 2, 3$). See (once again) Wald 1984, Ch. 8.

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result that Hamiltonian-generated dynamics of the usual quantum form can be recovered. In this setting, it seems that the Hilbert Space Conservative can recover as much of the Imperialist's dynamics as she cares to.

However, dynamical automorphisms are not always unitarily implementable. Torre and Varadarajan (1998) show, for a flat $\mathfrak{T}^n \times \Re$ spacetime with $n > 2$ ($\mathfrak{T}^n$ is the n-torus), that dynamical automorphisms between non-isometrically related Cauchy surfaces will not in general be unitarily implementable with respect to the standard vacuum state. What's more, they show that there exist Cauchy surfaces Σ and Σ' such that $\alpha_{\Sigma\Sigma'}$ fails to be unitarily implementable no matter what identification map is chosen! What stymies unitary implementability seems to be the fact that Σ and Σ' are not isometric.

Finally, consider a non-stationary globally hyperbolic spacetime $\Sigma \times \mathbb{R}$. Let t be a global time function with Cauchy surfaces $t = constant$. Suppose that for $t \leq t_-$, and for $t \geq t_+$ (where $t_+ > t_-$) the metric is static, so that the spatial 3-metrics $\gamma^+_{\alpha\beta}$ and $\gamma^-_{\alpha\beta}$ are independent of t. Helfer (1996) shows that even when Σ is compact, and even when $\gamma^+_{\alpha\beta} = C\gamma^-_{\alpha\beta}$, $C = constant$ (for instance, a case of nocturnal doubling), the evolution from $\Sigma : t = t_-$ to $\Sigma' : t = t_+$, with the identification map given by following along the trajectories of $\partial/\partial t$, is not unitarily implementable with respect to Hadamard states.

7.3. Morals? One moral these results suggest is that in exactly those spacetimes whose symmetries furnish principles by appeal to which the Hilbert Space Conservative might privilege a unitary equivalence class of representations, dynamical automorphisms induced by privileged identification maps between Cauchy surfaces *are* unitarily implementable with respect to those representations. But in more general settings, or in the same settings but with underprivileged identification maps, the Imperialist's dynamical automorphisms are not unitarily implementable in the Hilbert space framework. Taking non-unitarily implementable quantum field dynamics seriously, we may wish to revisit the sanctity of unitarity in other quantum theories. If dynamics needn't be unitary to be quantum, the complexion of the traditional quantum measurement problem changes dramatically. Lest I fan false hope for a simple solution to this problem, let me hasten to add that dynamical automorphisms keep pure states pure. So casting dynamics in terms of algebraic automorphisms, even non-unitarily implementable ones, is not on its own going to accomplish the pure-to-mixed state evolution at issue in standard statements of the measurement problem.

An aggressive Imperialist could try to put non-unitarily implementable dynamics to further use, by using them to undermine Conservative accounts of physical equivalence and Conservative criteria of privilege. This

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aggressive Imperialist would begin his attempt by announcing a *principle of dynamical closure:*

> If φ is a possible state of a system, and $\varphi \rightarrow \varphi'$ is a transition allowed by the theory's dynamical laws, then φ' is a also a possible state of the system.

Construing unitary equivalence as necessary to physical equivalence, the aggressive Imperialist would suggest, violates this principle. Let ω be an algebraic state, which gives rise to the GNS representation $(\mathcal{H}, \pi)$. And let α be a dynamical automorphism which is not unitarily implementable in this representation. The states ω and $\omega' = \omega \circ \alpha$ are dynamically connected. But the corresponding representations $(\mathcal{H}, \pi)$ and $(\mathcal{H}', \pi')$ are unitarily inequivalent. If unitary equivalence is necessary for physical equivalence, physical possibility is picked out by (a possibly proper subclass of) a unitary equivalence class of representations. And if physical possibility is picked out by (a possibly proper subclass of) a unitary equivalence class of representations, it can't be the case that states from both representations $(\mathcal{H}, \pi)$ and $(\mathcal{H}', \pi')$ are physically possible. But such states are dynamically connected. If we take non-unitarily implementable dynamics seriously, the principle of dynamical closure contradicts unitary equivalence as a criterion for physical equivalence.

Taking dynamical closure seriously also plays havoc with the "degrees of possibility" blue print for reconciling algebraic and Hilbert space approaches. The blueprint makes the idea, that the "most possible" states reside in the folium of the actual state of the universe ω, the foundation of reconciliation. Extending most possible status to states connected to ω by non-unitarily implementable dynamical automorphism, the closure principle undermines this foundation. Embracing the closure principle, we can't understand the set of most possible algebraic states as a quantum state space in the Hilbert space sense.

Respecting the principle of dynamical closure, the Imperialist would regard algebraic states connected by non-unitarily implementable dynamical automorphisms as possible relative to one another, even if they give rise to unitarily inequivalent representations. Plying the principle of dynamical closure, the Imperialist can constitute a state space for QFT more generous than any the Hilbert Space Conservative recognizes, and do so on principles the Conservative can't dismiss as crudely operationalist. What's more, Helfer's result reveals a tension between this principle of dynamical closure and the Hadamard condition, by which the Conservative would privilege a unitary equivalence class of vacuum states in a closed spacetime. For Helfer's result shows that even with respect to closed universes—where all Hadamard vacuum states are unitarily equivalent—there are dynamical automorphisms that carry an algebraic state which is

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Hadamard to one which is not. That is to say that the set of Hadamard states is not closed under dynamical automorphism. This would seem to make the Hadamard condition less attractive as a condition on physical reasonableness.

The stalwart Conservative need not crumble in the face of this onslaught. Citing the result that if a state of a quantum field is Hadamard on a thin sandwich around a Cauchy surface, it's Hadamard through all of spacetime (Fulling, Sweeny, and Wald (1978)), she can question whether the dynamical closure the Hadamard condition violates is an attractive principle to begin with. The Fulling et al. result establishes that if the state of the universe is ever Hadamard, it's always Hadamard: there is no evolution, in the instantaneous state to instantaneous state sense, from a Hadamard state to a non-Hadamard one. The states converted from Hadamard to non-Hadamard by Helfer's dynamical automorphism are global states, states-for-all-time; the "evolution" at issue is not time development within a universe, but the history-to-history map made available by the deterministic dynamics of the classical theory. That is to say, *ω and $\omega' = \omega \circ \alpha$ are not states of the same universe at different times, but states of different universes.* Now, the stalwart Conservative might urge, our intuitions about dynamical closure are intuitions about the state-to-state variety of dynamics, not the history-to-history sort. Maintaining that the principle has no legitimacy when framed by dynamics in the latter sense, she would seek to repel the Imperialist's onslaught by maintaining the Hadamard condition, notwithstanding the availability of history-to-history dynamics connecting Hadamard states to non-Hadamard ones. Degrees of possibility reconcilers of algebraic and Hilbert space approaches could likewise resist the Imperialist.

In response, the Imperialist might retreat, but not surrender. Consider a universe in algebraic state ω, and a dynamical automorphism $\alpha^{\iota}_{\Sigma\Sigma'}$ which fails to be unitarily implementable. Here, the Imperialist might contend, he can do something which the Conservative cannot: identify observables over time, in the Heisenberg-picturesque sense of identifying the observables on Cauchy slice Σ' which are "evolutes" of Σ observables. This is dynamical work performed by the automorphism $\alpha^{\iota}_{\Sigma\Sigma'}$, dynamical work without analog in the Conservative's picture.

Hajicek (1978) suggests a Conservative rejoinder: only in the case that Cauchy surfaces are related by a timelike isometry, he claims, does it even make sense to identify observables over the time lapsed between them. Pursuing other ends, he offers no exhaustive elaboration of this claim, but perhaps something like the following will do. Identity over time is something whose criteria should be staunch. Where Σ and Σ' are not isometrically related, there are as many ways $\alpha^{\iota}_{\Sigma\Sigma'}$ to associate Σ observables with Σ' ones as there are ways to draw identity maps between the surfaces—

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and no grounds whatsoever for distinguishing any one of these associations as *the* relation of identity over time for observables. Staunch criteria of identity are lacking. But where Σ and Σ' are isometrically related, there is a distinguished identity map between them—the one obtained by following the orbit of the timelike isometry—and so an association $\alpha^{i}_{\Sigma\Sigma'}$ between earlier and later observables staunch enough to be identity. Taking Hajicek's point that only automorphisms induced by privileged identification maps serve to identify observables over time, the Conservative can joyfully remind us that these automorphisms are unitarily implementable—so that there is no dynamical work that the Imperialist can do but that she can not.

Of course, it remains the case that there is, so to speak, *kinematical* work that the Hilbert Space Conservative must do which the Algebraic Imperialist need not. I mean the work of privileging a unitary equivalence class of representations of Weyl relations as the state space of the theory. Where substantial spacetime symmetries are present, the Conservative can use them to do this work—and also to parry rearguard Imperialist assaults on the utility of her notion of dynamics. But in the absence of such symmetries, it is hard to see how the Conservative might single out a unitary equivalence class of representations as physical. In closed universes, perhaps the Hadamard condition will do. But there is no more reason to restrict attention to spacetimes with compact Cauchy surfaces than there is to restrict attention to spacetimes with global time-like isometries. In general spacetime settings, the extreme Conservative has no good answer to the problem of privilege.

8. Conclusion. By shameless use of the rhetorical device of those fictional extremists, the Algebraic Imperialist and the Hilbert Space Conservative, I've tried to illustrate some of the questions of interpretation which arise in connection with QFT. They include questions about how to characterize the state space of the theory, questions about which physical magnitudes belong in the set of observables the theory wields, and questions about how to implement the dynamics of the theory. I've also tried to sketch, albeit impressionistically, moderating responses to these questions. One moderating proposal takes the Algebraist's state space to pick out physical possibilities in a broad sense, and takes the elements of his algebra to correspond to a core constituency of observables. It then uses the GNS construction for the actual state of the universe ω to identify a more narrow space of physical possibility (states in the folium of ω) and to supplement the core constituency of observables with specially appointed ones (self adjoint operators on the GNS construction for ω). Thus algebraic notions identify possibilities in the broad sense and the core set of observables; Hilbert space notions characterize possibility in the narrow sense

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and the supplemented set of observables; and part of what we've learned from QFT is that narrowing possibility and supplementing the core set of observables are state-dependent matters. Although possibly dubious intuitions about dynamical closure could undo this reconciliation, other moderating suggestions—for instance, to narrow the full space of algebraic possibility by imposing the Hadamard condition—are in the offing. We can be interpreters of QFT without being extremists.

Confronting the questions presented here, we confront interpretative issues not prominent in philosopher's discussion of ordinary quantum mechanics. We also confront extrinsic constraints which (by hypothesis of QFT's situation in the network of physical theories being different from the situation of ordinary quantum mechanics) we might miss were we to confine our attention to the interpretation of non-relativistic quantum mechanics. Philosophers of physics more subtle than my extremists should turn—they are turning—their attention to such questions.

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