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On the note emitted from a mug while mixing instant coffee

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We have recently observed a curious phenomenon, undoubtedly well known, while mixing instant coffee in a mug. If the bottom of the mug was tapped repeatedly with the spoon as the powder was stirred into the water, the note emitted could be heard to rise in pitch by over an octave. This rise occurred in a matter of seconds, about the same time as it took the mug to heat up. Further experimentation showed that almost any powder could produce the effect in cold or hot water, but water alone would not. Vigorous stirring did not seem essential. A full mug showed as pronounced an effect as a half-filled one.

Most of the explanations that come to mind can be easily dismissed using this experimental evidence. Any thermal explanation is ruled out by the observations with cold water. Theories depending on wave scattering from suspended particles are untenable in view of the small ratio of particle size to wavelength, if any reasonable sound velocity is assumed. Appeals to the non-linear nature of the fluid motion equations fail because the coffee does not need to be agitated violently.

We have been able to construct a mechanism that readily accounts for all the observations; it relies upon the tiny bubbles trapped on the powder, which are subsequently released into the water. A homogeneous mixture of bubbles in water constitutes a peculiar fluid with essentially the density of water and a compressibility, k_m, of approximately

$$k_m = v_b k_a,$$

where v_b is the fraction of the volume filled with bubbles and k_a is the compressibility of air. If $v_b = 0{\cdot}01$, the velocity of sound in the mixture is 30 times lower than that in ordinary water. For such a mixture the wavelengths of audible disturbances can become comparable with the dimensions of the mug, and large changes in the eigenfrequencies of the system are to be expected. Calculations on a simple model indicate that the bubble effect is in the correct direction (i.e. an increase in frequency as the bubbles disappear) and of the right magnitude.

Consider an infinitely long cylindrical shell, radius r_0, wall thickness h, density ρ_1 containing a compressible fluid with a density ρ_2 and sound velocity c. Flexing modes in the shell are coupled into the fluid. The equation governing small radial displacements ξ from an equilibrium position r_0 is

$$\rho_1 h \frac{\partial^2 \xi}{\partial t^2} = \frac{Eh^3}{12(1-\sigma^2) r_0^4} \frac{\partial^4 \xi}{\partial \theta^4} + P.$$

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Here E and σ are the Young's modulus and Poisson's ratio of the shell, θ is the angle coordinate measured around the axis, and P the pressure term due to the contained fluid. A similar problem is discussed by Landau and Lifshitz ((1), p. 114). In the fluid, introducing the velocity potential ϕ, we have the usual wave equation:

$$\nabla^2\phi = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}.$$

Note that:

$$P = \rho_2\frac{\partial\phi}{\partial t}.$$

We seek solutions harmonic in time (radian frequency ω) and periodic in the angle coordinate with $2n$ nodes per revolution. After applying the boundary condition that the normal velocity be continuous at $r = r_0$, we obtain the following equation for the non-dimensional eigenfrequency

$$n^4 = \Omega^2 + \alpha\Omega\Omega_1\frac{J_n(\Omega/\Omega_1)}{J_n'(\Omega/\Omega_1)}$$

J_n is the Bessel function and the dimensionless frequencies are defined as:

$$\Omega = \omega/\omega_0, \quad \Omega_1 = c/r_0\omega_0, \quad \alpha = \rho_2 r_0/\rho_1 h,$$

where the characteristic frequency associated with the shell, ω_0, is given by

$$\omega_0^2 = \frac{Eh^2}{12(1-\sigma^2)\rho_1 r_0^4}.$$

Note that ω_0 and α are independent of c and thus variations in c affect Ω_1 alone.

We have computed numerical solutions to the eigenfrequency equation for the gravest mode with $n = 2$ and α ranging from 1 to 10. The value of α was not critical, and the solutions for various values of this parameter differed only slightly: a typical number is about 3·5. We find the eigenfrequency Ω increases with the sound velocity in the fluid, tending to a finite limit, Ω_∞, for the incompressible case. Figure 1 shows the normalized solution for the range $0 \leqslant \Omega_1 \leqslant 1{\cdot}5$; for values larger than this Ω is sensibly constant. With reasonable numbers for the elastic and geometric constants $\Omega_1 = 5$ for ordinary water so that water alone behaves essentially as an incompressible fluid. With bubbles producing a thirty fold decrease in c, $\Omega_1 = 0{\cdot}16$ and it can be seen from the figure that the eigenfrequency drops by about two octaves.

We must emphasize that the behaviour of the $n = 2$ gravest mode is merely a typical example of an effect which every eigenmode will exhibit in this ideal system. We expect similar behaviour from an actual coffee mug, although it is difficult to determine exactly how the acoustical properties of mixture vary with time.

One final piece of experimental work supports our theory. The most striking results are achieved by tapping the bottom of a freshly poured glass of beer. As the cloud of bubbles disperses the ringing tone rises, reaching a constant pitch when the beer is clear.

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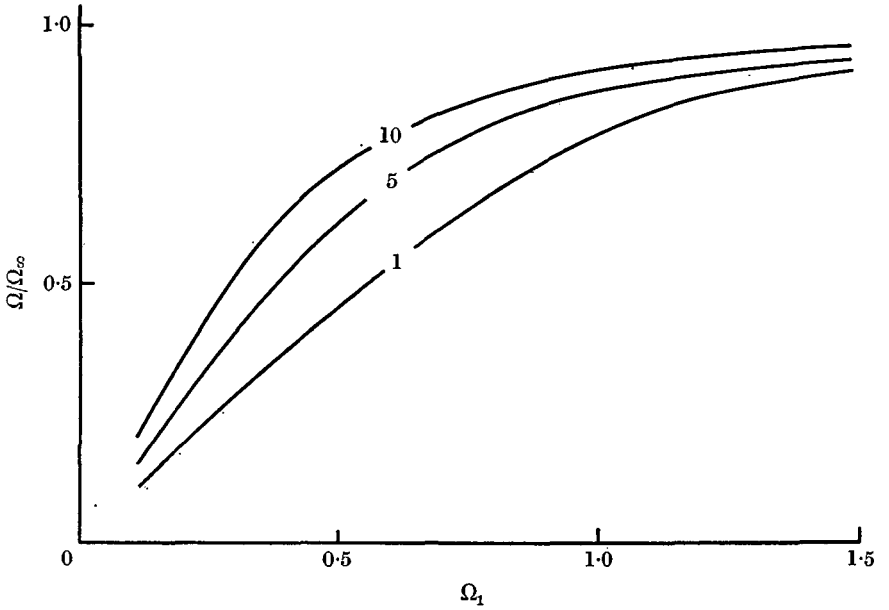


Fig. 1. Relative change in frequency of gravest mode of the ideal system for $n = 2$ as sound velocity of mixture varies. Ω_∞ is the eigenfrequency for an incompressible fluid, and $\alpha = 1, 5, 10$.

REFERENCE

(1) Landau, L. D. and Lifshitz, E. M. *Theory of elasticity*. (Pergamon Press; London, 1959).

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