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Multiscale structures in a two-temperature relativistic electron–positron–ion plasma

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Abstract. A relativistically hot electron, positron and ion (e–p–i) plasma relaxes to a triple curl Beltrami (TCB) field. The TCB field being the superposition of three Beltrami fields is characterized by three scale parameters and hence there exist multiscale structures in the system. It is shown that temperatures of the plasma constituents strongly affect the scale parameters. Generally, the scale parameters associated with the TCB field may be a combination of real and complex roots. The numerical results show that for given Beltrami parameters, an increase in the thermal energy of plasma particles could transform the real eigenvalues to complex ones. It is also observed that one component is more strongly affected relative to other components on increasing temperatures of plasma species. Two different vortices become the same at higher thermal energies. This suggests that it is possible to create high β (kinetic to magnetic pressure ratio) and fully diamagnetic plasma configurations. The study has a potential relevance to space, astrophysics and laboratory plasmas.

1. Introduction

The steady-state force-free relaxed equilibrium of a stationary magnetized plasma is achieved when the magnetic field and its vorticity are aligned and magnetic energy becomes minimum. The relaxed state is described by the eigenvalue equation $\nabla \times \mathbf{B} = \lambda \mathbf{B}$, where $\mathbf{B}$ is the eigenfunction and λ is a scalar constant. A vector field which satisfies this eigenvalue equation is called the Beltrami field. The force-free Beltrami field is an exact free-decay solution to the magnetohydrodynamics (MHD) equations (Chandrasekhar and Kendall 1957). Taylor (1974, 1986) derived the force-free state (also called the Taylor state) by invoking the variational principle under appropriate constraints. Theoretical studies of plasma relaxation have been pursued extensively to generalize and make it more practicable. The models developed for the relaxation of multifluid plasmas show that it is possible to create a non force-free relaxed equilibrium (Steinhauer and Ishida 1997; Mahajan and Yoshida 1998). The relaxed state of multifluid plasmas can be interpreted as a superposition of multiple Beltrami fields. For example, the relaxed state in Hall MHD represents the coupling of two Beltrami fields (Yoshida and Mahajan 1999), whereas the inclusion of inertial effects leads to a relaxed state composed of three Beltrami fields (Bhattacharyya et al. 2003; Iqbal and Shukla 2011). In contrast to MHD relaxation, the relaxed equilibria of multifluid plasmas are characterized by strong flow and pressure gradients (Iqbal et al. 2001; Iqbal 2005). The steady state of two- or three-fluid magnetized plasmas is governed by both the macroscopic (of system size) and microscopic (of the order of electron or ion skin depth) scales while single fluid MHD relaxation has only one macroscopic scale. The relaxation of multifluid plasmas admits very novel and complex structures and could be used to understand a variety of physical phenomena occurring in laboratory and space, for instance, high beta relaxed states in tokamak, eruptive events in space plasma and dynamo mechanism (Yoshida et al. 2001, Iqbal and Shukla 2012a).

Recently, it is shown that relativistically hot electron, positron and ion (e–p–i) plasmas relax to a triple curl Beltrami (TCB) field – a combination of three eigenfunctions of the curl operator (Iqbal and Shukla 2012b). This work deals with the creation of multiscale structures and the effect of relativistic thermal energies on the eigenstates developed through the relaxation of a hot e–p–i plasma. The three-component (e–p–i) plasmas are found in the pulsar magnetosphere and can be generated by the propagation of intense short laser pulses through a plasma (Berezhiani et al. 1992). Such plasmas are also found in experiments when positrons are used to study transport in tokamaks (Surko and Murphy 1990; Tinkle et al. 1994). In this paper, we will show that multiscale structures are strongly affected

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when the relativistic temperatures of plasma species parameterized by an 'effective mass' are varied. At higher relativistic thermal energies of plasma species, the scale parameters characterizing the length scales of relaxed structures in a three-component (e–p–i) plasma become complex. This feature suggests that fully diamagnetic field configurations could be obtained at higher thermal energies of plasma components. It is in contrast to the result where all the scale parameters become real when a relativistic pair plasma gets ultrahigh relativistic temperature (Iqbal et al. 2008). One of the three scale parameters is strongly affected and two of the relaxed structures become approximately equal at higher thermal energies.

We describe the macroscopic normalized evolution equations for electrons, positrons and ions comprising the relativistically hot plasma and evaluate the relaxed state of the system in Sec. 2. The dependence of scale parameters on Beltrami parameters and temperatures of the species is described analytically in Sec. 3 and the character of roots is also shown graphically for non-relativistic and relativistic temperatures. Sec. 4 is devoted to describe different features of the relaxed structures. The eigenvalues go from real to complex and large vortices could be created when thermal energy increases. In Sec. 5, a summary of the results is given.

2. Model

When the thermal energy of the plasma particles becomes of the order of or greater than the rest mass energy, the plasma is assumed to be hot. For a local relativistic Maxwellian distribution of particle velocities, the relativistic temperature effect can be expressed by the factor $G_\alpha(z_\alpha) = K_3\left(1/z_\alpha\right)/K_2\left(1/z_\alpha\right)$, where K values are modified Bessel functions, and $z_\alpha = T_\alpha/m_{0\alpha}c^2$; $m_{0\alpha}$ and T_α are the particle invariant rest mass and temperature, respectively, and c is the speed of light. For non-relativistic temperatures $T_\alpha \ll m_{0\alpha}c^2$ and $G_\alpha = 1 + 5T_\alpha/2m_{0\alpha}c^2$; and for highly relativistic temperatures $T_\alpha \gg m_{0\alpha}c^2$ and $G_\alpha = 4T_\alpha/m_{0\alpha}c^2$. The hydrodynamics of the fluids of species α ($\alpha = e, p, i$) is given as (Berezhiani et al. 2007; Iqbal et al. 2008):

$$\frac{\partial}{\partial t}(G_\alpha m_{0\alpha}\gamma_\alpha \mathbf{V}_\alpha) + m_{0\alpha}c^2\nabla(G_\alpha\gamma_\alpha) = e_\alpha\mathbf{E} + \mathbf{V}_\alpha \times \boldsymbol{\Omega}_\alpha, \quad (1)$$

with the generalized vorticity $\boldsymbol{\Omega}_\alpha = \nabla \times (G_\alpha m_{0\alpha}\gamma_\alpha \mathbf{V}_\alpha) + e_\alpha \mathbf{B}/c$, where $\mathbf{B}$ is the magnetic field, $\mathbf{V}_\alpha$, the velocity of fluid, $\gamma_\alpha = \left(1 - \mathbf{V}_\alpha^2/c^2\right)^{-1/2}$, the relativistic factor and e_α is the charge of the α species. The thermal pressure $P_\alpha\ \left(= n_\alpha T_\alpha/\gamma_\alpha\right)$ appears in (1) through the temperature-dependent factor G_α defined by $\gamma_\alpha \nabla P_\alpha = m_{0\alpha}c^2 n_\alpha \nabla G_\alpha$, where n_α is the density in the laboratory frame of the fluid of species α. We consider a constant density quasi-neutral plasma where the velocities of fluid elements are non-relativistic $\left[\gamma_\alpha \simeq 1 + \mathbf{V}_\alpha^2/2c^2\right]$. We assume $T_e = T_p = T_1\ \left(G_e = G_p = G_1\right)$ and $T_i = T_2\ (G_i = G_2)$ so that temperatures of hot electrons and positrons are equal and constant, whereas the temperature of ions is taken to be different from that of electrons and positrons. Taking $\mathbf{E} = -c^{-1}\partial\mathbf{A}/\partial t - \nabla\phi$, the above equation can be rewritten as

$$\frac{\partial \hat{\mathbf{P}}_\alpha}{\partial t} = \hat{\mathbf{V}}_\alpha \times \hat{\nabla} \times \hat{\mathbf{P}}_\alpha - \hat{\nabla}\psi_\alpha, \quad (2)$$

where $\hat{\mathbf{P}}_\alpha$ represents the fluid momentum, $\hat{\mathbf{P}}_e = G_1\hat{\mathbf{V}}_e - \hat{\mathbf{A}}$, $\hat{\mathbf{P}}_p = G_1\hat{\mathbf{V}}_p + \hat{\mathbf{A}}$, $\hat{\mathbf{P}}_i = G_2\hat{\mathbf{V}}_i + \mu\hat{\mathbf{A}}$, where $\mu = m_{0e}/m_{0i}$, $\psi_e = -\hat{\phi} + \hat{c}^2 G_1$, $\psi_p = \hat{\phi} + \hat{c}^2 G_1$ and $\psi_i = \mu\hat{\phi} + \hat{c}^2 G_2$, where $\hat{c} = c/V_0$. In evaluating the above equation, the following set of normalizations is employed: $\mathbf{x} = \lambda_e\hat{\mathbf{x}}$, $t = \left(\lambda_e/V_0\right)\hat{t}$, $\mathbf{V} = V_0\hat{\mathbf{V}}$ and $\mathbf{B} = B_0\hat{\mathbf{B}}$, where $\lambda_e = c/\omega_{pe} = c\sqrt{m_{0e}}/\sqrt{4\pi n_{0e}e^2}$ is the electron skin depth, $V_0 = B_0/\sqrt{4\pi n_{0e}m_{0e}}$ is equivalent to the Alfvén velocity of electrons and B_0 is a constant magnetic field. Taking the curl of (2), we obtain

$$\frac{\partial \hat{\boldsymbol{\Omega}}_\alpha}{\partial t} = \hat{\nabla} \times (\hat{\mathbf{V}}_\alpha \times \hat{\boldsymbol{\Omega}}_\alpha). \quad (3)$$

The generalized vorticities are

$$\hat{\boldsymbol{\Omega}}_e = \hat{\nabla} \times G_1\hat{\mathbf{V}}_e - \hat{\mathbf{B}}, \quad (4)$$

$$\hat{\boldsymbol{\Omega}}_p = \hat{\nabla} \times G_1\hat{\mathbf{V}}_p + \hat{\mathbf{B}}, \quad (5)$$

$$\hat{\boldsymbol{\Omega}}_i = \hat{\nabla} \times G_2\hat{\mathbf{V}}_i. \quad (6)$$

In evaluating the above equations, we have taken $m_{0e} = m_{0p}$. As the rest mass of ions is greater than that of electrons and $\mu \ll 1$, we have neglected the term involving μ in (6). Making use of Ampere's law ($c\nabla \times \mathbf{B} = 4\pi\mathbf{J}$), the normalized expression for current density reads as

$$\hat{\nabla} \times \hat{\mathbf{B}} - N_i\hat{\mathbf{V}}_i = N_p\hat{\mathbf{V}}_p - \hat{\mathbf{V}}_e, \quad (7)$$

where $N_i = n_i/n_e$ and $N_p = n_p/n_e$. In what follows, ignoring the normalization sign on variables, one of the simplest possible steady-state solutions to (3) for electrons, positrons and ions, respectively, can be written as

$$\nabla \times G_1\mathbf{V}_e - \mathbf{B} = a\mathbf{V}_e, \quad (8)$$

$$\nabla \times G_1\mathbf{V}_p + \mathbf{B} = a\mathbf{V}_p, \quad (9)$$

$$\nabla \times G_2\mathbf{V}_i = b\mathbf{V}_i. \quad (10)$$

The constant parameters a and b are the Beltrami parameters. It shows that the flow of electrons and positrons is proportional to their respective generalized vorticities, whereas the vorticity of ions becomes parallel to their flow. The equilibrium, therefore, satisfies the 'Beltrami conditions' due to the alignment of vorticities to corresponding flows of different constituents of the hot electron, positron and ion plasma. Multiplying (9) by N_p, and subtracting (8) from the resulting equation, we obtain

$$\nabla \times G_1\left(N_p\mathbf{V}_p - \mathbf{V}_e\right) + \left(N_p + 1\right)\mathbf{B} = a\left(N_p\mathbf{V}_p - \mathbf{V}_e\right). \quad (11)$$

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Considering $N_p = n_p/n_e \ll 1$, that is positrons density is much less than the density of electrons, we ignore N_p in comparison with unity. The above equation can be written as

$$\nabla \times G_1 \left(N_p \mathbf{V}_p - \mathbf{V}_e\right) + \mathbf{B} = a \left(N_p \mathbf{V}_p - \mathbf{V}_e\right). \quad (12)$$

From (7) and (12), after eliminating flows of electrons and positrons, we obtain

$$G_1 \nabla \times \nabla \times \mathbf{B} - a \nabla \times \mathbf{B} + \mathbf{B} = G_1 N_i \nabla \times \mathbf{V}_i - a N_i \mathbf{V}_i. \quad (13)$$

Eliminating $\nabla \times \mathbf{V}_i$ from (10) and (13), the velocity of ions is given by

$$\mathbf{V}_i = \chi^{-1} \left(G_1 \nabla \times \nabla \times \mathbf{B} - a \nabla \times \mathbf{B} + \mathbf{B}\right), \quad (14)$$

where $\chi = N_i \left(bG_1 - aG_2\right)/G_2$. Putting the value of ion flow from the above equation into (10), we obtain a TCB equation in $\mathbf{B}$ as

$$\nabla \times \nabla \times \nabla \times \mathbf{B} - k_1 \nabla \times \nabla \times \mathbf{B} + k_2 \nabla \times \mathbf{B} - k_3 \mathbf{B} = 0, \quad (15)$$

where $k_1 = \left(a/G_1\right) + \left(b/G_2\right)$, $k_2 = \left(1/G_1\right) + ak_3$ and $k_3 = b/\left(G_1 G_2\right)$.

3. Roots analysis

Equation (15) is equivalent to the linear sum of three Beltrami fields with different eigenfunctions and eigenvalues of the curl operator. Suppose $\mathbf{B}_j$ and λ_j (where $j = 1, 2, 3$) represent the eigenfunctions and eigenvalues respectively and satisfy the condition $\nabla \times \mathbf{B}_j = \lambda_j \mathbf{B}_j$, then the TCB field expressed as

$$\mathbf{B} = \sum_{j=1}^{3} C_j \boldsymbol{B_j}, \quad (16)$$

where C_j values are arbitrary constants, is the solution of (15). Chandrasekhar–Kendall functions (Chandrasekhar and Kendall 1957) and the ABC flow (Yoshida 2010) in cylindrical and Cartesian coordinates respectively are the examples of such eigenfunctions of the curl operator. The eigenvalues of the curl operator (λ_j) which are the solutions of the cubic equation

$$\lambda^3 - k_1 \lambda^2 + k_2 \lambda - k_3 = 0, \quad (17)$$

and dimensionally they are the inverse of length, may be real or complex (Yoshida and Giga 1990). The scale parameters are

$$\lambda_1 = s - \tau + \frac{k_1}{3}, \quad (18)$$

$$\lambda_2 = -\frac{1}{2}(s - \tau) - i\frac{\sqrt{3}}{2}(s + \tau) + \frac{k_1}{3}, \quad (19)$$

$$\lambda_3 = -\frac{1}{2}(s - \tau) + i\frac{\sqrt{3}}{2}(s + \tau) + \frac{k_1}{3}, \quad (20)$$

where $\tau = (0.5q + \sqrt{D})^{1/3}$, $s = -[0.5q - \sqrt{D}]^{1/3}$, $p = (3k_2 - k_1^2)/3$ and $q = (k_1 k_2/3) - (2k_1^3/27) - k_3$. The factor $D = (q/2)^2 + (p/3)^3$ determines the character of roots. All roots are real when $D < 0$, and two of the roots (λ_j) are complex conjugate while the third one is real for $D > 0$. All the roots of (17) are real and at least two are equal when $D = 0$. In terms of eigenvalues (λ_j), the values of k_j can be expressed as

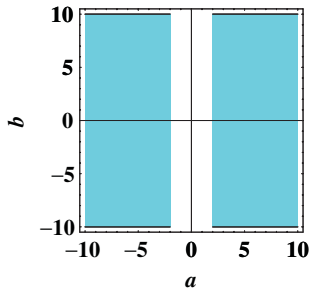


Figure 1. (Colour online) The eigenvalues as a function of Beltrami parameters a and b for non-relativistic temperatures ($G_1 = G_2 = 1$). The colored/shaded and clear regions show the real and complex roots, respectively.

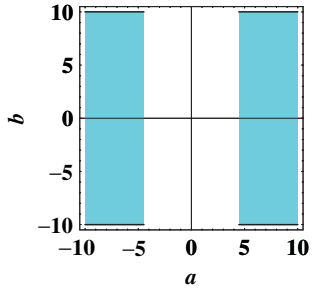


Figure 2. (Colour online) The eigenvalues as a function of Beltrami parameters a and b for relativistic temperatures ($G_1 = G_2 = 5$). The colored/shaded and clear regions show the real and complex roots, respectively.

$$k_1 = \lambda_1 + \lambda_2 + \lambda_3, \quad (21)$$

$$k_2 = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3, \quad (22)$$

$$k_3 = \lambda_1\lambda_2\lambda_3. \quad (23)$$

The aim of the present study is to show the effect of two thermal energies (G_1 and G_2) on the character of roots (λ_j) which define the microscopic and macroscopic structures on the relaxation of two-temperature electron–positron and ion plasma. In Fig. 1, we have plotted the values of roots of the cubic equation (17) as a function of Beltrami parameters a and b for the non-relativistic temperatures $G_1 = G_2 = 1$. The roots are real in the colored/shaded region of the graph whereas two of the eigenvalues are complex conjugate while the third one is real in the clear region of the graph. The figure illustrates that roots are complex when the Beltrami parameter $a < |2|$; otherwise, we will have real roots for non-relativistic thermal energies.

Figure 2 shows the character of roots when the temperatures of the plasma species are relativistic. The thermal energies are taken to be the same in this case ($G_1 = G_2 = 5$). We observe that complex roots are increased on increasing the temperature of species as the shaded/colored region has been reduced in Fig. 2. The

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roots are complex when $a < |4.7|$ and become real when $a > |4.7|$ for any value of the Beltrami parameter b. The complex roots introduce modified Bessel functions which always increase away from the center. Therefore, it seems possible to have highly diamagnetic large structures at higher relativistic temperatures.

4. Characteristics of relaxed structures

The relaxed equilibrium structures formed in two-temperature electron, positron and ion plasma depend on two Beltrami parameters a and b, and two temperatures G_1 and G_2. The Beltrami parameter a is the ratio of electrons and positrons generalized vorticities to their flows respectively, whereas b is the ratio of ions vorticity to their flow. When $a = 0$, the curl of canonical momenta of both the electrons and positrons becomes zero. The steady-state relaxed equation still remains TCB equation but values of k_1 and k_2 change while k_3 remains the same; k_1 and k_2 read as $k_1 = b/G_2$ and $k_2 = 1/G_1$. When $b = 0$, the vorticity of ions becomes zero and the cubic equation modifies as

$$\lambda^3 - k_1\lambda^2 + k_2\lambda = 0, \tag{24}$$

where $k_1 = a/G_1$ and $k_2 = 1/G_1$. In this case, $k_3 = 0$ and one of the roots is zero while other two roots are solutions of the quadratic equation

$$\lambda^2 - k_1\lambda + k_2 = 0, \tag{25}$$

and can be expressed

$$\lambda = \frac{k_1}{2} \pm \frac{\sqrt{k_1^2 - 4k_2}}{2}. \tag{26}$$

If $k_1^2 = 4k_2$, both roots are degenerate and real, and when $k_1^2 \gg 4k_2$, one of the roots is zero while other one equals to k_1 and both are real. But when $k_1^2 \ll 4k_2$, the two roots become complex conjugate.

When $a = b = 0$, then $k_1 = k_3 = 0$ and $k_2 = 1/G_1$. The cubic equation reduces to

$$\lambda^3 + k_2\lambda = 0. \tag{27}$$

Again, one of the roots is zero and other two roots being pure imaginary are $\lambda = \pm i\sqrt{k_2}$. The formation of relaxed structures now depends only on G_1 – the thermal energy of electrons and positrons. The ions vorticity and the curl of canonical momenta of both the lighter species (electrons and positrons) become zero. Consequently, electron and positron vorticities become aligned to the magnetic field.

When $a = \sqrt{G_1}$ and $b = -G_2/\sqrt{G_1}$, then $k_1 = k_2 = 0$ and $k_3 = -(G_1)^{-3/2}$. The cubic equation now reads as

$$\lambda^3 - k_3 = 0. \tag{28}$$

All three roots are real and equal to $(-1/\sqrt{G_1})$. It shows that equilibrium structures depend on the temperature of electrons and positrons.

If $b = -aG_2/G_1$, then $k_1 = 0$, $k_2 = (1/G_1) - (a/G_1)^2$ and $k_3 = -aG_1^{-2}$. The cubic equation becomes

$$\lambda^3 + k_2\lambda - k_3 = 0. \tag{29}$$

For a special case, when both the temperatures are non-relativistic, that is $G_1 = G_2 = 1$, then for $b = -a$, $k_1 = 0$, $k_2 = 1 - a^2$ and $k_3 = -a$. If we further assume that $a = \pm 1$, then $b = \mp 1$, $k_1 = k_2 = 0$ and $k_3 = \pm 1$, and the cubic equation becomes

$$\lambda^3 \mp 1 = 0. \tag{30}$$

The two roots will be complex conjugate and one will be real. For $b = -G_2/a$, the factor k_2 becomes zero, $k_1 = (a/G_1) - a^{-1}$ and $k_3 = -(aG_1)^{-1}$. The cubic equation can be written as

$$\lambda^3 - k_1\lambda^2 - k_3 = 0. \tag{31}$$

If we suppose that both the temperatures are equal, that is $G_1 = G_2 = G$, then $k_1 = (a + b)/G$, $k_2 = \left(1 + abG^{-1}\right)/G$ and $k_3 = b/G^2$. For $G = 1$, that is both the temperatures are non-relativistic, and $b = 0$, we have $k_1 = a$, $k_2 = 1$ and $k_3 = 0$. The cubic equation reduces to

$$\lambda^3 - a\lambda^2 + \lambda = 0. \tag{32}$$

One of the roots is zero (real) while other roots are the solution of the quadratic equation $\lambda^2 - a\lambda + 1 = 0$ and are given by

$$\lambda = \frac{1}{2}\left(a \pm \sqrt{a^2 - 4}\right). \tag{33}$$

The roots are complex if $a < |2|$ and will be real when $a \geqslant |2|$. A glimpse of this result can be seen in Fig. 1. For equal thermal energies of plasma particles ($G_1 = G_2 = G$) and $b = 0$, the coefficients of the polynomial are $k_1 = a/G$, $k_2 = 1/G$ and $k_3 = 0$. The cubic equation becomes

$$\lambda^3 - \frac{a}{G}\lambda^2 + \frac{1}{G}\lambda = 0. \tag{34}$$

Again, one root is real (equal to zero) and the remaining can be found by solving the quadratic equation $G\lambda^2 - a\lambda + 1 = 0$. The roots are

$$\lambda = \frac{1}{2}\left(\frac{a}{G} \pm \sqrt{\left(\frac{a}{G}\right)^2 - 4/G}\right). \tag{35}$$

Thus, the roots are real when $a \geqslant \pm 2\sqrt{G}$ and complex for $a < \pm 2\sqrt{G}$, as is evident from Fig. 2.

Figure 3 shows how the roots are changed on varying the thermal energy of electrons G_1 for $a = 5.0$ and $b = 0.9$. The thermal energy of ions in this case is taken as constant $G_2 = 1.5$. The scale parameters λ_j are real and distinct when the thermal energy of lighter species is non-relativistic; their values are $\lambda_1 = 0.2168$, $\lambda_2 = 0.5380$ and $\lambda_3 = 5.1453$. Two of the roots become complex conjugate ($1.0665 \pm 0.0749i$) while the third one is real (0.2083) when the thermal energy of electrons is $G_1 = 2.52$. Figure 4 shows the plot of scale parameters

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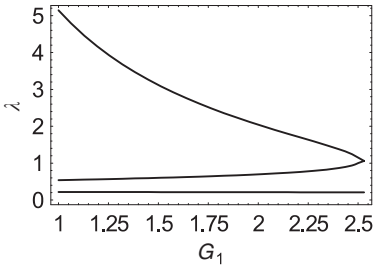


Figure 3. Character of the scale parameters as a function of the thermal energy of electrons G_1. The Beltrami parameters are $a = 5.0, b = 0.9$ and $G_2 = 1.5$.

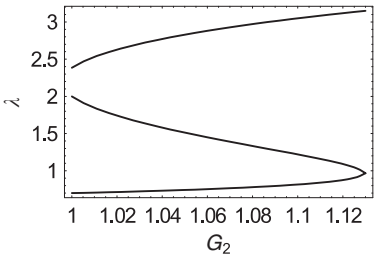


Figure 4. Character of the scale parameters as a function of the thermal energy of ions G_2. The Beltrami parameters are $a = 2.1, b = 4.0$ and $G_1 = 1.2$.

versus thermal energy of ions G_2 for a constant value of Beltrami parameters ($a = 2.1$ and $b = 4.0$) and thermal energy of electrons ($G_1 = 1.2$). The figure illustrates that initially for $G_2 = 1$, all three roots are real and distinct ($\lambda_1 = 0.6990$, $\lambda_2 = 2.0$ and $\lambda_3 = 2.3843$). An increase in G_2 changes the roots, and at $G_2 = 1.13$ two roots are complex conjugate ($0.9672 \pm 0.0364i$) and the remaining one is real (3.1490). Figures 3 and 4 show that an increase in the thermal energy of the species can make the scale parameters which are real at the initial stage to change to complex roots. It is also evident that one small structure at non-relativistic temperature changes to a large structure at higher relativistic temperature. In Fig. 3, we observe that one component with a small-scale parameter λ shows a large-scale structure (system size) while other two components with a large λ value correspond to small size structures in the system at higher thermal energies. Figure 4 illustrates that there are two components which are approximately equal and get macroscopic structures (system size) while the third component appears with a small scale of the order of electron skin depth at relativistic temperature.

Figures 5 and 6 show the effects of the temperature of plasma species on scale parameters λ_j for Beltrami parameters $a = 3.5$ and $b = 1.1$. In Fig. 5, the thermal velocity of ions G_2 is taken as 1.2 and kept constant. Cubic equation (17) is plotted for different values of G_1 – temperatures of lighter species (electrons and positrons). It is evident that for $G_1 = 1$, 1.3 and 1.4, all three scale parameters are real and distinct because the curve crosses the x-axis at three different points. When $G_1 = 1.5$, the curve crosses the x-axis only once. It shows that only one root is real while

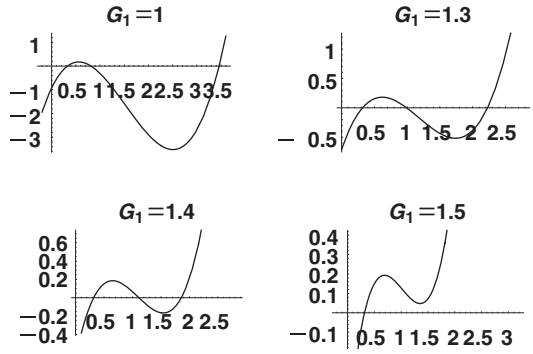


Figure 5. Variation of scale parameters λ_j for different values of G_1. The Beltrami parameters are $a = 3.5, b = 1.1$ and the thermal energy of ions is taken as $G_2 = 1.2$.

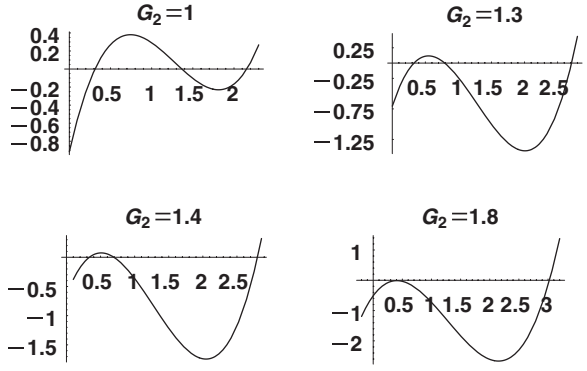


Figure 6. Variation of scale parameters λ_j for different values of G_2. The Beltrami parameters are $a = 3.5$, $b = 1.1$ and the thermal energy of electrons and positrons is taken as $G_1 = 1.2$.

other two roots are complex conjugate. Similarly, we plot the cubic equation (17) for different values of ion temperatures G_2 while $G_1 = 1.2$ and taken as constant. We observe that for $G_2 = 1$, 1.3 and 1.4, all the roots of the cubic equation are real and distinct but for $G_2 = 1.8$, two scale parameters are complex conjugate and the third parameter is real. Figures 5 and 6 illustrate that on increasing the temperature of plasma species, it is possible to have one real and two complex roots.

5. Summary

The hot e–p–i plasma relaxes to a TCB field which is a superposition of three Beltrami fields. Therefore, multiscale structures can be obtained when the hot e–p–i plasma attains the steady-state equilibrium. The relaxed structures strongly depend on Beltrami parameters a and b, the ratios of the curl of the canonical momentum of different fluids to their flow, and the temperature-dependent mass of the species G_1 and G_2, where G_1 is the thermal energy of electrons and positrons, and G_2 is the temperature of ions. The scale parameters λ_j characterize the size of structures formed in the system as they are dimensionally equal to the inverse of length. The eigenvalues of the Beltrami function λ_j depend on Beltrami parameters and thermal energies (G_1 and G_2); therefore, the microscopic as well as

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macroscopic structures can be obtained by varying the Beltrami parameters and temperatures of the plasma species. It is shown that an increase in thermal energy causes the transitions of roots from real to complex. It is found that one scale parameter is highly influenced by an increase in thermal energy and, as a result, the small size structure at non-relativistic temperature is transformed into a large size structure at relativistic temperature. Two real scale parameters become complex conjugate at higher temperatures. The complex eigenvalues give rise to highly diamagnetic structures and therefore high β relaxed equilibrium states could be accessible. We expect that desired relaxed configurations can be obtained by controlling the thermal velocities of plasma species. One can easily attain any arbitrary value of thermal energies by injecting electron and/or ion beams into plasma or exposing the system to ultrarelativistic laser beams. Our results could also be used to study large-scale relativistic jets found in radio galaxies, quasars, or radio pulsars.

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