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Elastic electron–ion collisions in a Lorentzian plasma

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Abstract. Plasma screening effects on elastic electron–ion collision processes in a Lorentzian (kappa)-distribution plasma are investigated using the eikonal method. The electron–ion interaction potential is obtained by introducing a plasma dielectric function ε_κ. The semiclassical straight-line trajectory method is applied to the path of a projectile electron in order to investigate the variation of the eikonal phase as a function of impact parameter and spectral index κ. In the first-order eikonal approximation, the dynamic plasma screening effect on the eikonal phase is found to be a static screening effect, and depends only on the Debye length and spectral index. The position of the maximum of the differential elastic eikonal cross-section recedes from the ion core with increasing spectral index. The plasma screening effect on the eikonal cross-section increases with decreasing spectral index.

1. Introduction

Electron–ion scattering processes (Janev et al. 1985; Jung 1995a, b; Jung and Cho 1995; Hong and Jung 1996) in plasmas are of great interest since they can be used for plasma diagnostics. Dynamic screening effects have been investigated in the context of electron–ion collisional excitation (Jung 1997a) and Coulomb bremsstrahlung (Jung 1997b) processes in dense plasmas using the dynamic interaction potentials obtained from the plasma dielectric functions. Many of the characteristic features of plasmas can be understood by knowing the velocity or energy dependence. The general equilibrium velocity distribution function of a collisionless plasma is known to be Maxwellian. A Maxwellian plasma is in thermal equilibrium, which implies that it does not contain any free energy, and hence there are no energy exchange processes between the particles in the plasma. It is then clear that the velocities of the particles can be assumed to be distributed around the average velocity. However, in general, most plasmas exist in a state very far removed from thermal equilibrium. In laboratory plasmas, the coupling of the injected external energy with the target most often generates superthermal electrons that have escaped from the Maxwellian distribution (Lamoureux 1993; Baumjohan and Treumann 1996).

The quantum mechanical method known as the eikonal approximation (Joachain 1983; Burke and Joachain 1995; Jung 1996, 1997c) has been widely used in many atomic and nuclear collision processes. Recently, elastic

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electron–ion collision processes in strongly coupled plasmas (Jung 1996) and in weakly coupled plasmas (Jung 1997c) have been investigated using the eikonal approximation. This method has the great advantage that we can obtain the general continuum wave functions for the electron in a dynamically screened Coulomb field in terms of the eikonal phase, including dynamic screening effects. Once the eikonal phase has been obtained, the quantum mechanical wave function for the continuum-state electron can also be obtained, since the phase includes all physical information. Then, using the quantum mechanical wave functions, we can readily calculate and investigate scattering cross-sections for atomic transition phenomena such as electron-impact excitation, ionization and bremsstrahlung processes in plasmas. Hence, in this paper, we investigate the dynamic plasma screening effects on elastic electron–ion collisions in a non-Maxwellian electron velocity distribution plasma described by a Lorentzian (kappa)-distribution model, since theoretical atomic spectroscopy is essential in the study of non-Maxwellian plasmas. The semiclassical straight-line (SL) trajectory method is applied to the path of the projectile electron in order to investigate the variation of the eikonal phase χ as a function of the impact parameter b, i.e. the separation between the projectile electron and the target ion.

In Sec. 2, we discuss the eikonal method and derive a closed form of the eikonal phase in elastic electron–ion collisions using the SL trajectory method. In Sec. 3, we obtain the eikonal differential elastic scattering (EDES) cross-section and the total cross-section in a Lorentzian (kappa)-distribution plasma using the dynamic interaction potential described by the plasma dielectric function. We also investigate the plasma screening effects on the EDES and total cross-sections for several values of the spectral index. The results show that the EDES cross-section substantially decreases with increasing kinetic energy of the projectile electron. The dynamic plasma screening effect on the eikonal phase in the first-order eikonal approximation is found to be independent of the velocity of the plasma electron, and depends only on the Debye length and spectral index. The maximum position of the eikonal cross-section recedes from the ion core with increasing spectral index. As the projectile energy is increased, the position of the maximum shifts towards the target nucleus. It is also found that the plasma screening effect on the elastic scattering cross-section is more significant for smaller spectral index. Finally, in Sec. 4, our conclusions are given.

2. Eikonal wave function

We use the first-order theory arising from the Lippmann–Schwinger equation (Friedrich 1991)

$$\Psi(\mathbf{r}) = \phi(\mathbf{r}) + \int d^3\mathbf{r}'\, G_0(\mathbf{r}-\mathbf{r}')\, U(\mathbf{r}')\, \Psi(\mathbf{r}'). \tag{1}$$

Here $U(\mathbf{r}) = 2mV(\mathbf{r})/\hbar^2$ is the reduced interaction potential, m is the electron rest mass, and $\Psi(\mathbf{r})$, $\phi(\mathbf{r})$ and $G_0(\mathbf{r}-\mathbf{r}')$ are the solution of the Schrödinger

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equation, the solution of the homogeneous equation and the Green function respectively, such that

$$(\nabla^2 + k^2)\,\Psi(\mathbf{r}) = U(\mathbf{r})\,\Psi(\mathbf{r}), \tag{2}$$

$$(\nabla^2 + k^2)\,\phi(\mathbf{r}) = 0, \tag{3}$$

$$(\nabla^2 + k^2)\,G_0(\mathbf{r}-\mathbf{r}') = \delta^3(\mathbf{r}-\mathbf{r}'), \tag{4}$$

where $k = (2mE/\hbar^2)^{1/2}$ is the wavenumber of the projectile electron, E is the kinetic energy of the projectile electron and $\delta^3(\mathbf{r}-\mathbf{r}')$ is the three-dimensional Dirac delta function. Choosing the z axis in the direction of the incident wave vector $\mathbf{k}_i$ and using the expansion technique on the free-particle outgoing Green function $G_0^{(+)}$ (Jung 1996), we then obtain the eikonal wave function $\Psi^{E}(\mathbf{r})$ as

$$\Psi^{E}(\mathbf{r}) = (2\pi)^{-3/2}\exp\left[i\mathbf{k}_i\cdot\mathbf{r} - \frac{i}{2k_i}\int_{-\infty}^{z} dz'\,U(\mathbf{b},z')\right], \tag{5}$$

where $\mathbf{b}$ is the impact parameter and the integration is along a straight line parallel to the incident wave vector $\mathbf{k}_i$. The condition for the validity of the eikonal approximation is known to be $|V|/E \ll 1$ (Joachain 1983), where $|V|$ is a typical strength of the interaction potential. If we adopt a cylindrical coordinate system such that $\mathbf{r} = \mathbf{b} + z\hat{\mathbf{n}}$, $\hat{\mathbf{n}}$ being a unit vector perpendicular to the momentum transfer $\boldsymbol{\Delta} \equiv \mathbf{k}_i - \mathbf{k}_f$, and integrate over the variable z, the elastic eikonal scattering amplitude f^E becomes

$$f^E = -\frac{1}{4\pi}\int d^3\mathbf{r}\,e^{i\Delta\cdot\mathbf{r}}U(\mathbf{r})\exp\left[-\frac{i}{2k_i}\int_{-\infty}^{z} dz'\,U(\mathbf{b},z')\right]. \tag{6}$$

The total elastic scattering cross-section in the eikonal approximation is then

$$\sigma^E = \int d\Omega\,|f^E|^2, \tag{7a}$$

$$= 2\pi\int db\,b\left|\exp\left[\frac{i}{k}\chi(b)\right]-1\right|^2, \tag{7b}$$

where the differential solid angle $d\Omega$ has been replaced by $d^2\Delta/k^2$ since $|\mathbf{k}_i| = |\mathbf{k}_f| = k$ in elastic collision processes. For a symmetric potential $V(r)$, the EDES cross-section is then given by

$$\frac{d\sigma^E}{db} = 2\pi b\left|\exp\left[\frac{i}{k}\chi(b)\right]-1\right|^2, \tag{8}$$

where the first-order eikonal phase χ is given by

$$\chi(b) = -\frac{m}{\hbar^2}\int_{-\infty}^{\infty} dz\,V(r), \tag{9}$$

with $r^2 = b^2 + z^2$. A higher-order correction to the eikonal phase has been proposed within the impact parameter formalism (Wallace 1973; Burke and Joachain 1995). A recent investigation (Jung 1996) of elastic electron–ion collisions in strongly coupled plasmas has shown that the higher-order corrections to the eikonal phase are found to be less than 1 % for high projectile

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energies $\bar{E} \equiv E/Z^2Ry \geqslant 10$, where $Ry = me^4/2\hbar^2 \approx 13.6$ eV is the Rydberg constant. Hence, in this paper, we neglect the higher-order correction to the first-order eikonal phase $\chi(b)$ since we are interested in high-energy elastic collisions. The general quantum mechanical continuum wave functions for the projectile electron in arbitrarily symmetric potentials $V(r)$ can be obtained by the eikonal wave function (5) with the eikonal phase (9). Thus, the eikonal wave function (5) can be widely used for atomic processes in plasmas.

3. Eikonal cross-section

In a plasma, the dynamic interaction potential between a projectile electron and a target ion with charge Z can be obtained as

$$V(\mathbf{r}) = (2\pi)^{-3} \int d^3\mathbf{q}\, e^{-i\mathbf{q}\cdot\mathbf{r}} \tilde{V}(\mathbf{q}), \tag{10}$$

where $\mathbf{r}$ is the position of the projectile electron with respect to the ion core and $\tilde{V}(\mathbf{q})$ is the interaction potential in momentum space $\mathbf{q}$ (Thompson 1962),

$$\tilde{V}(\mathbf{q}) = -\frac{4\pi Ze^2}{q^2\varepsilon(q,\omega)}, \tag{11}$$

where $\varepsilon(q,\omega)$ is the dielectric function of the plasma. After some straightforward manipulations, the dynamic interaction potential can then be written as

$$V(\mathbf{r}) = -\frac{Ze^2}{\pi} \int_0^\infty dq_\perp \frac{q_\perp J_0(q_\perp b)}{q_\perp^2 + q_\parallel^2} \int_{-\infty}^\infty dq_\parallel \frac{e^{-iq_\parallel z}}{\varepsilon(q_\perp, q_\parallel, q_\parallel v)}, \tag{12}$$

where we set $\mathbf{q} = \mathbf{q}_\perp + q_\parallel \hat{\mathbf{z}}$ and $d^3\mathbf{q} = d^2\mathbf{q}_\perp\, dq_\parallel$. In obtaining (12), we have used the following identity for the zeroth-order Bessel function (Abramowitz and Stegun 1965):

$$\int_0^{2\pi} d\varphi\, e^{-iq_\perp b\cos\varphi} = 2\pi J_0(q_\perp b), \tag{13}$$

where φ is the angle between $\mathbf{q}_\perp$ and $\mathbf{b}$. Then the first-order eikonal phase becomes

$$\frac{1}{k}\chi(\bar{b}) = \frac{1}{\pi\bar{E}} \int_{-\infty}^\infty d\bar{z} \int_0^\infty dQ_\perp\, Q_\perp J_0(Q_\perp \bar{b}) \int_{-\infty}^\infty dQ_\parallel \frac{e^{-iQ_\parallel \bar{z}}}{(Q_\perp^2 + Q_\parallel^2)\,\varepsilon(Q_\perp, Q_\parallel, a_\Lambda)}, \tag{14}$$

where we have introduced the scaled parameters

$$Q_\perp \equiv q_\perp a_z, \qquad Q_\parallel \equiv q_\parallel a_z, \qquad \bar{z} \equiv \frac{z}{a_Z},$$

and $\bar{b} \equiv b/a_Z$ is the scaled impact parameter.

Important departures from equilibrium can be expected in many laboratory plasmas due to the strong external actions generating them. This is the case for plasmas of interest for fusion, which are confined by magnetic or laser fields, and for plasmas generated from gaseous or solid targets through electric

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discharges or electron beams. In laboratory plasmas, the interaction of the injected external energy with the target system most often generates hot electrons. These so-called superthermal electrons escape the Maxwellian distribution corresponding to the bulk of electrons, and can be modelled more effectively by a generalized Lorentzian (or kappa) distribution (Meng et al. 1992). Hasegawa et al. (1985) showed theoretically that the equilibrium plasma distribution function in the presence of a superthermal radiation field resembles the kappa distribution. Thus we assume that the plasma has a high-energy tail for the particle distribution described by the Lorentzian (kappa) distribution function with spectral index κ of the form

$$f_\kappa(v) = n\left(\frac{m}{2\pi\kappa E_\kappa}\right)^{3/2}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\left(1+\frac{mv^2}{2\kappa E_\kappa}\right)^{-\kappa^{-1}}, \tag{15}$$

where n is the number density, m is the electron mass, v is the velocity of the plasma electron, $E_\kappa = kT(1-3/2\kappa)$ is a characteristic energy and Γ is the gamma function. This distribution function has some important features: first, at high velocities, the distribution follows an inverse power law (Jung 1994), and secondly, for all velocities, in the limit as the special index $\kappa \to \infty$, the distribution becomes a simple Maxwellian distribution function:

$$f_{\kappa\to\infty}(v) \to f_M(v) = n\left(\frac{m}{2\pi E_M}\right)^{3/2}\exp\left(-\frac{mv^2}{2E_M}\right), \tag{16}$$

where $E_M \equiv E_{\kappa\to\infty} = kT$ is the Maxwellian characteristic energy. For smaller $\kappa > 1$, the distribution (15) possesses a high-velocity tail. Here we discuss the dielectric function in this Lorentzian model, with the electron distribution function being assumed to satisfy the Vlasov equation for an unmagnetized plasma (Sitenko and Malnev 1995; Brambilla 1998):

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \frac{e}{m}\mathbf{E}\cdot\frac{\partial f}{\partial \mathbf{v}} = 0. \tag{17}$$

By linearizing this Vlasov equation, the dielectric function in the Lorentzian plasma has been obtained by Summers and Thorne (1991) and Meng et al. (1992):

$$\varepsilon_\kappa = 1 + (q\Lambda)^{-2}\frac{\kappa}{\kappa-\frac{3}{2}}\left[\frac{\kappa-\frac{1}{2}}{\kappa} + \xi\mathbf{Z}_\kappa(\xi)\right], \tag{18}$$

where

$$\xi = \frac{\omega}{\theta_\kappa q},$$

$$\theta_\kappa = \left(\frac{2\kappa-3}{\kappa}\right)^{1/2}\left(\frac{T_e}{m}\right)^{1/2}$$

is the electron thermal velocity in a Lorentzian plasma, $\mathbf{v}$ is the velocity of the projectile electron, and the spectral index κ satisfies the condition $\kappa > \frac{3}{2}$. Here $\mathbf{Z}_\kappa(\xi)$ is the modified plasma dispersion function in a Lorentzian-distribution plasma (Summers and Thorne 1991):

$$\mathbf{Z}_\kappa(\xi) = \frac{1}{\pi^{1/2}\kappa^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\int_{-\infty}^{\infty}\frac{dx}{(x-\xi)\,(1+x^2/\kappa)^{\kappa+1}}. \tag{19}$$

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Here (18) involves the non-zero frequency $\omega = \mathbf{q}\cdot\mathbf{v}$; it is therefore said to take account of dynamic screening (Lifshitz and Pitaevskii 1981). Since in the limit as $\kappa \to \infty$, the Lorentzian distribution becomes the Maxwellian distribution, the dielectric function (18) reduces to the form for the Maxwellian (Jung 1997a). The plasma dielectric function in a Lorentzian-distribution plasma can be rewritten in the form

$$\varepsilon_\kappa(Q_\perp, Q_\parallel, a_\Lambda) = 1 + \frac{a_\Lambda^2}{Q_\perp^2 + Q_\parallel^2} \frac{\kappa - \frac{1}{2}}{\kappa - \frac{3}{2}} [1 + \Sigma_\kappa(\xi)], \tag{20}$$

where $a_\Lambda \equiv a_Z/\Lambda$ is the scaled reciprocal Debye length and $\Sigma_\kappa(\xi)$ is the dynamic polarization function:

$$\Sigma_\kappa(\xi) = \frac{z}{\pi^{1/2}\kappa^{1/2}(\kappa - \frac{3}{2})} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa - \frac{1}{2})} \int_{-\infty}^{\infty} \frac{dx}{(x-\xi)\,(1 + x^2/\kappa)^{\kappa+1}}, \tag{21}$$

with

$$\xi = \xi(Q_\perp, Q_\parallel, \tilde{\theta}_\kappa) = \frac{Q_\parallel}{\tilde{\theta}_\kappa (Q_\perp^2 + Q_\parallel^2)^{1/2}} \tag{22}$$

and $\tilde{\theta}_\kappa \equiv \theta_\kappa/v$. After some straightforward manipulations, the first-order eikonal phase is found to be

$$\frac{1}{k}\chi_\kappa(b) = \frac{2}{\bar{E}^{1/2}} \int_0^\infty dQ_\perp \frac{J_0(Q_\perp \bar{b})}{Q_\perp\, \varepsilon_\kappa(Q_\perp, 0, a_\Lambda)}, \tag{23}$$

where

$$\varepsilon_\kappa(Q_\perp, 0, a_\Lambda) = 1 + \frac{a_\Lambda^2}{Q_\perp^2} \frac{\kappa - \frac{1}{2}}{\kappa - \frac{3}{2}}$$

which is independent of the ratio $\tilde{\theta}_\kappa$ of the electron thermal velocity to the projectile velocity. Using the definition of the zeroth-order modified Bessel function K_0 (Whittaker and Watson 1927),

$$K_0(\alpha) = \int_0^\infty dt \frac{t J_0(\alpha t)}{1 + t^2}, \tag{24}$$

the first-order eikonal phase for elastic electron–ion collisions in a Lorentzian plasma is then found to be

$$\frac{1}{k}\chi_\kappa(\bar{b}) = \frac{2}{\bar{E}^{1/2}} K_0\left(\left(\frac{\kappa - \frac{1}{2}}{\kappa - \frac{3}{2}}\right)^{1/2} a_\Lambda \bar{b}\right). \tag{25}$$

It is quite interesting to point out that the first-order eikonal phase (25) obtained from the dynamic interaction potential $V(\mathbf{r})$, (12), is found to be independent of the ratio of the electron thermal velocity to the projectile velocity, $\tilde{\theta}_\kappa = \theta_\kappa/v$, and depends only on the Debye length Λ and the spectral index κ. This is due to the symmetric property of the interaction potential and the SL trajectory method. Hence the scaled EDES cross-section for high-energy electron projectiles per unit scaled impact parameter is obtained as

$$\frac{d\sigma_\kappa^E}{d\bar{b}} \Big/ \pi a_Z^2 = 2\bar{b} \left| \exp\left[\frac{2i}{\bar{E}^{1/2}} K_0\left(\left(\frac{\kappa - \frac{1}{2}}{\kappa - \frac{3}{2}}\right)^{1/2} a_\Lambda \bar{b}\right)\right] - 1 \right|^2. \tag{26}$$

This EDES cross-section is also independent of the velocity of the plasma electron, θ_κ. In order to explicitly investigate the plasma screening effect on the

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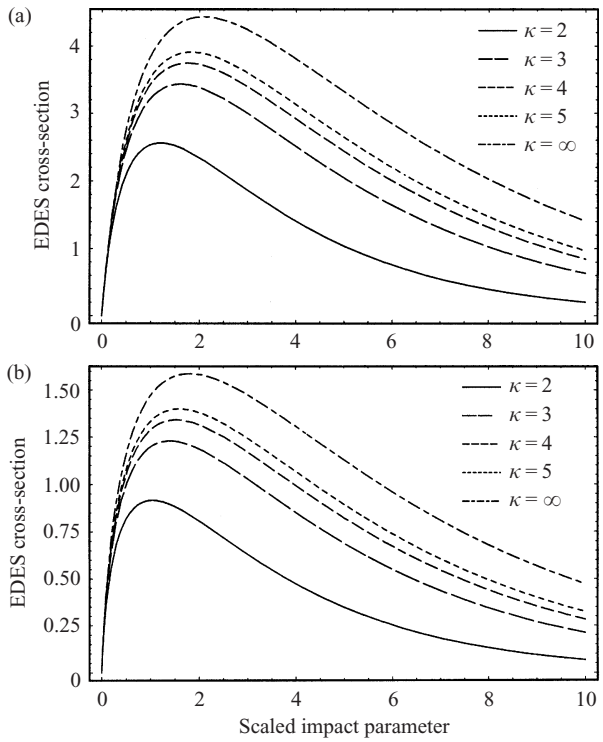


Figure 1. The scaled eikonal differential elastic scattering cross-sections (20) in units of πa_Z^2 as functions of the scaled impact parameter $\bar{b} \equiv b/a_Z$ for several values of the spectral index κ with $a_\Lambda = 0.1$: (a) $\bar{E} = 10$; (b) $\bar{E} = 30$.

EDES cross-section, specifically, we consider five cases of the spectral index ($\kappa = 2$, 3, 4, 5 and ∞) and two cases of the projectile energy ($\bar{E} \equiv E/Z^2 Ry = 10$ and 30), since the eikonal method is known to be valid under the high-energy condition, i.e. $E \gg |V|$. Figure 1 shows the scaled EDES cross-sections for electron–ion collisions in a Lorentzian plasma for $a_\Lambda = 0.1$. As we can see in this figure, the EDES cross-section substantially decreases with increasing projectile energy. The dynamic plasma screening effect on the elastic cross-section in the first-order eikonal approximation turns out to be a static screening effect, and depends only on the Debye length and spectral index, i.e. it is independent of the electron thermal velocity in a Lorentzian plasma. It is also found that the plasma screening effect on the eikonal differential cross-section is found to be more significant for smaller spectral indices. Hence the EDES cross-section increases with increasing spectral index. The maximum values of the EDES cross-sections are listed in Table 1. The positions of the maxima are also indicated in parentheses. The position of the maximum of the EDES cross-section, i.e. the position at which the elastic collision process takes place, recedes from the target ion with increasing spectral index for a given projectile energy. It is also found that the maximum position is shifted towards the target ion with increasing projectile energy. Since the argument of the first-order eikonal phase is proportional to $a_\Lambda \bar{b}$, the position of the maximum of the eikonal cross-section increases with increasing Debye length Λ.

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Table 1. The maximum values of the scaled eikonal differential elastic scattering cross-sections (20) for several values of the spectral index κ with $a_\Lambda = 0.1$.

κ	$\frac{d\sigma_\kappa^E(\bar{E}=10)}{d\bar{b}\,\pi a_Z^2}$†	$\frac{d\sigma_\kappa^E(\bar{E}=30)}{d\bar{b}\,\pi a_Z^2}$‡
2	2.56007 (at $\bar{b}=1.2$)	0.91607 (at $\bar{b}=1.0$)
3	3.43463 (at $\bar{b}=1.6$)	1.22958 (at $\bar{b}=1.4$)
4	3.74698 (at $\bar{b}=1.8$)	1.34148 (at $\bar{b}=1.5$)
5	3.90919 (at $\bar{b}=1.9$)	1.39994 (at $\bar{b}=1.6$)
∞	4.43407 (at $\bar{b}=2.1$)	1.58738 (at $\bar{b}=1.8$)

† The scaled EDES cross-sections for $\bar{E}=10$.
‡ The scaled EDES cross-sections for $\bar{E}=30$.

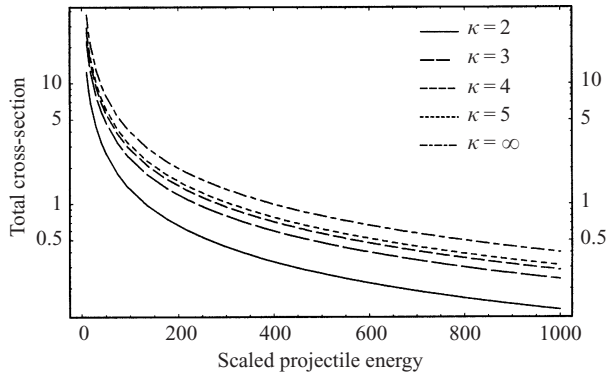


Figure 2. The scaled total electron–ion elastic scattering cross-sections (27) in units of πa_Z^2 as functions of the scaled projectile energy $\bar{E} \equiv E/Z^2 Ry$ for several values of the spectral index κ with $a_\Lambda = 0.1$.

From (7b) and (25), the scaled total elastic electron–ion collision cross-section $\sigma_{\mathrm{tot},\kappa}^E$ in units of πa_Z^2, is given by

$$\frac{\sigma_{\mathrm{tot},\kappa}^E}{\pi a_Z^2} = 2\int_0^\infty \bar{b}\,d\bar{b}\left|\exp\left[\frac{2i}{\bar{E}^{1/2}}K_0\left(\left(\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\right)^{1/2}a_\Lambda\bar{b}\right)\right]-1\right|^2. \tag{27}$$

Figure 2 shows the scaled total electron–ion elastic scattering cross-sections for $\bar{E} \geqslant 10$ for several values of the spectral index κ. As we can see in this figure, the plasma screening effect is also found to be more significant for smaller spectral index.

4. Conclusions

We have investigated elastic electron–ion collisions in a Lorentzian (kappa)-distribution plasma using the eikonal method. The dynamic electron–ion interaction potential has been obtained by the introduction of the plasma

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dielectric function. The semiclassical straight-line trajectory method has been applied to the path of the projectile electron in order to visualize the variation of the eikonal differential cross-section as a function of the impact parameter, Debye length and spectral index. In the first-order eikonal approximation, the dynamic plasma screening effect on the eikonal phase becomes a static screening effect and depends only on the Debye length and spectral index. The elastic differential elastic scattering cross-section decreases with increasing projectile energy. It is found that the plasma screening effects on the eikonal differential cross-section are more significant for smaller spectral index. Hence the minimum screening effect on the elastic cross-section is given for $\kappa \to \infty$, i.e. a Maxwellian plasma. The eikonal cross-section increases with decreasing spectral index. The position of the maximum of the eikonal cross-section recedes from the ion core with increasing spectral index. With an increase in the projectile energy, the position of the maximum is shifted towards the target nucleus for a given spectral index. It is also found that the plasma screening effect on total scattering cross section increases with decreasing spectral index. These results provide a general description of elastic electron–ion collisions in a Lorentzian plasma.

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