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Craig on the actual infinite

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Abstract: **In a series of much discussed articles and books, William Lane Craig defends the view that the past could not consist in a beginningless series of events. In the present paper, I cast a critical eye on just one part of Craig's case for the finitude of the past – viz. his philosophical argument against the possibility of actually infinite sets of objects in the 'real world'. I shall try to show that this argument is unsuccessful. I shall also take a close look at several considerations that are often thought to favour the possibility of an actual infinite, arguing in each case that Craig's response is inadequate.**

In a series of much discussed articles and books, William Lane Craig has vigorously defended the view that the past could not consist in a beginningless series of events.[1] Craig's goal, of course, is to make a strong case for the existence of God. If the past has a beginning, then so does the universe, and a familiar line of argument suggests that there must be a First Cause.[2] In the present paper, I cast a critical eye on just one part of Craig's case for the finitude of the past – viz. his philosophical argument against the possibility of actually infinite sets in the 'real world'.[3] If this argument were to succeed, then an actually infinite series of past events would have been proved impossible, and we could go on to ask about the cause of the very first event. However, I do not believe that Craig has succeeded in proving that actually infinite sets are impossible. As far as this particular line of argument is concerned, I shall try to show that it remains an open question whether the past could consist in a beginningless series of events. I shall also take a close look at several considerations that are often thought to favour the possibility of an actual infinite, arguing in each case that Craig's response is inadequate.

'Absurd implications'

Craig's main line of argument against the possibility of an actual infinite charges that 'various absurdities' would result 'if an actual infinite were to be

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instantiated in the real world'.[4] For example, there would be no logical bar to the existence of an 'infinite library' in which every other book has a red cover and the rest have black covers. Such a library would have some quite remarkable – and, to Craig's way of thinking, utterly impossible – properties. One could add as many books as one liked without increasing the number of books in the library. One could remove any finite number of books without decreasing the size of the library. And since the library would have no more red- and black-covered books together than black-covered ones alone, one could remove all the red-covered books without decreasing the number of books in the collection.

Doubtless this feature of the infinite library is 'weird' and 'paradoxical'. But is it genuinely absurd and impossible? Is it a metaphysically necessary truth that the number of elements in any legitimate set must be greater than the number of elements in any of its proper subsets? This, I believe, is the central issue, and in the next section, I shall consider what we should say about it. But there are a number of other supposedly 'absurd' implications of the actual infinite that Craig also likes to emphasize. The rest of this section will be devoted to them. For example, Craig claims that the gaps created by the removal of all the red-covered books from the shelves of our imaginary library could be filled without adding any new books. 'The cumulative gap created by the missing books would be an infinite distance, yet if we push the books together to close the gaps, all the infinite shelves will remain full'.[5]

Craig's way of putting this point is a bit misleading, since we obviously cannot 'push' any of the books together without creating some new shelf space. If we were to 'push' even two books together, we would have created space for at least one new book. What *is* true, however, is that the infinitely many remaining books could – at one stroke, so to speak – be assigned to new spaces in such a way that no gaps would be left on the shelves. All parties to the present dispute would presumably agree that such a monumental reassignment is physically impossible, but Craig holds something stronger – viz. that there is no possible world in which such a 'closing of the gaps' by way of 'infinite reassignment' could be accomplished. God Himself could not pull off such a thing.

Even if Craig is right about this, and it really is absurd to suppose that the 'cumulative gap' created by the removal of the red books could be 'closed' by cleverly moving the books to different locations, it does not immediately follow that infinite sets *in general* are impossible. Before drawing so sweeping a conclusion, we need to consider what it is in the example that produces the (allegedly) absurd implication. The answer, I think, can be found in the way in which the number of elements in the set interacts with other features of the example. A library is a collection of *coexistent* objects (books and shelves) whose physical relationship to one another can be *changed*. It is only when these features are *combined* with the property of having infinitely many elements that we get this particular sort of implication. If the infinitely many books and spaces for books did not exist at the

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same time, there could be no thought of rearranging them. And a fortiori, if it were (metaphysically) impossible to move the books to different locations, it would again be impossible to rearrange them.

The upshot is that even if it really is 'absurd' to suppose that one could close the empty spaces in Craig's infinite library merely by rearranging the books on the shelves, this particular sort of 'absurdity' cannot be duplicated for just any actually infinite set.[6] Significantly, it cannot be duplicated for an infinite series of past events. Events that have happened are fixed in their temporal locations. They cannot be changed or rearranged in such a way as to open or close temporal locations. Even if it should turn out that *other* absurdities afflict the idea of an infinite past,[7] this cannot be one of them.

Yet another twist to the infinite library scenario that Craig likes to emphasize goes like this. If an infinite library existed, its books could be numbered sequentially, beginning with 0, in such a way that every natural number is printed on the spine of exactly one book. In that case, Craig claims, it would be impossible to *add* any books to the library, since all the numbers for such additions would already have been 'taken'. It would be necessary to create a new number – something that is obviously impossible. Since one can always add a book to any real library, Craig thinks it follows that an actually infinite library is impossible.

You might think that the books in an infinite library could simply be renumbered so as to release a number for the new book. For example, book 0 might be renumbered as book 1, book 1 as book 2, and so on. Zero would then be available. But Craig rejects this possibility, on that ground that it would violate 'the initial conditions laid down in the argument', according to which '[w]e are to imagine a series of consecutively numbered books beginning at 0 and increasing infinitely, not a series of books numbered from some finite number'. 'Once the objects are numbered as stipulated', Craig says, 'reassigning the numbers to begin with the proposed addition seems impossible'.[8]

It is not immediately clear why Craig thinks such a reassignment would contradict the 'initial conditions' of the example. What we were asked to suppose was that the books in the library were numbered from zero onwards *before* any new books were added – *not* that the numerals on the spines of the books could not be changed to accommodate new books. We would be violating the 'initial conditions' of the example only if we supposed that the books were not *initially* assigned to the natural numbers, beginning with zero.

Craig appears to think that the set of books in the library must be just as complete and unalterable as the set of natural numbers itself:

> **Is it possible to add a new integer to the series of natural numbers? Of course not, for the natural number series is determinate and complete … . But just as we cannot add more numbers to the natural numbers, so we cannot add more books to an infinite collection of books each of which stands in a one-to-one correspondence with the natural numbers … . Therefore, just as the collection of**

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> **natural numbers could not be added to, neither could an actually infinite collection of books – which is absurd.**[9]

This argument is deeply flawed. It is true, of course, that one cannot create a new natural number and add it to the set of all natural numbers. That is because the set of natural numbers is the 'determinate and complete' set *of all possible natural numbers*. But an infinite library – even one whose volumes have been placed in one-to-one correlation with the series of natural numbers – is not to be confused either with the *complete* set of *all possible books*, or even with the set of all actual books. So, contrary to what Craig seems to assume, there is no reason at all to think that the way in which the books are correlated with the natural numbers (via the numerals on the spines of the books) must share the immutability of the natural numbers themselves. Nor, consequently, is there any reason to deny that the books in the infinite library could be renumbered in such a way as to make room for a new one.

It is true, of course, that *we* could never complete the process of replacing the numerals on the spines of the books, since *we* could only renumber finitely many books at a time. But that is merely a reflection of our limited power, and has nothing to do with the possibility of an infinite set. Presumably God would have enough power to do the job – 'at one stroke', so to speak – without having to start out on an endless series of renumberings.

Analogous errors are present in another of Craig's arguments – this one directed specifically against the infinite past. He asks us to imagine someone 'who claims to have been counting from eternity, and now … is finishing: -5, -4, -3, -2, -1, 0'.[10] Craig argues that this is impossible, on the ground that the count should have been finished before now. Indeed, he says, if the past is really beginningless the count should have been finished before any given time in the past. Since the count must always already have been finished, there in no time at which such a person could be finishing his count. The moral: just as no one could have been counting numbers 'from eternity', so too there cannot have been 'events from eternity'.[11]

The parallel with the previous argument should be obvious. Just as Craig thinks all the numbers have been 'used up' in our infinite library, so too he apparently thinks that at any stage of his count, our infinite counter must already have 'used up' all the numbers, so that no number is left 'free' for use at that stage.

But this is the wrong way to look at the infinite counter's story. It is true that at any moment in the past, the man had already counted off infinitely many numbers, but it does not follow that he had already counted off *all* the numbers or that he had already reached zero. Perhaps that *could* have been the way the man's count went. But it was not, and so zero is still available today. Ironically, it is Craig who is here guilty of tampering with the 'initial conditions' of the example!

Another point worth noting is that, unlike the numerals on the spines of the books in our infinite library, which *can* at least in principle be changed, the series

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of past 'countings' cannot be changed precisely because they are *past*. If, one minute ago, the man did in fact reach -1 then zero is still available and there is nothing Craig or anybody else can do about that. It's no use trying to 'renumber' the man's past in such a way that zero is already taken!

Yet another supposedly 'absurd' implication of the actual infinite that Craig likes to stress concerns inverse mathematical operations. Here, he says, the actual infinite entails out-and-out 'logical contradictions', making the case against the actual infinite absolutely 'decisive'.[12]

Precisely what 'logical contradictions' does Craig have in mind here? As nearly as I can tell, they are supposed to emerge in the following passage, in which Craig once again illustrates his point by reference to the infinite library.

> **Suppose books 1, 3, 5, … are loaned out. The collection has been depleted of an infinite number of books, and yet we are told that the number of books remains constant … . But suppose we were to loan out books 4, 5, 6, … . At a single stroke the collection would be virtually wiped out, the shelves emptied, and the infinite library reduced to finitude. And yet, we have removed *exactly the same number* of books this time as when we removed books 1, 3, 5 … .**[13]

Well, yes, it's true that if all the odd-numbered books were removed, infinitely many books would remain, whereas if all the books numbered 4 and higher were removed, only four books would remain in the library. {0, 1, 2, 3} has only four members, whereas {0, 2, 4, …} is denumerably infinite. But where is the contradiction?

Craig appears to be assuming that certain familiar arithmetical operations can be performed on the number of elements in any legitimate set. Given this assumption, perhaps we can see what the 'contradiction' is supposed to be. Let m = the number of books in our infinite library, n = the number of odd-numbered books, and o = the number of books numbered 4 or higher. Then perhaps Craig's argument goes like this:

$(m-n)$ = infinity, whereas $(m-o) = 4$.

But,

$n = o$ (since both n and o are infinite).

It follows that we get inconsistent results subtracting the same number from m.

Or do we? If we say that $(m-n)$ = infinity, but that $(m-o) = 4$, we are not actually subtracting *numbers* at all. What we are doing instead is imagining various subsets of books in our infinite library being 'removed' from the library, and then determining the cardinality of the subset that 'remains'. When the set of books, {4, 5, 6, …}, is 'removed', {0, 1, 2, 3} 'remains'. Its cardinality is 4. When {1, 3, 5, …} is 'removed', {0, 2, 4, …} 'remains'. Its cardinality is $\aleph_0$. There is no logical inconsistency so far. But what if we insist on subtracting the *numbers*, n and o, respectively, from m? Even then, we will not get inconsistent results. For no matter how

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($\aleph_0 - \aleph_0$) is defined, both ($m - n$) and ($m - o$) will produce exactly same 'remainder', since m, n, and o just are $\aleph_0$.

But of course there is no definition for ($\aleph_0 - \aleph_0$) in Cantor's system, and in mathematics generally, 'infinity minus infinity' is left undefined.[14] Craig thinks this just goes to show that there is something wrong with infinite sets. But why should we accept Craig's assumption that ordinary subtraction *must* apply to the number of elements in *any* set that can be instantiated in 'the real world?' Why think that what holds true for finite sets must also hold for infinite ones? Here is Craig's explanation:

> **It may be said that inverse operations cannot be performed with the transfinite numbers – but this qualification applies to the mathematical world only, not the real world. While we may correct the mathematician who attempts inverse operations with transfinite numbers, we cannot in the real world prevent people from checking out what books they please from our library.**[15]

And elsewhere he writes: 'mathematicians recognize that the notion of an actually infinite number of things leads to self-contradictions unless you impose some wholly arbitrary rules to prevent this'.[16] Craig here claims that it is 'arbitrary' to deny that inverse operations apply to infinite sets, leaving 'infinity minus infinity' undefined. 'In the real world', he says, addition and subtraction must always be possible. If a person 'checks out' a book from a library, he has 'subtracted' one book. If he 'checks out' all the odd-numbered books, he has 'subtracted' infinitely many books. And so on. So if a mathematician chooses to define a transfinite number (such as Cantor's $\aleph_0$) for which this operation is not possible, he is free to do so – but no set having that number of elements can then be instantiated 'in the real world.'

This argument is badly confused. If a person 'checks out' one or more books, he does indeed remove them from the library – but he is not 'subtracting' them in the arithmetical sense. And even if ordinary arithmetical subtraction is undefined for transfinite numbers, it does not follow that physically removing books from an infinite library is similarly 'undefined', much less that removing books from it is impossible. What follows is only that, since subtraction is undefined for infinite quantities, we cannot automatically assume that the *number* of books is smaller after some of them have been removed. That is indeed a characteristic of the actual infinite, but it is hardly a 'logical contradiction'.

This point can be generalized as follows. Addition and subtraction of *numbers* is one thing; constructing a new set by adding in new members or removing old ones is quite a different thing. Operations of the second sort may be possible even when operations of the first sort make no sense or are undefined. It is only by confounding the two sorts of operation that Craig can imagine that he has derived a 'logical contradiction' from the actual infinite.

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A deeper analysis

So far, we have seen that Craig argues that the instantiation of an actual infinite in reality would lead to absurdity, and even to logical contradiction. But he wants to dig deeper. What is it about the actual infinite, he asks, that generates all of these absurdities? Here is his answer: 'It seems to me that the surd problem in instantiating an actual infinite in the real world lies in Cantor's Principle of Correspondence. The principle asserts that if a one-to-one correspondence between two sets can be established, the sets are equivalent'.[17] Given the Principle of Correspondence, we are forced to say that 'a proper subset of an infinite set is equivalent to the whole set'. Craig finds this result 'strange' because he thinks it runs counter to what he refers to as 'Euclid's maxim'[18] – viz. the intuitively plausible principle that the whole must be greater than a part. As it applies to sets, Craig supposes the maxim says that a set must have a larger number of members than any of its proper subsets.

Craig acknowledges that in mathematics 'Euclid's maxim holds only for finite magnitudes, not infinite ones'.[19] But he is not impressed by this way of dealing with the problem: 'But surely the question that then needs to be asked is, How does one know that the Principle of Correspondence does not also hold for finite collections, but not for infinite ones? Here the mathematician can only say that it is simply defined as doing so.'[20] In Cantor's theory, Craig points out, equivalent sets are 'simply defined' as 'sets having a one-to-one correspondence'. As he sees it, this supports his own contention that the infinite sets of Cantorian set theory cannot be instantiated in the real world. The Principle of Correspondence, he says, 'is simply a convention adopted for use in the mathematical system created by the mathematicians'.[21]

At this point, one might have the impression that Craig has serious doubts about the Principle of Correspondence – at least as applied to sets in the 'real world'. But that is apparently not how he wants to be understood. A few sentences further on, he asserts that both the Principle of Correspondence and Euclid's maxim are 'intuitively obvious'. And since 'counter-intuitive situations' result when they are 'applied to the actual infinite', he concludes that 'the most reasonable approach is simply to regard both principles as valid in reality and the existence of an actual infinite as impossible'.[22]

Craig is a bit difficult to interpret at this point. But I think it is clear that he holds that both the Principle of Correspondence and Euclid's maxim are valid for sets instantiated in the 'real world'. And what he wants to prove is that no infinite sets can be instantiated in 'the real world'. With this understood, we can see how Craig's main line of argument must go. 'In the real world', it says, both of the following principles are true:

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(1) If there is a one-to-one correspondence between two sets, they must have the same number of elements. (This is the Principle of Correspondence.)
(2) The number of elements in a set must be greater than the number of elements in any proper subset of that set.[23] (This is Euclid's maxim, as Craig thinks it applies to set theory.)

But it is also true that:

(3) If there were an infinite set in the 'real world', its elements could be placed in a one-to-one correspondence with a proper subset of itself.

From these premises, it follows that:

(4) There are no infinite sets in the 'real world'.

This way of putting Craig's argument has the merit of making it clear just what the principal issue is. On Cantor's theory, premise (2) is true for finite sets, but not for infinite ones. Craig, on the other hand, insists that (2) would have to be true of any set that is instantiated 'in the real world'. So if infinite sets cannot be squared with premise (2), so much the worse for such sets – they cannot be instantiated 'in reality'.

It should be emphasized that Craig's argument requires the truth of *both* Euclid's maxim (suitably interpreted) *and* the Principle of Correspondence. Without the latter, he cannot derive the (supposedly) 'absurd' consequences from an actual infinite; and without the former, he has no reason for thinking them absurd.

What is there to say in support of the way Craig applies Euclid's maxim to 'real world' sets? Why should we suppose that premise (2) applies to *all* legitimate sets? Craig's stock answer is to point once again to the intuitive 'absurdity' of infinite libraries and hotels and the like.[24] But we need to be careful here. Many of these supposedly 'absurd' implications are so only because they appear to violate Euclid's axiom by leaving open the possibility of sets having no 'more' members than various proper subsets of those sets. If, for example, we think it is absurd to suppose that we can add or remove books from our imaginary library without increasing or decreasing the 'size' of an infinite library, that is because we are already committed to the view that in the 'real world' *all* sets (even infinite ones) are subject to Euclid's maxim. Craig's examples *illustrate* the violation of this principle, and they doubtless stimulate 'anti-infinitist' intuitions in some readers, but they do not *establish* the truth of premise (2) of the argument.

There is also quite a lot to be said *against* the way Craig applies Euclid's maxim to sets. Even if we agree to treat sets as 'wholes' and their proper subsets as 'parts', Euclid's maxim need not be interpreted to mean that the *number* of elements in the whole is greater than the *number* of elements in the part. There is, for example, an obvious sense in which Craig's imaginary library is 'greater' than any of its parts,

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and this is so despite the fact that it does not have a greater *number* of books than they. For instance, the library as a whole is 'greater' ('larger') than the part of the library containing only books numbered 3 and higher simply in virtue of the fact that it contains books numbered 0, 1, and 2 as well as all the higher numbered books. This is all by itself a perfectly legitimate sense of the word 'greater' – one that is logically independent of the question, 'What is the *number* of books in the two sets?'[25]

There is, then, a fairly intuitive sense in which any set – even an infinite one – is 'greater' than any of its proper subsets. Not because the *number* of elements in the greater set is necessarily larger than the number of elements in the lesser one – but merely in virtue of the fact that it 'contains' all the elements in the lesser set plus some others that the lesser one does not contain. That, all by itself, and without any reference to the *number* of elements in either set, is sufficient to make one 'greater' than the other. When it is understood this way, an actually infinite set does *not* violate the principle that the 'whole' is greater than its 'part.'

It is true, of course, that the number of elements in any *finite* set is necessarily greater than the number of elements in any of its proper subsets. But why think this must hold for *all* sets in the 'real world'? Euclid's maxim about wholes and parts may be 'intuitively' obvious; but as we have just seen, this provides little, if any, support for premise (2). I would even go so far as to say that it is 'intuitively obvious' that Craig's infinite library is 'greater' than any of its proper parts, even though it does not have a greater number of books than some of those parts.

Admittedly, if it could be established on independent grounds that only finite sets can exist in the real world, it would follow that Craig's premise (2) holds for *all* real world sets. But that observation is of no use to us here, since the very question we are trying to settle is whether it is true that only finite sets can exist in the real world. To derive the impossibility of the actual infinite from premise (2) (together with some other premises), while at the same time deriving premise (2) from the claim that the real world can contain only finite sets, would be to argue in a very tight – and vicious – circle.

Where does this leave us? The logical situation would seem to be as follows. Premises (1), (2), and (3) (at least in their full generality) are logically incompatible with the possibility of an actual infinite. Something has to go. Craig thinks it is the possibility of an actual infinite that must be rejected. But since he has given no independent argument for affirming (1), (2), and (3), the friends of the actual infinite have just as much right to say that one of those propositions should be rejected. And if, on the basis of Cantor's theory of transfinite numbers, they say that (2) holds only for finite sets, it will be difficult for Craig to come up with an objection that does not beg the question against them. So far, at any rate, he has not done so.

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Existence in reality vs existence in God's mind

In spite of the absurdities – and even 'contradictions' – Craig claims to have found in the idea of an infinite set, he says that he has no intention of 'trying to drive mathematicians from their Cantorian paradise'. He claims only that infinite sets cannot be instantiated 'in reality'.[26]

The first part of this claim is rather puzzling. It is hard to see why mathematicians should be any less concerned about genuine contradictions than anyone else. The rules that govern the 'mathematical realm' are surely not that much more relaxed than those that govern reality!

But the second part of Craig's claim also deserves attention. What, exactly, does he mean by 'real' when he says that infinite sets cannot exist 'in the real world'? Here is his explanation: 'When I say that an actual infinite cannot exist, I mean ''exist in the real world'' or ''exist outside the mind'' … [w]hat I am arguing is that an actual infinite cannot exist in the real world of stars and planets and rocks and men.'[27] It seems, then, that when Craig denies that there is an actual infinite 'in reality', he is denying merely that there is an actual infinite 'outside the mind'. Since there are infinitely many numbers and properties and other abstract entities, it follows that such things can exist only 'in thought' or 'in a mind'. Platonism is out, and conceptualism is in.

This might seem to leave Craig with the following problem. It is obvious that there are infinitely many abstract entities that no human person has ever thought of. In what sense do these things exist 'in the mind'? How can they have even 'conceptual reality' on Craig's view? Craig puts the problem this way.

> **In addition to tangible objects like people and chairs and mountains and trees, philosophers have noticed that there also appear to be abstract objects, like numbers and sets and propositions and properties. These sorts of things seem to have a conceptual reality rather like ideas. And yet it's obvious that they're not just ideas in some human mind. So what is the metaphysical foundation for such abstract entities?**[28]

For a theist, Craig thinks there is an easy solution. He can simply say that abstract entities are 'grounded in the mind of God'.[29] Craig endorses the following statement by Alvin Plantinga: 'It seems plausible to think of numbers as dependent upon or even constituted by intellectual activity. But there are too many of them to arise as a result of human intellectual activity. We should therefore think of them as … the concepts of an unlimited mind: a divine mind.'[30] It seems, then, that numbers (to say nothing of the all other abstract entities in Plantinga's ontology) are to be thought of as ideas or concepts in an infinite mind. For numbers and other abstract entities, *to be* is *to be conceived by God*.

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Are there infinitely many abstract entities in God's mind?

Whatever the merits of Craig's view of the ontological status of abstract entities, it is difficult to see why an actual infinite existing in God's mind should be thought less objectionable than one existing apart from God's mind. For even if 'Cantor's paradise' is located in God's understanding, it seems that many of the paradoxes Craig finds so objectionable can still be generated inside God's mind.

Craig is well aware of this problem, but he thinks it is easily solved. 'In the first place', he says, 'one need not be conceptualizing consciously all that one knows. I know, for example, the multiplication table up to 10 although I am not consciously entertaining any of its individual equations, so that my knowledge of the multiplication table does not imply that I have 102 ideas'.[31] True enough. One need not have, consciously present to one's mind, a distinct idea for each of the operations that one's knowledge of the multiplication table enables one to perform. On the other hand, one must be *capable* of having each of the '102 ideas'. They must be at least *potentially* present within one's mind.

Is that what Craig wants to say about God? That God knows all of mathematics by virtue of having some sort of 'super-disposition' that makes Him *capable* of answering every possible mathematical question? I doubt it. Knowledge that is merely potential in us must somehow (I think Craig would agree) be actual in God. But since the number of mathematical truths (to say nothing of all the other eternal truths concerning properties and propositions and the like) is clearly infinite, it follows – does it not? – that an *actual* infinity is present in God's knowledge.

Not at all, says Craig. The divine conceptualist can consistently deny that there is any multiplicity at all in God's knowledge. How could this be? Craig explains:

> **... the Conceptualist may avail himself of the theological tradition that in God there are not, in fact, a plurality of divine ideas; rather God's knowledge is simple and is merely represented by us finite knowers as broken up into knowledge of discrete propositions and a plurality of divine ideas.**[32]

This new twist in Craig's argument is puzzling, to say the least. First, we are told that the conceptualist would be well advised to hold that abstract entities are ideas in God's mind, since there are simply *too many* of them for any finite mind to contain them all. Then we are informed that there is no multiplicity at all in God's mind – not even a multiplicity of ideas or concepts. How, then, are we to understand the relation between God's knowledge, which is supposed to be simple, and the many abstract entities that are supposed to exist 'in' this simplicity? How is God's awareness of numbers and properties and propositions and other abstract entities to be understood?

One possibility would be to distinguish sharply between God's awareness and the object of His awareness. God's act of awareness might be simple and internally undifferentiated, even if it embraces many different objects. On this picture, it

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would not be right to say that God's mind contains many distinct ideas, but it would be right to say that it contains a single idea *of* many different things.

If this were what Craig meant, it would not help him avoid the actual infinite. Even if God has just one 'idea' of everything, it will have to be an idea *of* something that is complex enough to include all abstract entities (numbers, properties, propositions, and the rest). But in that case, the *object* of God's knowledge will contain an actual infinity of logically distinct items – and this will be so whether or not they all somehow 'depend' on His awareness.

So Craig must have something else in mind. What could it be? Craig offers few details. He cites Aquinas, who holds that God knows everything by knowing His own simple self. Since Craig does not himself accept the full Thomist doctrine of simplicity, he refers the reader to a well-known article by William P. Alston,[33] where it is supposed to have been established that subscribing to the simplicity of God's knowledge 'does not commit one to a full-blown doctrine of divine simplicity'.[34]

In the essay cited by Craig, Alston does not defend divine conceptualism – he is concerned only to argue that God does not have beliefs. The Thomist view of divine knowledge comes into the discussion because Alston takes it to be a paradigmatic example of a non-propositional view of God's knowledge – a view on which it would make no sense to represent God as having beliefs. Although Alston does not recommend the Thomist conception of God's knowledge, he does what he can to make sense of the suggestion that the content of God's knowledge does not consist in a plurality of propositions. However – and this is of critical importance – nothing in Alston's discussion can reasonably be construed as a defence of the claim that there is no other sort of multiplicity either within God's intellect or in the object of God's knowledge. On the contrary, as we are about to see, Alston's best effort to account for the unity of God's knowledge seems implicitly to view it as a unity *in* diversity.

To give us some idea of what it could mean to say that God's knowledge consists in a single non-propositional awareness of everything, Alston suggests that we 'think of divine knowledge as like our initial visual perception of a scene, where we have not yet begun the job of extracting separately statable facts …'.[35] But Alston quickly points out that the analogy is inadequate, for the obvious reason that in human beings non-propositional awareness is far lower in cognitive value than propositional knowledge. So Alston tries another tack, borrowing a pair of categories from F. H. Bradley. On Bradley's account, the immediacy of pure 'feeling' is contrasted with discursive knowledge, and the ultimate goal of thought – not realized in any human being, but only in the Absolute – is a sort of Hegelian synthesis of these two. Alston's idea is that God's knowledge has 'all the richness and articulation' of discursive thought, held together in a 'unity' that is 'as tight and satisfying as that of pure immediacy'.[36] 'Strangely enough', he writes, 'this bit of British Hegelianism serves rather well as a model for the Thomist conception of divine knowledge and of the way in which it compares with human knowledge'.[37]

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On Alston's proposal, then, God's knowledge is certainly not chopped up into a plurality of *propositional states*. On the other hand, it is said to have 'all the richness and articulation' of discursive thought. Even if this 'richness and articulation' does not consist in a multiplicity of *propositional beliefs*, it must surely involve some sort of distinction and variation and multiplicity within the divine intellect. However 'tight and satisfying' the unity of God's knowledge, it must be thought of as a unity within a multiplicity – a one in a many.

It seems, then, that Alston's (tentative) suggestion has no clear bearing on the question whether there must be an actual infinite within God's intellect to 'accommodate' all the abstract entities. Even if God's thought does not contain infinitely many *concepts* and *propositions*, it could still be infinitely articulated. And if, as Craig believes, *all* concepts and propositions that could ever be entertained by a finite mind (a potential infinite, if there ever was one!) are somehow embedded in a single divine thought, it might seem that this thought *must* be infinitely articulated. To be sure that this is not so, we would need to know quite a lot more about the 'articulation' of God's thought and about the way in which all the particular concepts and propositions and truths of human knowledge are related to it.

It will doubtless be said that on any theory of God's knowledge, it is bound to be utterly beyond our understanding – and that our inability to answer all the hard questions about its relation to particular concepts and propositions is not therefore a special mark against the simplicity theory. There may be something to this. In the present context, however, such a response would be completely inadequate. For Craig has endorsed a *theory* of the nature of abstract entities. Following Plantinga, he says that they have *conceptual reality* in the mind of God. Numbers, properties, propositions, and the rest are said to exist insofar as they are all somehow present in God's understanding. Craig needs to tell us what this claim means when it is combined with the view that there is no multiplicity within God's understanding.

On Plantinga's view, there is no such problem. Properties and numbers are God's concepts. Propositions are God's thoughts. In a paper favourably cited by Craig, Plantinga writes:

> **… a proposition exists because God thinks or conceives it. For propositions, as I see it, are best thought of [as] the thoughts of God … . As we know, serious difficulties attend the claim that propositions are *our* thoughts; these difficulties fall away for the claim that propositions are *God's* thoughts.**[38]

On the face of it, Plantinga's divine conceptualism is inconsistent with Alston's suggestion that God's thought is non-propositional. And in view of the vast multitude of logically distinct properties and numbers and propositions, it also seems inconsistent Craig's claim that there is no multiplicity in God's intellect.

Clearly, Craig has some explaining to do. He needs to come up with a version of divine conceptualism that makes sense of the idea that properties and numbers

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and propositions are somehow embedded in God's simple, non-propositional thought. And he must do this without introducing any sort of multiplicity into God's knowledge. It's a tall order, and a blanket appeal to the 'otherness' and mystery of God's knowledge is plainly insufficient to fill it.

An infinite body of truth about the future?

A related area in which I think it is difficult for Craig to avoid the actual infinite concerns the future. Craig thinks that God (and we) will live forever. So there is a clear sense in which he is committed to the view that the series of future events is infinite. Craig argues that this is not an *actual* infinite, on the ground that the future is not real. Future events, he says, are not *real* until they happen. While there is no limit to the number of new events that will be added to those that have already taken place, we never arrive at a time at which *all* of them have happened. Since we never arrive at a completed infinity, Craig thinks the future is only *potentially* infinite.

This does not completely eliminate the problem, however, since Craig also maintains that there is (already) a *complete body of truth* about the future. According to him, there are no truth value gaps for future contingents. God already knows everything that will ever happen.[39]

Suppose, then, that there is an angel whose job is simply to 'count minutes'. At the moment of creation, the angel said 'one'. A minute later, he said 'two', and so on. The angel will continue in this manner forever. (Perhaps he is a 'fallen' angel, and this is his punishment.) From this supposition, it follows that there are infinitely many logically distinct 'truths' about what the angel will say at a given moment in the future. These truths constitute an *actual* (and not merely a potential) infinity, since they are *already* true. It is not just that the angel *can* always count another minute – it is already true that he *will* do so.

This does not mean, of course, that there will ever be a time at which the angel has finished his count. But the fact remains that each of his infinitely many 'countings', not only *can*, but *will* take place – one for each of the natural numbers greater than the one at which he has just arrived. For each of those infinitely many minutes there is *already* a distinct *truth* about what the angel will be saying at that time. And this is sufficient to bring back the main feature of the actual infinite that Craig finds so objectionable.

To see this, one need only observe that the number of 'truths' concerning the angel's use of even numbers in his 'count' is equal to the number of 'truths' concerning his use of odd-or-even numbers. But the former is a proper subset of the latter. So if we insist on applying both the Principle of Correspondence and Euclid's maxim (as Craig understands it) to this situation, we fall into contradiction.

Craig's solution to this problem is identical to the one he proposes for the

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seemingly infinite number of abstract entities. Due to our human limitations, we have to break Truth up into distinct propositions. Since *we* never arrive at a *complete* body of truths about the future, there is (for us) never more than a *potential* infinity of such truths. But God's knowledge is not broken up into an actually infinite set of distinct propositions. He does not know the future by knowing infinitely many distinct 'truths' about what is going to happen. God knows Truth, not 'truths'. Neither in God's case nor in ours, then, is there an *actual* infinity of truths.[40]

I find this 'solution' very perplexing. First, there is the problem, already discussed in the previous section of this paper, of saying how the many particular truths are supposed to be related to God's knowledge. Are there any natural points of division within the one Truth? If so, how have we avoided the actual infinite? If not, then how is it that the many truths are embedded within it? But now we can see that it's actually worse than that. How, within the compass of a single internally undifferentiated Truth – a Truth containing no multiplicity – could there be any distinction between different sorts of truth? For example, between necessary truth and contingent truths? Or between contingent truths about the future and contingent truths about the past?

Nor is this all. For theists who, like Craig, hold a dynamic theory of time, God's knowledge of time must undergo continual change. Which events are still future, which are present, and which are now past? According to the dynamic theory, the correct answer is constantly changing, as more and more future events become present, present events become past, and past events sink farther and farther into the past. Since 'the body of tensed facts is constantly changing', Craig concludes that

> **… a being which only knew all tenseless facts about the world, including which events occur at any date and time, would still be completely in the dark about tensed facts. He would have no idea at all of what is now going on in the universe, of which events are past and which are future. On the other hand, any being which *does* know tensed facts cannot be timeless, for his knowledge must be in constant flux, as the tensed facts known by him change.**[41]

Assuming, then, that God is omniscient, His knowledge must somehow embrace all the facts, including 'tensed' ones. It follows that – in a certain small but important respect – His knowledge undergoes continual change. I do not see any way to square this claim of Craig's with the alleged 'simplicity' of God's intellect. How, if there is no multiplicity in God's knowledge, can we distinguish between what does and does not change within it? Nor do I see any way to square the 'simplicity' thesis with the claim that God's knowledge is non-propositional. How, otherwise, could we distinguish between the 'part' of God's knowledge that concerns the past and the 'part' that concerns the future?

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The infinite divisibility of space and the actual infinite

Whatever may be said about the 'reality' of abstract entities or about the number of 'truths' about the future, few philosophers would deny that space exists in the 'real world'. So if it could be shown that space has properties that entail the existence of an actual infinite, we would have good reason to believe that an actual infinite not only can, but does, exist in reality. For example, Michael Tooley has pointed out that if space is real and continuous, there must be infinitely many different finite sub-segments in a given chunk of extension. This is not a merely potential infinite, since all the infinitely many sub-segments exist in reality.[42]

Craig replies by denying that space is 'continuous in the sense of being composed of … infinitely many points'.[43] But it is not at all clear that Tooley's argument presupposes that space is composed of points. Even if a region of space is not composed of points, it may still be true that there are infinitely many finite sub-segments within it.

Craig might reply that this misses his point. The fact that space is not composed of points entails that space does not come already 'chopped up' into sub-regions. There are, so to speak, no natural points for division within a region of pure space. It is divided up into two or more parts only when someone (at least in thought) makes that division. Any such region is, of course, infinitely divisible – but the 'parts' into which it can be divided are not 'there' until someone (at least in thought) marks them out. And since no one could complete *all* the possible divisions, they are only *potentially* 'there'. So what we have here is, after all, only a *potential*, and not an actual, infinite.

This is probably the best reply available to Craig, but I do not find it convincing. It seems to me that what follows from the lack of natural boundaries within a region of space is not that the infinitely many sub-regions are not actually 'there', but only that they are not 'there' *apart from a specified way of dividing things up*.

It is not difficult to come up with a specification relative to which the number of coexistent sub-regions is infinite. Just as we can specify the set of natural numbers all at once by the single rule, 'starting with one, add one to the previous sum ad infinitum', so too, I suggest that we can specify all the sub-regions of a given region *R relative to the following rule*: 'starting with *R*, divide the results of the previous division by half ad infinitum'. We do not have to rely on *natural* points of division within *R* to apply this rule to *R*. Nor do we need to *complete* the series of divisions in order to know that, *relative to this rule*, there is an *actual* – and not merely a potential – infinity of sub-regions. At least that's how it seems to me.

But even if I am wrong about this, it is interesting to note that Craig's reply to the 'infinite divisibility' objection is available only when the 'divisible' entity is completely homogenous – or at least when there is not an infinity of natural points for division within it. Space may be as good a candidate for this sort of treatment as we are likely to see. But as we saw above, God's knowledge is not. Even if it is

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concentrated in a single thought, God's knowledge will probably have to include enough internal multiplicity to support the claim that all abstract entities and all truths are somehow embedded in it.

Could space have been Euclidean?

As we have seen, Craig's main line of argument against the actual infinite relies heavily on Euclid's maxim ('the whole is greater than a part'). But there is another Euclidean intuition that seems to tell us that space is infinite. Euclid's second postulate ('a finite straight line can be extended continuously in a straight line') seems to entail that there can be no 'end' to space. A line cannot be 'extended' unless there is somewhere for it to 'go', and in non-curved, Euclidean space, at any rate, it cannot retrace its steps. So it seems that Euclidean space must be infinite. The line itself is only potentially infinite (it 'can' always be extended), but the space in which it can always be extended must be an actual infinite.

But now we know better, you may say. Space is not Euclidean and it is not infinite. Fair enough. But Euclidean intuitions are real – real enough to have made non-Euclidean geometries a very hard sell. Indeed, non-Euclidean geometry remains extremely counterintuitive. To see this, imagine yourself travelling in a straight line. At the level of raw, untutored intuition, does it not seem impossible that you could ever arrive at the 'end of space' – at a 'here' beyond which there is no 'there'?

What this shows, I suggest, is that even very strong 'intuitions' sometimes have to be given up or qualified. The friends of the actual infinite may well think that Euclid's maxim about wholes and parts (as interpreted by Craig) is a case in point. However plausible it may seem at first glance, further reflection shows that it applies only to finite sets, and not to infinite ones.

But this is not all. Even if space is not in fact Euclidean, it seems obvious that it could have been.[44] There are possible worlds, so to speak, in which parallel straight lines never meet and in which finite straight lines can be extended indefinitely. In such worlds, space *is* actually infinite. So an actually infinite space is at least possible. I do not say that space *is* infinite. But I see no good reason to deny that it could have been.

In view of Craig's response to the infinite divisibility problem, you might expect him simply to deny that Euclidean space is infinite in the sense that is at issue here. If space is not composed of points – if there are no natural points of division within it – then Craig might claim that it does not contain an actually infinite set of sub-regions. We can in principle map out as many distinct, non-overlapping sub-regions as we please, but since (1) our mapping is only potentially infinite, and (2) the regions are not 'already there', prior to the mapping, it follows that the number of these sub-regions is only potentially infinite.

I do not think that this move is any more effective in defusing the present

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problem than it was with regard to the problem posed by infinite divisibility. It seems to me that the number of sub-regions of a Euclidean space would be actually infinite *relative to* any consistent way of dividing it up into distinct, finite, and non-overlapping sub-regions.

But even if I am wrong about this, it is hard to see why, if space had no boundaries, it could not have been filled with distinct objects of finite size. The number of distinct coexistent objects would then be infinite – and an actually infinite set would exist 'in reality', contrary to what Craig supposes possible.

As it happens, however, Craig does *not* respond to the problem posed by Euclidean intuitions about space by arguing that infinite space does not embrace an infinite set of sub-regions. What he does instead is simply deny that space could have been Euclidean. 'I would deny that physical space could be Euclidean in the sense of being actually infinite because the notion of an actual infinite ultimately results in self-contradictions.'[45] The only 'self-contradiction' Craig mentions is one we have already discussed and dismissed. What is 'infinity minus infinity?', he asks, 'Well, mathematically you get self-contradictory answers. This shows that infinity is just an idea in your mind, not something that exists in reality.'[46] Apparently, Craig thinks no experimental evidence at all should have been needed to demonstrate that Euclid – and Newton – were wrong about space. It should have been enough merely to observe that actual infinity leads to self-contradiction!

Summing up

Craig has recently opined that 'any normal adult whose intuitions have not been jaded by the common textbook assertions that actual infinities are wholly unobjectionable' will find his argument against the actual infinite 'extremely plausible'.[47] I do not share this assessment of Craig's various arguments against the actual infinite.[48] Some of them appear to be question-begging, and others involve deep conceptual confusion. At the heart of the controversy is Craig's attempt to apply Euclid's maxim about wholes and parts to sets. While this principle (as interpreted by Craig) is uncontroversially true of all finite sets, I do not believe that we have been given any good reason to think that it *must* be true of *all* 'real world' sets.

Could there be any infinite sets in reality? I have suggested that space could have been Euclidean, in which case there could have been an infinite set of distinct, non-overlapping spatial regions.

Are there in fact any infinite sets in reality? Three considerations may be thought to favour an affirmative answer. (1) The infinite divisibility of any region of space strongly suggests that there is an actual infinity of (overlapping) sub-regions within any such region. (2) If numbers and properties and propositions and other abstract entities exist (even in God's mind), then there are infinitely many of them. (3) If, as Craig believes, there is a complete body of truths about an

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endless series of *future* events, then there are infinitely many truths about the future. Even if, as Craig suggests, all abstract entities and all truths are somehow 'embedded' in a single divine idea, it is hard to avoid the conclusion that this idea is infinitely articulated – that it is a 'one' in an infinite 'many'.

What about the past? Have there been infinitely many different past events? Or does the past have a beginning? I have not taken a position on this question. For all I know, the past *may* have a beginning and there *may* have been a First Event. But Craig's argument against the possibility of the actual infinite does not persuade me that this *must* be so.[49]

Notes

1. See especially William Lane Craig (with Quentin Smith) *Theism, Atheism, and Big Bang Cosmology* (Oxford: Oxford University Press, 1993), chs 1 and 3.
2. Matters are considerably more complicated than this, however. See Wes Morriston 'Must the beginning of the universe have a personal cause? A critical examination of the *kalam* cosmological argument', *Faith and Philosophy*, **17** (2000), 149–169.
3. Craig also argues that even if an actual infinite were possible, the series of past events could not be actually infinite since it is 'formed by successive addition', and no series formed in this way can be infinite. For critical evaluation of this argument, see Wes Morriston, 'Must the past have a beginning?', *Philo*, **2** (1999), 5–19.
4. Craig *Theism, Atheism, and Big Bang Cosmology*, 12.
5. *Ibid*., 15. As Craig points out, this is 'Hilbert's Hotel in reverse'. It will be recalled that 'Hilbert's Hotel' is the hotel with infinitely many rooms envisaged by David Hilbert. The hotel starts out full, but 'room' is made for infinitely many new guests by creatively reassigning the old ones to different rooms. For example, the guest in room 1 might move to room 2, the guest in room 2 to room 4, and so on.
6. The logic of the situation is as follows. If a set S is impossible because an absurd implication follows from features *a*, *b*, and *c* of set S, it does not follow that no set having feature *a* is possible. What follows is only that no set combining all three features is possible.
7. For example, if infinitely many days have passed by, then infinitely weeks must also have passed by. So an infinite past would not contain a greater number of days than weeks. Whether this sort of implication is genuinely absurd will be the main issue discussed in the next section below.
8. Craig *Theism, Atheism, and Big Bang Cosmology*, 96.
9. *Ibid*.
10. William Lane Craig 'Philosophical and scientific pointers to *creation ex nihilo*', in R. Douglas Geivett and Brendan Sweetman (eds) *Contemporary Perspectives on Religious Epistemology* (New York NY and Oxford: Oxford University Press, 1992), 189.
11. *Ibid*., 190.
12. Craig *Theism, Atheism, and Big Bang Cosmology*, 98.
13. *Ibid*., 15.
14. A slight qualification should be made. There are highly esoteric systems of transfinite *ordinal* arithmetic. But these systems do not entail contradictions. For the details, see Graham Oppy 'Inverse operations with transfinite numbers and the kalam cosmological argument', *International Philosophical Quarterly*, **35** (1995), 219–221.
15. Craig *Theism, Atheism, and Big Bang Cosmology*, 15.
16. William Lane Craig (with Michael Tooley), *A Classic Debate on the Existence of God* (November 1994, University of Colorado at Boulder). (http://www.leaderu.com/offices/billcraig/docs/craig-tooleyo.html).
17. Craig *Theism, Atheism, and Big Bang Cosmology*, 23.
18. Actually, this is the Fifth Axiom of Euclid, which is not to be confused with the much discussed Fifth Postulate of Euclid.

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19. Craig *Theism, Atheism, and Big Bang Cosmology*, 23.
20. *Ibid*., 23–24.
21. *Ibid*., 23.
22. *Ibid*., 24.
23. That is, if one set contains all the elements in another plus some additional ones, then the number of elements in the first set is greater than the number of elements in the second.
24. See, for example, William Lane Craig 'Professor Mackie and the kalam cosmological argument', *Religious Studies*, **20** (1985), 367–375.
25. Paul Draper has an excellent discussion of this point. See his 'The kalam cosmological argument', in Louis Pojman (ed.) *Philosophy of Religion*, 3rd. edn (Belmont, CA: Wadsworth, 1997), 42–47.
26. Craig *Theism, Atheism, and Big Bang Cosmology*, 12.
27. Craig 'Philosophical and scientific pointers to *creation ex nihilo*', 187.
28. Craig *A Classic Debate on the Existence of God*.
29. In *ibid*., this is one of the six arguments for the existence of God that Craig presents. God, he says, 'provides the best explanation for the existence of abstract entities'.
30. Alvin Plantinga 'Two dozen (or so) theistic arguments', paper presented at the 33rd Annual Philosophy Conference, Wheaton College, October 23–25 1986.
31. William Lane Craig 'A swift and simple refutation of the *kalam* cosmological argument?', *Religious Studies*, **35** (1999), 61.
32. *Ibid*.
33. William Alston 'Does God have beliefs?', *Religious Studies*, **22** (1986), 287–306.
34. Craig 'Swift and simple refutation of the *kalam* cosmological argument?', 61.
35. Alston 'Does God have beliefs?', 290.
36. *Ibid*.
37. *Ibid*., 291.
38. Alvin Plantinga, 'How to be an anti-realist', *Proceedings and Addresses of the American Philosophical Association*, **56** (1982), 70.
39. For an interesting version of this argument, see William F. Lawhead, 'The symmetry of the past and the future in the *kalam* cosmological argument', in William Lane Craig and Mark S. McLeod (eds) *The Logic of Rational Theism: Exploratory Essays* (Lewistorn/Queenston/Lampeter: The Edwin Mellen Press, 1990), 99–111. See also Erik Sotnak 'The kalam cosmological argument and the possibility of an actually infinite future', *Philo*, **2** (1999), 41–52.
40. See William Lane Craig *Divine Foreknowledge and Human Freedom* (Leiden: E. J. Brill, 1992), 288, n. 12. See also *idem* 'Classical apologetics', in Steven B. Cowan (ed.) *Five Views of Apologetics* (Grand Rapids MI: Zondervan, 2000), 50–51, n. 27.
41. William Lane Craig *Time and Eternity: Exploring God's Relationship to Time* (Wheaton IL: Crossway Books, 2001), 99.
42. Craig *A Classic Debate on the Existence of God*.
43. *Ibid*.
44. See Tooley *A Classic Debate on the Existence of God*.
45. *Ibid*.
46. *Ibid*.
47. Craig 'Classical apologetics', 52.
48. This is so, despite the fact that I have scant acquaintance with the textbooks in question! I will leave it to others to determine whether I am a 'normal adult'.
49. I want to thank Barbara Morriston for listening to me read countless different versions of various parts of this paper, and saving me from countless errors.

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