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A survey of qualitative spatial representations

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Abstract

Representation and reasoning with qualitative spatial relations is an important problem in artificial intelligence and has wide applications in the fields of geographic information system, computer vision, autonomous robot navigation, natural language understanding, spatial databases and so on. The reasons for this interest in using qualitative spatial relations include cognitive comprehensibility, efficiency and computational facility. This paper summarizes progress in qualitative spatial representation by describing key calculi representing different types of spatial relationships. The paper concludes with a discussion of current research and glimpse of future work.

1 Introduction

Spatial representation and reasoning plays an essential role in human daily life. Although quantitative approaches can provide the most precise information, numerical information is often unnecessary or unavailable at human level. For example, when people describe relationships between objects, they usually cannot give an accurate numerical description, but something like 'the coffee is in the cup, the cup is in the room', which is quite enough to give a reasonable deduction 'the coffee is in the room'. The qualitative approach for spatial reasoning, known as QSR (qualitative spatial reasoning; Cohn, 1997; Vieu, 1997; Cohn & Hazarika, 2001; Liu *et al.*, 2004; Cohn & Renz, 2007) thus becomes a promising way to process spatial information at this level and has prevailed in artificial intelligence, Geographical Information Systems, database and multimedia communities for its understandability, high efficiency and computational facility.

It is quite natural to represent spatial situations by the qualitative relationship between the considered objects and therefore qualitative spatial relationship becomes a basic way of representing and reasoning with spatial knowledge. Thus, dozens of formalisms of qualitative spatial relations have been proposed for describing various aspects of space. This paper presents a much broader range of qualitative spatial calculi, including not only the most representative calculi in the previous surveys by Cohn (1997), Vieu (1997), Cohn and Hazarika (2001), Liu *et al.* (2004), Cohn and Renz (2007) but also the new results for the existing calculi, new calculi and comparisons between them. A more systematic survey of direction relation calculi is given than that in the previous surveys. Representative figures are provided to facilitate the understanding of the calculi. Calculi modeling relationships between moving objects are presented that were not part of the previous surveys. Two types of combination of spatial calculi are presented, where loose combination looks more practical

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than tight combination, as it is unlikely to develop a single universal qualitative spatial representation language (Cohn & Hazarika, 2001; Cohn & Renz, 2007). Another novel aspect of this paper is the material describing the well-known implementations for QSR. Finally, a perspective of future research directions is presented, such as the combination of different aspects of spatial relationships, spatial relationships over complex objects in 3D or discrete space, and exploiting the hierarchical structure of spatial relationships according to their granularity.

The structure of the paper is as follows: Section 2 introduces the main notions and terminologies about qualitative spatial representation. Section 3 provides a categorized survey of representative spatial relationship formalisms. Section 4 summarizes current publicly available implementations using qualitative spatial reasoning techniques. Section 5 proposes some future works for qualitative spatial representations.

2 Preliminaries

Spatial relationships specify how some spatial entities are related in space to others. It includes abstractions of physical spatial entities and the structure of the space. The space the entities embedded in ranges from one to three dimensions (or even four dimensions if space-time histories are considered), acyclic or cyclic (e.g. regions on a surface or a sphere), discrete (raster or grid) to continuous (vector), all of which result in a variety of formalisms fit for different kinds of space. $\mathbb{R}^2$ and $\mathbb{Z}^2$ are the most common approaches modeling the continuous and discrete 2D surface, respectively. Moreover, they are the space of the most studied approaches in QSR. Traditionally, in mathematical theories, points are considered as primary primitive spatial entities and perhaps lines and planes. Extended spatial entities such as regions are of particular interest; if necessary, they are defined as a set of points on a plane (or a higher dimensional space). In some formalism, regions are divided into simple regions (disk-like and with connected boundaries and interior) and complex regions (multi-part regions possibly with holes) according to their connectivity.

Formally, a relation R over the sets $X_1, \ldots, X_k$ is a subset of their Cartesian product, written as $R \subseteq X_1 \times \ldots \times X_k$, and each element of R is a tuple $(x_1, \ldots, x_k)$, where x_i is a member of corresponding domain of X_i. When specified to spatial relations, the considered domains that are identical, namely, the set of spatial entities, such as points, lines or regions. Usually, the considered domain is infinite, and therefore the spatial relations contain infinitely many tuples. However, fortunately, the set of relations over the spatial entities is finite, and is usually a JEPD (Joint Exhaustive and Pairwise Disjoint) relation set, that is, for arbitrary spatial entities there is one and only one specific relation in the set that can be satisfied. JEPD relations are also called base or basic relations and represent definite relationships between spatial entities. Indefinite information can be expressed by the union of these base relations, which can possibly hold. If no information is known, then all base relations are possible.

Although this paper focuses on qualitative spatial representation, it also involves some parts of QSR, such as constraint satisfaction problems (CSPs), composition, converse and so on. Often the information between spatial entities is expressed as constraints, such as unary constraint 'this room is 5 meters long and 3 meters wide', or the binary constraint 'the table is in front of the desk', or even the ternary constraint 'the trolley is between the chair and the desk'. Although there are many different spatial reasoning tasks, much of the research on QSR has focused on one particular reasoning problem, the consistency problem, that is, whether the given information consistent or inconsistent. Usually, it is abstracted as a CSP. As many spatial constraints are binary, this problem is transformed to whether the first-order logic expression $\exists x_1 \ldots \exists x_n \bigwedge_{i,j} R_{i,j}(x_i, x_j)$ has an instantiation, where $x_1, \ldots, x_n$ are the variables over the domain of spatial entities, $R_{i,j}(x_i, x_j)$ is the binary spatial relation constraint between x_i and x_j, $\mathcal{A}$ is the set of all available base relations and $R_{i,j} \in 2^{\mathcal{A}}$ (i.e., $R_{i,j}$ is a base relation or a disjuction of base relations). If the domain of the variables is finite, the CSPs can be solved by backtracking over the ordered domains of the single variables, but it is not very helpful when the domain of the variables is infinite. The domain of spatial entities happens to be such an infinite range and hence it needs another method.

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Relation algebra (Tarski, 1941) is one way of dealing with the CSPs with infinite domains (Ladkin & Maddux, 1994). The power set of a set of binary base relations is a relation algebra if and only if it is closed under specific relation operations: union ($\cup$), intersection ($\cap$), complement ($^-$), converse ($^{-1}$) or composition ($\circ$) and three particular relations: the empty relation that cannot hold between any two members of the domain; the universal relation that holds between any two members of the domain, that is, the disjunction of all base relations; and the identity relations hold between each member of the domain and itself. The formal definitions of the first five of these operations in relation algebra are:

- $\forall x \forall y\colon \; x(S \cup T)y \leftrightarrow x\ S\ y \vee x\ T\ y$,
- $\forall x \forall y\colon \; x(S \cap T)y \leftrightarrow x\ S\ y \wedge x\ T\ y$,
- $\forall x \forall y\colon \; x(S \circ T)y \leftrightarrow \exists z : x\ S\ z \wedge z\ T\ y$,
- $\forall x \forall y\colon \; x\ \overline{T}\ y \leftrightarrow \neg(x\ T\ y)$,
- $\forall x \forall y\colon \; x\ T^{-1}y \leftrightarrow y\ Tx$

Of these operations, composition is of particular interest, which is correlated with a local consistency, *path consistency*. As the consistency of CSPs with infinite domains is generally undecidable (Hirsch, 1999), path consistency is of particular attention. A CSP is path consistent if and only if for every consistent instantiation of two variables it is always possible to find an instantiation for any third variable such that the three variables together are consistent. Accordingly, the algorithm to get a set of path-consistent constraints is called the path-consistency algorithm. However, in many cases of QSR, composition cannot be computed and only weak composition ($\diamond$), that is, algebraic closure is available. If $S \diamond T = \{R_1, \ldots, R_n\}$ then $\forall x \forall y \forall z(xSy \wedge yTz) \rightarrow (xR_1z \vee \ldots \vee xR_nz)$. Whereas composition is based on existence, algebraic closure is based on consistency. It is obvious that algebraic closure is less constrained than composition (also called strong composition, compared with weak composition) and the result relation set includes composition. For further information about weak composition, see Bennett *et al.* (1997), Renz and Ligozat (2005), Cohn and Renz (2007). A special subset of the power set $2^{\mathcal{A}}$ is of particular interest, a maximal tractable subset, that is, if all the relational constraints are limited to this subset then the CSP over this subset is tractable (can be decided in polynomial time). Moreover, the CSP over its superset is intractable.

3 Aspects of qualitative spatial relations

This section will review a set of representative spatial relationship formalisms. These formalisms can be divided into the aspects of mereotopology, direction, distance and shape, the relationships between static spatial entities; with the increasing availability of data involving mobile entities, relationships between moving objects have become an increasingly important new focus of QSR. Moreover, as many applications need to represent and reason about multiple aspects of space, integrations of different formalisms is also a problem of much interest. Finally, we note that uncertainty is another vital aspect of spatial relationships.

3.1 Mereotopology

Mereotopology, the integration of mereology (the theory of parthood) and topology, is the most studied theory in QSR, concerning the invariant properties that are under continuous deformations of objects, including translating, rotating and scaling. As it can only make qualitative distinctions, this kind of relationship (also called topological relations) is perhaps the most fundamental spatial relationship. Although mathematical topology has influenced various qualitative spatial theories, the wholesale importation is undesirable for a number of reasons (Gotts *et al.*, 1996). RCC (region connection calculus; Cohn *et al.*, 1997; Renz, 2002) and n-intersections (Egenhofer & Franzosa, 1991, 1995; Egenhofer & Herring, 1991; Egenhofer & Sharma, 1993; Egenhofer *et al.*, 1994a, 1994b; Egenhofer, 2005; Egenhofer & Vasardani, 2007) are the two best-known approaches for representing and reasoning with topological relations; most existing approaches are extensions or improvements of them.

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RCC is based on a reflexive and symmetric primitive relationship between spatial regions $C(x, y)$. The intended topological interpretation of $C(x, y)$ is that two regions x and y are connected if and only if their topological closures share a common point where the spatial regions are non-empty regular subsets of some topological space. It does not require that the regions be simple ones, that is, they might consist of (multiple) disconnected pieces. Of the number of relations defined by $C(x, y)$, some relations are of particular interest: $DC(x, y)$, $P(x, y)$, $PP(x, y)$, $EQ(x, y)$, $O(x, y)$, $PO(x, y)$, $DR(x, y)$, $EC(x, y)$, $TPP(x, y)$, $NTPP(x, y)$ and P^{-1}, PP^{-1}, TPP^{-1} and $NTPP^{-1}$, the converse of non-symmetrical relation P, PP, TPP and NTPP, respectively. Their formal definitions and the meaning under the intended interpretations, denoted after the double slash, are the following (Randell *et al.*, 1992b; Renz, 2002):

$$
\begin{array}{rll}
DC(x, y) & \equiv_{def} \neg C(x, y) & //x \text{ is disconnected from } y \\
P(x, y) & \equiv_{def} \forall z(C(z, x) \rightarrow C(z, y)) & //x \text{ is a part of } y \\
PP(x, y) & \equiv_{def} P(x, y) \wedge \neg P(y, x) & //x \text{ is a proper part of } y \\
EQ(x, y) & \equiv_{def} P(x, y) \wedge P(y, x) & //x \text{ equals } y \\
O(x, y) & \equiv_{def} \exists z(P(z, x) \wedge P(z, y)) & //x \text{ overlaps } y \\
PO(x, y) & \equiv_{def} O(x, y) \wedge \neg P(x, y) \wedge \neg P(y, x) & //x \text{ partially overlaps } y \\
DR(x, y) & \equiv_{def} \neg O(x, y) & //x \text{ is discrete from } y \\
EC(x, y) & \equiv_{def} C(x, y) \wedge \neg O(x, y) & //x \text{ is externally connected with } y \\
TPP(x, y) & \equiv_{def} PP(x, y) \wedge \exists z(EC(z, x) \wedge EC(z, y)) & //x \text{ is a tangential proper part of } y \\
NTPP(x, y) & \equiv_{def} PP(x, y) \wedge \neg \exists z(EC(z, x) \wedge EC(z, y)) & //x \text{ is a non-tangential proper part of } y \\
P^{-1}(x, y) & \equiv_{def} P(y, x) & //y \text{ is a part of } x \\
PP^{-1}(x, y) & \equiv_{def} PP(y, x) & //y \text{ is a proper part of } x \\
TPP^{-1}(x, y) & \equiv_{def} TPP(y, x) & //y \text{ is a tangential proper part of } x \\
NTPP^{-1}(x, y) & \equiv_{def} NTPP(y, x) & //y \text{ is a non-tangential proper part of } x
\end{array}
$$

By adding new primitive relations and functions, a much larger number of different relations can be defined upon the C relation (Cohn & Hazarika, 2001). This paper only presents a small fraction of the relations that can be expressed by RCC theory. If some of the above relations can form a set of JEPD relations and are closed under composition, then they generate a relation algebra; thus, the reasoning about these relations can be done by the path-consistency algorithm mentioned in Section 2. DC, EC, PO, EQ, TPP, NTPP, TPP^{-1} and $NTPP^{-1}$ are the eight JEPD relations of the constraint language RCC8,[1] as shown in Figure 1. They are of particular interest, for they form the smallest set of base relations that allows topological distinctions rather than just mereological, that is, being expressed by part–whole relationship (Randell *et al.*, 1992b; Renz, 2002) RCC5 is the coarser edition of RCC8, containing five base relations DR, PO, EQ, PP and PP^{-1}, and does not take the boundary of a region into account.

A series of extensions and completions of RCC theory have been made: the density axiom of the RCC has been shown to be redundant and each RCC model leads to a Boolean algebra (Düntsch *et al.*, 2001). Three maximal tractable subsets of RCC8: $\hat{\mathcal{H}}_8$, $\mathcal{C}_8$ and $\mathcal{Q}_8$ have been given in Renz (2002). Each RCC model is a consistent model of the RCC8 CT (the consistency-based composition table of RCC8) and no RCC model can be interpreted extensionally anyway, thus giving a negative answer to Bennett's conjecture, which is to remove the universal region from the domain of possible referents of the region constants (Li & Ying, 2003). A generalization of RCC (GRCC; Li & Ying, 2004) is introduced, which accommodates both discrete and continuous

[1] It is important to distinguish between RCC as a general theory of space, that is, as an axiomatization in first-order logic, and the various constraint languages with varying numbers of JEPD relations (such as RCC5 and RCC8); in these languages the JEPD relations are themselves taken as primitives rather than the C relation. Terms such as RCC8 are usually regarded as referring to the constraint language, though in some presentations the first-order theory interpretation is actually meant.

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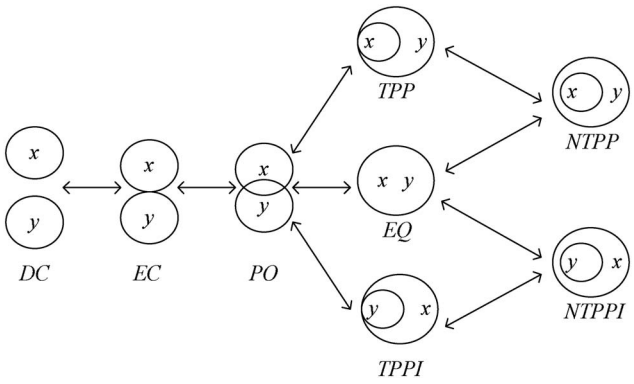


Figure 1 The eight jointly exhaustive and pairwise disjoint relations of region connection calculus (RCC8). The arrows show which relation is the next relation a configuration would transit to, assuming the continuous movements or deformations. This structure has been called a continuity network (Cohn *et al.*, 1997: 295) or a conceptual neighborhood (Cohn & Hazarika, 2001: 7; Cohn & Renz, 2007: 564)

spatial information. Each countable RCC model (compared with the 'standard RCC in continuous space') can be constructed hierarchically from a sequence of finite GRCC models, and every finite GRCC model can be isomorphically embedded in any RCC model; this bridges the gap between qualitative and quantitative approaches for spatial information (Li *et al.*, 2005). Consistency w.r.t. RCC theory is equivalent to consistency w.r.t. topology, and an $O(n^3)$ algorithm for generating a realization of any path-consistent network of RCC8 base relations has been proposed, proving that any consistent RCC8 network has a realization in the digital plane and in any RCC model (Li, 2006b). Any consistent RCC8 binary constraint network can be consistently extended (Li & Wang, 2006), which removes the doubts about the complexity analysis of RCC8, that is, whether the reduction method proposed in Renz and Nebel (1999) works for all calculi using weak compositions, given that the composition table of RCC8 is a weak one.

N-intersections are an alternative representation framework of mereotopology based on point-set topological theory. An object can be seen as a point set embedded in a specified space. For example, a region is a homogeneous 2D point set x embedded in $\mathbb{R}^2$ related to three point sets: the interior (x°) is the union of all open sets in x, the boundary (∂x) is the intersection of the closure of x and the closure of the exterior of x, and the exterior (x^-) is the set of all points not contained in x. In the simplest case, only the interior (x°) and the boundary (∂x) are considered. The relationship between any two (simply connected) 2D regions x and y can be characterized by a 2×2 matrix called the four-intersection matrix (4IM; Egenhofer & Franzosa, 1991):

$$\mathrm{R}(x, y) = \begin{bmatrix} x^\circ \bigcap y^\circ & x^\circ \bigcap \partial y \\ \partial x \bigcap y^\circ & \partial x \bigcap \partial y \end{bmatrix}$$

After taking into account the constraints imposed by the physical reality of planar space and some specific assumptions about the regions, it turns out that there are exactly eight valid matrices that correspond to the RCC8 and RCC5 relations, as shown in Figure 2. Egenhofer *et al.* (1994a) have shown that the topological relationship between regions with holes can be classified by not only the relationship between each pair of regions but also the relationship each hole of each region has w.r.t. the other region and each of its holes, where the topological relationships between basic elements (regions and holes) are the eight basic relationships discerned by 4IM. The shortcomings of this approach are that the resulting relations are dependent on the number of holes, as the number of holes increasing this approach will be inefficient and painstaking. A method independent of holes is proposed in Vasardani and Egenhofer (2009), which is based on the 23 JEPD relations between a region and a region with a hole (Egenhofer & Vasardani, 2007).

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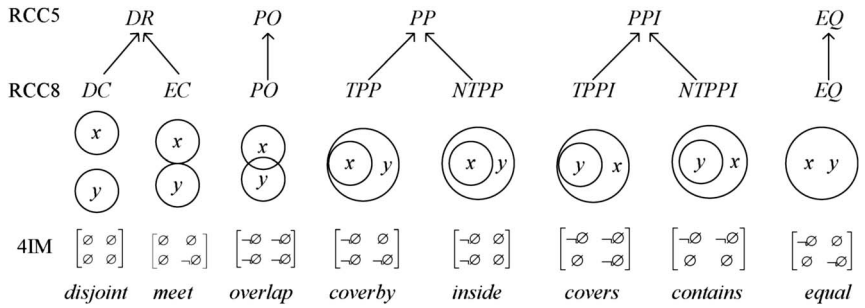


Figure 2 The corresponding basic relations of 4-intersection matrix (4IM), region connection calculus (RCC8) and RCC5

It shows that there are 152 basic topological relations (Vasardani & Egenhofer, 2008) between single-holed regions.

Considering the exterior (x^-), the 4IM is extended into 9IM; one can use this calculus to classify the relationship not only between pairs of regions but also between all combinations of lines, points and regions (Egenhofer & Herring, 1991; Egenhofer *et al.*, 1994b). Dividing the exterior of a concave region into two parts, the inside and outside of the convex hull, the 9IM is extended to 16IM (Ouyang *et al.*, 2009b), which is a refinement of RCC23 (Cohn, 1995), a calculus discriminating the topological relationship between concave regions:

$$\mathrm{R}(x, y) = \begin{bmatrix} x^\circ \bigcap y^\circ & x^\circ \bigcap \partial y & x^\circ \bigcap y^- \\ \partial x \bigcap y^\circ & \partial x \bigcap \partial y & \partial x \bigcap y^- \\ x^- \bigcap y^\circ & x^- \bigcap \partial y & x^- \bigcap y^- \end{bmatrix}$$

Different calculi with more JEPD relations can be derived by changing the assumptions about what kind of space the entities are in and what information the cells of the matrix represent. For example, three new relations can be identified between the regions on the spherical surface (Egenhofer, 2005). If the exterior of the entity is replaced by its voronoi-region in the 9IM, a more comprehensive calculus V9I (Voronoi-based 9-intersection; Chen *et al.*, 2001) is obtained. One can derive a calculus for representing and reasoning about regions in $\mathbb{Z}^2$ rather than $\mathbb{R}^2$, in which case the base relations in $\mathbb{R}^2$ is a strict subset of those in $\mathbb{Z}^2$ because of the one-pixel extent the boundary of a raster region has (Egenhofer & Sharma, 1993). Alternatively, one can extend the representation in each matrix cell by the dimension of the intersection rather than emptiness and non-emptiness, which is called the dimension entended method (DEM). To handle the large number of possible relationships identified by the DEM, the calculus-based method has been proposed to group all possible cases into five meaningful topological relationships: disjoint, touch, in, overlap and cross (Clementini *et al.*, 1993). Besides the dimension of the non-empty intersection, the types (touching, crossing and different refinements of crossings), the relationships w.r.t. the exterior neighborhoods and sequence can also be used to discriminate fine-grained relationships and evaluate the topological similarity between any two pairs of spatial objects, which have the same topological relation (Egenhofer & Franzosa, 1995).

It should be noticed that the above calculi are mostly concerned about topological relationships that are discernible between simple spatial objects such as points, lines or disk-like regions in 2D space[2]. However, in practice, spatial objects are the deformations or aggregations of these primary elements or entities in 3D space, such as complex points, compound regions or bodies. 9IM is the most popular calculus to reason the topological relationships between these objects. Compared with the

[2] RCC is actually formulated for an arbitrary, fixed, dimension, though most often has been applied in the 2D case.

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eight possible 9IM matrices over simple regions, it has been proved that only 43 (out of $2^9 = 512$) matrices are identified as realizable over general regions, where a general region is a non-empty, proper, regular, closed subset of the Euclidean plane (Li, 2006a). According to 25 rules that encode the constraints imposed by physical space and assumptions relating to spatial objects in 3D space, only 69 valid 9IM matrices remain and can discriminate 279 different relationships between all types of combinations of objects (Zlatanova, 2000). Given the definitions of complex points (finite collections of single points), complex lines (finite collections of one-dimensional curves) and complex regions (multi-part regions possibly with holes), one can use 9IM to classify 248 exclusive relations for all type combinations (Schneider & Behr, 2006). A nested matrix, called the 9+-intersection matrix that extends each matrix cell by a sub matrix, gives a systematic way representing different topological relations in planar space, cubic space, circle and spherical surface (Kurata, 2008).

Ever since Allen (1983) gave the composition table for the Interval Algebra (he actually called it the transitivity table), building composition tables has become a major challenge for current QSR approaches (Randell *et al.*, 1992a). El-Geresy and Abdelmoty (2004) present a general method for automatic derivation of such tables between different types of objects according to the space division theory inside the 9IM.

3.2 Direction

Direction relations describe where a spatial entity is placed relative to one another, which is more constrained than topological information but less constrained than metrical information. It usually involves three primary elements: the target object, the reference object and the reference frame; thus, in contrast to the binary relationships usually found in mereotopological calculi, direction relations can be binary (if the reference frame is implicit), ternary or with even more arguments. Depending on the dimension of the objects involved, direction calculi can be divided into two categories: one based on the points and the other based on extended objects.

The two most common binary point-based calculi are cone-shaped direction (Frank, 1991) and projection-based (Frank, 1996) direction, shown in Figures 3 and 4, respectively. The basic relations of the cone-shaped calculus are based on the four or eight disjoint sectors of the space divided by the lines going through the reference point, whereas the basic relations of the projection-based calculus are decided by the horizontal and vertical lines across the reference point. Their combination with a coarser version of double-cross (Freksa, 1992) calculus leads to a more expressive calculus $c\mathcal{COA}$ (Isli *et al.*, 2003), which claims to integrate directions in geography and those of human perception. For oriented points (an abstraction of objects with an intrinsic direction), the Oriented Point Algebra ($\mathcal{OPRA}$) (Moratz, 2006) can be used to describe the relative direction information, which can be seen as an extension of the cone-shaped calculus. Scalable granularity is used in $\mathcal{OPRA}_m$ (Mossakowski & Moratz, 2012) dealing with information with different granularities in one frame, where m is the number of lines going through the points. As shown in Figure 5, $x_4 \angle^3_{13} y$ denotes that oriented point x lies on the third part of the space divided by the lines going through oriented point y, whereas y lies on the 13th part decided by x, where 4 means that there are four lines going through points and divide the space around the points equally.

The CYCORD (short for CYClic ORDering) system is a ternary relation calculus based on the cyclic ordering relation CYCORD (x, y, z), which is true when x, y and z are clockwise in 2D space. It shows that a number of qualitative calculi can be translated into the CYCORD system, but reasoning in it is NP complete (Röhrig, 1994, 1997). $\mathcal{CYC}_t$ is a refinement of CYCORD system, which leads to a ternary relation algebra containing 24 basic relations, as shown in Figure 6. A basic relation expresses for triples of orientations[3] whether each of the three orientations is

[3] Orientation is an inherent character of objects. For example, an oriented point can be seen as an abstraction of a car, since cars have heads and tails intrinsically. The main difference from direction is that orientation often does not involve reference objects, thus orientation is not a relative relation.

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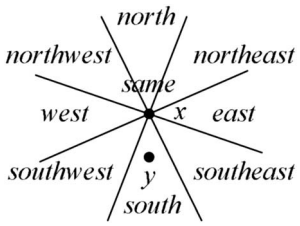


Figure 3 Cone-shaped direction relations (Isli *et al.*, 2003: 7)

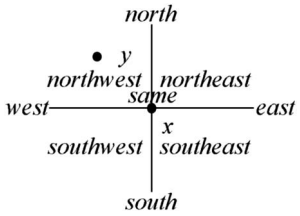


Figure 4 Projection-based direction relations (Isli *et al.*, 2003: 7)

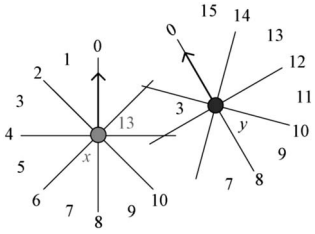


Figure 5 A basic relation $x_4 \angle^3_{13} y$ in $\mathcal{OPRA}_4$ (Mossakowski & Moratz, 2012: 37)

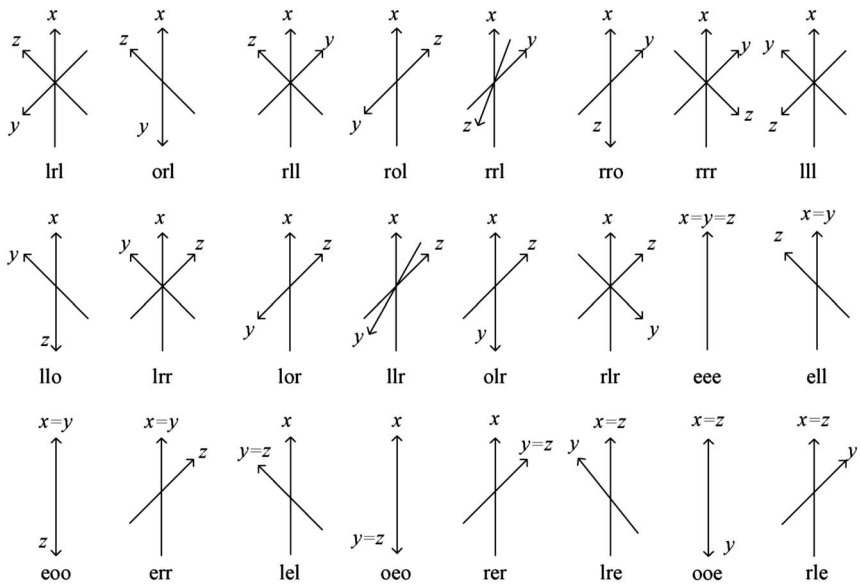


Figure 6 Pictorial examples of the 24 $\mathcal{CYS}_t$ basic relations, where l, e, o and r stands for to the left of, equal to, opposite to and to the right of, thus orl denotes that orientation y is opposite to x, z is to the right of y and z is to the left of x (Isli & Cohn, 2000: 156)

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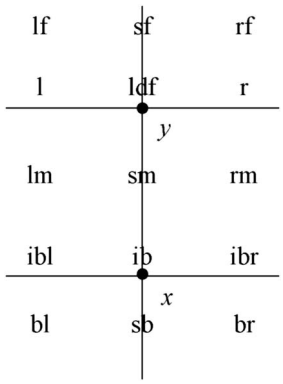


Figure 7 Fifteen base relations in the double-cross direction calculus, where l, s, r, f, i, b and n stand for left, straight, right, front, identical, back and neutral (Isli *et al.*, 2003: 7)

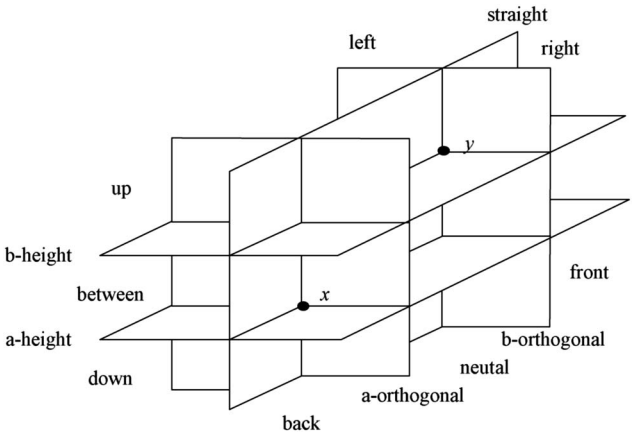


Figure 8 Double-cross direction calculus in 3D space (Pacheco *et al.*, 2002: 91)

equal to, to the left of, opposite to or to the right of each of the other two orientations. The constraint propagation procedure is proved to be polynomial and complete for a subset including all atoms; it also delineates those situations when the consistency checking problem is tractable or NP complete (Isli & Cohn, 2000).

Motivated by cognitive considerations and based on relative direction information about spatial environments, the concept of view point is introduced in the ternary double-cross calculus (Freksa, 1992), as shown in Figure 7. A relation in the double-cross calculus can be seen as the relationship of a target point c w.r.t. a vector ab, denoted by $ab:c$, as there is no difference between view point and reference point. A binary composition operation and unary operations are defined: INV (derive $ab:c$ from $ba:c$), HM (map $ab:c$ to $bc:a$) and SC (derive $ac:b$ from $ab:c$; Zimmermann & Freksa, 1996) to compute all the possibilities between the three points. More relations can be derived when the double-cross calculus is extended to 3D space, as shown in Figure 8; this naturally increases the complexity of the calculus. Three models under different granularity are proposed to reduce the complexity and dealing with coarse direction information: length coarse model, height coarse model and general coarse model, where an accurate model can be derived from the integration of the above three coarse models (Pacheco *et al.*, 2002).

A ternary point-based calculus is RST (the initials of rotation, scaling and translation) (Scivos & Nebel, 2004) if the relations are invariant when all points are mapped by rotations, scalings or translations. Examples for such RST calculi are the double-cross (Freksa, 1992), the flip-flop (Ligozat, 1993) (also called $\mathcal{LR}$ in Scivos and Nebel (2004), as shown in Figure 9) and the Ternary Point Configuration Calculus (TPCC; Moratz & Ragni, 2008). However, it has been proved

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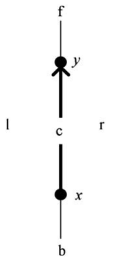


Figure 9 Pictorial illustration of seven base relations in the $\mathcal{LR}$ calculus when $x \neq y$. l, f, r, b and c stand for left, further, right, back and closer (Scivos & Nebel, 2004: 285)

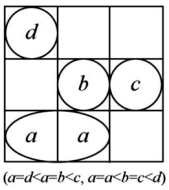


Figure 10 An example of a 2D-string (Chang *et al.*, 1987: 414)

(Wolter & Lee, 2010) that a qualitative calculus expressive enough to distinguish from left to right of including flip-flop (Ligozat, 1993), double-cross (Freksa, 1992), dipole (Moratz *et al.*, 2000), $\mathcal{OPRA}$ (Moratz, 2006) and TPCC (Moratz & Ragni, 2008), existing relation algebraic approach is too weak for deciding consistency problems and all reasonable sub-algebras remain NP-hard, that is, directional relation calculi are inherently intractable.

The above is a brief summary of point-based direction. Direction relations between extended objects are more complex, for extended objects often have intrinsic directions and shapes in themselves. Therefore, to simplify the process, MBRs (Minimal Bounding Rectangles) are often used as the approximation of such objects. This approach translates the direction relation between the MBRs into a pair of interval relations, where the intervals are the projections of MBRs on each axis. Examples of such approaches are: 2D-String (Chang *et al.*, 1987) as shown in Figure 10, its extension 2D-CString (Lee & Hsu, 1990) and 2D-GString (Chang *et al.*, 1996), the two-dimensional extension of Interval Algebra (Allen, 1983), as shown in Figure 11, the Rectangle Algebra (Balbiani *et al.*, 1998, 1999a), as shown in Figure 12, and the three-dimensional extension the Block Algebra (Balbiani *et al.*, 1999b). It should be noticed that these string-based and interval algebra-based methods can represent not only the direction information but also topological information[4]. A string-based model only dealing with topological information is presented in Li and Liu (2010).

The direction information described by MBR-based calculi is relatively coarse and cannot express the direction influenced by the shape of the objects in precise reasoning. The cardinal direction matrix (direction matrix for brevity; Goyal & Egenhofer, 2000, 2001) and Cardinal Direction Calculus (CDC; Skiadopoulos & Koubarakis, 2004, 2005) are the two most well-known binary direction relation calculi.

A direction matrix is a 3×3 matrix (Goyal & Egenhofer, 2000, 2001), whose elements have the same topological organization as the nine direction tiles formed by the corresponding MBR of the reference object, as shown in Figure 13. The content of each entry of a direction matrix is decided by whether the target object intersects with it. After taking into account the constraints imposed

[4] If regions are abstracted to MBRs, then the actual topological relations between the actual regions will not in general be correctly reflected. As shown in Figure 13 x DC y is satisfied, although using MBRs to abstract the region x and y then MBR(x) PO MBR(b) holds.

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relation	symbol	inverse	meaning
preceeds	p	pi	*xxxx* *yyyy*
meets	m	mi	*xxxx* *yyyy*
overlaps	o	oi	*xxxx* *yyyy*
starts	s	si	*xxxx* *yyyyyyy*
during	d	di	*xxxx* *yyyyyyy*
finishes	f	fi	*xxxx* *yyyyyyy*
equals	e	e	*xxxx* *yyyy*

Figure 11 The 13 basic relations of the Interval Algebra (Allen, 1983: 835)

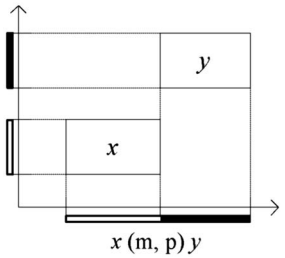


Figure 12 An example of the basic rectangle algebra relation x (m, p) y

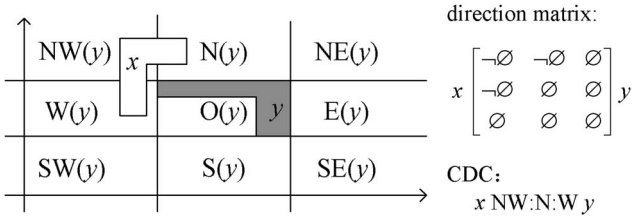


Figure 13 A pictorial example of the nine direction tiles divided by the Minimal Bounding Rectangle (MBR) of reference object y and the corresponding representations in direction matrix and Cardinal Direction Calculus (CDC)

by simple regions, it turns out that only 218 of the $2^9 - 1 = 511$ matrices are realizable. When dealing with objects with different dimensions, the deep direction relation matrix (Goyal & Egenhofer, 2000) is used, where each cell of the matrix is a nine-bit neighbor code recording the information about intersections with the direction tiles and the neighboring boundary. Instead of the Boolean value of the intersection, the cell of the matrix can also represent the percentage of how much the target object intersects with each direction tile, which is useful for similarity assessment (Goyal & Egenhofer, 2001). In a topological calculus, usually each basic relation corresponds to only one basic relation when inverted: for example, the converse relation of PP (x, y) is PP^{-1} (y, x). However, in the direction matrix calculus, one direction matrix may have several valid converse matrices; for example, the matrix representing the direction relation NE:E and E are both valid converses of the matrix of W. It has been proved (Cicerone & Felice, 2004) that of the 218^2 possibilities, only 2004 pairs of matrices are realizable over simple regions, although this is still a huge number of relations.

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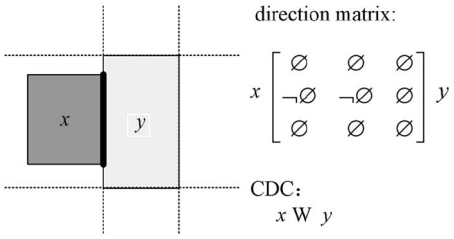


Figure 14 A pictorial example of the difference between the direction matrix representation and Cardinal Direction Calculus (CDC): x and y intersect on y's left border, but due to the definitions, the direction relations between x and y are different in direction matrix and CDC (Chen *et al.*, 2010a: 280)

In contrast to direction matrices, an arbitrary basic CDC relation (Skiadopoulos & Koubarakis, 2004, 2005) is a binary relation involving a target object and a reference object, and a symbol that is a non-empty subset of nine atomic relations, which has similar semantics to the nine direction tiles in direction matrix. The most important difference between the two calculi is the semantics of intersections of the target object and the direction tiles (Chen *et al.*, 2010a). In the direction matrix, the only question is whether there is an intersection, so the intersecting part can be a point, a line or a region. But in the CDC, the intersecting part must at least be a sub region of the target object, as shown in Figure 14.

An $O(n^5)$ and an improved cubic consistency checking algorithm for the basic CDC constraints are given in Skiadopoulos and Koubarakis (2005) and Liu *et al.* (2010), respectively. It has been proved that composition in CDC is weak composition (Skiadopoulos & Koubarakis, 2004) and consistency checking a set of unrestricted CDC constraints is NP-complete (Skiadopoulos & Koubarakis, 2005; Liu *et al.*, 2010). However, by introducing the rectangle algebra into CDC, a tractable subset including 36 base rectangular relations has been identified (Navarrete & Sciavicco, 2006); the rectangle-based inverting rules (Chen *et al.*, 2010a) and composition (Chen *et al.*, 2010b) are given, and these can be easily extended to cubic space. Neither the direction matrix nor CDC can handle direction information between overlapping and contained regions properly. ICD (Internal Cardinal Direction; Liu *et al.*, 2005) solves this problem partially, but the notable inherent shortcoming is that the ICD relations are not closed under converse. For example, if x and y have a specific ICD relation, the relationship between y and x will not be in ICD anymore. The interior-boundary direction calculus settles this problem (Du *et al.*, 2008a).

However, it can be argued that the direction partition in the direction matrix and CDC is unnatural. Typically, people tend to organize surrounding space using lines with angles similar to the cone-based model, and hence the cone-based partition is arguably more intuitive and descriptive (Huttenlocher *et al.*, 1991; Franklin *et al.*, 1995). An alternative family of direction relation models (Skiadopoulos *et al.*, 2007) is proposed, where the reference object is approximated by its MBR as with the direction matrix and CDC, but the space around the reference object is partitioned into five zones using the cone-based model; thus, the family contains an infinite number of models identified by the unique value for $\varphi(0^\circ < \varphi < 90^\circ)$ defining the original angle of the space partitioning lines, as shown in Figure 15. For a given φ, the five partitioned zones correspond to five atomic relations, and result in $2^5 - 1 = 31$ basic relations in the calculus, which is significantly smaller than the respective sets of CDC or direction matrix, which contains 511 basic relations, respectively.

The projective direction calculus is the most prominent ternary calculus. This calculus is built on a partition of the plane into five separate zones: before, between, after, left side and right side, which are obtained from projective properties of two reference objects as shown in Figure 16. Then, by considering the empty/non-empty intersections of a primary object with these zones, the model is able to distinguish 34 different projective relations (Clementini & Billen, 2006). Its extension to a spherical surface is addressed in Clementini (2008). The converse, rotation and composition rules are discussed in Clementini *et al.* (2010).

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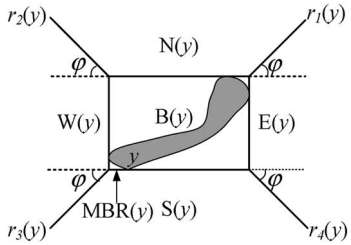


Figure 15 The five direction tiles formed by the Minimal Bounding Rectangle (MBR) of reference object y and the four lines starting at the vertices of the MBR with a horizontal angle of φ in the cone-shaped Cardinal Direction Calculus (CDC; Skiadopoulos *et al.*, 2007: 1118)

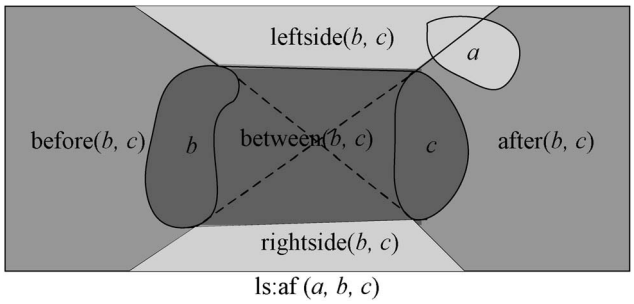


Figure 16 A pictorial example of five separate zones in ternary projective relations, where a is on the left side and after b and c, denoted by ls:af (a, b, c) (Clementini & Billen, 2006: 805)

Although above direction calculi partially support complex objects such that the target object is a multi-part region, they cannot always yield precise or may give counterintuitive results for certain spatial scenarios. For example, according to the CDC model, Argentina lies in partially same location and partially south of Brazil; however, in human intuition, Argentina lies partially to the southwest and west of Brazil. The main reason is that for complex objects one needs to consider the influence of shape in a much better manner. A two-phase model, called the Objects Interaction Matrix (OIM; Schneider *et al.*, 2012), is proposed to solve these problems and determine the cardinal direction even between complex regions. It consists of a tiling and interpretation phase. In the tiling phase, a tiling strategy first determines a bounding grid called objects interaction grid. Then for each grid cell it stores the information on how the region objects intersect with the corresponding tile, which leads to the OIM. In the subsequent interpretation phase, an interpretation method is used to the OIM and determines the detail of cardinal directions.

3.3 Distance[5]

The spatial representation of distance can be classified into two main groups: absolute and relative. Absolute approaches usually measure the distance between two objects, such as 'the distance between A and B is 100 meters' or 'A is close to B'. Relative approaches often involve the comparison to a third object. For example, 'A is closer to B than that to C'. Absolute approaches can be qualitative or quantitative, whereas relative approaches are in generally qualitative.

Distance relations alone are not enough in reasoning, normally direction relations are needed too: for instance, given the distance from A to B and from A to C, to determine the distance

[5] In general, the distance relation has meaning only when combined with direction relations; strictly speaking, it should be classified into the combination of different spatial calculi as said in section 3.6. But such distance relations are quite common in daily life, so it is selected as a single category.

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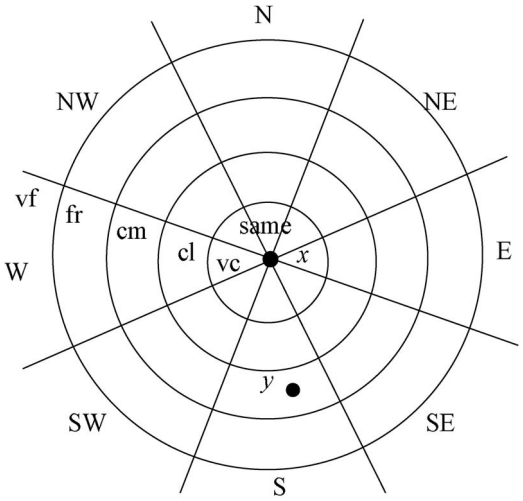


Figure 17 An iconic representation of the combination of cone-shaped direction and absolute distance: very close(vc), close(cl), commensurate(cm), far(fr) and very far(vf) (Clementini *et al*., 1997: 324)

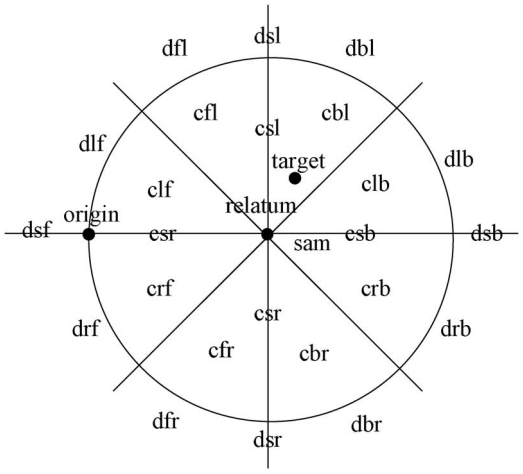


Figure 18 A pictorial representation of 27 basic relations in Ternary Point Configuration Calculus (TPCC), where f, b, l, r, s, d, c stand for front, back, left, right, straight, distant and close (Moratz & Ragni, 2008: 79)

from B to C, the direction relations between these three points are necessary. A straightforward idea is to combine directions as represented by segments of the compass with a separate distance metric, which leads to a unified framework across different levels of granularities in Clementini *et al*. (1997), as shown in Figure 17. One approach combines the Delta calculus (Zimmermann & Freksa, 1996) with point-based directions, but only a restricted set of distance distinctions can be identified (Zimmermann, 1993). Another approach combines qualitative distance with qualitative direction angles and then formulates a set of qualitative inference rules called qualitative trigonometry and qualitative arithmetic (Liu, 1998). Several examples are illustrated as to how to use this method in qualitative spatial constraints analysis, such as reducing the search space in a simulated annealing-based quantitative value assignment problem. TPCC is proposed in Moratz and Ragni (2008), where direction knowledge is represented in the double-cross calculus (Freksa, 1992) and distance measurements are based on the two of the three points. It has 27 atomic JEPD relations as shown in Figure 18 and hence gives finer distinction than in previously published calculi.

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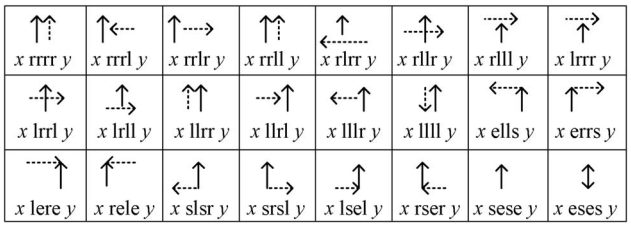


Figure 19 Basic relations of the Dipole calculus, where l, r, s, e stand for left, right, start point and end point, and the oriented line segment x and y are denoted by solid and dotted line separately (Moratz *et al.*, 2000: 235)

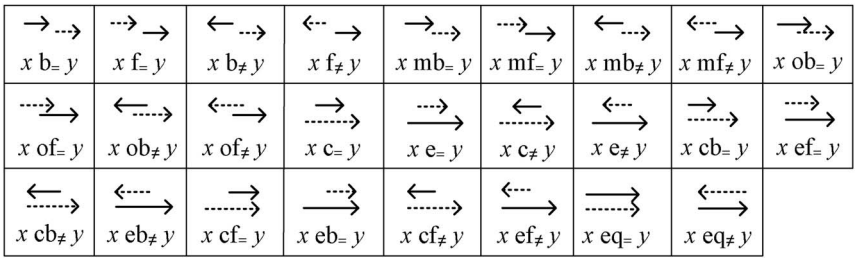


Figure 20 Basic relations of directed interval algebra (DIA), where b, f, m, o, c, e, eq stand for behind, in-front-of, meets, overlaps,contained in, extends and equals, respectively. = denotes that both intervals have the same orientation and $\neq$ for opposite. x is represented by a solid arrowed interval while y is by a dotted arrowed interval (Renz, 2001: 52)

3.4 Moving objects

With the popularization of mobile positioning and communication, moving objects have become a new point of focus for QSR. To facilitate information collection and processing, moving objects are often abstracted as points, and the focus is mainly on the relationships between the trajectories formed by these moving points. Although certain direction information is included for moving objects, the essential difference is that this kind of calculus commonly describes the relative motion between objects but not the relative direction between the static objects at any given time point.

One simple method approximates trajectories as oriented line segments and then expressing movements by the relative position between the segments, which leads to several segment-based calculi. One approach abstracts such oriented line segments as dipoles constituted by the start points and end points. It distinguishes the location and orientation of different dipoles according to whether a point lies to the left, to the right or shares the same point with the start or the end point of the referring dipole. Then it forms a relation algebra called the Dipole calculus (Moratz *et al.*, 2000) including 24 basic relations; however, reasoning over these relations is NP-hard. For example, 'x ells y' as shown in Figure 19 denotes the case that the start point of x shares the same point with the end point of y, the end point of x lies to the left of y, whereas the start point of y lies to the left of x and the end point of y and shares the same point with the start point of x. Another approach is based on interval algebra (Allen, 1983). By considering whether the direction of two intervals is same or not, the interval algebra can be extended to the directed interval algebra (DIA) (Renz, 2001). DIA can be seen as the Cartesian product of the interval algebra with $\{=, \neq\}$, which represent whether the orientations of two intervals are same or not; thus, the calculus includes 26 basic relations, as shown in Figure 20. Path consistency decides the consistency for a certain subset of DIA, which contains all basic relations. The major disadvantage of the DIA is that it is only suitable for movements in one dimension; free or restricted 2D movements are not supported by the DIA. A corresponding refinement of DIA on a road network is introduced in Wang *et al.* (2005).

Rather than comparing relative movements over a pair of fixed intervals, the family of QTC (Qualitative Trajectory Calculus) calculi compares trajectories point by point by considering

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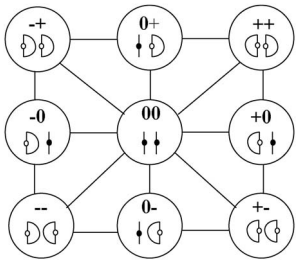


Figure 21 Basic relations of basic Qualitative Trajectory Calculus (QTC_B) in a conceptual neighborhood diagram. The solid dots represent the stationary objects and the open dots represent the moving objects (de Weghe *et al.*, 2004: 1103)

instantaneous relative motions. In these calculi, the point-wise relative motions between different spatial entities are mapped to one of the qualitative values $\{-, +, 0\}$, denoting toward, away from and stable. In the basic Qualitative Trajectory Calculus (QTC_B; de Weghe *et al.*, 2004), the direction frame is compass-like and the base relations can be represented by tuples of qualitative values as shown in Figure 21. For example, $(+, 0)$ means that the first object is moving away from the second object's position and the second object is stable with respect to the first object's position. QTC_B is expressive enough to describe movements such as a carnivore hunting a prey; however, for movements such as vehicles traversing crossroads, the complex double-cross direction (Freksa, 1992) frame is needed, and therefore the QTC Double-Cross (QTC_C; de Weghe *et al.*, 2005a) has been defined. QTC_C includes 81 base relations and the rules for composing relations are given in de Weghe *et al.* (2005b).

According to Moreira *et al.* (1999), there are two types of moving objects: objects that have a completely free trajectory, which is only constrained by the dynamics of the object itself (e.g. a bird flying through the sky), and objects that have a constrained trajectory (e.g. a car on a road network). QTC_B and QTC_C can describe two objects moving freely in a plane but not objects with constrained trajectories in a network. The network version of QTC, QTC_N (Bogaert *et al.*, 2006), is proposed where the movement toward or away from is defined along the shortest path in a network between two objects, rather than the Euclidean distance as in QTC_B. Whereas the topological structure of the network can change with time, the QTC on changing networks QTC_{DN} (Delafontaine *et al.*, 2008) is derived from QTC_N. A prototype of QTC-based information system has been implemented in Delafontaine *et al.* (2011).

3.5 Shape

Shape is perhaps one of the most important characteristics of an object, but it is also particularly difficult to describe qualitatively. Topological information can express limited shape information, such as whether there is a hole in a region or the interior is connected; this information is too weak for finer grained distinctions for shape information. Although direction and distance have already added something to pure mereotopology, these calculi and models are not directly applicable for shape description.

Region-based methods focus on the interior of objects. A hierarchical representation approach is presented in Cohn (1995). It uses two primitive notions: that of two regions connecting and the convex hull of a region, computing the topological relationships between the concavities in the convex hull and number of concavities, and then a wide variety of concave shapes can be distinguished as shown in Figure 22. However, the predicate to detect whether two concavities i_1 and i_2 in the convex hull of region x lie on the same side of x, SameSide (i_1, i_2, x) in Cohn (1995) does not always give the correct answer. This problem is solved in Ouyang *et al.* (2009a) by an improved SameSide* predicate definition and an algorithm based on detecting concave concavities and transforming them to convex concavities, which are amenable to the revised SameSide predicate definition. The convex hull is a

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Figure 22 Concave regions with different shapes distinguishable using convex hulls (Cohn, 1995: 313)

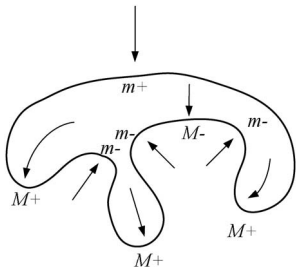


Figure 23 A pictorial example of Process-Grammar (Leyton, 1988: 220)

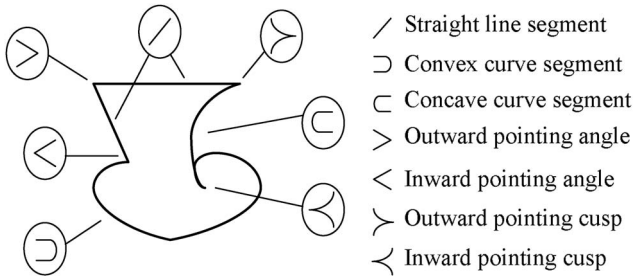


Figure 24 A picture of the seven qualitative curvature types (Galton & Meathrel, 1999: 1063)

powerful primitive and it has been shown that any pair of bounded, regular regions in 2D space can be distinguished by the three primitives: external connection, proper part of and being convex, if and only if they are not related by an affine transformation (Davis *et al.*, 1999).

Most boundary-based methods distinguish shapes mainly by the sequence of different types of curvature extrema along its contour or the boundary segments. Galton and Meathrel (1999) generalized much of this work. Process-Grammar (Leyton, 1988) expresses the smooth shape evolution of a smooth 2D curve in terms of transitions at five extrema: M+, m+, M−, m− and 0 corresponding to actions: protrusion, squashing, internal resistance, indentation and zero-curvature, respectively, as shown in Figure 23.

In contrast to the smooth outline assumption in Leyton (1988), a representation of outlines by means of strings over an alphabet of seven qualitative curvature types (as shown in Figure 24) is proposed in Galton and Meathrel (1999) that forms a formal language for the qualitative representation of two-dimensional outlines. $\mathcal{TLT}$ (Tripartite Line Tracks; Gottfried, 2003a, 2003b) considers three end-point-connected lines then the medial line determines an orientation grid, which has 15 partitions. And the two endpoints are described with respect to the medial line by their position in the orientation grid. If only considers the general position (i.e., not considering the position on the lines), then it at most can one discern 36 different cases. Removing all the symmetrical relations of $\mathcal{TLT}_{36}$, there remain 12 distinguishable relations denoted by $\mathcal{TLT}_{12}$, where the subscript is the number of possible relations. Omitting the length information encoded in $\mathcal{TLT}_{12}$, only six relations are left. Thus, $\mathcal{TLT}$ can describe 2D polygonal outlines by a sequence of different types of $\mathcal{TLT}$ under different granularities (Figure 25).

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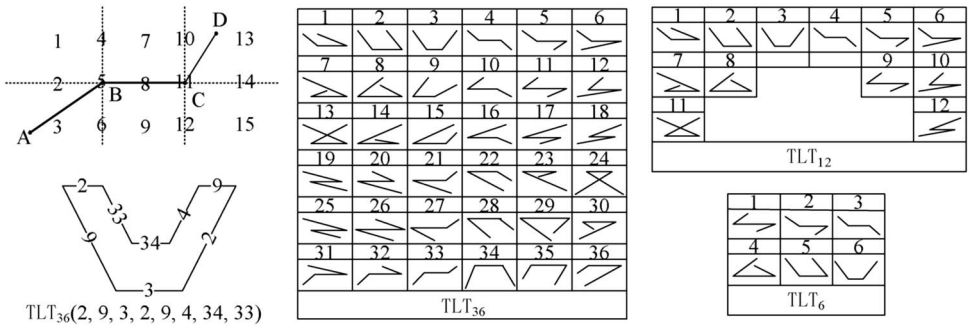


Figure 25 Left: The orientation grid formed by line segment BC and an example polygon described by $\mathcal{TLT}_{36}$; Middle: Iconic representations of $\mathcal{TLT}_{36}$; Right: Iconic representations of $\mathcal{TLT}_{12}$ and $\mathcal{TLT}_{6}$ (Gottfried, 2003b: 103, 104, 106)

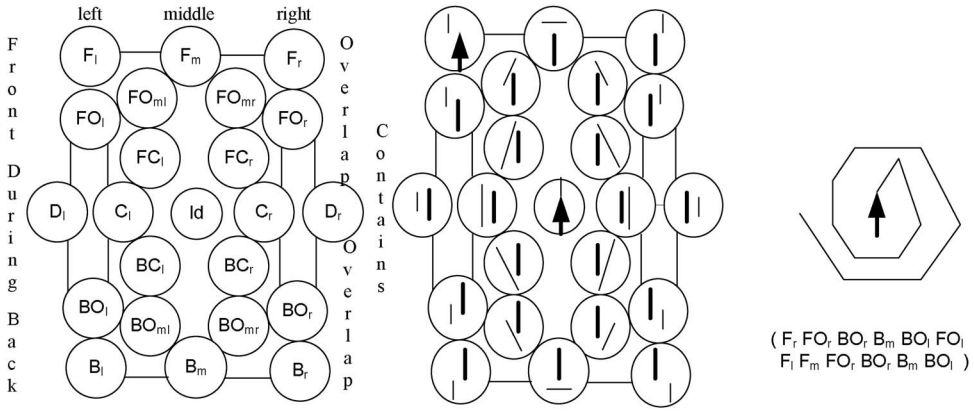


Figure 26 Left: Conceptual neighborhood of $\mathcal{BA}_{23}$ (right); Middle: iconic instantiations of all relations; Right: an example polygonal outlines described by $\mathcal{BA}_{23}$, the bold vertical segment being the reference segment which is oriented upwards (Gottfried, 2006: 114, 119)

Instead of considering three end-point-connected line segments, the $\mathcal{BA}$ (Bipartite Arrangements) Calculus (Gottfried, 2004) describes arbitrary pairs of line segments and has 23 different base relations as shown in Figure 26. If one line of a polygon is made the basis, then the position of every other line can be described relative to it, using the relations of $\mathcal{BA}$. Consequently, the list of $\mathcal{BA}$ relations so formed gives a qualitative shape representation, which is used in similarity assessment (Gottfried, 2006, 2008).

6.3.6 Combinations of binary qualitative calculi

Many types or aspects of spatial relations are formed by making different abstractions of the objective (metric) spatial relations. Topology, distance, direction and shape are classified by the subjective motivations of how people observe the world. These aspects can be integrated. Compared with single-aspect spatial relations, integrating multi-aspect spatial relations has more potential for practical application, as expressiveness is increased and has become a focus in QSR research.

In QSR, existing calculi can be combined to create new formalisms (Wölfl & Westphal, 2009); for example, the rectangle algebra (block algebra) can be defined as a specific twofold (threefold) product of the interval algebra. Compared with such orthogonal cases (where there is no interaction between the component algebra), it is more challenging to combine calculi with interdependent semantics such as the work (Gerevini & Renz, 2002) combining RCC8 and Point Algebra (Vilain *et al.*, 1990). Two possible combining strategies (Wölfl & Westphal, 2009) can be

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found in these non-orthogonal combinations: loose combining and tight combining. Loose combining focuses more on reasoning and interactions between the component calculi, whereas tight combining in general leads to a new constraint language and can be more expressive.

The combinations of RCC8 with either qualitative or metric-size information are obvious examples of loose combinations (Gerevini & Renz, 2002). It has been shown that, in each of the three maximal tractable subsets of RCC8, qualitative-size information does not increase reasoning complexity. However, with respect to the metric size, even when the set of topological relations is restricted to the eight basic relations, constraint network satisfaction is intractable; however, if some basic relation(s) are not used, then deciding the consistency becomes polynomial; for example, if PO is not permitted, which is a realistic assumption for applications where the variables represent physical objects. If we assume that all changes are continuous, then relations adjacent in time must be neighbors in the conceptual neighborhood diagram, and one can formalize temporal information and integrate it with topological and size information (Wang *et al*., 2003).

Other loose combination examples concentrate on integrating topological information with region-based direction calculi. In Li (2007), it is shown that, for the three maximal tractable subsets of RCC8, the satisfiability of a joint network of RCC8 and DIR_9 (MBR-based direction relations) is decided by the two satisfiability problems in RCC8 and rectangle algebra. In Liu *et al*. (2009), it is proven that only considering basic relations, the joint network of RCC8 and rectangle algebra is tractable, but that of RCC8 and CDC is NP-complete. Furthermore, in Li and Cohn (2012), the bipath-consistency algorithm in Gerevini and Renz (2002) is proven to be able to separate the topological constraints in polynomial time from directional ones, when the topological constraints are taken from two tractable subclasses of RCC8 ($\hat{\mathcal{H}}_8$, $\mathcal{C}_8$), and directional constraints are from a sub-algebra of the rectangle algebra, termed DIR_{49}. Considering the mutual influence between topological and directional relations, it is natural to derive topological relations from direction relations, when topological information is unavailable for computing topological relations. In Guo and Du (2009), it presents such computation methods for deriving topological relations from one and two CDC relations. In turn, because topological relations are less constrained than direction information; it makes little sense to derive direction information from topological relations. If the topological information is that x and y share a same part, then x must share a same part with MBR(y); therefore, only base CDC relation O can be exactly derived. If no same part is shared between x and y, then no useful direction information can be derived.

Tight combinations commonly form a new set of relations that can express multiple aspects of spatial information. In Sistla and Yu (2000), a set of qualitative relations and reasoning rules are presented to describe both topological and directional information. The set of basic relations includes *left_of*, *right_of*, *behind*, *in_front_of*, *above*, *below*, *inside*, *outside* and *overlaps*, and it has been proven that its deductive system is complete for 3D space but is not complete for 2D space. The $\mathcal{INDU}$ calculus (Pujari *et al*., 1999) can be seen as a tight combination of Interval Algebra with Point Algebra, that is, each basic relation includes not only the interval relations but also the size information represented by the point algebra, and it forms a JEPD relation set including 25 basic relations, as shown in Figure 27. The detailed analysis of its consistency is in Balbiani *et al*. (2006). Extended rectangle relations (Chen *et al*., 2010c), the twofold product of $\mathcal{INDU}$ relations, can be used for the integrating coarse representing and reasoning of the topological, directional and size information over MBRs.

3.7 Uncertainty

Uncertainty is an inherent characteristic of almost every aspect of knowledge representation in the real world. Although qualitative approaches can be seen as a technique dealing with uncertainty in some degree, as precise quantitative information may be unavailable or unnecessary; in practice, some situations cannot be properly represented by qualitative relations. For example, it is hard to define the accurate boundaries of regions. One reason is the limitation of observations: the object may have an explicit boundary but cannot be measured accurately. Another reason is intrinsic to

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Basic Interval Relation	Converse Relation	Meaning	Basic Interval Relation	Converse Relation	Meaning
$p^{<}$	$pi^{>}$	*xxxx* *yyyyyyy*	$o^{<}$	$oi^{>}$	*xxxx* *yyyyyyy*
$p^{=}$	$pi^{=}$	*xxxx* *yyyy*	$o^{=}$	$oi^{=}$	*xxxx* *yyyy*
$p^{>}$	$pi^{<}$	*xxxxxxx* *yyyy*	$o^{>}$	$oi^{<}$	*xxxxxxx* *yyyy*
$m^{<}$	$mi^{>}$	*xxxx* *yyyyyyyy*	$d^{<}$	$di^{>}$	*xxxx* *yyyyyyy*
$m^{=}$	$mi^{=}$	*xxxx* *yyyy*	$s^{<}$	$si^{>}$	*xxxx* *yyyyyyy*
$m^{>}$	$mi^{<}$	*xxxx* *yyyyyyy*	$f^{<}$	$fi^{>}$	*xxxx* *yyyyyyy*
$e^{=}$	$e^{=}$	*xxxx* *yyyy*			

Figure 27 25 basic relations of $\mathcal{INDU}$ calculus (Pujari *et al.*, 1999: 294)

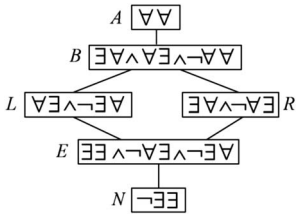


Figure 28 The value lattice (Yu *et al.*, 2004: 613)

the objects themselves, such as the boundary between a canyon and a river, or between mountains and flatlands: there is no exact boundary, but one would still like to represent that the mountains are adjacent to the flatlands.

Existing techniques for representing and reasoning about such uncertainty can be mainly classified into two categories: one approach extends existing spatial calculi specifically to express the indeterminacy, whereas the other applies mathematical tools, such as fuzzy sets and rough sets, to express the vagueness of the spatial objects or the spatial relationships.

Approaches in the former category often approximate the vagueness of regions by two nested crisp regions or a broad boundary. A vague region with indefinite boundary is represented by egg-yolk pairs (Cohn & Gotts, 1996), a pair of nested traditional crisp regions. The yolk represents a minimum extension of the indefinite region, whereas the egg represents its maximum extension. The topological relations between the indefinite regions are represented by a quadruple of RCC relations between the component crisp regions. For example, $F(yolk(x), egg(y))$ means the RCC5 relation between the minimum extension of x and the maximum extension of y. A total of 46 different mereological relationships are identified by the quadruples. A generalization of the egg-yolk calculus (Roy & Stell, 2001) is based on Lukasiewicz algebra, where the indeterminate regions include the crisp ones as a special case. A refinement of the egg-yolk model (Yu *et al.*, 2004) is based on the three predicates: P(x, y), C(x, y) and I(x, y), which mean that x has a part of y, x contains y and y contains x, respectively. Owing to the uncertainty, the value of the predicates is no longer a Boolean value but a value in a lattice, as shown in Figure 28. Value A means that all regions satisfy the predicate, N means there are no regions satisfying the predicate; $\{B, L, R, E\}$ are refinements of the value M, that is, there may exist regions satisfying the predicate. Only considering boundary-insensitive RCC5, this approach can discriminate 14 different topological relations when the values of predicates are in $\{A, M, N\}$ and 51 different topological relations, whereas the range of the predicates is $\{A, B, L, R, E, N\}$.

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$$\begin{pmatrix} A^{\circ} \cap B^{\circ} & A^{\circ} \cap \Delta B & A^{\circ} \cap B^{-} \\ \Delta A \cap B^{\circ} & \Delta A \cap \Delta B & \Delta A \cap B^{-} \\ A^{-} \cap B^{\circ} & A^{-} \cap \Delta B & A^{-} \cap B^{-} \end{pmatrix}$$

Figure 29 9-intersection matrix with broad boundary (Clementini & Felice, 1997b: 179)

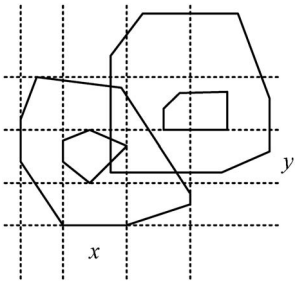


Figure 30 A pictorial example of 5×5 direction tiles (Cicerone & Felice, 2000: 17)

The extension of the nine-intersection models (Clementini & Felice, 1997) looks very similar to the egg-yolk calculus but is not equivalent, as only 44 relations are discerned rather than the 46 relations in egg-yolk theory, and the conceptual neighborhood graph is different from that in egg-yolk theory; this difference arises from different assumptions about the nature of the regions. In the latter, it is assumed that the broad boundary is not so broad as to accommodate the other region. The indefinite boundary is approximated as a broad boundary (ΔA) in place of a sharp boundary (∂x), as shown in Figure 29. It is shown in Clementini and Felice (1997) that this extension can be used to reason not only about regions with indeterminate boundaries but also can be specialized to cover various regions such as the convex hulls of regions, buffer zones, MBRs and raster images. An extension of broad boundaries to complex regions offers a solution to the problem of representing the uncertainty commonly affecting the boundaries of spatial objects (Clementini & Felice, 2001). An alternative four-tuple equivalent representation of the extended nine-intersection relations and composition table are presented in Du *et al.* (2008b) that aids in transforming the topological relationships between broad boundary regions into four tuples of the topological relationships between the crisp regions composing of the broad boundary regions, that is, the topological relations of lower crisp w.r.t. lower crisp, lower crisp to upper crisp, upper crisp to lower crisp and upper crisp to upper crisp. However, it should be mentioned that this quadruple representation is inherently different from that in egg-yolk theory. The first difference is that both the yolk and the white are closed sets, whereas the interior in Du *et al.* (2008b) is open and the broad boundary is closed. The second difference lies in the semantic groups of the base relations. A third difference is that the four relations in the quadruples of Du *et al.* (2008b) can express topological relations between objects with different dimensions, whereas egg-yolk theory requires that both objects have the same dimension.

Similar to the approaches handling uncertainty in topological relations, one approach to handling uncertainty in directional relations partitions the space by the pair of MBRs of minimum and maximum extension of an indeterminate region, and obtains 25 direction tiles, as shown in Figure 30, represented by a 5×5 direction matrix. In contrast to the empty and non-empty values of the 9-intersection, each entry of the matrix stores one of the four values, expressing the four different situations the target objects can have when intersecting with the direction tiles, which give rise to a huge number (4^{25}) of potential base relations (Cicerone & Felice, 2000). Another approach expresses the direction relation between broad boundary regions as a four-tuple of direction relations (Figure 31) between the crisp regions, which are the direction relations of lower crisp w.r.t. lower crisp, lower crisp to upper crisp, upper crisp to lower crisp and upper crisp to upper crisp (Dong *et al.*, 2011).

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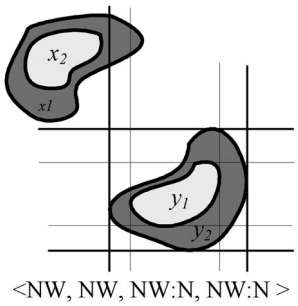


Figure 31 An example of the Cardinal Direction Calculus (CDC) quadruple between the broad boundary regions (Dong *et al.*, 2011: 331)

When treating uncertainty numerically, fuzzy sets and rough sets are the main tools to express the vagueness of the spatial entities and their relations. In Zhan's approach (Zhan, 1998; Zhan & Lin, 2003), a vague region is treated as a fuzzy set, which can be decomposed into three parts in: the core, the indeterminate boundary and the exterior. The indeterminate boundary is approximated by a set of α-boundaries, each of which corresponds to the boundary of an α-cut-level region; thus, a binary topological relation between fuzzy regions only belongs to a prototypical topological relation to some degree. The fuzzy extension of the nine-intersection calculus (Du *et al.*, 2005) is quite similar to Zhan's approach. According to the definitions of the membership functions, the topological space is divided into the fuzzy exterior, fuzzy boundary and fuzzy interior. Therefore, the domain of each element in nine-intersection matrix is a value in the interval [0, 1], rather than a Boolean value. The uncertainty of topological relations in fuzzy nine-intersection calculus is indicated by a membership degree. Unlike the previous discussion pertaining to continuous space, a finite resolution fuzzy spatial algebra (Schneider, 2000, 2003) is introduced achieving a uniform treatment of continuous and discrete space, and applied into the fuzzy extension of nine-interesection calculus. A fuzzification of the well-known RCC framework adapted for the finite discrete space domain is presented in Palshikar (2004).

Contrasting with approaches focusing on the fuzzy representation of regions, the fuzzy RCC (Schockaert *et al.*, 2006, 2008, 2009) only requires that the fuzzy version of the C relation is reflexive and symmetric, as in standard RCC. It does not impose any constraints on how regions are represented, nor how connection should be interpreted. Therefore, it is applicable in a wide variety of contexts, including those where space is used in a metaphorical way.

Rough set-based approaches often relate to the partition of the plane. In Bittner and Stell (2000, 2002), the indefiniteness also relates to locations. The rough location of a spatial object within a regional partition is characterized by a set of relations between parts of the object and parts of regions forming the regional partition, whereas the exact location is the region of space taken up by the object. For example, the rough location of boundary-insensitive approximations are defined by functions from the partition to the value set $\{fo, po, no\}$, which means the region covers all, some but not all and none of the cells defined by the partition. Thus, the indefiniteness is expressed by the pairs of greatest minimal and least maximal regions. Another way of expressing this is that the vague region defined by rough location consists of the three concentric regions: core, wide boundary and exterior, which coincide with the idea approximating vagueness by a broad boundary and pair of crisp regions in the former category of approaches dealing with uncertainty (Cohn & Gotts, 1996; Clementini & Felice, 1997; Cicerone & Felice, 2000; Roy & Stell, 2001; Dong *et al.*, 2011). When applying this rough location extension to the nine-intersection matrix, an algebra with 249 base relations emerges (Wang *et al.*, 2003). A combined description of topological and direction relations between broad boundary regions is given in Du *et al.* (2006).

Granularity is another concept worth mentioning relating to rough set approaches dealing with uncertainty. It characterizes the scale or level of detail (partition of the space), and therefore reflects the common phenomenon in knowledge stratification that knowledge can be organized

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into a hierarchical structure according to different granularities. Uncertainty can be managed by selecting appropriate granularities. Temporal and spatial granularity techniques are proposed, respectively, in Bittner (2002), Bittner and Smith (2001) and applied in knowledge discovery (Bédard *et al.*, 2001; Roddick & Spiliopoulou, 2002). A qualitative extent for spatio-temporal granularity is addressed in Stell (2003), which shows how the three spatial extents (everywhere, somewhere and nowhere) can be generalized to the granular description of spatio-temporal regions. The framework in Li and Nebel (2007) hierarchically represents and reasons about topological information, which implements jumping between different granularities of the topological relations.

4 Implementations

In contrast to the large number of qualitative spatial calculi (both those cited above, and others in the literature), the number of publicly available implementations using QSR techniques is still relatively small. QAT[6] (the Qualitative Algebra Toolkit; Condotta *et al.*, 2006) is a JAVA constraint programming library developed at the University of Artois, which aims at providing an open and generic tool to handle qualitative algebras and constraint networks on these algebras. The core of QAT includes three packages: the algebra package allows user to define qualitative algebras (including non-binary algebras) in a simple XML file; the QCN (Qualitative Constraint Network) package contains tools for defining the constraint networks; and the Solver package provides various methods solving the problems of interests on QAT, such as consistency checking, finding solutions and minimal networks.

SPARQ[7] (SPAtial Reasoning done Qualitatively; Dylla *et al.*, 2006; Wallgrün *et al.*, 2006, 2010) is a QSR toolbox developed at University of Bremen, which supports the most common tasks: qualification, computing with relations and constraint-based reasoning over binary and ternary calculi. SPARQ is written in LISP and libraries are implemented in the C language. The qualification module transforms the quantitative geometric scene description into a qualitative description according to one of the supported calculi. The computing relations module applies the relational operations defined in the calculus specification. Finally, the constraint reasoning module performs computations on constraint networks, such as path-consistency, scenario-consistency and back tracking search. According to the latest version of the manual (Wallgrün *et al.*, 2010), SPARQ supports 20 calculi in total.

GQR[8] (Generic Qualitative Reasoner; Gantner *et al.*, 2008) is a solver for binary QCNs, developed at the University of Freiburg. GQR is written in C++ and its aim is to implement a fast and extensible generic solver, while preserving the efficiency of calculus-specific solver as much as possible, which contrasts with the focus on QAT and SPARQ. It takes a calculus specification and several constraint networks as input, and tries to solve the networks by path-consistency and (heuristic) back tracking methods. New calculi can be added into the system by the specifications in text or XML format.

Spatial knowledge is essential component in human knowledge, and therefore several well-known Knowledge Bases and reasoning engines have introduced relations from qualitative spatial calculi into their ontology. For example, RCC has been introduced into the CYC[9] spatial ontology (Grenon, 2003). SNARK[10] (Stickel *et al.*, 2000; Waldinger *et al.*, 2003), an automated theorem prover once used as the reasoning component of SRI's High-Performance Knowledge Base (HPKB) system, has a built-in version of Allen's Interval calculus and RCC. Pellet Spatial[11] (Stocker & Sirin, 2009) is a QSR engine implemented on the top of OWL reasoner Pellet (Sirin & Parsia, 2004), which is capable of consistency checking and query answering over spatial data represented with RCC.

[6] http://www.cril.univ-artois.fr/~saade/QAT/

[7] http://www.sfbtr8.uni-bremen.de/project/r3/sparq/

[8] http://sfbtr8.informatik.uni-freiburg.de/R4LogoSpace/Tools/gqr.html

[9] http://www.opencyc.org/doc/

[10] http://www.ai.sri.com/~stickel/snark.html

[11] http://clarkparsia.com/pellet/spatial/

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5 Final discussion

In this paper, we have surveyed some of the representative results in the literature about qualitative spatial relationships, but space has certainly not allowed an exhaustive survey. As has already been seen in many other fields of knowledge representation, a single universal language for spatial representation is unlikely to appear. The best we can hope is that a series of representation and reasoning systems and the criteria for the applications will be developed (Cohn, 1997; Cohn & Hazarika, 2001; Cohn & Renz, 2007). It is shown in this survey that there is already a rich set of various spatial relationship calculi concerning the different aspects of space. We have outlined some of the key calculi according to the different type of spatial relationships (topology, direction, etc.) and have classified them in terms of the objects considered (region-based and point-based, etc.).

Most existing calculi focus only on one aspect of space, but many applications concern more than just one aspect of space; thus, integrations of spatial calculi need to be considered. The combination is not only confined to the combination of different aspects of calculi, such as topology and direction, or direction and distance but also the combination of qualitative calculi with quantitative information: qualitative and quantitative approaches are two complementary aspects of knowledge representation and reasoning.

In many situations, spatial entities cannot be abstracted as simple objects, such as simple regions, simple lines or points, but the deformations or integration of these basic components, which we called complex objects, for example, groups of points or compound regions. The spatial relationships between these objects will be more complicated but potentially more applicable in real-world applications. Calculi should not be confined to planar space but should also give frameworks for representation and reasoning in 3D space.

Another aspect to be noted is the abstraction of space. Continuous and discrete space (vector and raster space) result in different spatial relationship calculi, see, for example, the difference of 9IM in vector and raster space. Moreover, most spatial information obtained from physical recording devices nowadays is invariably digital in form and therefore implicitly uses a discrete representation of space. How to bridge the gap between the two types of space or to develop a unified framework is worthy of further research.

Hierarchy and granularity is fundamental to human cognition (Langacker, 1987). People can conceptualize the world at different levels and switch among these levels freely, and this is fundamental to human intelligence and flexibility (Hobbs, 1985). It reflects the process from coarse to fine, from simple to complex, from superficial to essential. Usually, the hierarchical structure consists of a number of distinct levels with respect to different levels of abstractions, where these levels can be decided by the semantics, granularities or resolutions. For example, as the distance between the observers and the observed decreases, more and more details can be captured by the observers. The same observed objects may be abstracted as either points or regions of increasing shape complexity at different resolutions; therefore, the calculi modeling the spatial relationships between them may need to be adapted to reflect the finer resolution; to be able to reason across these different levels, one needs a unified calculus or a way of propagating knowledge across levels.

Acknowledgments

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