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Dust acoustic instability with non-extensive distribution

SAN QIU LIU[1] and HUI BIN QIU[2]

[1]Department of Physics & School of Materials Science and Engineering, Nanchang University, Nanchang, 330047, China (sqlgroup@ncu.edu.cn)
[2]Department of Physics, Nanchang University, Nanchang, 330047, China

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Abstract. The instability of dust acoustic waves driven by electrons and ions with different drift velocities in dusty non-extensive plasma is investigated based on the kinetic theory. The non-extensivity parameters of non-extensive distribution for three plasma components are different from each other. The instability growth rate is shown to be dependent on the non-extensivity parameters as well as on the ion–electron number density ratio. In the extensive limit ($q = 1$), the result in Maxwellian distribution plasma is recovered. The instability growth rate is found to decrease as the population of suprathermal electrons and dust grains increases, but it enhances when the number of suprathermal ions increases and electron density decreases.

1. Introduction

Recently, there is an increasing evidence that the non-extensive statistical mechanics can be considered as a more appropriate basis of theoretical framework to describe complex systems whose properties cannot be exactly described by the Boltzmann–Gibbs statistical mechanics (Gell-Mann and Tsallis 2004). For $q < 1$, it gives the distribution having suprathermal power-law tails at high energies (Leubner 2002), which have been proved to be equivalent to kappa distribution (Livadiotis et al. 2009); for $q > 1$, it derives the distribution suitable for the description of systems containing only low-speed particles; and when the parameter $q \to 1$, Maxwellian distribution is recovered (Tsallis 1988). Much attention has been paid to the physics of non-extensive distributed plasma. For instance, such non-extensive statistics has been used to analyze plasma oscillations (Lima et al. 2000), Stellar plasma and solar neutrinos (Lavagno and Quarati 2000), and non-equilibrium plasma systems with Coulombian long-range interactions (Du 2004). Some theoretical studies on ion-acoustic mode (Liu and Du 2008), electro-acoustic mode (Bains 2011), relativistic oscillation (Liu and Chen 2011a,b), and arbitrary amplitude kinetic Alfvén solitons (Liu et al. 2011) in non-extensive plasma have been reported, as well as experimental laboratory observations (e.g. Tang et al. 2009.

Dusty plasmas appear in quite a large number of diverse environments such as planetary rings, cometary tails, interstellar media, Earth's ionosphere, stellarator, tokomak (e.g. Goertz 1989; Mendis and Rosenberg 1994; Horanyi 1996; Shukla and Mamun 2002; Sergeev 2006; Krasheninnikov et al. 2007), and so on. It has been shown both theoretically and experimentally that the presence of these extremely massive and highly charged dust grains in a plasma can modify the behavior of usual plasma waves and instabilities or introduce new eigenmodes (Tribeche and Bacha 2010). The dust acoustic wave is one new mode of a dusty plasma wave in which the massive negatively charged dust particles actually participate in wave motion (Thomas et al. 2007). The dust acoustic mode was first discovered theoretically by Rao et al. (1990). They derived the dispersion relation for small-amplitude dust acoustic waves. The linear dispersion relation of dust acoustic waves was also contained in the general dispersion relation of the low-frequency electrostatic waves (D'Angelo 1990). Four years later, observations of large-amplitude, low-frequency oscillations in dusty plasma were reported (Chu et al. 1994), and these observations were interpreted as very likely being dust acoustic waves (D'Angelo 1995). Subsequently, dust acoustic waves in strongly coupled dusty plasmas (Rosenberg and Kalman 1997), boundary effects on dust acoustic waves in collisional dusty plasmas (Shukla and Rosenberg 1999), modulation of dust acoustic waves with non-adiabatic dust charge fluctuations (Xue 2003), and influence of a polarization force on dust acoustic waves (Khrapak 2009) have been investigated. In recent years, studies on dusty plasmas in non-extensive distribution have been reported (e.g. Amour and Tribeche 2010; Gong and Du 2012).

Among many plasma wave phenomena, instability/Landau damping is the most well-known process in plasma physics. The Landau damping of dust acoustic waves is well known for a Maxwellian velocity distribution (Shukla and Mamun 2002), and instability of dust acoustic waves driven by drifting ions and electrons for a Maxwellian velocity distribution was reported (Rosenberg 1993). The Lorentzian plasma has also been considered. The Landau damping of dust acoustic waves with kappa distribution (Lee 2007), dust

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acoustic instability driven by drifting ions and electrons, which have the same drift velocity (Liu and Du 2009), and a more general case in which ions and electrons have different drift velocities (Liu and Li 2011) have been studied. However, to our knowledge, the instability of dust acoustic waves in dusty plasma with non-extensive distribution has not been investigated yet. In this paper, an effort is made to obtain the kinetic modes of dust acoustic instability driven by weak ion and electron drifts in unmagnetized, collisionless, and isotropic dusty plasma. The plasma components (electrons, ions, and dust particles) are assumed to be described by the non-extensive distribution with different q-indices, and the ions and electrons have different drift velocities.

Liu and Li (2011) investigate dust acoustic instability with kappa distribution, but the kappa distribution is only a part of non-extensive distribution. In this paper we expand the kappa distribution to the non-extensive distribution and give a correct three-dimensional (3D) non-extensive distribution that is different from that given by Silva et al. (1998).

The paper is organized as follows. In Sec. 2, we obtain dispersion relation and instability growth rate of dust acoustic waves. In Sec. 3, numerical results are given and discussed. A summary of our results and conclusions is given in Sec. 4.

2. Instability growth rate of dust acoustic waves

Conventionally, the 3D equilibrium, non-extensive distribution function, which is different from that given by Silva et al. (1998), can be taken as

$$f_0(\mathbf{v}) = \frac{n_0 A_q}{\pi^{3/2} v_T^3}\left[1-(q-1)\frac{\mathbf{v}^2}{v_T^2}\right]^{\frac{2-q}{q-1}}, \tag{2.1}$$

where n_0, A_q, and q are the particle number density, the constant of normalization, and non-extensive parameter characterizing universality classes of non-additivity (Tsallis 2009) respectively; $v_T = \sqrt{2\kappa_B T/m}$ is the effective thermal speed, where κ_B, T, and m are the Boltzmann constant, temperature, and mass respectively. As one may check, for $q \leqslant -1$, the distribution is unnormalizable; it has power-law tails for $-1 < q < 1$ and exhibits a thermal cutoff on the maximum value allowed for the velocity of the particles for $q > 1$, given by $v_{\max} = \sqrt{v_T^2/(q-1)}$; in addition, in the limit of $q \to 1$, Maxwellian distribution is recovered. This means that the range of integration over v_x, v_y, and v_z containing the term $f_0(\mathbf{v})$ should be from $-\infty$ to $+\infty$ when $-1 < q \leqslant 1$, and from $-v_{\max}$ to $+v_{\max}$ when $q \geqslant 1$. The constant of normalization $A_q = \sqrt{1-q}\frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q}-\frac{1}{2})}$ when $-1 < q \leqslant 1$; $A_q = \frac{1+q}{2}\sqrt{q-1}\frac{\Gamma(\frac{1}{q-1}+\frac{1}{2})}{\Gamma(\frac{1}{q-1})}$ when $q \geqslant 1$. Here Γ is the gamma function.

Without loss of generality, we assume the plane wave to propagate in z-direction, and the distribution functions for electrons, ions, and dust particles are given by the drifting non-extensive distribution,

$$f_{q,\alpha}(\mathbf{v}) = \frac{n_{\alpha 0} A_{q_\alpha}}{\pi^{3/2} v_\alpha^3}\left[1-(q_\alpha-1)\frac{v_x^2+v_y^2+(v_z-u_{\alpha 0})^2}{v_\alpha^2}\right]^{\frac{2-q_\alpha}{q_\alpha-1}}, \tag{2.2}$$

where $\alpha(=e,i,d)$ denotes electron (e), ion (i), or dust particle (d), the drift velocities of electrons and ions are $u_{e0} = u_{ed}$ and $u_{i0} = u_{id}$ respectively ($u_{ed} \neq u_{id}$), dust particles are at rest at $u_{d0} = 0$, and u_{ed} and u_{id} are the relative speeds of electrons and ions to dust grain respectively.

Here a collisionless, isotropic, and unmagnetized dusty plasma with electrons, singly charged ions, and negatively charged dust particles has been taken into account to obtain a dust acoustic waves dispersion relation. From the kinetic theory, the longitudinal dielectric function can be written as (Li 2004)

$$\varepsilon_k^l = 1 + \sum_\alpha \frac{4\pi e_\alpha^2}{k^2}\int \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v} + i\varphi}\cdot\left(\mathbf{k}\cdot\frac{\partial f_\alpha^R}{\partial \mathbf{p}}\right)d\mathbf{p}, \tag{2.3}$$

where e_α is the charge of component α, $i\varphi$ comes from the Landau rules ($\varphi \to 0^+$) (Lifshitz and Pitaevskii 1981), and f_α^R is the particle equilibrium distribution function.

As we have chosen the z-axis along the direction of the wave vector $\mathbf{k}$, (2.3) becomes

$$\varepsilon_k^l = 1 + \sum_\alpha \frac{4\pi e_\alpha^2}{k^2}\int \frac{k}{\omega - k v_z + i\varphi}\cdot\left(\frac{\partial f_{\alpha,p_z}^R}{\partial p_z}\right)dp_z, \tag{2.4}$$

where f_{α,p_z}^R is the 1D non-extensive momentum distribution function

$$f_{\alpha,p_z}^R \equiv \int f_\alpha^R dp_y dp_z = \frac{n_{\alpha 0} A_{q_\alpha}}{\pi^{1/2} m_\alpha v_\alpha} \times\left[1-(q_\alpha-1)\frac{m_\alpha^2(v_z-u_{\alpha 0})^2}{m_\alpha^2 v_\alpha^2}\right]^{\frac{1}{q_\alpha-1}}. \tag{2.5}$$

Substituting (2.5) into (2.4), we obtain a generalized dispersion equation,

$$\varepsilon_k^l = 1 + \sum_\alpha \frac{2\omega_{p\alpha}^2}{k^2 v_\alpha^2}\left[\frac{1+q_\alpha}{2} - Z(x_\alpha)\right], \tag{2.6}$$

where $\omega_{p\alpha} = \sqrt{4\pi e_\alpha^2 n_{\alpha 0}/m_\alpha}$ is the plasma frequency of component α, $x_\alpha = \omega/k v_\alpha$ is the phase velocity normalized to the thermal velocity v_α, and $Z(x_\alpha)$ is the dispersion function,

$$Z(x_\alpha) = \frac{A_{q_\alpha}}{\pi^{1/2}}\int_{-\xi_{\alpha\max}}^{\xi_{\alpha\max}} \frac{x_\alpha}{x_\alpha - (\xi_\alpha - u'_{\alpha 0}) + i\varphi} \times\left[1-(q_\alpha-1)(\xi_\alpha-u'_{\alpha 0})^2\right]^{\frac{2-q_\alpha}{q_\alpha-1}} d(\xi_\alpha - u'_{\alpha 0}), \tag{2.7}$$

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where $\xi_\alpha = v_z/v_\alpha$ and $u'_{\alpha 0} = u_{\alpha 0}/v_\alpha$. $\xi_{\alpha\max} = +\infty$, when $-1 < q_\alpha \leqslant 1$; $\xi_{\alpha\max} = 1/\sqrt{(q_\alpha - 1)}$, when $q_\alpha > 1$. For dust acoustic waves, the phase velocity is much larger than the thermal velocity of dust but much less than those of electrons and ions, namely $v_d \ll \omega/k \ll v_i \ll v_e$. From the dimensionless parameter $x_\alpha = \omega/kv_\alpha$, one gets $x_d \gg 1$ and $x_e \ll x_i \ll 1$.

Using the Plemelj formula (Li 2004),

$$\frac{1}{z \pm i0} = \wp\frac{1}{z} \mp i\pi\delta(z), \tag{2.8}$$

where $\wp$ denotes the principal value, and the ξ_d integral in equation (2.7) becomes

$$\begin{aligned}\int_{-\xi_{d\max}}^{\xi_{d\max}} \frac{x_d}{x_d - \xi_d + i\varphi}\left[1-(q_d-1)\xi_d^2\right]^{\frac{2-q_d}{q_d-1}} d\xi_d \\ = x_d\wp\int_{-\xi_{d\max}}^{\xi_{d\max}} \frac{1}{x_d-\xi_d}\left[1-(q_d-1)\xi_d^2\right]^{\frac{2-q_d}{q_d-1}} d\xi_d \\ - i\pi x_d\left[1-(q_d-1)\xi_d^2\right]^{\frac{2-q_d}{q_d-1}}\end{aligned} \tag{2.9}$$

Making the large argument expansion for dispersion function of dust grains, integrating and substituting the result into the longitudinal dielectric function of dust grains, we can obtain

$$\begin{aligned}\varepsilon_k^{d(l)} \approx 1 + \frac{2\omega_{pd}^2}{k^2v_d^2}\Bigg\{ -\frac{1}{2}\frac{k^2v_d^2}{\omega^2} - \frac{3}{2(3q_d-1)}\frac{k^4v_d^4}{\omega^4} \\ + i\sqrt{\pi}A_{q_d}\frac{\omega}{kv_d}\left[1-(q_d-1)\frac{\omega^2}{k^2v_d^2}\right]^{\frac{2-q_d}{q_d-1}}\Bigg\}.\end{aligned} \tag{2.10}$$

Similarly, making the small argument expansion for dispersion function of electrons and ions, the longitudinal dielectric function of electrons and ions is obtained,

$$\begin{aligned}\varepsilon_k^{\alpha(l)} \approx 1 + \frac{2\omega_{p\alpha}^2}{k^2v_\alpha^2}\Bigg\{\frac{1+q_\alpha}{2} - \frac{(1+q_\alpha)(3-q_\alpha)}{2}\left(\frac{\omega-ku_{\alpha d}}{kv_\alpha}\right)^2 \\ + i\sqrt{\pi}A_{q_\alpha}\frac{\omega-ku_{\alpha d}}{kv_\alpha}\Bigg\}, (\alpha = e, i).\end{aligned} \tag{2.11}$$

Combining (2.10) and (2.11), we obtain the dispersion equation of longitudinal oscillation,

$$\begin{aligned}\varepsilon_k^l = 1 + \sum_{\alpha=i,e}\frac{2\omega_{p\alpha}^2}{k^2v_\alpha^2}\Bigg[\frac{1+q_\alpha}{2} - \frac{(1+q_\alpha)(3-q_\alpha)}{2} \\ \times\left(\frac{\omega-ku_{\alpha d}}{kv_\alpha}\right)^2 + i\sqrt{\pi}A_{q_\alpha}\frac{\omega-ku_{\alpha d}}{kv_\alpha}\Bigg] \\ + \frac{2\omega_{pd}^2}{k^2v_{Td}^2}\Bigg[-\frac{1}{2}\frac{k^2v_d^2}{\omega^2} - \frac{3}{2(3q_d-1)}\frac{k^4v_d^4}{\omega^4} \\ + i\sqrt{\pi}A_{q_d}\frac{\omega}{kv_d}\left[1-(q_d-1)\frac{\omega^2}{k^2v_d^2}\right]^{\frac{2-q_d}{q_d-1}}\Bigg]\end{aligned} \tag{2.12}$$

In order to obtain the dispersion relation of dust acoustic wave, we write $\varepsilon_k^l = Re\varepsilon_k^l + iIm\varepsilon_k^l$, where $Re\varepsilon_k^l$ and $Im\varepsilon_k^l$ are respectively the real and imaginary parts of the dielectric permittivity; we also write $\omega = \omega_r + i\gamma$, where ω_r and γ are the real and imaginary parts of the frequency respectively. In the case of $|Re\varepsilon_k^l| \gg |Im\varepsilon_k^l|$ and $|\omega_r| \gg |\gamma|$, the real and imaginary parts of dielectric permittivity can be written as

$$Re\varepsilon_k^l \approx 1 + \sum_{\alpha=i,e}\frac{2\omega_{p\alpha}^2}{k^2v_{T\alpha}^2}\left(\frac{1+q_\alpha}{2}\right) - \frac{\omega_{pd}^2}{\omega_r^2}, \tag{2.13}$$

$$\begin{aligned}Im\varepsilon_k^l = \sum_{\alpha=i,e}\frac{2\omega_{p\alpha}^2}{k^2v_\alpha^2}\frac{\omega-ku_{\alpha d}}{kv_\alpha}\sqrt{\pi}A_{q_\alpha} + \frac{2\omega_{pd}^2}{k^2v_{Td}^2}\frac{\omega}{kv_d}\sqrt{\pi}A_{q_d} \\ \times\left[1-(q_d-1)\frac{\omega^2}{k^2v_d^2}\right]^{\frac{2-q_d}{q_d-1}}.\end{aligned} \tag{2.14}$$

According to the longitudinal dispersion relation

$$Re\left[\varepsilon_k^l(\omega_r, k)\right] = 0, \tag{2.15}$$

we can obtain the dispersion relation of dust acoustic waves,

$$\frac{\omega_r}{\omega_{pd}} = k\lambda_{De}\left(k^2\lambda_{De}^2 + \frac{1+q_e}{3q_e-1} + \delta\frac{T_e}{T_i}\frac{1+q_i}{3q_i-1}\right)^{-1/2}, \tag{2.16}$$

where the charge neutrality condition $\chi_d Z_d n_{d0} + n_{i0} - n_{e0} = 0$ has been used, and $\chi_d = +1(-1)$ are for positively (negatively) charged dust particles. $\lambda_{De} = v_e/\omega_{pe}$ is the Debye length of electrons, $\delta = n_{i0}/n_{e0}$ is the ion–electron density ratio, and Z_d is the number of charges of the dust grain. The instability growth rate reads as

$$\gamma_q = -\left.\frac{Im\varepsilon_k^l}{\partial Re\varepsilon_k^l/\partial\omega}\right|_{\omega=\omega_r}. \tag{2.17}$$

Substituting (2.13) and (2.14) into (2.17), we obtain

$$\frac{\gamma_q}{\omega_{pd}} = \Lambda_q\left[B_{q1}\left(1 - D_q\frac{u_{ed}}{v_e}\right) + B_{q2}\left(1 - D_q\frac{u_{id}}{v_e}\right) + C_q\right], \tag{2.18}$$

where

$$\begin{aligned}\Lambda_q = -\frac{k\lambda_{De}\sqrt{\chi_d Z_d(1-\delta)}}{2}\sqrt{\frac{1}{3q_e-1}} \\ \times\left(k^2\lambda_{De}^2 + \frac{1+q_e}{3q_e-1} + \delta\frac{T_e}{T_i}\frac{1+q_i}{3q_i-1}\right)^{-2},\end{aligned} \tag{2.19}$$

$$B_{q1} = \frac{2\sqrt{\pi}}{3q_e-1}A_{q_e}\left(\frac{m_e}{m_d}\right)^{1/2}, \tag{2.20}$$

$$B_{q2} = \sqrt{\frac{3q_e-1}{2}}\left(\frac{2}{3q_i-1}\right)^{3/2}A_{q_i}\delta\left(\frac{m_i}{m_d}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2}, \tag{2.21}$$

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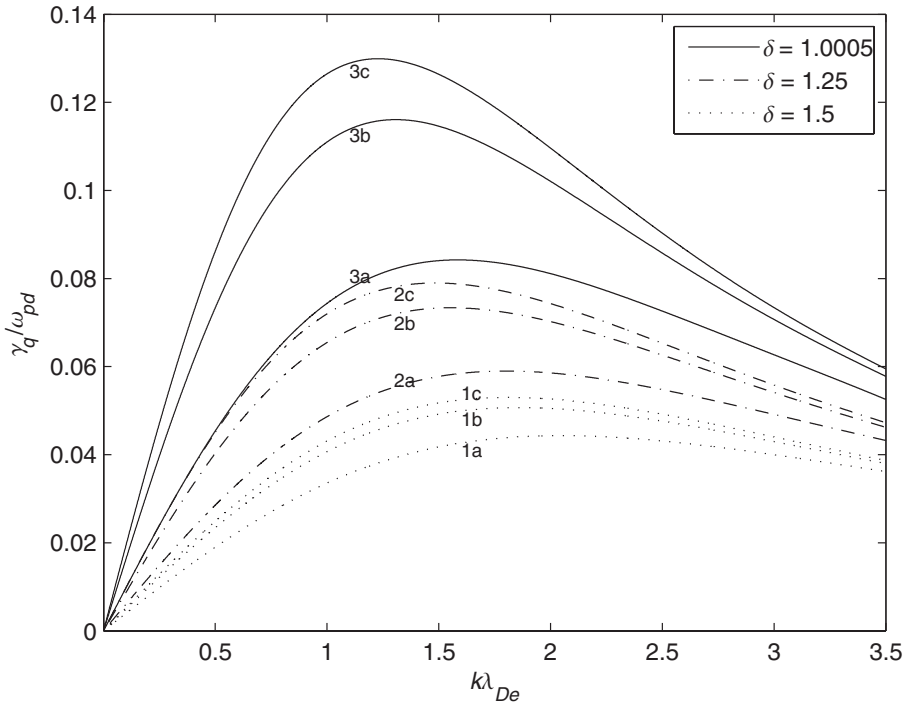


Figure 1. Normalized instability growth rate of dust acoustic waves as a function of the normalized wave number $k\lambda_{De}$ for different values of q_e (a corresponds to $q_e = 0.5$, b to $q_e = 0.75$, and c to $q_e = 1$) when δ have different values.

$$C_q = \sqrt{\frac{3q_e - 1}{2}} \left(\frac{2}{3q_d - 1}\right)^{3/2} A_{q_d} \chi_d Z_d (1-\delta) \left(\frac{T_e}{T_d}\right)^{3/2} \times \left[1 + \frac{1-q_d}{3q_d - 1}\frac{T_e}{T_d}\chi_d Z_d (1-\delta) \times \left(k^2\lambda_{De}^2 + \frac{1+q_e}{3q_e - 1} + \delta\frac{T_e}{T_i}\frac{1+q_i}{3q_i - 1}\right)^{-1}\right]^{\frac{2-q_d}{q_d - 1}}, \quad (2.22)$$

$$D_q = \frac{\sqrt{2}}{\sqrt{\chi_d Z_d (1-\delta)}}\sqrt{\frac{m_d}{m_e}} \times \left(k^2\lambda_{De}^2 + \frac{1+q_e}{3q_e - 1} + \delta\frac{T_e}{T_i}\frac{1+q_i}{3q_i - 1}\right)^{1/2}. \quad (2.23)$$

As seen from (2.18), the instability growth rate of dust acoustic waves depends not only on the mass, temperature, ion–electron number density ratio, the dust charge state, and drift velocities of components but also on the non-extensive parameter, q_α. When $q_\alpha = 1 - \frac{1}{\kappa_\alpha}$ (κ_α is the spectral index of kappa distribution), γ_q reduces to growth rate in kappa distribution (Liu and Li 2011); and in the limit $q_\alpha \to 1$, the result in Maxwellian distribution plasma is recovered.

3. Numerical results and discussions

In this section, we will numerically investigate the effects of non-extensive parameters and the ion–electron number density ratio on instability growth rate in dust plasma with non-extensive distributed electrons, ions, and dust grains. According to (2.18), the instability growth rate is related to mass, temperature, and number density ratios between the particles of components, dust charge state, and the drifting-phase velocity ratios. We choose nominal values (Goertz et al. 1988; Goertz 1989) in Saturn's E-ring or G-ring, which are $Z_d \sim 7 \times 10^4$, $m_e/m_d \sim 10^{-8}$, and $T_e = 2T_i = 10T_d$, $\chi_d = -1$, $u_{ed}/v_e = 1/60$ are assumed. With these values, we plot the curves of the normalized instability growth rate γ_q/ω_{pd} as a function of the normalized wave number $k\lambda_{De}$ with selected values of the non-extensive parameters q_α in the cases of $\delta = 1.0005$, 1.25, and 1.5 in Figs. 1–4 respectively. Considering the charge neutrality condition, we find that $\delta = u_{ed}/u_{id}$. Because the experimental results point to a class of non-extensive distribution described by non-extensive parameters smaller than unity (Lima et al. 2000; Silva et al. 2005), the selected values of the non-extensive parameters are 0.5, 0.75, and 1 (Maxwellian).

Figure 1 shows the effect of non-extensive parameter of electron q_e and ion–electron number density ratio δ on the instability growth rate. The calculation is based on (2.18), where $q_i = q_d = 1$ is assumed. We find that for a fixed value of δ, as q_e increases, the instability growth rate increases and has the maximum value at $q_e = 1$ (Maxwellian). So we conclude that the non-extensive distribution of electrons in plasma has a stabilized effect on dust acoustic waves. The dust acoustic waves will be more unstable for bigger q_e than for smaller q_e, and this effect is more obvious for smaller $k\lambda_{De}$ than for larger $k\lambda_{De}$.

Figure 2 shows the effect of q_i and δ on the instability growth rate. Here $q_e = q_d = 1$ is assumed. It is found that for a fixed value of δ, as q_i increases, the instability

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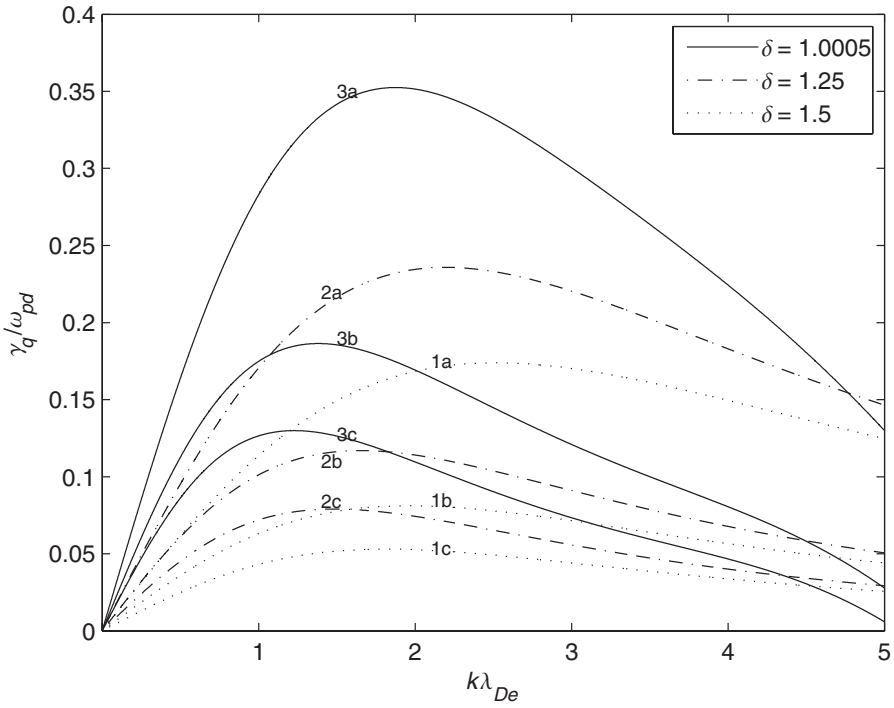


Figure 2. Normalized instability growth rate of dust acoustic waves as a function of the normalized wave number $k\lambda_{De}$ for different values of q_i (a corresponds to $q_i = 0.5$, b to $q_i = 0.75$, and c to $q_i = 1$) when δ have different values.

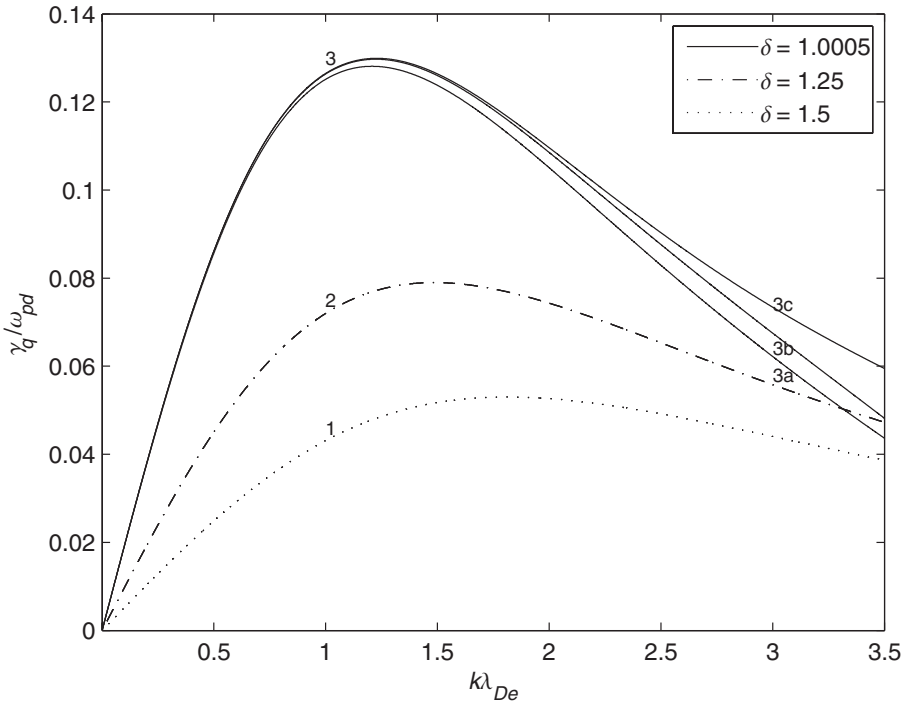


Figure 3. Normalized instability growth rate of dust acoustic waves as a function of the normalized wave number $k\lambda_{De}$ for different values of q_d (a corresponds to $q_d = 0.5$, b to $q_d = 0.75$, and c to $q_d = 1$) when δ have different values.

growth rate decreases and has the minimum value at $q_i = 1$ (Maxwellian). So we conclude that the non-extensive distribution of ions in the plasma has a stabilized effect on dust acoustic waves. The dust acoustic waves will be more unstable for smaller q_i than for bigger q_i, and this effect is more obvious for smaller $k\lambda_{De}$ than for larger $k\lambda_{De}$.

In Fig. 3, instability growth rate curves are plotted for selected values of q_d and δ. Here $q_e = q_i = 1$ is assumed. This figure clearly shows that when ion–electron number density ratio δ is nearly unity, the growth rate increases with an increase in q_d at large normalized wave numbers $k\lambda_{De}$ and has the maximum value at $q_d = 1$ (Maxwellian; see $\delta = 1.0005$). This means that the non-extensive

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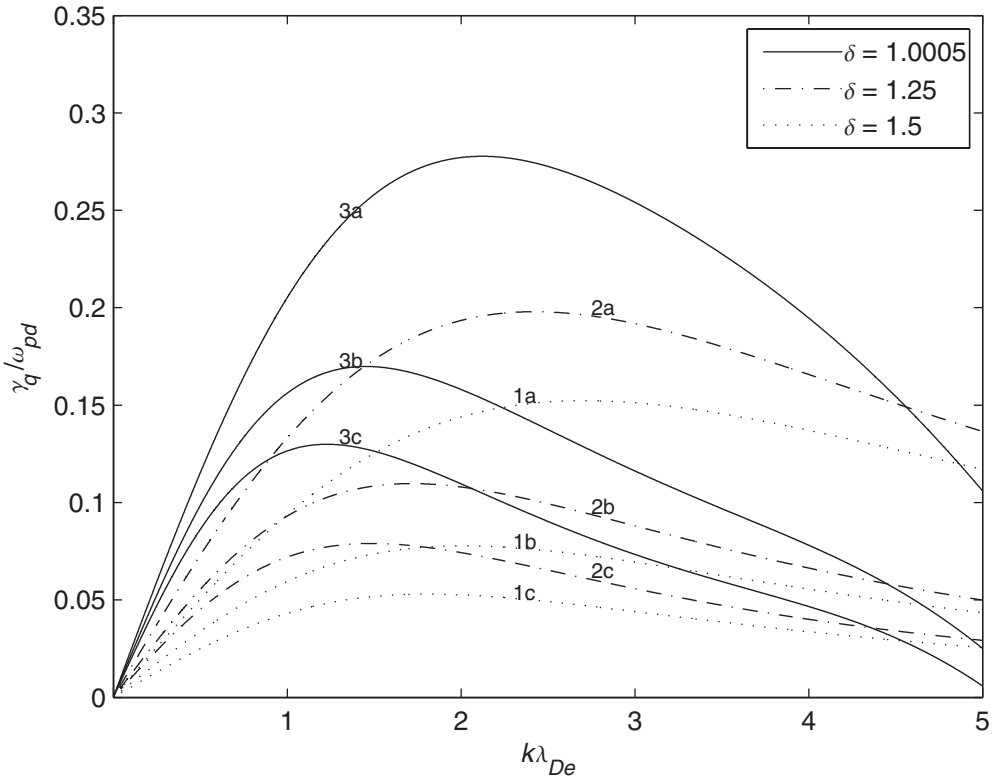


Figure 4. Normalized instability growth rate of dust acoustic waves as a function of the normalized wave number $k\lambda_{De}$ for different values of q (a corresponds to $q = q_e = q_i = 0.5$, b to $q = q_e = q_i = 0.75$, and c to $q = q_e = q_i = 1$) when δ have different values.

distribution of dust particles has a stabilizing effect on dust acoustic waves. When $\delta = 1.25$ or 1.5, all three curves are almost the same. So we conclude that dust acoustic waves will be more unstable for bigger q_d than for smaller q_d, but this effect is not so essential when δ is large.

Finally, we take this discussion a step further. We let $q_e = q_i = q$ and $q_d = 1$ in (2.18) so as to investigate joint effects on instability. Figure 4 presents a set of curves for γ_q/ω_{pd} against q at three fixed values of ion–electron number density ratio: $\delta = 1.0005$, 1.25, and 1.5. We find that for each case the instability growth rate decreases with an increase in q and has the minimum value at $q = 1$. Thus, we conclude that the non-extensive distribution of electrons and ions has an instability effect on dust acoustic waves in plasma, and this effect is more obvious for smaller $k\lambda_{De}$ than for larger $k\lambda_{De}$.

As we have seen from Figs. 1–4, when q_α is fixed and δ increases, the instability growth rate decreases. This means that decreasing of electron density will be useful for stability.

4. Summary and conclusion

We have considered the instability growth rate of dust acoustic waves propagating in a collisionless, unbounded, unmagnetized, isotropic dusty plasma modeled by non-extensive distribution with different q-parameters for electrons, ions, and dust grains. The drift velocities of electrons and ions are assumed to be different, and the dust grains are at rest. Based on the kinetic theory, we obtain the dispersion relation and the instability growth rate of dust acoustic waves. In the case of $q = 1 - \frac{1}{\kappa}$, the results based on kappa distribution are recovered; and in the limit $q \to 1$, the results reduced to the case of Maxwellian distribution. We found that the normalized instability growth rate γ_q/ω_{pd} decreases with an increase in ion–electron number density ratio δ. Meanwhile, we found that when δ is fixed, the non-extensive parameters of electrons and dust grains have a stabilizing effect, but the non-extensive parameter of ions has an instability effect on dust acoustic waves.

Since dust grains appear to be almost omnipresent in the space, our results can be useful for understanding the non-extensive effects on dispersion relation and instability growth rate of dust acoustic waves in dusty plasma which can be found in regions of planetary rings, cometary tails, interstellar media, tokomak, stellarator, and so on.

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