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Adaptive Impedance Control of Robot Manipulators based on Function Approximation Technique

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SUMMARY

This paper presents an adaptive impedance control scheme for an n-link constrained rigid robot manipulator without using the regressor. In addition, inversion of the estimated inertia matrix is also avoided and the new design is free from end-point acceleration measurements. The dynamics of the robot manipulator is assumed that all of the matrices in robot model are unavailable. Since these matrices are time-varying and their variation bounds are not given, traditional adaptive or robust designs do not apply. The function approximation technique is used here to represent uncertainties in some finite linear combinations of the orthogonal basis. The dynamics of the output tracking can thus be proved to be a stable first order filter driven by function approximation errors. Using the Lyapunov stability theory, a set of update laws is derived to give closed loop stability with proper tracking performance. A 2 DOF planar robot with environment constraint is used in the computer simulations to test the efficacy of the proposed scheme.

KEYWORDS: Impedance control; Adaptive control; Function approximation.

1. INTRODUCTION

The impedance control[1] of robot manipulators is to maintain a desired dynamic relationship between the end-effector and the environment. A second order mass-spring-damper system is used to specify the target dynamics. It gives a unified approach for controlling the robot in both free space and constrained motion control. Following the work of Hogan, several studies of the impedance control have been proposed. Anderson and Spong[2] combined the impedance control with the hybrid control. Goldenberg[3] used feedback and feedforward compensation for both force and impedance control. Mills and Liu[4] proposed a impedance control method to control the generalized contact force and position. Gonzalez and Widmann[5] presented a hybrid impedance control scheme which uses force commands to replace desired trajectory. Yoshikawa[6] surveyed the force control for robot manipulators.

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In practical applications, the dynamics of the robot manipulator and the environment inevitably contains various uncertainties and disturbances. Under this circumstance, one of the effective ways to deal with this difficulty is to apply the adaptive strategy to the impedance control. Slotine and Li[7] extended the adaptive free motion control to the constrained manipulators. Kelly et al.[8] suggested two adaptive impedance controllers to reduce model uncertainties. Lu and Meng[9] presented a concept of target-impedance reference trajectories to deal with the problems of imperfect sensor feedback and uncertain robot parameters. Based on singular motion robot representation, Carelli and Kelly[10] designed an adaptive position/force controller for constrained robots to achieve global stability results. Colbaugh et al.[11] proposed a direct adaptive impedance control scheme without the knowledge of the structure or the parameters of the robot dynamics. Since joint acceleration is difficult to measure precisely, Zhen and Goldenberg[12] designed an adaptive impedance controller without requiring measurements or estimates of acceleration. To compute efficiently, Wedeward and Colbaugh[13] presented a position-based adaptive impedance controller and a model reference adaptive impedance controller that did not need to calculate the robot inverse dynamics or inverse kinematic transformation. Using the camera-in-hand, Mut et al.[14] proposed an adaptive impedance tracking controller with visual feedback.

For these adaptive impedance control approaches, most of them require some information of the system model such as the regressor matrix and bounds of the uncertainty. In this paper, we would like to propose an adaptive impedance controller based on the function approximation technique[15–19] without using the regressor matrix. In addition, this approach does not require joint acceleration measurements or inversion of the estimated inertia matrix.[20,21] The basic idea is to represent system uncertainties using finite combinations of orthogonal basis with some unknown constant weightings. Output error dynamics can thus be derived as a stable first order filter driven by parameter error vectors. Appropriate update laws for the weighting vectors can be selected so that the time derivative of some Lyapunov function candidate can be proved to be negative semi-definite. Effects of the approximation error on system performance can then be investigated.

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This paper is organized as follow: In section 2, a regressor–free adaptive impedance controller is designed with rigorous proof of closed loop stability. Section 3 presents computer simulation results of the proposed controller on a 2-D constrained robot. Section 4 concludes the paper.

2. MAIN RESULTS

The dynamics of an n-link constrained rigid robot manipulator can be described by

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} - \mathbf{J}^T\mathbf{F}_{ext} \tag{1}$$

where $\mathbf{q} \in \Re^n$ is the generalized coordinate, $\boldsymbol{\tau} \in \Re^n$ is the applied joint torque, $\mathbf{D}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ is an n-vector of centrifugal and Coriolis forces, and $\mathbf{g}(\mathbf{q})$ is the gravity vector. $\mathbf{J}(\mathbf{q}) \in \Re^{n\times n}$ is the Jacobian matrix, which is assumed to be nonsingular, and $\mathbf{F}_{ext} \in \Re^n$ is the external force at the end-effector. The system model can also be represented in the Cartesian space as

$$\mathbf{D}_x(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{C}_x(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}_x(\mathbf{x}) = \mathbf{J}^{-T}\boldsymbol{\tau} - \mathbf{F}_{ext} \tag{2}$$

where $\mathbf{x} \in \Re^n$ is the position vector of the end-effector in the Cartesian space, and

$$\mathbf{D}_x(\mathbf{x}) = \mathbf{J}^{-T}\mathbf{D}(\mathbf{q})\mathbf{J}^{-1} \tag{3a}$$

$$\mathbf{C}_x(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{J}^{-T}(\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{D}(\mathbf{q})\mathbf{J}^{-1}\dot{\mathbf{J}})\mathbf{J}^{-1} \tag{3b}$$

$$\mathbf{g}_x(\mathbf{x}) = \mathbf{J}^{-T}\mathbf{g}(\mathbf{q}). \tag{3c}$$

In the traditional impedance control,[22] all system parameters are given and a controller is designed so that the closed-loop system behaves like a target impedance

$$\mathbf{M}_m(\ddot{\mathbf{x}} - \ddot{\mathbf{x}}_d) + \mathbf{B}_m(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) + \mathbf{K}_m(\mathbf{x} - \mathbf{x}_d) = -\mathbf{F}_{ext} \tag{4}$$

where $\mathbf{x}_d \in \Re^n$ is the desired trajectory, and $\mathbf{M}_m \in \Re^{n\times n}$, $\mathbf{B}_m \in \Re^{n\times n}$, and $\mathbf{K}_m \in \Re^{n\times n}$ are diagonal matrices representing the desired apparent inertia, damping, and stiffness, respectively. Equation (4) implies that, in the free space tracking phase of the operation, i.e. $\mathbf{F}_{ext} = 0$, the system trajectory converges to the desired trajectory asymptotically. On the other hand, in the constrained motion phase, equation (4) represents a stable 2nd order LTI system driven by the external force. Conceptually, we may regard the impedance controller as a model reference controller. The target impedance plays the role of the reference model and the impedance controller drives the robot to follow the dynamics of the reference model in both the free space tracking and constrained motion phases.

Suppose some of the system parameters are not available and hence the traditional impedance controller cannot be derived. We would like to design an adaptive impedance controller without requiring the knowledge of the regressor.[18] In addition, inversion of the inertia matrix is avoided and the new design is free from acceleration measurements. To this end, instead of (4), we consider the target impedance

$$\mathbf{M}_m(\ddot{\mathbf{x}}_m - \ddot{\mathbf{x}}_d) + \mathbf{B}_m(\dot{\mathbf{x}}_m - \dot{\mathbf{x}}_d) + \mathbf{K}_m(\mathbf{x}_m - \mathbf{x}_d) = -\mathbf{F}_{ext} \tag{5}$$

where $\mathbf{x}_m \in \Re^n$ is the state vector of the reference model

$$\mathbf{M}_m\ddot{\mathbf{x}}_m + \mathbf{B}_m\dot{\mathbf{x}}_m + \mathbf{K}_m\mathbf{x}_m = \mathbf{M}_m\ddot{\mathbf{x}}_d + \mathbf{B}_m\dot{\mathbf{x}}_d + \mathbf{K}_m\mathbf{x}_d - \mathbf{F}_{ext}.$$

If an adaptive controller can be designed such that $\mathbf{x} \to \mathbf{x}_m$ asymptotically, the target impedance (5) converges to (4) as desired. Let's consider the case first when the robot model is precisely known, and then the case with the presence of uncertainties.

2.1. Impedance control of robots with known dynamics[8]

Define an error vector $\mathbf{s} = \dot{\mathbf{e}} + \Lambda\mathbf{e}$ where $\mathbf{e} = \mathbf{x} - \mathbf{x}_m$ is the state error in the Cartesian space and $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$ with $\lambda_i > 0$ for all $i = 1, \ldots, n$. Rewrite the robot model (2) into the form

$$\begin{aligned}\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{g}_x + \mathbf{D}_x\ddot{\mathbf{x}}_m - \mathbf{D}_x\boldsymbol{\Lambda}\dot{\mathbf{e}} + \mathbf{C}_x\dot{\mathbf{x}}_m - \mathbf{C}_x\Lambda\mathbf{e} \\ = \mathbf{J}^{-T}\boldsymbol{\tau} - \mathbf{F}_{ext}.\end{aligned} \tag{6}$$

Suppose the robot model is known, then we may pick the controller as

$$\begin{aligned}\boldsymbol{\tau} = \mathbf{J}^T[\mathbf{F}_{ext} + \mathbf{g}_x + \mathbf{D}_x\ddot{\mathbf{x}}_m - \mathbf{D}_x\boldsymbol{\Lambda}\dot{\mathbf{e}} + \mathbf{C}_x\dot{\mathbf{x}}_m \\ - \mathbf{C}_x\Lambda\mathbf{e} - \mathbf{K}_d(\dot{\mathbf{e}} + \Lambda\mathbf{e})]\end{aligned} \tag{7}$$

where $\mathbf{K}_d$ is a positive definite matrix. Hence, the closed loop system becomes

$$\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{K}_d\mathbf{s} = \mathbf{0}. \tag{8}$$

To justify the feasibility of the controller (7), a Lyapunov function candidate is designed as

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{D}_x\mathbf{s}. \tag{9}$$

Taking the time derivative of V along the trajectory of (8), we have

$$\dot{V} = -\mathbf{s}^T\mathbf{K}_d\mathbf{s} + \frac{1}{2}\mathbf{s}^T(\dot{\mathbf{D}}_x - 2\mathbf{C}_x)\mathbf{s}. \tag{10}$$

Since $\dot{\mathbf{D}}_x(\mathbf{x}) - 2\mathbf{C}_x(\mathbf{x}, \dot{\mathbf{x}})$ is skew-symmetric, (10) becomes

$$\dot{V} = -\mathbf{s}^T\mathbf{K}_d\mathbf{s} \le 0. \tag{11}$$

It is easy to prove that $\mathbf{s} \in L_\infty \cap L_2$, and $\dot{\mathbf{s}} \in L_\infty$. Hence, $\mathbf{s} \to \mathbf{0}$ as $t \to \infty$, or we may say that $\mathbf{e} \to \mathbf{0}$ as $t \to \infty$. It is worth to mention that, in realizing the controller (7), there is no need to measure the acceleration nor does it need to compute the inversion of the inertia matrix.

2.2. Impedance control of robots with uncertainties

Now, let us consider the case when $\mathbf{D}_x$, $\mathbf{C}_x$, and $\mathbf{g}_x$ are not known. This implies that controller (7) is not realizable. We propose the controller

$$\begin{aligned}\boldsymbol{\tau} = \mathbf{J}^T(\mathbf{F}_{ext} + \hat{\mathbf{g}}_x + \hat{\mathbf{D}}_x\ddot{\mathbf{x}}_m - \hat{\mathbf{D}}_x\Lambda\dot{\mathbf{e}} + \hat{\mathbf{C}}_x\dot{\mathbf{x}}_m \\ - \hat{\mathbf{C}}_x\Lambda\mathbf{e} - \mathbf{K}_d(\dot{\mathbf{e}} + \Lambda\mathbf{e}))\end{aligned} \tag{12}$$

where all items with hats are estimations of their corresponding quantities. Substituting (12) into (2) and after

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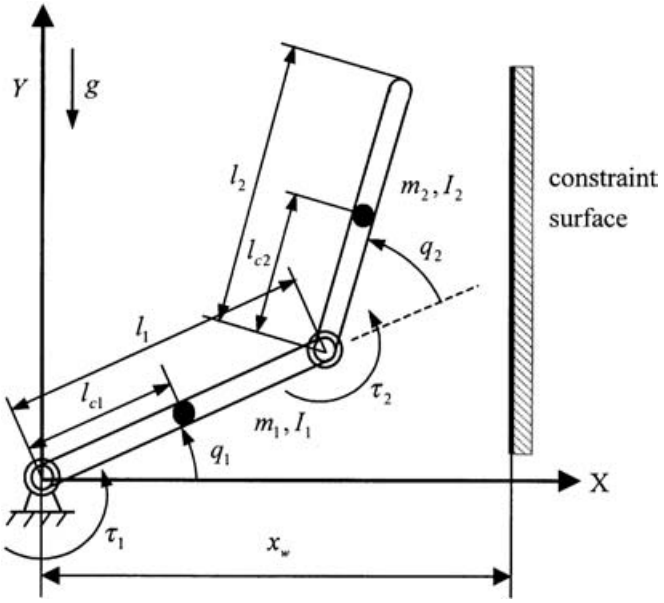


Fig. 1. Two-link planar robot.

some rearrangements, we may have the closed loop dynamics

$$\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{K}_d\mathbf{s} = (\hat{\mathbf{D}}_x - \mathbf{D}_x)(\ddot{\mathbf{x}}_m - \Lambda\dot{\mathbf{e}}) + (\hat{\mathbf{C}}_x - \mathbf{C}_x)(\dot{\mathbf{x}}_m - \Lambda\mathbf{e}) + (\hat{\mathbf{g}}_x - \mathbf{g}_x). \tag{13}$$

If we may design appropriate update laws such that $\hat{\mathbf{D}}_x \to \mathbf{D}_x$, $\hat{\mathbf{C}}_x \to \mathbf{C}_x$, and $\hat{\mathbf{g}}_x \to \mathbf{g}_x$, then (13) becomes (8) and hence (12) can give proper closed loop performance. To design the update laws, let us apply the function approximation representation[15]

$$\mathbf{D}_x = \mathbf{W}_\mathbf{D}^T\mathbf{Z}_\mathbf{D} \tag{14a}$$

$$\mathbf{C}_x = \mathbf{W}_\mathbf{C}^T\mathbf{Z}_\mathbf{C} \tag{14b}$$

$$\mathbf{g}_x = \mathbf{W}_\mathbf{g}^T\mathbf{Z}_\mathbf{g} \tag{14c}$$

where $\mathbf{W_D} \in \Re^{n^2\beta_D \times n}$, $\mathbf{W_C} \in \Re^{n^2\beta_C \times n}$ and $\mathbf{W_g} \in \Re^{n\beta_g \times n}$ are weighting matrices and $\mathbf{Z_D} \in \Re^{n^2\beta_D \times n}$, $\mathbf{Z_C} \in \Re^{n^2\beta_C \times n}$ and $\mathbf{Z_g} \in \Re^{n\beta_g \times 1}$ are matrices of basis functions. The number $\beta_{(\cdot)}$ represents the number of basis functions used. Effects of the approximation error will be considered later in this paper. Using the same set of basis functions, the corresponding estimates can also be represented as

$$\hat{\mathbf{D}}_x = \hat{\mathbf{W}}_\mathbf{D}^T\mathbf{Z}_\mathbf{D} \tag{14d}$$

$$\hat{\mathbf{C}}_x = \hat{\mathbf{W}}_\mathbf{C}^T Z_\mathbf{C} \tag{14e}$$

$$\hat{\mathbf{g}}_x = \hat{\mathbf{W}}_\mathbf{g}^T\mathbf{Z}_\mathbf{g}. \tag{14f}$$

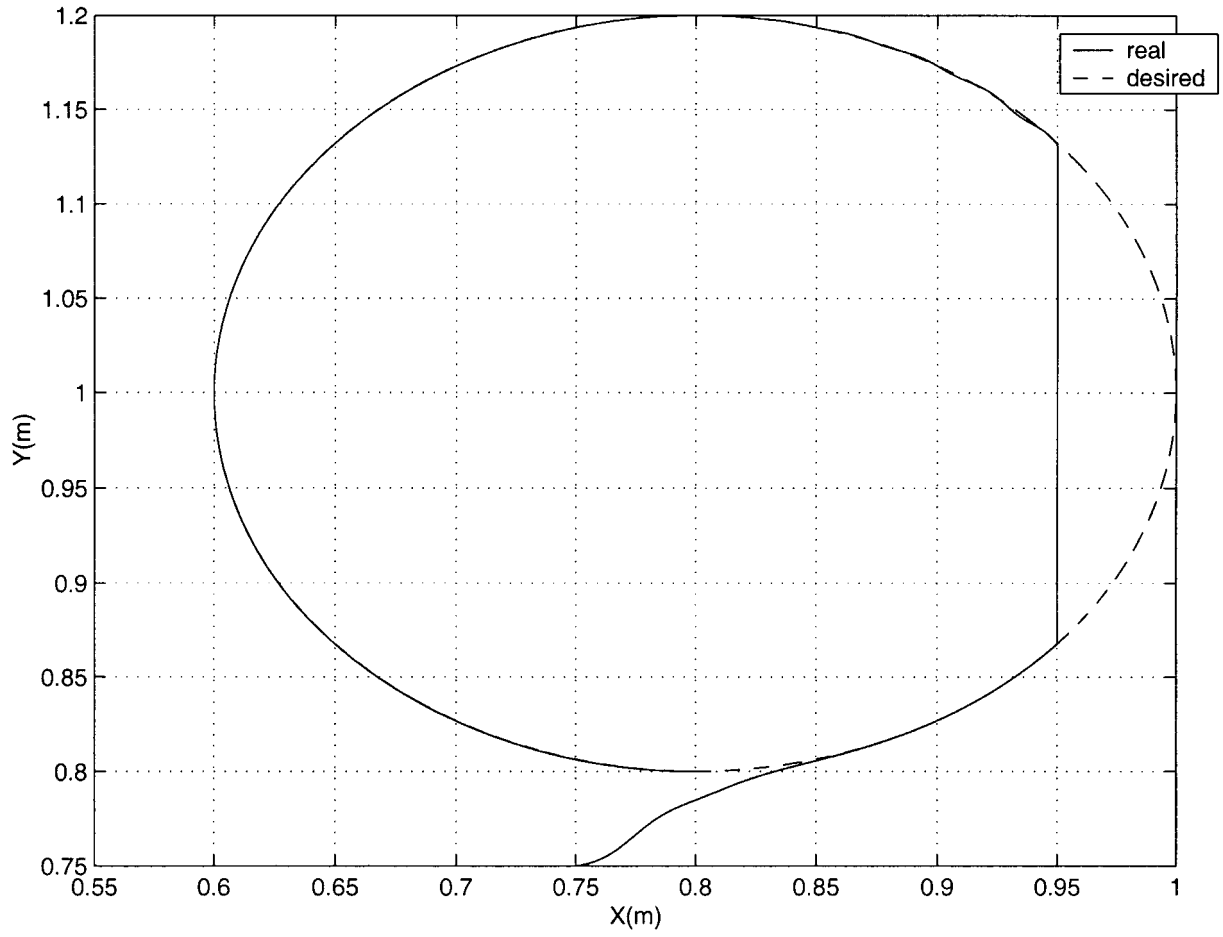


Fig. 2. Tracking performance of the end-point and the desired trajectory. The end-point smoothly contacts the surface, although we do not know much knowledge for the system.

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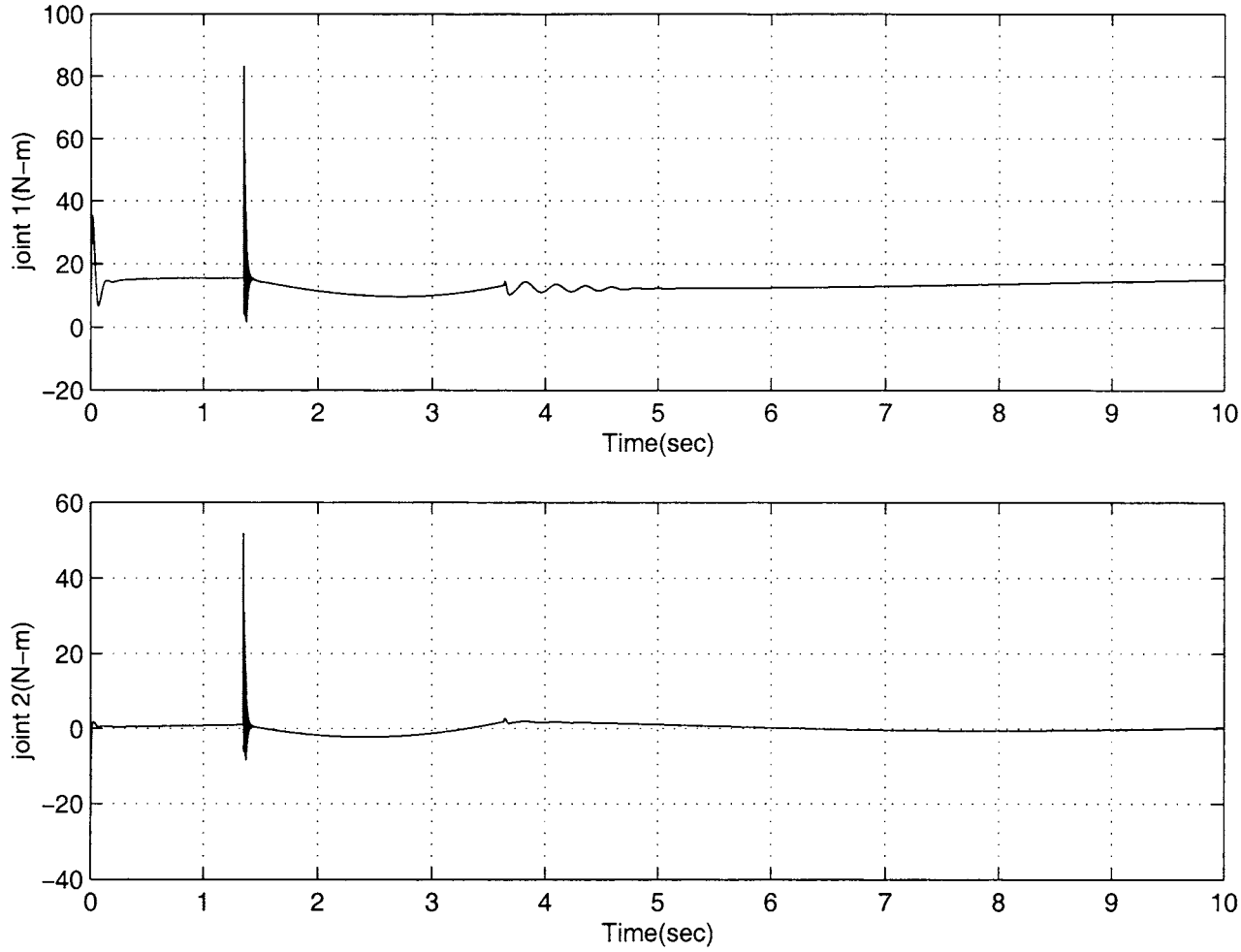


Fig. 3. Control inputs.

With these representations, equation (13) becomes

$$\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{K}_d\mathbf{s} = \tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Z}_{\mathbf{D}}(\ddot{\mathbf{x}}_m - \Lambda\dot{\mathbf{e}}) + \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Z}_{\mathbf{C}}(\dot{\mathbf{x}}_m - \Lambda\mathbf{e}) + \tilde{\mathbf{W}}_{\mathbf{g}}^T Z_{\mathbf{g}} \quad (15)$$

where $\tilde{\mathbf{W}}_{(\cdot)} = \hat{\mathbf{W}}_{(\cdot)} - \mathbf{W}_{(\cdot)}$. Since $\mathbf{W}_{(\cdot)}$ are constant vectors, their update laws can be easily found by proper selection of the Lyapunov function. Let us consider a candidate

$$V(\mathbf{s}, \tilde{\mathbf{W}}_{\mathbf{D}}, \tilde{\mathbf{W}}_{\mathbf{C}}, \tilde{\mathbf{W}}_{\mathbf{g}}) = \frac{1}{2}\mathbf{s}^T\mathbf{D}_x\mathbf{s} + \frac{1}{2}Tr\big(\tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Q}_{\mathbf{D}}\tilde{\mathbf{W}}_{\mathbf{D}} + \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Q}_{\mathbf{C}}\tilde{\mathbf{W}}_{\mathbf{C}} + \tilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Q}_{\mathbf{g}}\tilde{\mathbf{W}}_{\mathbf{g}}\big) \quad (16)$$

where $\mathbf{Q}_{\mathbf{D}} \in \Re^{n^2\beta_{\mathbf{D}} \times n^2\beta_{\mathbf{D}}}$, $\mathbf{Q}_{\mathbf{C}} \in \Re^{n^2\beta_C \times n^2\beta_C}$ and $\mathbf{Q}_{\mathbf{g}} \in \Re^{n\beta_g \times n\beta_g}$ are positive definite matrices. The notation $Tr(.)$ denotes the trace operation of matrices. The time derivative of V along the trajectory of (15) can be computed as

$$\dot{V} = \mathbf{s}^T\big[-\mathbf{C}_x\mathbf{s} - \mathbf{K}_d\mathbf{s} + \tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Z}_{\mathbf{D}}(\ddot{\mathbf{x}}_m - \Lambda\dot{\mathbf{e}}) + \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Z}_{\mathbf{C}}(\dot{\mathbf{x}}_m - \Lambda\mathbf{e}) + \tilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Z}_{\mathbf{g}}\big] + \frac{1}{2}\mathbf{s}^T\dot{\mathbf{D}}_x\mathbf{s} + Tr\big(\tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Q}_{\mathbf{D}}\dot{\hat{\mathbf{W}}}_{\mathbf{D}} + \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Q}_{\mathbf{C}}\dot{\hat{\mathbf{W}}}_{\mathbf{C}} + \tilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Q}_{\mathbf{g}}\dot{\hat{\mathbf{W}}}_{\mathbf{g}}\big). \quad (17)$$

Using the fact that $\mathbf{s}^T[\dot{\mathbf{D}}_x(\mathbf{x}) - 2\mathbf{C}_x(\mathbf{x}, \dot{\mathbf{x}})]\mathbf{s} = 0$ for all $\mathbf{s} \in \Re^n$ and choosing the update laws to be

$$\dot{\hat{\mathbf{W}}}_{\mathbf{D}} = -\mathbf{Q}_{\mathbf{D}}^{-1}\mathbf{Z}_{\mathbf{D}}(\ddot{\mathbf{x}}_m - \Lambda\dot{\mathbf{e}})\mathbf{s}^T \quad (18a)$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{C}} = -\mathbf{Q}_{\mathbf{C}}^{-1}\mathbf{Z}_{\mathbf{C}}(\dot{\mathbf{x}}_m - \Lambda\mathbf{e})\mathbf{s}^T \quad (18b)$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{g}} = -\mathbf{Q}_{\mathbf{g}}^{-1}\mathbf{Z}_{\mathbf{g}}\mathbf{s}^T \quad (18c)$$

then equation (17) becomes

$$\dot{V} = -\mathbf{s}^T\mathbf{K}_d\mathbf{s} \le 0. \quad (19)$$

Therefore $\mathbf{s}, \tilde{\mathbf{W}}_{\mathbf{D}}, \tilde{\mathbf{W}}_{\mathbf{C}}, \tilde{\mathbf{W}}_{\mathbf{g}}$ are uniformly bounded and $\int_0^\infty \mathbf{s}^T\mathbf{K}_d\mathbf{s}\,dt \le -\int_0^\infty V dt < \infty$. Hence we can conclude that $\mathbf{s} \in L_\infty \cap L_2$. From (15) $\dot{\mathbf{s}}$ is bounded; as a result, asymptotic convergence of $\mathbf{s}$ can easily be shown by the Barbalat's lemma. This further implies that the error $\mathbf{e}$ converges, even if $\mathbf{D}_x$, $\mathbf{C}_x$, and $\mathbf{g}_x$ are all unknown. To implement the controller (12) and update law (18), we do not need to inverse the inertia matrix, nor do we measure the acceleration. The convergence of the parameters, however, can be proved to depend on the persistent excitation condition of the input.

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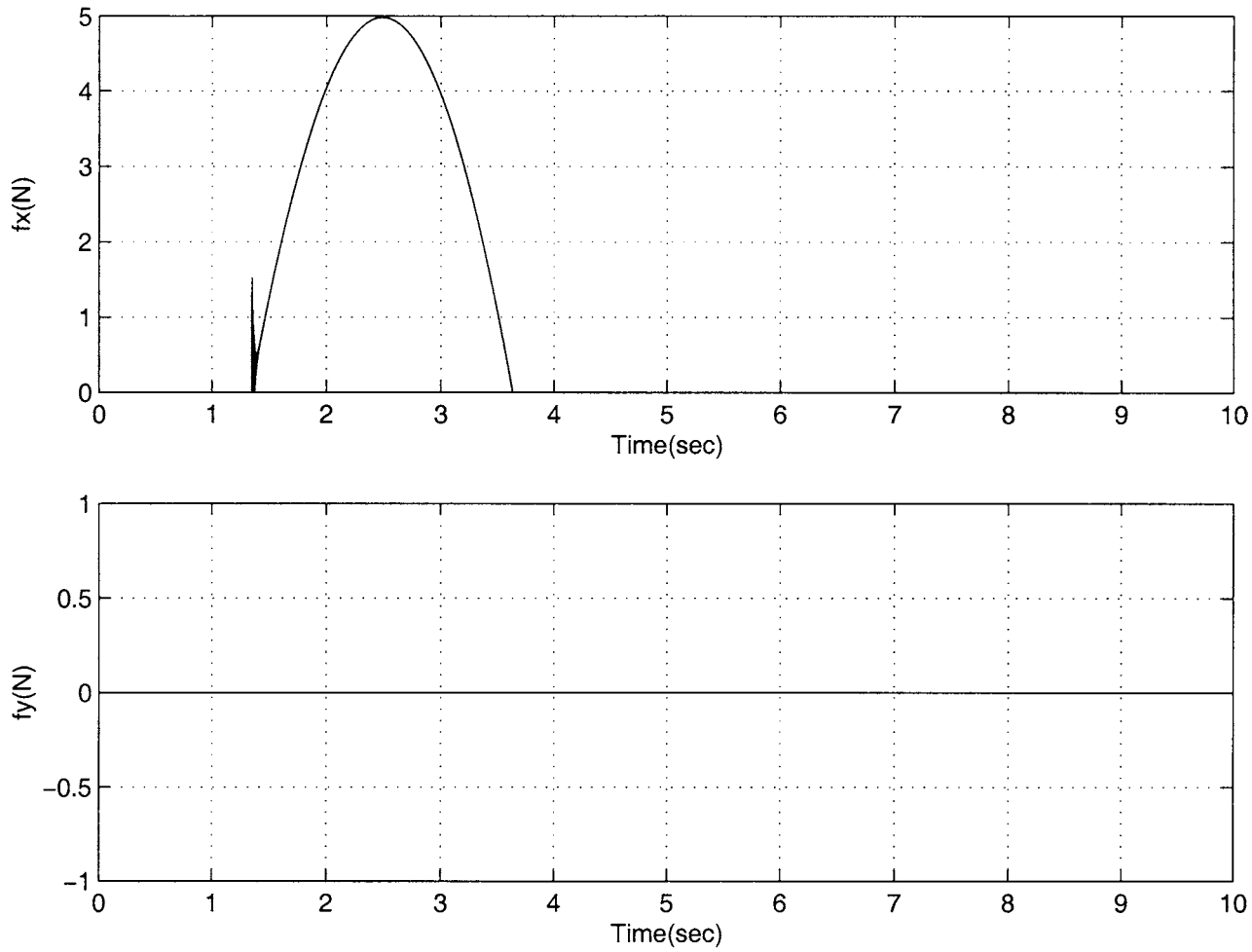


Fig. 4. External forces.

2.3. Consideration of the approximation error

In the above derivation, we ignored the approximation error by assuming that a sufficient large number of orthonormal functions were needed. It is, however, very important to know the effect of the approximation error on the performance of the closed loop system. Now, instead of (14), $\mathbf{D}_x$, $\mathbf{C}_x$, and $\mathbf{g}_x$ are represented as

$$\mathbf{D}_x = \mathbf{W}_\mathbf{D}^T \mathbf{Z}_\mathbf{D} + \boldsymbol{\varepsilon}_\mathbf{D} \tag{20a}$$

$$\mathbf{C}_x = \mathbf{W}_\mathbf{C}^T \mathbf{Z}_\mathbf{C} + \boldsymbol{\varepsilon}_\mathbf{C} \tag{20b}$$

$$\mathbf{g}_x = \mathbf{W}_\mathbf{g}^T \mathbf{Z}_\mathbf{g} + \boldsymbol{\varepsilon}_\mathbf{g} \tag{20c}$$

where $\boldsymbol{\varepsilon}_{(\cdot)}$ are approximation error matrices. Therefore, the closed loop dynamics (15) can be rewritten as

$$\begin{aligned}\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{K}_d\mathbf{s} = \tilde{\mathbf{W}}_\mathbf{D}^T \mathbf{Z}_\mathbf{D}(\ddot{\mathbf{x}}_m - \Lambda\dot{\mathbf{e}}) \\ + \tilde{\mathbf{W}}_\mathbf{C}^T \mathbf{Z}_\mathbf{C}(\dot{\mathbf{x}}_m - \boldsymbol{\Lambda}\mathbf{e}) + \tilde{\mathbf{W}}_\mathbf{g}^T \mathbf{Z}_\mathbf{g} + \boldsymbol{\varepsilon}\end{aligned} \tag{21}$$

where $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\boldsymbol{\varepsilon}_\mathbf{D}, \boldsymbol{\varepsilon}_\mathbf{C}, \boldsymbol{\varepsilon}_g, \mathbf{s}, \ddot{\mathbf{x}}_d)$ is a lumped approximation error. By selecting the update laws (18), the time derivative of V in (16) along the trajectory (21) can be computed to be

$$\dot{V} = -\mathbf{s}^T \mathbf{K}_d \mathbf{s} + \mathbf{s}^T \boldsymbol{\varepsilon}. \tag{22}$$

Owing to the existence of $\boldsymbol{\varepsilon}$, the definiteness of $\dot{V}$ cannot be determined. Let us proceed by representing (22) in the form

$$\dot{V} \leq (-\lambda_{\min}(\mathbf{K}_d)\,\|\mathbf{s}\| + \|\boldsymbol{\varepsilon}\|)\,\|\mathbf{s}\|\,. \tag{23}$$

If we select a suitable $\mathbf{K}_d$ and a proper set of orthonormal function, then $\dot{V} \leq 0$ whenever $\mathbf{s} \in \{\boldsymbol{\sigma} \,|\, \|\boldsymbol{\sigma}\| > \frac{\|\boldsymbol{\varepsilon}\|}{\lambda_{\min}(\mathbf{K}_d)}\}$. This concludes that the state error is uniformly ultimately bounded.

2.4. The case when the bound of the approximation error is available

Suppose, in some cases, the bound of the approximation error is available i.e. there exists a positive constant $\delta > 0$ such that $\|\boldsymbol{\varepsilon}\| \leq \delta$. To cover the effect of this bounded approximation error, the controller (12) is modified to be

$$\begin{aligned}\boldsymbol{\tau} = \mathbf{J}^T(\mathbf{F}_{ext} + \hat{\mathbf{g}}_x + \hat{\mathbf{D}}_x\ddot{\mathbf{x}}_m - \hat{\mathbf{D}}_x\Lambda\dot{\mathbf{e}} + \hat{\mathbf{C}}_x\dot{\mathbf{x}}_m \\ - \hat{\mathbf{C}}_x\Lambda\mathbf{e} - \mathbf{K}_d(\dot{\mathbf{e}} + \Lambda\mathbf{e})) + \boldsymbol{\tau}_{robust}\end{aligned} \tag{24}$$

where $\boldsymbol{\tau}_{robust}$ is the robust term to be designed. Let us consider the Lyapunov function candidate (16) and the update law (18) again. The time derivative of V can be computed as

$$\dot{V} \leq -\mathbf{s}^T \mathbf{K}_d \mathbf{s} + \delta\|\mathbf{s}\| + \mathbf{s}^T \boldsymbol{\tau}_{robust}. \tag{25}$$

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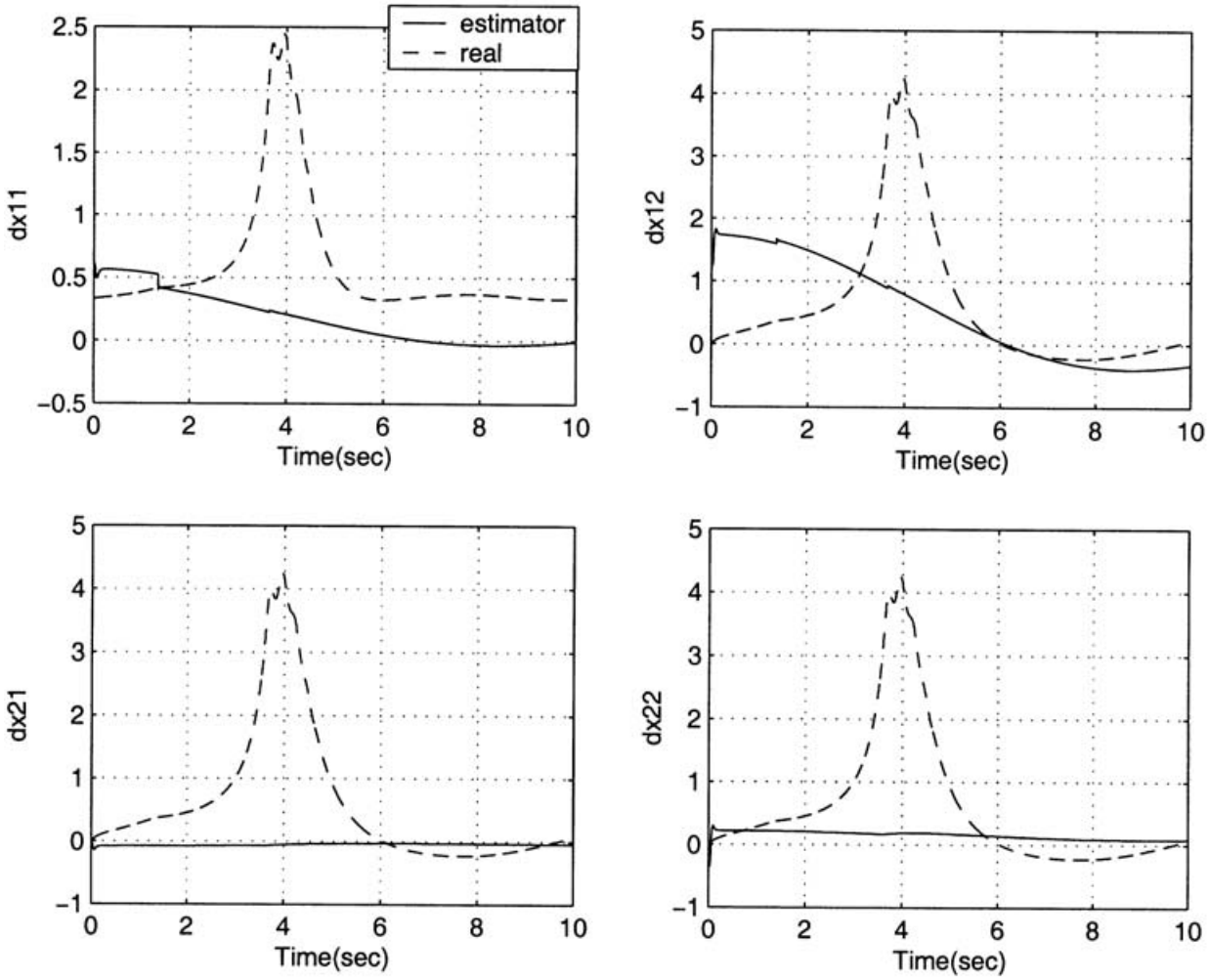


Fig. 5. Approximation of $\mathbf{D}_x(\mathbf{x})$. Since the reference input does not satisfy the persistent excitation condition, all estimates do not converge to their actual values but remain bounded as desired.

By picking $\boldsymbol{\tau}_{robust} = -\delta[\,\mathrm{sgn}(s_1)\cdots\mathrm{sgn}(s_n)\,]^T$, where $s_i, i = 1,\ldots,n$ are the i-th element of the vector $\mathbf{s}$, we may have $\dot{V} \leq 0$, and asymptotic convergence of the state error can be concluded by the Barbalat's lemma.

3. SIMULATION STUDY

Consider a planar robot (Figure 1) with two rigid links and two rigid revolute joints represented by the differential equation (1). The quantities m_i, l_i, l_{ci} and I_i are mass, length, gravity center distance and inertia of link i, respectively. Actual values of these parameters in the simulation are[23] $m_1 = 2\,kg, m_2 = 1\,kg, l_1 = l_2 = 0.75\,m, l_{c1} = l_{c2}$=0.375 m, $I_1 = 0.09375\,kg\text{-}m^2$, and $I_2 = 0.046975\,kg\text{-}m^2$. The inertia matrix $\mathbf{D}(\mathbf{q})$ is defined as

$$\mathbf{D}(\mathbf{q}) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

where d_{ij}, $i = 1, 2$ and $j = 1, 2$ are parameters depending on physical properties of the robot. Their exact forms can be derived as

$$d_{11} = m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + m_2 l_{c2}^2 + I_2 + 2m_2 l_1 l_{c2} \cos q_2$$

$$d_{12} = d_{21} = m_2 l_{c2}^2 + I_2 + m_2 l_1 l_{c2} \cos q_2$$

$$d_{22} = m_2 l_{c2}^2 + I_2.$$

The Coriolis and centripetal matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ and the gravitation vector $\mathbf{g}(\mathbf{q})$ are, respectively, modeled as

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -m_2 l_1 l_{c2} \dot{q}_2 \sin q_2 & -m_2 l_1 l_{c2} (\dot{q}_1 + \dot{q}_2) \sin q_2 \\ m_2 l_1 l_{c2} \dot{q}_1 \sin q_2 & 0 \end{bmatrix},$$

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} (m_1 l_{c2} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2) \\ m_2 l_{c2} g \cos(q_1 + q_2) \end{bmatrix}.$$

The Jacobian matrix is defined as

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}.$$

Initial condition of the end-point and reference states are $\mathbf{x} = [\,0.75\ 0.75\ 0\ 0\,]^{\mathrm{T}}$ and $\mathbf{x}_m = [\,0.8\ 0.8\ 0\ 0\,]^{\mathrm{T}}$, respectively. We would like the end-point to track a 0.2 m-radius circle centered at (0.8 m, 1.0 m) in 10 seconds. In this simulation, $\mathbf{D}$, $\mathbf{C}$ and $\mathbf{g}$ are assumed to be unknown, and the regressor is unavailable; therefore, traditional adaptive strategies may not

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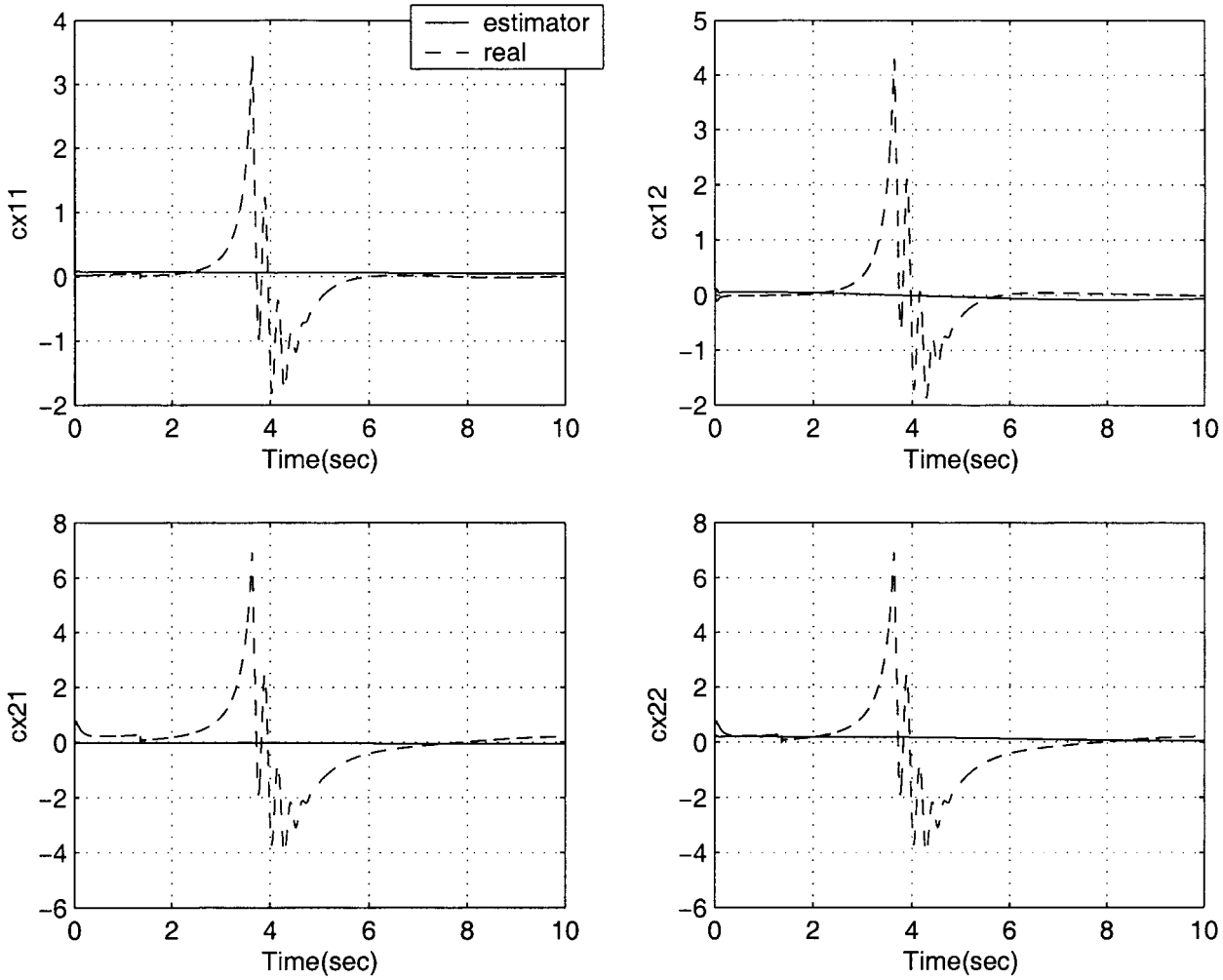


Fig. 6. Approximation of $\mathbf{C}_x(\mathbf{x}, \dot{\mathbf{x}})$.

be applied. Let us consider the controller in (12) and update laws in (18). The controller gains are selected as

$$\mathbf{K}_d = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \quad \mathbf{\Lambda} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}.$$

Matrices in the target impedance are designed to be

$$\mathbf{M}_m = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} (\text{kg}),$$

$$\mathbf{B}_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\text{kg.sec/m}),$$

$$\mathbf{K}_m = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} (\text{kg/m}).$$

Each element of $\mathbf{D}$, $\mathbf{C}$, and $\mathbf{g}$ is approximated by the first 41 terms of the Fourier series. Hence, $\tilde{\mathbf{W}}_\mathbf{D}$ and $\tilde{\mathbf{W}}_\mathbf{C}$ are both 164×2 matrices and the dimension of $\tilde{\mathbf{W}}_\mathbf{g}$ is 82×2. Initial weighting vectors of the basis functions are chosen arbitrarily as

$$\hat{\mathbf{w}}_{d_{11}}(0) = \hat{\mathbf{w}}_{c_{11}}(0) = [\,0.05\ 0 \cdots 0\,]^T \in \Re^{41\times 1},$$
$$\hat{\mathbf{w}}_{d_{12}}(0) = \hat{\mathbf{w}}_{c_{12}}(0) = [\,-0.05\ 0 \cdots 0\,]^T \in \Re^{41\times 1}$$
$$\hat{\mathbf{w}}_{d_{21}}(0) = \hat{\mathbf{w}}_{c_{21}}(0) = [-0.05\ 0 \cdots 0\,]^T \in \Re^{41\times 1},$$
$$\hat{\mathbf{w}}_{d_{22}}(0) = \hat{\mathbf{w}}_{c_{22}}(0) = [\,0.1\ 0 \cdots 0\,]^T \in \Re^{41\times 1}$$
$$\hat{\mathbf{w}}_{g_1}(0) = \hat{\mathbf{w}}_{g_2}(0) = [\,0\ 0 \cdots 0\,]^T \in \Re^{41\times 1}.$$

The adaptive gain matrices used in (18) are selected with

$$\mathbf{Q}_\mathbf{D}^{-1} = \mathbf{Q}_\mathbf{C}^{-1} = diag(q_1, \ldots, q_{164}), \quad q_i = 1, \forall i = 1, \ldots, 164$$
$$\mathbf{Q}_\mathbf{g}^{-1} = diag(q_{g_1}, \ldots, q_{g_{82}}), \quad q_{g_i} = 100, \forall i = 1, \ldots, 82.$$

The constraint surface is smooth and can be modeled as a linear spring [24]

$$f_{ext} = k_w(x - x_w) \tag{26}$$

where f_{ext} is the force acting on the surface, $k_w = 50000$ N/m is the environmental stiffness, x is the coordinate of the end-point in the X direction, and $x_w = 0.95\,m$ is the position of the surface. Hence, the external force vector in (21) becomes $\mathbf{F}_{ext} = [\,f_{ext}\ 0\,]^T$. The simulation results are shown in Figure 2 to 7. Figure 2 shows the tracking performance of the end-point and the desired trajectory in the Cartesian space. It is observed that the end-point compliantly contacts the constraint surface and the tracking error is very small in the

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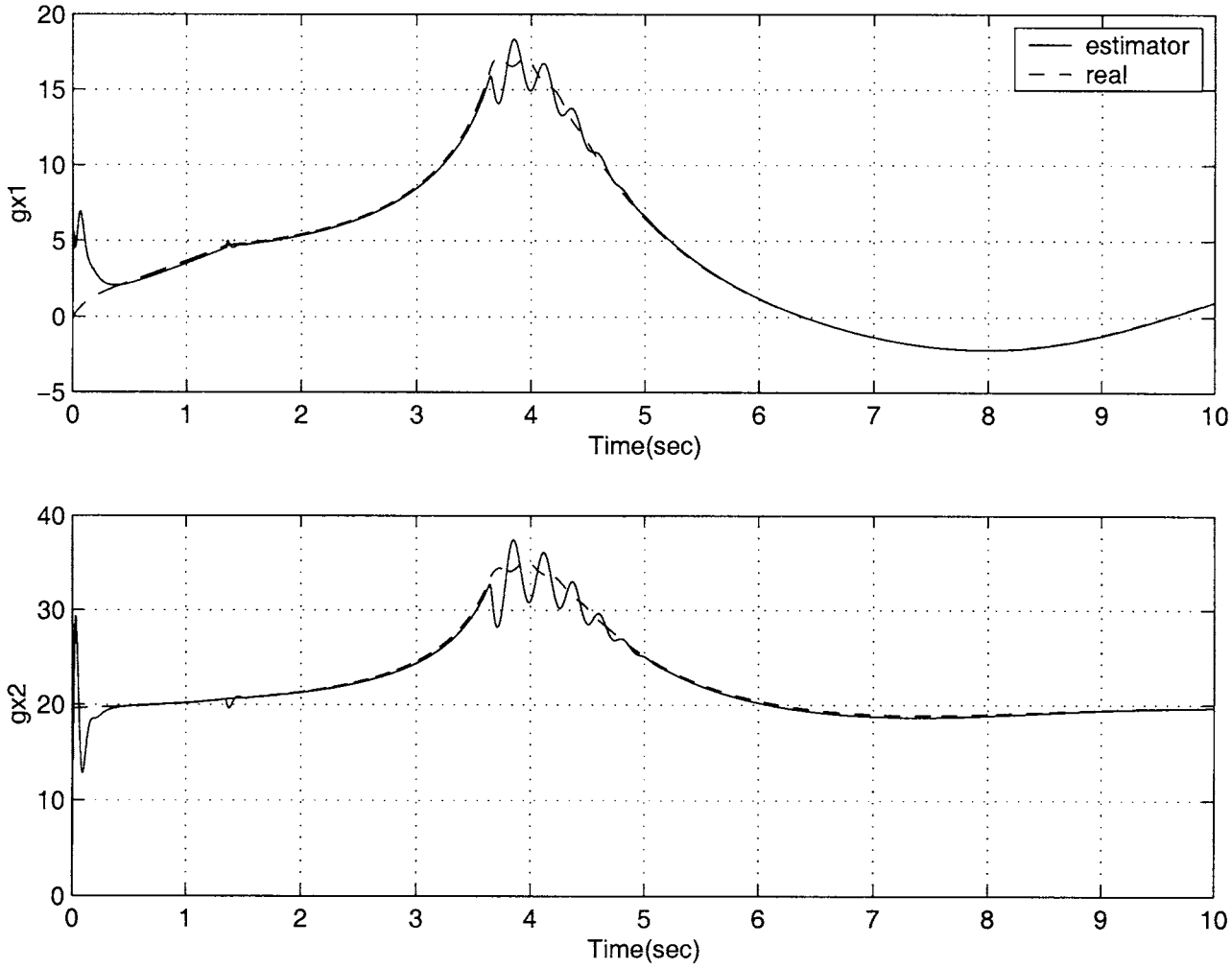


Fig. 7. Approximation of $\mathbf{g}_x(\mathbf{x})$.

free space. Figure 3 is the control torque in N-m. The spike at about 1.3 seconds is a result of the contact of the end-point and the constraint surface. Figure 4 is the external force in N. Figure 5 to 7 are the performance of function approximation for **D**, **C** and **g**, respectively. Since the reference input does not satisfy the persistent excitation condition, all estimates do not converge to their actual values but remain bounded as desired. It is worth to note that in designing the controller we do not need to know much knowledge of the system, to measure the acceleration, or to compute the inverse of the inertia matrix. All we have to do is to pick some controller parameters and some initial weighting matrices. As for the number of terms of the basis functions, we firstly try an arbitrary number. If the performance is satisfactory, the number is reduced; otherwise, it is increased. Finally, the number can be determined based on the compromise between computation efficiency and tracking performance. Therefore, the proposed controller is very easy to implement.

4. CONCLUSIONS

In this paper, we have proposed a function approximation based adaptive impedance controller for a rigid constrained robot manipulator without using the regressor. The proposed controller does not have to compute the inverse of the estimated inertia matrix and do not need to feedback the acceleration of end-point. Analysis of the closed loop stability has been investigated with consideration of the approximation error. For its practical implementation, the control strategy does not require much knowledge about the system model as long as proper sets of basis functions are used. Computer simulation results justify its feasibility of giving satisfactory tracking performance on a rigid robot.

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