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REVIEW

Ponzi schemes: a review

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Abstract

Ponzi schemes are financial frauds that are pervasive throughout the world. Since they cause serious harm to society, it is of interest to study them so that they can be prevented. Typically, a Ponzi scheme is instigated by a promoter who promises above-average investment returns. He uses funds from the early investors to pay his later investors. These scams can occasionally last a long time, but they are ultimately unsustainable. This paper describes some well-known Ponzi schemes and identifies their common characteristics. We also review some of the approaches used to model Ponzi schemes.

Keywords: financial crime; Madoff; Ponzi scheme; SIR model

1. Introduction

Misconduct by financial advisors is all too common in modern society (Egan et al., 2019), and Ponzi schemes are among the most notorious examples. The promoter of a Ponzi scheme cheats, lies, and steals his clients' money. He concocts a plausible investment story and promises an attractive rate of return to dupe his clients.[1] Usually, the money collected is not invested but used to fund his lifestyle or to promote the fraud. As long as he can expand his investor base, making sure that the incoming funds exceed the outflows from withdrawals, the promoter is able to sustain a positive cash flow. Since the investor base cannot expand indefinitely to generate sufficient cash inflow, the scheme will eventually run out of money and collapse. Alternatively, the scam may be detected by the regulators. Depending on the jurisdiction and the circumstances, the victims may receive a portion of their funds back or nothing at all.

Throughout history, Ponzi schemes have occurred in all types of economies, ranging from the least advanced to the most sophisticated (Lewis, 2012, 2015). Their prototypes can be found in literary masterpieces earlier than their eponym, Charles Ponzi's scheme.[2] More recently, the

[1]We use the pronoun "he/his" because the promoters of Ponzi schemes are predominantly male.

[2]The term Ponzi scheme comes from Charles Ponzi, who was a charismatic salesman as well as a serial fraudster. In 1920, he established a notorious scam in Boston by claiming that trading in international reply coupons would generate superior profits for his investors. Before Ponzi, such schemes were foreshadowed by two famous fictional con artists from the second half of the nineteenth century. In Charles Dickens' book, *Little Dorrit* (Dickens, 1857, Book II, Chapter 13), the scheming banker, Mr. Merdle, had acquired a reputation for financial brilliance. Dickens satirically wrote, "*There never was, there never had been, there never again should be such a man as Mr. Merdle. Nobody knew what he had done, but everybody knew him to be the greatest that had appeared.*" Agustus Melmotte was another fictional swindler, created by Anthony Trollope in the novel *The Way We Live Now* (Trollope, 1875). Melmotte was viewed as a financial wizard and fleeced his investors by selling shares on a railway from Salt Lake City to Vera Cruz. Melmotte burnished his reputation by hobnobbing with royalty and getting elected to Parliament. Similar reputation-building ploys have been used by subsequent Ponzi promoters.

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attention to Ponzi schemes ensued from the negative externalities they impose on society, and their adverse consequences are well-documented in the literature. For example, Carvajal et al. (2009) identify several negative impacts of Ponzi schemes, including misallocation of investment capital, undermining of confidence in financial markets, and tarnishing of the reputation of the authorities and regulators. According to Bhattacharya (2003), the collapse of the notorious Albanian Ponzi scheme annihilated the savings of one-sixth of the country's population. In the case of the Finnish Ponzi scheme WinCapita, Knüpfer et al. (2021) demonstrate that victims experienced an increased rate of job loss, lower labor income, and increased divorce rates compared to a matched sample.

In view of the destructive impact of Ponzi schemes on society, it is important to study them so as to prevent their occurrence. Section 2 describes representative Ponzi schemes, where we selected two types of schemes for discussion. The first type tends to have unique characteristics: they usually involve a large number of investors and can last a long time. Such schemes include the Madoff scam, the Allen Stanford fraud, and the Finnish WinCapita Ponzi scheme. The second type has common features and occurs in waves. As an example, we can cite peer-to-peer (P2P) schemes that flourished in China during the second decade of the 21st century. We end the section with a summary of three studies that provide empirical evidence on important aspects of Ponzi schemes. The first paper by Tennant (2011) uses Jamaican data to examine the behaviors of investors in these scams. The second paper by Hofstetter et al. (2018) uses data from Columbia to document the economic and social costs inflicted by Ponzi schemes. The third study by Stephen *et al.* (2021) provides information on the characteristics of samples of US Ponzi schemes.

Section 3 discusses a number of important features of Ponzi scams and contrasts them with *bona fide* investment firms. Since the Ponzi promoter wants to represent his scheme as above board, Ponzi schemes share some features with legitimate investment companies, but there are also revealing differences. A Ponzi scheme is initiated by a promoter who normally has some financial experience. He promotes a plausible investment story and often promises a guaranteed return. In contrast, legitimate investment firms do not usually promise a guaranteed return. The promised return is a key feature of the promoter's strategy. It cannot be too low since it has to entice investors; it cannot be too high, as it will attract suspicion. If the promised return seems to be too good to be true, it usually is.

The Ponzi promoter has to convince investors of his competence and integrity and will employ various trust-building techniques to do so. To survive, the scheme has to be designed to avoid detection, and various strategies have been used to accomplish this. Most Ponzi schemes just have a single strategy, but a few of the more long-lived ones have been able to evolve and innovate using new stratagems. Madoff's fraud is a case in point, where the promoter was able to modify his strategies over time. Any successful Ponzi scheme also needs an effective marketing strategy to attract new clients, and a wide variety of methods have been used. We illustrate these features by referring to the Ponzi schemes discussed in Section 3. Certain features, which are more likely to be prominent in Ponzi schemes than in law-abiding firms, can act as red flags.

Section 4 introduces a simple theoretical model of a Ponzi scheme where the promoter maximizes his welfare by optimally choosing some key variables. The decision variables of interest include the promised return, the growth rate of cash inflow, and the promoter's consumption. This approach is in the spirit of investment models where the manager's actions are designed to prolong the scheme and maximize his benefits. The value of this model is that it provides useful insights into the interactions among the variables and shows how the promoter's different choices impact the duration and profitability of the scheme.

Specifically, the constraints in the simple model can be modified to capture the implicit contract – in terms of resource sharing – between the promoter and his investors. Based on different assumptions on the dynamics of the cash flows, which may imply a demographic process, we describe in Section 5 several Ponzi-type models that have appeared in the literature. The final section contains a brief summary of the paper.

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2. Some well-known Ponzi schemes

This section discusses several Ponzi schemes. They are classified into two main types, depending on whether they share the same investment story. We term schemes in the first type as *heterogeneous*, since each scheme has a unique investment story. These include two of the largest North American schemes, operated by Bernard L. Madoff and Robert Allen Stanford. The third scheme is the Finnish WinCapita case. Schemes of the second type are labeled as *homogeneous* and share a common investment story. They tend to be very similar in other aspects (e.g., the products they market have common features). They include a cluster of Ponzi schemes in Jamaica, Columbia, and the US, as well as the so-called peer-to-peer (P2P) lending schemes that were once popular in China (Lo & Kan, 2023).

2.1 Heterogenous Ponzi schemes

2.1.1 Madoff's Ponzi scheme

We now describe Madoff's Ponzi scheme. Madoff began his investment operations in his early twenties when he established the firm of Bernard L. Madoff Investment Securities (BLM). Anticipating the potential of automatic trading, he became an early adopter of this technology, establishing BLM as an innovative leader in the over-the-counter market as a broker-dealer. Battalio (1997) noted in the *Journal of Finance* that after BLM entered the market as a broker-dealer, "the quoted bid-ask spreads tightens," indicating increased market efficiency.

Madoff became recognized as a pioneer in the industry, and he began to play an active and increasingly important role in the securities industry. He served on various committees of the National Association of Securities Dealers (NASD), rose to become a member of the NASD board, and was later elected Chairman of NASDAQ. These activities enabled him to establish and maintain good relations with the Securities and Exchange Commission (SEC). From this vantage point, Madoff was able to anticipate, influence, and exploit regulatory changes.

While establishing his broker-dealer business, Madoff also provided investment advice to his family and friends. This advisory business, which started as a side activity, eventually grew into the largest Ponzi scheme in history. His early investors included his father-in-law, Saul Alpern, and Alpern's clients. By enabling four of his early Jewish clients to become very rich, Madoff's reputation spread by word of mouth, expanding his client base to Jewish groups, wealthy individuals, and charities. Madoff claimed he was able to make profitable trades because his trading experience as a broker-dealer gave him an edge.[3]

In the early 1990s, Madoff claimed to use a new trading strategy that enabled him to deliver stable returns. The claim of steady returns was used by financial institutions as a marketing tool to impress clients and potential investors. Thus, it helped raise vast amounts for Madoff's Ponzi scheme. The strategy, known as the split-strike conversion strategy, involved sophisticated option trades. It consists of a long position in a stock portfolio, a long put option on the Index, and a short call of similar maturity, also on the Index. It was advertised that, with the help of the two options, the two extreme tails of the stock returns would be trimmed. Bernard & Boyle (2009) and Clauss et al. (2009) provided an in-depth analysis of this strategy and showed that the returns reported by Madoff's split-strike portfolios were implausible.[4]

[3]It is generally assumed that Madoff's advisory business started out aboveboard but at some point morphed into a Ponzi scheme. We do not know exactly when Madoff started his Ponzi scheme, but some have suggested it was in the mid-1970s. As to why he started the fraud, it is possible that he used new money from his investors to make up for losing trades and that he originally intended to pay it back.

[4]The strategy requires going long on blue-chip companies in the S&P 500, selling out-of-the-money call options on the Index, and buying out-of-the-money puts on the Index. When replicating the strategy, both Bernard & Boyle (2009) and Clauss et al. (2009) assumed that the portfolio is rebalanced monthly with European puts and calls of one-month maturity. In month τ, the value of the strategy at equals $V_\tau = S_\tau + e^{r\tau}(C_0 - P_0) + P_\tau - C_\tau$, where S_τ is the index level, r is the risk-free rate, C_0 and P_0 are the initial value of the call and put, and C_τ and P_τ are the values of the call and put, priced by the

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The split-strike strategy turned out to be beneficial to each of the key parties involved. It was appealing to the hedge funds who collected funds from the investors. These so-called feeder funds were handsomely compensated through the fees they charged. The fees consisted of a fixed percentage of the asset value plus a performance fee. By the time Madoff's scheme collapsed in 2008, there were 15 feeder funds. The investors were attracted to the strong, steady stream of returns, which the feeder funds used as a powerful marketing gimmick. As for Madoff himself, even though he appeared to charge very little, he was presumably very satisfied having access to an increasing flow of new funds, as long as he could meet the outflow.

Madoff's Ponzi scheme collapsed in 2008 as a result of the financial crisis. The turmoil, which caused panic selling, led to massive stock price markdowns and huge waves of redemptions that exhausted the cash in the scheme. On December 11, 2008, Madoff confessed to the massive fraud and was later sentenced to 150 years in prison.[5] After the scam was exposed, Irving Picard was appointed the trustee under the *Securities Investment Protection Act*. From an investor's viewpoint, this final stage tended to be a bitter experience. For those who lost money with Madoff, their dream of outsized investment returns was dashed. For those who withdrew more than the amount invested, the excess was clawed back and used to pay those who lost money. Many of Madoff's victims felt a deep sense of shame, embarrassment, guilt, and remorse from their unfortunate experience. As of this writing, March 2025, the trustee has recovered $14.7 billion for the victims, and the Department of Justice recovered an additional $4.3 billion, with the recovery process still ongoing.[6] The average recovery over all investors is about 75% on the dollar.

In the Madoff case, the SEC has been deservedly criticized for its regulatory negligence. The SEC completely failed to uncover the scam despite receiving various warnings, including six substantive complaints.[7] In retrospect, the SEC investigation team should have engaged experts with strong quantitative skills, such as financial economists, financial engineers, or actuaries. The team also lacked seasoned equity and option traders with market nous who would have been able to analyze and dissect Madoff's trades. The lawyers on the exam team, to compound matters even more, were too young and inexperienced to detect such a sophisticated scam and tended to be overawed and intimidated by Madoff's reputation.

2.1.2 Stanford's Ponzi scheme

Robert Allen Stanford's scheme affected 280,000 investors and incurred a loss of around USD $8 billion. Compared with Madoff's case, it has received less attention in the economics and finance literature. Nevertheless, there is considerable documentation available on this scheme, much of which is contained in various SEC reports (Kotz, 2014).[8]

Stanford grew up in Texas and obtained a finance degree from Baylor University. His foray into business began with the purchase of fitness clubs in Texas. These clubs flourished for a few years but were in Chapter 11 following the economic collapse in 1982. In November 1984, Stanford

Black-Scholes formula. The strike prices of the call and the put are set to be $1+\kappa$ and $1-\kappa$ times S_0, the initial index level. Because of the presence of a volatility skew in option prices, puts are relatively more expensive than calls, and this reduces the returns of the split-strike strategy. Bernard & Boyle (2009) take the skew into account. Replications of this strategy reveal the implausibility of the high returns reported by Fairfield Sentry, a major feeder fund of the Madoff scheme.

[5]Madoff died in prison on April 14, 2021.

[6]To date, a trustee – Irving Picard and his law firm BakerHostetler – has been paid a total of some $1.5 billion for their legal services on the Madoff case.

[7]At the start of this century, several finance professionals and others became suspicious of the consistently abnormal returns reported by Madoff. In May 2001, Michael Ocrant wrote an article entitled "Madoff Tops Charts, Skeptics Ask How." In the same year, Erin Arvedlund wrote a short note entitled "Don't Ask, Don't Tell: Bernie Madoff is so secretive, he even asks his investors to keep mum." A financial analyst, Harry Markopolous, reverse-engineered the split-strike returns and found them implausible. Markopolous made several submissions to the SEC that were either downplayed or ignored.

[8]The Vanity Fair article by Bryan Burroughs, entitled *Pirate of the Caribbean*, provides insightful background on Stanford (Burrough, 2009).

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managed to get a bankruptcy court to clear his debts so he could start afresh. In 1985, he founded a bank on the Caribbean island of Monserrat, offering certificates of deposit (CDs) that paid 200 basis points above the market rates. As many Monserrat banks were involved in money laundering for drug companies, Stanford's operation also came under scrutiny. His Monserrat banking license was revoked in May 1991.

In 1990, Stanford moved his operations to Antigua, setting up the Stanford International Bank (SIB). Antigua proved to be a hospitable habitat for the continued operation of his Ponzi scheme. Stanford's bank in Antigua also offered CDs at unusually high yields. He marketed these investments aggressively through affiliate companies in the US and across Central and Latin America. Stanford and his team assured investors that the deposits were secure by falsely claiming that (i) the bank invested the funds in liquid, secure assets; (ii) the portfolio was monitored on an ongoing basis by a team of 20 analysts; and (iii) the investments were subject to annual audits by Antiguan regulators.

Stanford's scheme managed to survive for almost twenty years, and the reasons for its remarkable duration, as well as why Stanford was not indicted, are threefold. First, Antigua's financial regulation was weak and amenable to manipulation. While his bank was subject to local regulations, Stanford was well-positioned to outmaneuver the regulatory system. Not only was he on the regulatory body, but he was also in charge of overseeing Antigua's banking laws. He gamed the enforcement system by bribing the main officials. One of the bribees, Leroy King, was the administrator and chief executive officer of Antigua's Financial Services Regulatory Commission (FSRC). King accepted thousands of dollars per month as bribes and turned a blind eye to Stanford's Ponzi scheme. Moreover, King supplied Stanford with confidential information about the SEC's investigation. In February 2021, King was sentenced to ten years in prison by a Texas court. He admitted to receiving some $500,000 in cash as well as other benefits in kind from Stanford.

Second, the island's political leadership was corrupt. For example, Stanford made it his business to establish good connections with Vere Bird, who was prime minister of Antigua and Barbados from 1981 to 1994, and also with his son Lester Bird, who led the country from 1994 to 2004. In 1995, Stanford lent the government millions of dollars to pay salaries and pensions.

Thirdly, Stanford worked hard to create and maintain an image of integrity and success to ensure the continuation of his fraud. Stanford promoted himself as a prominent businessman and philanthropist in Antigua and, at one time, was the island's largest employer.[9] In addition to his proactive endeavors to maintain his reputation, Stanford also employed strong-arm tactics. He was known to sue journalists and others who impugned his reputation and paid a security firm to vigorously and aggressively defend his good name.

On the US side, inter-agency conflict, incompetence, and borderline corruption prevented an early shutdown of Stanford's scheme. In fact, at least three US agencies – the FBI, the SEC, and FINRA – had been monitoring Stanford for several years.[10] The SEC suspected that he was running a Ponzi scheme as early as 1997. In particular, the SEC's Fort Worth office conducted four reviews of Stanford's investment operations and concluded each time that his returns were highly unlikely. Regarding the Fort Worth inaction, Spencer Barasch, one of the senior actors who decided not to pursue Stanford, eventually ended up working for Stanford. As the head of the enforcement group, Barasch claimed that the case was complicated, noting that the group

[9]Stanford built the island's hospital and supported a wide range of sporting events, such as hosting polo matches involving the British Royal Family. He also rebuilt and renamed a cricket ground in Antigua and funded a series of international cricket matches. Moreover, Stanford made generous donations to politicians in Antigua and both political parties in the US. He received a knighthood from Antigua in 2006, but this honor was revoked after his scam was exposed.

[10]The FBI had kept tabs on Stanford since his Monserrat days on suspicions of money laundering. FINRA, which is the self-regulatory body of investment advisors in the US, received information between 2003 and 2005 from five different sources, claiming that Stanford's CDs were a potential fraud.

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would be rewarded based on the number of cases they resolved, not the complexity thereof. Later, in September 2006, Stanford hired Barasch – who at that time had left the SEC – to seek advice on how to deal with regulators. Barasch was later fined $50,000 for breaching conflict of interest guidelines.

2.1.3 WinCapita (Finland)

WinCapita was started in 2003 by Hannu Kailajärvi, who had a background in information technology but none in finance. The scheme was operated through a shell company domiciled in Panama and became the largest investment fraud in Finnish history. At first, it was claimed that the profits were generated by sports betting, but later, foreign currency was advertised as the purported investment medium. By promising investors that they would get rich quickly, the scheme swindled more than 10,000 victims between 2003 and 2008, representing 0.2% of the entire Finnish population (Rantala, 2019).

To create a sense of exclusivity, investors in WinCapita could only join the scheme via invitation from a sponsor. This created an affinity network that resembled the transmission of an epidemic (Huebscher, 2016). After the scheme's demise, Rantala (2019) obtained detailed information on the scheme's investors. Using this dataset, he showed that the investor engagement with the scheme was not transmitted evenly from one investor to a fixed number of peers; instead, a small fraction of investors (i.e., sponsors) at the hub of the system were responsible for most of the observed transmission. Rantala (2019) found that investors put in more funds if their sponsor had higher income, was older, or had higher education. Presumably, these attributes made the information more credible. He also found that sponsors typically invested more than non-sponsors.

The Finnish regulator, Oikeusrekisterikeskus (ORK), started investigating the scheme in 2008. The investigation turned out to be a time-consuming legal process. Investors who made profits and withdrew their earnings from the scheme were ordered to pay back their gains. Vironen (2019) reported that 626 former investors were ordered to pay EUR 64 million in illegal benefits to the Finnish state. By the autumn of 2020, ORK claimed it had received a total of EUR 34.2 million in illegal proceeds from the scheme.

According to Huebscher (2016), the promoter, Hannu Kailajärvi, received only a five-year sentence, the maximum duration under Finnish laws. Nevertheless, even this short sentence was later cut to two and a half years. Given the size of the scheme and the seriousness of the crime, the punishment seems very mild compared to comparable sentences in the US and China.

2.2 Homogenous Ponzi schemes

We now describe a number of homogeneous Ponzi schemes.

2.2.1 Ponzi schemes in Jamaica

Ponzi schemes were very popular in Jamaica in the early years of the twenty-first century. Tennant (2011) provides an accessible description of these schemes and conducts an empirical analysis based on a sample of the participants. The data examined includes the personal profiles of 402 respondents who invested in 17 Ponzi schemes in Jamaica. The main dependent variable is exposure, defined as the money invested in any Ponzi scheme divided by personal income. Tennant (2011) found that those who are relatively well-off treat their investment in Ponzi as experiments. The motivation is consistent with the case of MMM in Africa: it is those individuals who want to improve their economic situations that have the largest exposure. Moreover, people who got referrals from friends and received high payments from Ponzi schemes tend to increase their exposure. This finding alludes to repeated investing behavior in Ponzi schemes.

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2.2.2 Ponzi schemes in Columbia

Cortés et al. (2016) explored the consequences of the crackdown of 12 Ponzi schemes, which caused the Colombian government to declare a state of emergency. The serial insolvencies of these scams, which posed a huge disruption to the economy, started with the collapse of DRFE and DMG on November 12 and 15 of 2008. DMG sold prepaid cards with a 6-month maturity that promised yields ranging from 50% to 300%. DRFE (Dinero Rápido, Fácil y Efectivo – Money in Cash, Fast and Easy) promised monthly returns between 80% and 150%. While an overall estimate of the total damage was not available, the reported losses are already significant. According to Hofstetter et al. (2018), DFRE attracted USD $865.6 million as deposits, victimizing 153,878 investors, and DMG attracted USD $1, 191.3 million as deposits, victimizing 356,631 investors.

Hofstetter et al. (2018) found that prior to (subsequent to) the crackdown of DRFE and DMG, victims invested in DMG and DFRE enjoyed more (fewer) loans and better (worse) credit standings. Municipalities that were more affected by the collapse saw a larger decrease of deposits placed in the financial sector. Using data from all 50 municipalities in Columbia, Cortés et al. (2016) found that the crime rates of shoplifting and robbery increased significantly following the crackdown of the 12 schemes, especially in 2009Q4 and in municipalities with weak law enforcement and limited access to credit.

2.2.3 Other US Ponzi schemes

Stephen et al. (2021) analyze 376 Ponzi schemes in the US that were prosecuted by the SEC from 1988Q1 to 2012Q4. The study found that the median size of these collapsed schemes was USD $14.7 million, with the median proportion purloined by the promoter being 13.2%. The median duration of these schemes was 3.1 years, implying that most scams are short-lived. Similar to Tennant (2011), trust built by affinity – either through kinship, friendship, shared religious belief, or ethnic origin – was very important for the promoter to ensnare his victims, evinced by 74% of the schemes being perpetrated by local promoters. Meanwhile, trust built by advertising on mass media had a mixed effect – it was negatively related to the duration but positively related to the size of the scheme. However, the analysis did not show how the promised interest rate affects the duration or the size of the scheme.

Jordan Maglich presents data on 1108 schemes covering the period 2008–2023, of which the average size was over USD $70 million.[11] For schemes that ended between 2008 and 2022, there is a full record of the total losses. Based on available news reports and the regulator's documents, 768 perpetrators were sentenced to a fixed term of prison time. The average (median) length of their sentence was 130 months (96 months). The ratio of male versus female is 8.5:1, implying that nearly 90% of the schemes were initiated by men.

2.2.4 Ezubao and other P2P lending scams in China

We next turn to a class of Ponzi schemes that swept through China during the 2011–2019 period. Unlike the heterogeneous schemes discussed in Section 2.1, these scams appeared in clusters and closely resembled one another. They flourished because of a gap in the country's regulations and were shut down when the gap was closed.

Thanks to China's long period of economic growth, there were millions of households with excess funds to invest in the early years of this century. However, with the majority of the population possessing limited financial knowledge, individuals preferred simple fixed-income investments. This preference stemmed from their everyday experience with bank deposits, which used to be the only investment vehicle available. After years of credit expansion in the post-crisis period, various bond-like Ponzi schemes appeared, catering to people's low risk appetite.

[11] Maglich publishes this information on his blog (https://www.ponzitracker.com/). He stresses that his statistics are presented for educational purposes only and have not been independently verified.

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Table 1. Summary statistics of the P2P industry in China (2011–2019)

Panel A: Numbers of new, existing, and failed platforms										
						Year				
Number	Source	2011	2012	2013	2014	2015	2016	2017	2018	2019
New	wdzj				957	1916	756	731	556	0
Total (accumulated)	wdzj				1942	3858	5853	6584	6655	6655
	p2peye	50	148	523	1854	3844	4624	5685		
In operation	wdzj				1575	2595	2448	1931	1075	343
Failed	wdzj	10	6	7	277	1291	1713	645	1279	732
	p2peye	10	5	75	281	1000	1470	977		
Panel B: Duration, investment terms, and interest rates										
						Year				
Characteristics	Source	2011	2012	2013	2014	2015	2016	2017	2018	2019
Duration (failed platforms, mean)	wdzj			5.19	6.72	9.31	16.37			
Duration (failed platforms, median)	wdzj			4.00	4.00	7.00	15.00			
Maturity (all platforms, mean)	wdzj	6.90	5.98	4.73	6.12	6.81	7.89	9.16	12.65	15.71
Promised return (all platforms, %)	wdzj	18.90	19.13	21.25	17.86	13.29	10.45	9.45	9.81	9.46
# of lenders (in 10,000)	wdzj		2.5	5.1	116	586	1375	1713	1331	1104
# of borrowers (in 10,000)	wdzj		1.9	15	63	285	876	2243	1992	1191

Source: The statistics were obtained from www.wdzj.com, www.p2peye.com, and www.yingcanzixun.com in 2020, all of which have been closed down around 2021. The original sources include the Bluebook for China's Online Lending Industry published in 2013, Annual Report on China's Online Lending Industry (wdzj, 2014, 2015, 2016); P2P Industry Annual Report (p2peye, 2016). Notes: 1. Statistics on f*ailed* platforms are based on victims' reports. The actual number of failures may be larger than reported. 2. Durations and investment terms are measured in months. Platforms that last less than a month are assigned a duration of one month. 3. The promised rate of return is *annualized*.

Many of the investment schemes emerged as peer-to-peer (P2P) lending platforms. A P2P platform puts borrowers directly in contact with lenders without involving a formal financial institution (e.g., a bank). Since P2P lending was a new type of financial arrangement in China, the prevailing regulations were ineffective (Wei, 2015). In principle, P2P platforms were only allowed to play the role of an information agent, matching up lenders with eligible borrowers; see Li et al. (2016) for details. However, these online platforms became very aggressive in seizing lenders' funds.

We provide summary statistics of the Chinese P2P industry from 2011 to 2019 in Table 1. Panel A provides the numbers of these schemes, and the last two rows show the number of failures. These numbers, extracted from two sources, did not agree with each other due to the self-reported nature of the statistics. According to the website www.wdzj.com, by the end of 2019, only 343 platforms had survived, suggesting a high rate of failure. Panel B of the table displays several key characteristics, including the average duration of the schemes, the average term of investment (i.e., maturity), the average coupon rate, and the average number of lenders and borrowers. On average, the failed platforms have a short duration. As the first row of Panel B indicates, the mean duration equals 5.19 months in 2013 and 16.37 months in 2016. In 2016, the median duration was 15 months, suggesting that most platforms typically did not survive two years after establishment. From 2013 to 2019, the average promised return decreased from 21.25% to 9.46%.

Researchers have documented a negative relationship between the probability of failure and the promised interest rate. Li et al. (2016) examined 308 P2P lending platforms during 2011–2014, which included 104 failures and 204 non-failures from the website www.wdzj.com. Using a Cox proportional hazard model, they find that the risk of failure is positively related to the promised interest rate but negatively related to the registered capital. A similar survival analysis by Wang et al. (2016) studied a larger sample of 3407 P2P platforms, among which 1048 failed.

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They found that failed platforms were more likely to promise returns exceeding 20%. Also, platforms that do not make key information available, such as the interest rate and registered capital, are less likely to survive.

Among all P2P scams, Ezubao became the most well-known. Launched by the Yucheng Group in July 2014, Ezubao underwent a period of rapid growth (Albrecht et al., 2017). It offered six products, each of which had a promised return rate exceeding 9%. To attract small investors, each product only required a minimum investment of one RMB (approximately 1/7 USD). Ezubao claimed that it put investors' money into a subsidiary of its parent group, which professed to operate a series of financial leasing projects. The subsidiary was supposed to purchase pieces of equipment and lease them to several lessee companies. According to Ezubao, when the lessees paid their rent, the subsidiary would transfer the money back to investors registered on the platform.

To make it appear more credible, Ezubao worked diligently to bolster its reputation and create a positive image. For example, it spent a huge amount of money placing advertisements in various media outlets. In particular, it used prime time commercials on national television, China Central Television (CCTV), as well as several provincial channels (Li, 2016). Furthermore, Ezubao rented luxurious offices at the Shanghai Pilot Free Trade Zone and the Beijing Central Business District. The company also held its annual meeting in the Great Hall of the People in Beijing. These trust-building activities were designed to instill confidence in the investors and disguise the underlying sham. In reality, all of Ezubao's investment projects were fictitious.

In December 2015, Ezubao eventually attracted the attention of the regulators. By then, the platform had swindled RMB 58 billion (USD $8.2 billion) from 900,000 investors across China (Bai & Chen, 2016). The scheme collapsed in early 2016. This scandal made investors more suspicious of P2P lending platforms, causing regulators to increase surveillance of suspect companies. The demise of Ezubao and the trial of its management foreshadowed a large-scale crackdown on the P2P sector from 2018 to 2020. By the end of 2019, there were just 343 active platforms compared to 3,500 four years earlier. And this number fell to zero in November 2020.

3. Features of Ponzi scheme

Promoters of a Ponzi scheme naturally desire the scam to be viewed as a legitimate operation. Hence, they often use various devices to fool unwary investors. For example, a promoter may engage in philanthropy to impress potential clients, thus copying successful investors like Warren Buffet, who has made extensive charitable donations.

In this section, we identify various features of Ponzi schemes and contrast them with *bona fide* financial firms. This comparative approach enables us to identify where the key differences occur and can assist us in spotting red flags and performing due diligence. The key features of a Ponzi scheme that we discuss include (i) the promoter or founder; (ii) the plausible story; (iii) trust-building mechanisms; (iv) the investment promise; (v) control features; (vi) marketing strategies; (vii) regulation circumventions; (viii) agency issues; and (ix) how the scheme ends. Table A.1 in the Appendix summarizes the main defining features of Ponzi schemes and illustrates the extent to which different Ponzi schemes exhibit these features.

3.1 The promoter

Ponzi schemes are often founded by an individual who typically has a charismatic personality and some background in investment. In general, the scheme starts out as a small-scale operation and grows as the fame of the promoter spreads. The promoter will use one or more devices to attract clients, such as appealing to members of some social groups or incentivizing early investors to bring in their friends. To escape detection, the scheme operates off the grid in terms of registration and avoids the use of third-party service providers as much as possible. In contrast, legitimate investment firms follow applicable laws and tend to involve more than one person in

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the management team. A potential investor, therefore, should check out the qualifications and experience of the principal(s) and confirm that the firm meets its legal requirements. Of course, even with such due diligence, an individual can still lose money by investing with a legitimate firm, but at least their money will not have been stolen.

3.2 A plausible story

The promoter of a Ponzi scheme, like the principals of a legitimate firm, must provide a plausible story to convince investors. In the Ponzi case, the story often centers around a particular and sometimes non-conventional investment opportunity. The promoter may emphasize the uniqueness of the scheme, such as that it is "one of a kind," and underscore its relative scarcity by claiming that there was only a short window for investment. For example, Madoff had claimed that his fund had almost reached capacity, and there was little available space.

By claiming that the details are proprietary, the promoter can justify the limited disclosure provided about the scheme. They can assert that such information must be held back from the public, since otherwise a competitor might copy them and arbitrage the opportunity away. From an investment point of view, opacity is a convenient ploy that can appear both reasonable and legitimate. It resonates with how hedge funds, private equity firms, and family offices operate. On the other hand, to present the scheme as a *bona fide* firm, the promoter must still provide a description of the opportunity, likely in the form of a big-picture overview. Prudent investors should be wary if there seems to be excessive secrecy.

3.3 Trust-building mechanisms

Typically, a promoter engages in activities to acquire a reputation for competence and integrity. By winning investors' trust, the promoter aims to swindle as many people as possible and enrich himself. Promoters can use a number of methods to establish the three types of trust categorized by Zucker (1986): process-based, institution-based, and characteristics-based.

In process-based trust, information on trust is collected via expected or past exchanges, which can also be established through status and reputation. To do so, the promoter builds up a positive social image, presenting himself as a successful businessperson, donating to charities, sponsoring sports events, joining boards of governors, etc. For example, both Madoff and Stanford established prominent social identities.

In the case of institution-based trust, the trust is forged through formal institutions, such as accreditation earned from government agencies, industry organizations, and professional associations. As an example, Madoff became a member of prestigious industry groups in the US that influenced regulation. Another illustration would be Stanford's prominent role with the banking regulatory body in Antigua.

Characteristics-based trust is the most common type, where trust is derived from friends, family, and fellows/peers. By exploiting his existing connections within a particular group, the promoter can engage people with the same characteristics, such as those sharing the same workplace, belonging to the same religious group, and having the same ethnic background. For example, in the case of the Oversea Chinese Fund, the con artist, Weizhen Tang, manipulated and fleeced recent Chinese immigrants to Canada (Boyle et al., 2025). One important way to increase the trust and hence membership of the scheme is to convince early investors that they will make a profit. In Ezubao and other P2P Ponzi schemes, the promoter paid early investors on time, reassuring them that the investments were safe and reliable. Some of the early investors may spread the word to their connections and even add more funds to the scheme. These investors are known colloquially as *songbirds* (Lewis, 2015).

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3.4 The investment promise

A Ponzi scheme invariably promises a return above the risk-free rate (Li et al., 2016; Wang et al., 2016; Stephen et al., 2021). After all, if the promised return is set too low, the scheme would become unattractive, and investors can shift to other investments that are either safer or more lucrative. Of course, the promised rate cannot be too high. A moderately high rate implies that the investors' funds do not grow at excessively high rates, and thus, redemption requests are easier to meet. Otherwise, the amounts credited to earlier investors will grow at a very fast pace, such that any redemption requests from them will soon exhaust the pool of funds, causing the scheme to collapse quickly. From a regulator's perspective, a high promised return that clearly breaches the risk-return trade-off can act as a red flag.[12]

A reasonably high return, instead, can function as a screening tool that helps the promoter locate a group of homogeneous targets. This may follow from the observation that people who prefer the same range of returns are likely to share similar socioeconomic or demographic statuses. For instance, Allen Stanford successfully lured many well-off pensioners by setting his allegedly guaranteed returns above the levels offered by other banks.

Some Ponzi schemes often promise guaranteed returns. A warning from the SEC reads, "*Be highly suspicious of any 'guaranteed' investment opportunity*." This points to the fact that legitimate investment firms are loath to specify or guarantee a fixed return. Instead, they often state a target return that lies within a range. Even when a range is given, it is accompanied by some qualifying comments, such as that the range only applies in normal circumstances. An unusually high guaranteed return is a sure warning sign.

For long-duration Ponzi schemes (e.g., Madoff and Stanford's scams), steady returns with low volatility make the returns more attractive to long-term risk-averse investors. Madoff's split-strike strategy provides a good example and shows that these individuals are more motivated by fear than by greed. According to Bernard & Boyle (2009), Madoff's reported net return was only slightly higher than that of the market. In contrast, the low volatility of his reported returns resulted in an annual Sharpe ratio of 2.47, which was almost seven times larger than the market's Sharpe ratio.[13] The SEC complaint on Stanford's scheme provides a time series of the returns on the portfolio of SIB and the CD rates paid to its investors (SEC, 2009). The reported returns on SIB's portfolio ranged from 14.5% in 1992 to 12.0% in 2006 and always exceeded a floor of 11%. The returns paid on the bank's CDs, while lower than the portfolio returns, invariably exceeded the CD rates offered by other institutions.

3.5 Various control features

Many legitimate investment funds impose restrictions concerning withdrawals, such as specifying the timing and the maximum size of redemptions. For instance, hedge funds often mandate a minimum length of time, during which an investor is not allowed to withdraw funds (Kaiser, 2008). This period, known as the lock-up period, can vary in length, depending on characteristics of the fund's investment strategy, such as the liquidity of the investment. Obviously, if the underlying assets are illiquid, a *bona fide* fund will find it difficult to meet a huge flow of redemptions. As such, it is common to see private credit firms requiring investors to lock up their funds for a period because the assets of the fund are long-term private loans.

Unsurprisingly, this lock-up feature is also observed in many Ponzi schemes. Similar to hedge funds, mandating such a period helps the Ponzi scheme to survive its start-up stage and avoid an early collapse. For example, the WinCapita scheme in Finland had a lock-up period of six months.

[12]In fact, the SEC warns on its web page that "Every investment carries some degree of risk, and investments yielding higher returns involve more risk."

[13]The net annualized return of Madoff's Fairfield Sentry Fund was 10.59% over an 18-year period, with a standard deviation of 2.45%. Over the same period, the corresponding return on the S&P 500 was 9.64% with a standard deviation of 14.28%.

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Moreover, prolonging the lock-up period before its eventual demise may also give the scheme some terminal breathing space. Under the infamous MMM scheme, the promoter – Sergey Mavrodi – extended the lock-up period from 7 days to 14 days and again to 21 days (Magaji, 2016).

3.6 Marketing strategies

The perpetrator must attract an increasing flow of new entrants to sustain the scheme. Hence, developing a successful marketing operation is much more critical in a Ponzi scheme than in the case of a law-abiding fund. Ponzi schemes use a variety of methods to entice new clients. These include, but are not limited to, word-of-mouth marketing, social media marketing, and, most importantly, referrals. For the first two types, the sales pitch is based, in part, on the realized returns and the (false) promised future returns. A highly effective way to increase sales is to pay high commissions. Both Madoff's split-strike strategy and Stanford's CD business offered unusually high sales commissions. If an investment scheme is very aggressively marketed and the sales force receives very high commissions, these are red flags that may indicate a fraud.

3.7 Regulation circumventions

While legitimate investment schemes follow the applicable regulations, Ponzi scheme promoters use a variety of methods to circumvent regulations. One way is to set up a scheme in a jurisdiction with laxer regulations. For example, Stanford moved his scheme to Antigua, where he was able to bribe the key regulators. The promoter may also exploit loopholes in the regulatory regime. For example, we have seen that Madoff operated two investment entities: one was regulated by the SEC, and the other was an unregulated hedge fund. He was thus able to claim he was regulated by the SEC without disclosing that it was only his brokerage business that was regulated. In China, many P2P lending scams appeared during a regime of ineffective regulation. A simple due diligence check should be done to confirm that the fund conforms with applicable regulations. If a scheme relocates to a regime with lax regulation or similar schemes appear *en masse*, this can be a red flag.

3.8 Agency problems

To reduce conflicts of interest and other agency problems, legitimate investment firms employ third-party agents to provide assurance to investors. These service providers perform critical verification functions and conduct an independent check on the firm's activities. For example, prime brokers hold assets in a trust to safeguard them. Moreover, auditors are hired to verify the existence of the assets and issue opinions on the accuracy of the financial statements.

Ponzi schemes, on the other hand, tend to ignore or subvert these checks. For example, Madoff retained control and custody of the assets without using a custodian trust. His firm computed the returns without any independent verification. In addition, the scheme was audited by a tiny, obscure New York accounting firm that rubber-stamped the books without performing an audit.[14] These deviations from best practices should have been warning signs.

3.9 How a Ponzi scheme ends

A Ponzi scheme ends in one of two ways. First, the scheme might implode when it reaches a crisis point in its cash holdings. The promoter needs cash inflows to pay for certain items. These include withdrawals by earlier investors, the operating costs of the scheme, and the promoter's own expenses. When cash inflow from new investors cannot cover the shortfall, the scheme will

[14]The firm in question, Frehling & Horowitz, was neither registered with the Public Company Accounting Oversight Board (PCAOB) nor did it engage in peer review.

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experience a shortage of funds and eventually collapse. Second, the regulator may uncover the fraud and shut it down. But this scenario is fairly rare. As the case of WinCapita shows, a scheme may continue to operate despite an ongoing police investigation.

For victims, the consequences linger long after the scheme runs out of cash. When a scheme collapses, investors who still hold a position in the scheme are the victims of the fraud. These victims will generally only recover a fraction of their principal. The precise amount depends on a number of factors, such as the legal environment and the ability of the authorities to claw back profits from earlier investors. In the case of Ezubao, the court seized more than RMB 11 billion (USD $1.56 billion) worth of real estate and financial assets from the former promoters. In January 2020, four years after Ezubao collapsed, around 35% of the principal was returned to the victims. As of the present time, the average recovery for the Madoff victims is 75% of their invested principal. Besides financial loss, victims may feel betrayed and disillusioned, and they may take a long time to recover from the emotional trauma.

After the scheme is closed down, the perpetrators are tried and sent to prison. The sentences vary enormously by jurisdiction, with the US and China being extremely stringent. Madoff got 150 years, and Stanford received 110 years. The main architects of the P2P Ezubao scam, the two Ding brothers, were given life sentences and hefty fines. However, Hannu Kailajärvi, the WinCapita mastermind, only got five years, which was further reduced.

4. The economic lineage of Ponzi models

This section shows how a basic economic model can be modified to study Ponzi schemes. The starting point is a standard intertemporal consumption-savings problem described in Section 4.1, where the promoter maximizes his utility subject to a resource constraint. The resource constraints of the standard model can be reframed to account for the cash inflows and outflows that underlie a Ponzi scheme. Based on a more realistic modeling of the cash flow process, we illustrate a stylistic Ponzi model in Section 4.2. This prototype Ponzi model also helps us understand more advanced models in Section 5.

4.1 A standard economic model

We now describe the standard economic model that serves as our starting point. Specifically, we set out the basic elements of an intertemporal consumption-savings problem and explain how it can be adapted to model a Ponzi scheme. Following the standard convention, we denote ρ as the time preference and W_t as the cash balance (assets or savings) at the beginning of period t; I_t as the cash inflow (investment or income) at the end of period t; and C_t as the withdrawal (consumption) at the end of period t. The new cash balance, W_{t+1}, equals $(1 + r_t)$ times the previous balance, assuming the money has been invested at a rate of r_t, plus the inflow, I_t, and minus the withdrawal, C_t.[15]

Hence, the utility maximization problem facing the agent is

$$\max \sum_{t=1}^{T-1} \rho^t U(C_t) \qquad \text{s.t.} \quad W_{t+1} = (1 + r_t)W_t + I_t - C_t, \quad W_t \geq 0, \quad C_t \geq 0. \tag{1}$$

While the Euler equation of the problem, $U'(C_t) = \rho U'(C_{t+1})(1 + r_{t+1})$ for $1 \leq t \leq T - 2$, is easy to derive, the problem has no explicit solution unless we assume a special type of utility function. For the moment, I_t, the cash inflow process, is given as exogenous.

[15]By convention, in the macroeconomic literature, the values of I_t and C_t are taken at the end of period t, while that of W_t is taken at the beginning of period t.

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4.2 A stylistic Ponzi scheme

4.2.1 How Ponzi scheme models differ from the economic model

A Ponzi scheme model differs from Equation (1) in three major ways: capital accumulation, withdrawal processes, and uncertainty in the duration. First, the cash accumulation in the resource constraint, $(1 + r_t)W_t$, is incorrect, as the promised investment opportunity does not actually exist. This can be accounted for by setting $r_t = 0$, assuming no real investment occurs, or $r_t = r_f$, assuming the previous cash balance has been deposited at the risk-free rate.

Second, in standard economic models, the entire amount withdrawn, C_t, represents the agent's consumption. In a Ponzi scheme, only a portion of this amount goes to the promoter, and the rest goes to early investors who redeem from the scheme.[16] We denote the two streams of withdrawals, attributable to the promoter and investors, by C_t and O_t, respectively. Projecting the cash inflow, I_t, the outflows O_t, and the promoter's takeout, C_t, is key to capturing the evolution of W_t.

Third, the duration or decision horizon, T,, is not predetermined at the outset but depends largely on the evolution of the scheme. The finite duration property of a Ponzi scheme implies that the total withdrawal, $O_t + C_t$, eventually outpaces the inflow, I_t, exhausting all the cash balance. The duration can be manipulated somewhat using the marketing strategies described in Section 3.6. Due to this interdependence of T and other variables, to simplify the model, the utility maximization component is often dispensed with in Ponzi models.[17]

4.2.2 A simple, stylistic Ponzi model

Before delving into more advanced Ponzi models, we first discuss a simple discrete-time version. This model will highlight the basic differences mentioned above and provide some intuition. Suppose each new investment has a maturity of one period, with the promised rate of return being r. Due to the one-period lock-up, investors cannot withdraw any funds in period zero, i.e., $O_0 = 0$; the initial investment is $I_0 = K$, with K being a fixed number. Starting from $t = 1$, the cash balance is

$$W_{t+1} = W_t + I_t - O_t - C_t. \tag{2}$$

This last equation shows that the cash balance in period $(t + 1)$, W_{t+1}, can be broken down into four parts: (i) the previous balance, W_t; (ii) new investment, I_t; (iii) the withdrawal by investors, O_t; and (iv) the takeout by the promoter, C_t.

The amount invested, I_t, is assumed to grow at a constant rate g, so that after t periods, the amount of new investment becomes $I_t = K(1 + g)^t$. At the end of the t-th period, the total amount to be paid out equals $O_t = I_{t-1}(1 + r)$, where I_{t-1} is the investment in period $t - 1$. Since there is no real investment activity, the cash inflow and outflow attributable to investors evolve according to the following rules:

$$I_t = I_{t-1}(1 + g), \tag{3}$$

$$O_t = I_{t-1}(1 + r), \tag{4}$$

where the growth rate of cash inflow, g, and the promised rate, r, are assumed constant over time. These are, of course, strong assumptions made to simplify the analysis. In reality, both g and r can vary over time and need not be independent of one another.

[16] In a Ponzi scheme, the promoter's withdrawal is used to fund his consumption and lifestyle and various scheme-related expenses. These expenses include marketing costs to promote the scheme, commissions paid to the salesforce, and possibly bribes to neutralize the regulators.

[17] In the standard, finite-horizon economic model, T is given and known *a priori*; in a Ponzi scheme, T may also be a control variable that is subject to promoter manipulation. For example, T may be a function of r.

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Table 2. Duration T assuming different g and r pairs in the stylistic model ($b = 4$)

	$r = 0.06$	$r = 0.08$	$r = 0.10$	$r = 0.12$	$r = 0.15$
$g = 0.06$	9	8	7	7	6
$g = 0.08$	12	9	8	7	6
$g = 0.10$	∞	12	9	8	7
$g = 0.12$	∞	∞	13	9	8
$g = 0.15$	∞	∞	∞	∞	9

Suppose the promoter has quadratic utility; that is, his single-period utility function can be written as $U(C_t) = C_t - bC_t^2$, with $b > 0$ and $C_t \leq 1/(2b)$.[18] In addition, the promoter is assumed to have zero time preference, so that $\rho = 1$. It is convenient to assume that his takeout each period corresponds to the satiation point, so that $C_t = 1/(2b)$ for $1 \leq t \leq (T-1)$. Adding Equations (2), (3), and (4) recursively, the cash balance of the scheme at the beginning of period t is

$$W_t = \frac{K}{g}\left[(1+g)^{t-1}(g-r)+r\right] - \frac{t-1}{2b}. \tag{5}$$

It is easy to show that W_t is a decreasing function of r.

If the scheme lasts T periods, we must have $W_{T-1} > 0$ and $W_T \leq 0$. This requires the first-order derivative of W_t with respect to t to be negative at $t = T$; that is,

$$\left.\frac{\partial W_t}{\partial t}\right|_T = \frac{K}{g}\left[(g-r)(1+g)^{T-1}\ln(1+g)\right] - \frac{1}{2b} < 0. \tag{6}$$

This inequality holds for $g < r$, as both the first and second terms are negative. When $g = r$, the system corresponds to the balanced path where the cash inflow offsets the cash outflow, with $I_t = O_t$. A more interesting case is when $g > r$. With certain parameter combinations, $\partial W_t/\partial t < 0$ is attainable for small t, resulting in a scheme that wraps up quickly due to large, early outflows. When $g \gg r$, W_t can be explosive, such that the scheme can last forever. However, the assumption that the inflow can grow perpetually is unrealistic.

Numerically, the duration of the scheme can be obtained by solving the two inequalities, $W_{T-1} > 0$ and $W_T \leq 0$. Table 2 illustrates how different combinations of parameter values affect the duration, T. For the sake of simplicity, we standardize K to 1 and set $b = 4$. We can see that the duration of the scheme declines as r increases, holding g fixed. The duration of the scheme increases with g, holding r fixed. Both results are intuitive. When $r = g$, $T = (2b+1) = 9$. It is also noteworthy that even if $g > r$, we can obtain a finite duration in some cases. For example, when $r = 0.06$ and $g = 0.08$, the duration is 12.

5. Ponzi models as modifications to the economic model

In the simple stylistic model described in Section 4.2, we require the cash inflow and outflow in Equations (3) and (4) to grow geometrically at a rate of g and r, respectively, until the terminal point. By setting $g < r$, we are certain to obtain a scheme with finite duration. This requirement, nevertheless, is not necessary: termination can also happen when $g = r$ and sometimes when $g > r$.

In fact, the key for a Ponzi scheme to end in finite time is that the amount of the cash outflow plus the withdrawal of the promoter, $O_t + C_t$, outpaces that of the inflow, I_t. In extant models, there are three main ways to model the finite duration. They are (i) simplifying the resource constraints by using a balanced path, requiring the new investment $_I_t$ to cover O_t plus C_t exactly;

[18] We can require b to take a relatively large value to reflect that the promoter's takeout, C_t, does not take out a significant portion of the initial cash balance. When $b \to \infty$, we are essentially modeling an altruistic promoter, who mechanically follows the economic contract dictated by the resource constraints.

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(ii) introducing a fictitious account, F_t, that allows the outflow, O_t, to be proportional to F_t thus growing explosively and more quickly than I_t; or (iii) introducing a special pattern of inflow, allowing I_t to expand and shrink over time.

In this section, we summarize a few extensions and modifications of the basic Ponzi model corresponding to the three types of cash flows described above. These include (i) balanced path models with the inflows being covered by the outflows, which is explained in Section 5.1; (ii) the case when the real balance, W_t, depends on a fictitious balance, which we elaborate on in Section 5.2; and (iii) the case when the real balance is driven by demographic dynamics. The last alternative can be captured by a susceptible-infected-recovered (SIR) model where an explicit demographic structure is imposed. This class of models will be explained in Section 5.3.

5.1 A balanced path approach for cash flows

In the balanced path approach, beyond the initial time period, the cash inflow covers investors' withdrawal plus the promoter's takeout, such that

$$I_t = O_t + C_t. \tag{7}$$

And I_t and O_t can be set to follow a specific path. For example, both can follow the same geometric growth, in the same way as in Equations (3) and (4).

For this strand of models, we begin by describing the model introduced by Bhattacharya (2003). In his model, although participants know that the arrangement is a Ponzi scheme, they will rationally join it since they anticipate the government will bail them out when the scheme collapses. Despite its extreme assumptions, this model provides useful insights. Interestingly, the balanced-path cash flow assumption is similar to that of a common pension arrangement, known as the pay-as-you-go (PAYG) plan. Under a PAYG plan, the pension benefits are met by contributions from active workers. We show that despite some similarities, there are profound differences between PAYG plans and Ponzi schemes.

5.1.1 A too-big-to-fail Ponzi scheme: Bhattacharya (2003)

Bhattacharya (2003) constructs a model of a large Ponzi scam in a small economy. He explains how to design a Ponzi scheme that will attract investors, even though they know it will collapse: the likelihood of a bailout provides the incentive. Bhattacharya (2003) makes assumptions regarding the shared information in the economy, the nature of the population, the type of investments, the decisions of the promoter, the design of the bailout, and the behavior of the regulator. His model is rather complicated, and we will only be able to cover the main ideas.

We now provide a brief review of his model. The promoter expects the scheme to last for T periods before it winds up. In each period, he sells a one-period investment product priced at P and promises a rate of return r throughout. Formally, denote $n_t \in [0,\ 1]$ as the mass of individuals that join the scheme in the t-th period; for each additional period, this mass grows at rate g, i.e., $n_{t+1} = (1+g)n_t$. The gross growth rate $(1+g)$ is bounded above by D, the natural speed of information transmission.

The model rests on two strong assumptions, symmetric information and government bailout, which Bhattacharya admits are severe assumptions. First, under symmetric information, as soon as individuals realize it is a Ponzi scheme, they know when it began, which stage it is at, and when it will end. Second, since the scheme is considered too big to fail, it will end up with a government bailout.[19] Consequently, an individual in the economy has two choices: either they invest in the

[19]The bailout component is essential to the model. As Basu (2014) commented, if there is no bailout, then, by backward deduction, no one would invest, and the scheme cannot get off the ground. This result is also contained in Corollary 1 of Bhattacharya (2003).

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scheme and end up with some compensation when it collapses, or they do not invest in the scheme and end up defraying the cost of the bailout.

The period zero cash flows are complicated. To attract an initial mass of n_0 investors and receive n_0P, the promoter incurs a marketing expense of $f_0 + f_1(n_0)n_0P$ and retains f_2 to pay for a regulatory penalty when the scheme collapses. The promoter's takeout, therefore, equals $C_0 = n_0P - (f_0 + f_1(n_0)n_0P + f_2)$. Here, f_0 and f_2 are fixed, while $f_1(n_0)$ is increasing and convex in n_0, the initial investor mass.

The cash flows from period one onwards follow a *balanced path*. That is, in period $t = 1, \ldots, T$, the cash inflow, $I_t = n_tP$, is split between (i) investor's withdrawal, $O_t = (1 + r)n_{t-1}P$, and (ii) the promoter's takeout, $C_t = cn_tP$. This balanced path implies $n_tP = (1 + r)n_{t-1}P + cn_tP$, such that the proportion of promoter takeout equals $c = (g - r)/(1 + g)$, implying that $g > r$.

The regulator can detect the Ponzi game and close it down with probability θ, implying that the expected promoter takeout is $\mathrm{E}[C_t] = (1 - \theta)cn_tP$. Once detected, the promoter's takeout in that period and onward is confiscated and becomes zero. Therefore, the utility maximization problem facing the promoter reduces to find the promised return (r), growth rate of the investor population (g), price of the investment (P), initial population mass (n_0), and duration of the scheme (T) to maximize his expected utility:

$$\mathrm{E}\left[\sum_{t=0}^{T} \rho^t U(C_t)\right] = n_0P - \left(f_0 + f_1(n_0)n_0P + f_2\right) + \sum_{t=1}^{T} (1 - \theta)^t cn_tP, \tag{8}$$

where $\rho = 1$ and $U(C_t) = C_t$ are chosen to characterize a patient, risk-neutral promoter.

Bhattacharya (2003) uses a series of steps to characterize the optimal solution. It can be shown that the expression in Equation (8) is decreasing in r and n_T and increasing in g and P. Consequently, the optimal solution reduces to finding the lower bounds of r and n_T but the upper bound of g and P. To induce participation in period $T - 1$, the expected gain of participation in *per capita* terms must be greater than that of non-participation, such that $(1 - \theta)Pr + \theta(-P) \geq 0$, which yields $r \geq \theta/(1 - \theta)$. The optimal r is the lower bound, $\theta/(1 - \theta)$. Moreover, n_T must exceed n^*, a threshold that triggers a bailout. The optimal n_T, therefore, is n^*. Since $(1 + g)$ is bounded by D, the optimal g equals $1 - D$. Let the *per capita* loss for non-participation be α, then the total gain for a bailout – given only to the mass of n_T who invested – is $\alpha(1 - n_T)$. In period T, suppose the probability of the bailout is β, the expected total gain for a bailout is $\beta \cdot \alpha(1 - n_T)$. To induce participation in period T, the *per capita* loss in the Ponzi scheme plus the bailout gain must be greater than the loss from non-participation, i.e., $-P + \beta \cdot \alpha(1 - n_T)/n_T \geq \beta \cdot (-\alpha)$. This implies $P \leq \alpha\beta/n_T$.

Combining these results, the optimal end-period investor mass, investment price P, and the duration of the scheme are

$$n_T = n^*, \quad P = \frac{\alpha\beta}{n^*}, \quad T = \frac{\ln n^* - \ln n_0}{\ln D}, \tag{9}$$

where n_0 solves the first-order condition of Equation (8).[20]

As already noted, this model rests on assumptions about the wealth redistribution effect of the bailouts. The reason an individual joins the scheme is because if they do not and the scheme fails, they will be forced to contribute to the bailout. In the case of contemporary Ponzi schemes, the bailout provisions envisaged in this model appear highly unusual. Currently, compensation to victims is normally provided by clawing back funds from investors who withdrew more than they invested in the scheme; see, for example, the Madoff case.

[20]The first-order condition is $n_0m'(n_0) + m(n_0) + \left[(1 - \theta)D\right]^{(\ln(n^*/n_0)/\ln D)} \cdot \left[\ln(1 - \theta)/\ln D\right] = 0$.

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5.1.2 A balanced path pay-as-you-go pension system

Pay-as-you-o (PAYG) pension plans have sometimes been described by their critics as Ponzi schemes (Boccia, 2024). In both PAYG plans and Ponzi schemes, the benefits payable to successive cohorts are funded by the contributions of the previous cohort, essentially implying a balanced path of cash flows.

It is of interest to consider a simple model of a PAYG system. For the sake of convenience, we tweak the notation in Morsomme *et al.* (2025). The current contributions to a PAYG system can be considered as the inflow, I_t, expressed as

$$I_t = \pi_t \cdot S_t \cdot N_t^I, \tag{10}$$

where π_t is the *contribution rate*, S_t is the average salary, and N_t^I is the number of workers who contribute to the pension plan at time t. The total pension payments at time t, which resemble the cash outflows of a Ponzi scheme, are given by

$$O_t = P_t \cdot N_t^R, \tag{11}$$

where P_t is the average pension and N_t^R is the number of retirees.

In the ideal case, the total contributions are set equal to total benefits, such that $I_t = O_t$. This balanced path implies that the contribution rate equals

$$\pi_t = \frac{P_t}{S_t} \cdot \frac{N_t^R}{N_t^I} = \delta_t \cdot d_t, \tag{12}$$

where $\delta_t = P_t/S_t$ is the *replacement rate* and $d_t = N_t^R/N_t^I$ is the *dependency ratio*, which equals the number of retirees divided by the number of active workers.

Equation (12) provides a convenient tool for examining the dynamics of a PAYG system. We can see that, *ceteris paribus*, if the relative number of retirees increases, the contribution rate will have to rise; and if the life expectancy increases or the birth rate falls, the dependency rate will increase. PAYG systems can offset these adverse effects by reducing the benefit level, raising the contribution ratio, increasing the retirement age, or implementing a combination of these changes (Boccia, 2024).

Despite the similarity in balanced cash flows, there are profound differences between PAYG plans and Ponzi schemes: transparency and the tools available to manage the cash flows. First and most significantly, the operations of a PAYG system are completely transparent. Under a state-sponsored PAYG scheme, the rules can generally be changed by a democratic process. On the contrary, a Ponzi scheme is inherently fraudulent: only the promoter knows what is going on, and he lies to his clients and steals their money. The sustainability of the arrangement depends critically on having an adequate number of new entrants at each stage to fund the benefit payments.

Second, the administrator of a PAYG system has more tools in its arsenal to tackle cash flow problems than a Ponzi promoter. For example, in China, to ensure the solvency of the pension system, the government has made several fundamental changes to the population policy to influence people's fertility and retirement decisions, thereby dynamically adjusting the replacement rate and the dependency ratio.[21] In contrast, in a Ponzi scheme, if the net cash flows become negative and remain so, then as soon as any buffer of cash reserves is depleted, the scheme will collapse. Given the nature of a Ponzi scheme, it will be very difficult for the promoter to make any

[21] These include but are not limited to implementing a one-child policy in the late 1970s, which aimed to curb population growth. This policy helped boost the quality of the labor force and prepared the country for the salary increase two decades later. Over time, facing a fertility rate that is lower than the replacement level and a rapidly aging population, the government shifted to a full-bloom two-child policy in 2013 and then a three-child policy in 2021 (Zeng et al., 2017). Considering that the life expectancy increased significantly, the government decided to gradually increase the compulsory retirement age, starting from 2025 (Master, 2024).

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changes that will reverse this trend. For example, if he reduces the promised return, the number of new investors will decline even further.

Nevertheless, even with all its policy tools, a PAYG plan can still encounter liquidity issues when demographic changes become too extreme or adverse economic conditions result in a sharp cut in salary levels. In recent history, PAYG schemes have run into difficulties in several countries; see, for example, the cases of Italy and Russia (Forni & Giordano, 2001; Jensen & Richter, 2004). When this happens, the payments will be halted and the scheme restructured, thus preventing it from collapsing.

5.2 Introducing a fictitious balance

In modeling Ponzi schemes, some researchers include F_t, a *fictitious balance*. This refers to a balance that grows at the promised rate and records the notional value of the scheme. The fictitious balance is adjusted to reflect the cash inflows and the redemption outflows to the scheme as shown in Equation (14). The cash outflows due to investor redemptions serve to draw down this balance.

A simple version of the resource constraints, accounting for the two balances, is as follows:

$$W_{t+1} = (1 + r_f)W_t + I_t - O_t - C_t, \tag{13}$$

$$F_{t+1} = (1 + r_t)F_t + I_t - O_t - c_t C_t, \tag{14}$$

where W_t corresponds to the true cash balance and F_t corresponds to the fictitious balance; $c_t \in [0,\ 1]$ is a proportion of the takeout shown in the fictitious account. The promoter tends to tone down the actual takeout C_t in the fictitious balance by only presenting part of it.

Variations of this type of resource constraint in discrete time can be found in Zhu (2011), see Section 5.2.1 for details. She assumes the promoter picks the promised return and consumption to maximize his welfare. In her model, the duration of the scheme, T, is exogenous. She finds it convenient to model the evolution of a fictitious account F_t, which, in her formulation, represents the current amount owed to the scheme's investors.

The continuous-time counterpart of the dynamics can be written as

$$\dot{W}(t) = r_f W(t) + I(t) - O(t) - C(t), \tag{15}$$

$$\dot{F}(t) = r(t)F(t) + I(t) - O(t) - c(t)C(t), \tag{16}$$

and again, $c(t) \in [0,\ 1]$.

Variations of these dynamics can be found in Artzrouni (2009) and Clauss et al. (2009). The two models move away from utility maximization and use a statistical approach to capture the evolution of the scheme until its demise, i.e., when the promoter's available cash goes to zero. The rationale is that if the promoter is subject to a quadratic preference, he can manipulate the duration T, which enables him to set the takeout, C_t, at the satiation point. By removing the utility function, we also have more flexibility to explore the interdependence between the decision variables (e.g., the promised rate) and the decision horizon (i.e., the duration of the scheme).

5.2.1 A Ponzi scheme with a fictitious balance and no redemption withdrawal: Zhu (2011)

In her doctoral thesis, Zhu (2011) proposed a Ponzi model with two balances: the actual balance, W_t; and the fictitious balance, F_t, which the investors (wrongly) perceive as real. Denote the promised rate of return as r_t, the risk-free rate as r_f, and C_t as the takeout of the promoter.[22] Then, the two balances, W_t and F_t, evolve according to the following equations:

$$W_{t+1} = (1 + r_f)W_t + I_t - C_t, \tag{17}$$

$$F_{t+1} = (1 + r_t)F_t + I_t, \tag{18}$$

$$I_t = g(r_t)F_t, \tag{19}$$

[22] In Zhu's original model, C_t is the consumption at time t, and we change the timing to be at the end of t.

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where the cash inflow, I_t, grows in proportion to the fictitious balance, F_t; investors do not redeem their money until the terminal time, thus $O_t = 0$; the proportion of takeout shown on the fictitious balance is $c_t = 0$. The proportion of fictitious balance invested, g, is defined as

$$g(r_t) = g_0\Big[1 - \exp\big(-a(r_t - r_f)\big)\Big]. \tag{20}$$

As $r_t \to r_f$, the growth rate tends to zero; and as $r_t \to \infty$, the growth rate approaches the upper limit, g_0.

In each period, the regulator observes r_t and sets an arbitrary detection threshold $\tilde{r}$, which is a random draw from the normal distribution, with $\tilde{r} \sim N(\bar{r},\ \sigma)$. If the promised return r_t exceeds $\tilde{r}$, the regulator detects the Ponzi scheme and will force it to close. This would drive both the fictitious and actual balances to zero. Otherwise, if there is no regulatory detection, the Ponzi scheme continues.

Assume that the duration of the Ponzi game, T, is known to the promoter, and at the terminal time T, there is a bequest $B(W_T,\ T)$. Denote $U(C_t,\ t)$ as the promoter's utility function, and assume that the promoter chooses a consumption and promised rate pair $(C_t,\ r_t)$ in each period to maximize his expected utility. His optimal behavior can be obtained by solving the following Bellman equation:

$$J(W_t,\ F_t,\ t) = \max_{\{C_t,\, r_t\}} \mathrm{E}_t\Big[U(C_t,\ t) + \rho \cdot J(W_{t+1},\ F_{t+1},\ t+1)\Big], \tag{21}$$

subject to the resource constraints in Equations (17)–(19) and a non-detection condition $r_t < \tilde{r}$. For simplicity, the discount factor ρ can be set to 1. At the final period T, the value function reduces to the bequest function, i.e., $J(W_T,\ F_T,\ T) = B(W_T,\ T)$.

The model rests on a strong assumption: there is no withdrawal by the investors from the fund during its life, or $O_t = 0$. That is, all promised benefits stay in the fund. Hence, the fictional account F_t grows geometrically at a rate of $r_t + g(r_t)$. This condition, as well as the assumption of known duration, is unrealistic.

5.2.2 Continuous time models: Clauss et al. (2009)

Up to now, we have dealt with discrete-time models. The model proposed by Clauss et al. (2009) is formulated in continuous time. One advantage of continuous-time models is that obtaining solutions is generally easier.

Similar to Zhu (2011), Clauss et al. (2009) consider the evolution of two fund balances. As before, $W(t)$ is the real cash balance, which grows at a return of $r(t)$. Let $F(t)$ be the fictitious balance, and $\mu(t)$ be the gross return. Unlike Zhu (2011), Clauss et al. (2009) specifically model cash outflows related to fund redemptions and assume the scheme ends when the cash runs out. In their model, the promoter's takeout takes the form of a management fee, which equals the unit rate, $m(t)$, times the fictitious balance, $F(t)$. Note that this management fee is assumed to be disclosed to the investors and is publicly known.

Both the cash inflow and outflow are proportional to $F(t)$, with an instantaneous intensity of $\lambda^+(t)$ and $\lambda^-(t)$, respectively. Given the above assumptions, the fund dynamics are given by

$$\dot{W}(t) = r(t)W(t) + I(t) - O(t) - C(t), \tag{22}$$

$$\dot{F}(t) = \mu(t)F(t) + I(t) - O(t) - C(t), \tag{23}$$

$$I(t) = \lambda^+(t)F(t), \tag{24}$$

$$O(t) = \lambda^-(t)F(t), \tag{25}$$

$$C(t) = m(t)F(t), \tag{26}$$

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with the initial condition being $W_0 = F_0$. Apparently, the proportion of takeout shown on the fictitious balance is $c(t) = 1$. In other words, the management fee is an actual cash withdraw, of which the full amount is deducted from both the actual wealth in Equation (22) and fictitious wealth in Equation (23). The scheme collapses if $W(t) < 0$. One solution to the system is

$$\begin{pmatrix} W(t) \\ F(t) \end{pmatrix} = W_0 \cdot \exp\left(\int_0^t \mathbf{A}(s)\mathrm{d}s \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \text{where } \mathbf{A}_t = \begin{pmatrix} r(t) & \lambda^+(t) - \lambda^-(t) - m(t) \\ 0 & \lambda^+(t) - \lambda^-(t) + \mu(t) - m(t) \end{pmatrix} \tag{27}$$

However, it is difficult to set the value of $\lambda^+(t)$ and $\lambda^-(t)$, as either may evolve over time; in particular, the sudden release of negative news concerning a liquidity crisis may lead to a jump in $\lambda^-(t)$.

5.2.3 Auxiliary fictitious balance: Artzrouni (2009)

Artzrouni (2009) proposes a model based on a set of four differential equations to capture the four processes underlying a Ponzi scheme.[23] These equations specify the dynamics of (i) $W(t)$, the actual cash balance; (ii) $F(t)$, the fictitious cash balance, which only serves an illustrative purpose for investors; (iii) $I(t)$, the cash inflow process, driven by new investors; and (iv) $O(t)$, the cash outflow, attributable to previous investors who redeem their investment. The promoter's takeout, $C(t)$, is not modeled and taken to be zero. This assumption seems unrealistic. As before, r denotes the promised rate of return, only that it is now expressed as continuous.

Suppose the initial real cash balance is $W(0) = W_0 = F_0$. The four cash processes evolve according to

$$\dot{W}(t) = r_f W(t) + I(t) - O(t), \tag{28}$$

$$\dot{F}(t) = (r - \gamma)F(t) + I(t), \tag{29}$$

$$\dot{I}(t) = \beta I(t), \tag{30}$$

$$\dot{O}(t) = (r - \gamma)O(t) + \gamma I(t), \tag{31}$$

where r $(r - \gamma)$ is the net return to the investor; and there is no promoter takeout, i.e., $C(t) = 0$. The inflow, $I(t)$, brought in by new investors, grows at an exponential rate of β, with the initial condition $I(0) = I_0$. The outflow, $O(t)$, is subject to an initial condition, $O(0) = \gamma\left(K + I_0\right)$. Artzrouni (2009) was able to derive explicit solutions for $W(t)$ and $F(t)$.

It is debatable whether Artzrouni's model can be used as a candidate tool to study the dynamics of a social security system as the author claimed. The assumption that $I(t)$ follows exponential growth is not attainable in the long run, as it is equivalent to assuming an ever-increasing pool of new investors. As a result, without specifying a finite horizon, the real balance in Equation (28) can be explosive or follow a balanced path for some parameter values.[24]

Interestingly, the model suggests an auxiliary role of the fictitious balance, $F(t)$, as it does not depend on the actual outflow, $O(t)$. In other words, the promoter does not equate the withdrawal from the fictitious balance, $\gamma F(t)$, with the actual investor withdrawal, $O(t)$. This specification illustrates the fact that $F(t)$ only serves to show the rosy yet bogus fortune promised by a Ponzi scheme and thus can be dispensed with in modeling.

It is worth noting that the dynamic of O_t, while exponential, can be either explosive or contractual. This model is prescient of Ponzi models based on the a class models that integrate investor

[23] We reformulated Artzrouni (2009)'s original model by mapping the notations to ours; also, we recast the development into an equivalent system of differential equations.

[24] Based on this model, Cunha et al. (2014) conducted simulation studies to examine how different parameter values affect the duration. Without considering the implicit assumption on population growth, Cunha et al. (2014) erroneously concluded that "...(the social security system's) vulnerability to demographic fluctuations has nothing to do with Ponzi schemes or other fraudulent financial transactions."

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demographic dynamics. These models, by imposing hump-shaped dynamics on the inflow and outflows, are able to eliminate the unrealistic assumption of perpetual growth and focus on the actual cash balance, $W(t)$. We discuss them in the next section.

5.3 Introducing demographic dynamics: SIR-type Ponzi models

Susceptible-infected-recovered (SIR) models, which can be traced back to Kermack & McKendrick (1927), are useful in modeling the spread of epidemics. In general, an SIR model captures the transitions among three compartments within the population: S represents susceptible individuals that are exposed to the risk of the disease; I represents infected individuals; and R represents recovered (or deceased) individuals.

In Ponzi schemes, the three compartments can be mapped to three different groups: N^S denotes the number of individuals who are potential investors; N^I denotes the number of investors currently in the scheme; and N^R denotes the number of individuals who have redeemed their holdings. The transitions among the three compartments are governed by a system of linked differential equations: , , ,

$$\dot{N}^S = -\beta \cdot \tfrac{N^I}{N} \cdot N^S \tag{32}$$

$$\dot{N}^I = \beta \cdot \tfrac{N^I}{N} \cdot N^S - \gamma \cdot N^I \tag{33}$$

$$\dot{N}^R = \gamma \cdot N^I \tag{34}$$

where $\dot{X} = \mathrm{d}X/\mathrm{d}t$; $N^S + N^I + N^R = N$ is the total population, which is assumed to be fixed; β is the transmission rate of the epidemic; γ, the recovery rate, indicates that the average duration of the epidemic is γ^{-1}.[25]

SIR-type Ponzi models recognize the fact that money contributed or received by the three groups plus the cut taken by the promoters is a zero-sum game. In addition, when the promoter runs out of cash to cover the redemptions, the scheme collapses. Beyond this point, it does not make sense to project the SIR-type demographics. All existing investors, unable to withdraw any funds, will become victims. In other words, any projections beyond the zero balance point are hypothetical and unrealistic. Therefore, these models can help to circumvent the possibility of explosive and perpetual cash inflows, thus yielding more realistic cash dynamics within a finite duration.

However, adapting the SIR framework to Ponzi modeling requires more work than merely adding a real cash balance process. In this subsection, we introduce refinements to tackle some of the challenges in integrating SIR dynamics with Ponzi models and present a prototype model with a wealth process. Finally, we briefly review some variations and possible extensions.

5.3.1 SIR model with add-on assumptions

To get a serviceable model, we adopt a gradual approach and start with some refinements. These refinements help simplify the model structure while retaining the link with the model's economic lineage.

First, we impose restrictions on investor behavior. In many Ponzi schemes, an investor may redeem part of her promised benefits, add more funds, or re-enter the scheme after her previous redemption. Therefore, the boundaries between "invested" (I) and "redeemed" (R) may be blurred. To exclude this complication, we assume that once an investor redeems, she withdraws all her principal and accrued interest. She takes no further part in the scheme.

Second, we impose a lock-up period on the investment, which is an analog of the single-period investment products in the discrete model introduced in Section 4.2. The lock-up period also

[25]The differential equation for the recovered individuals, N^R, is redundant. This stems from the fact that Equation (34) can be derived from the other two equations using $\dot{N}^S + \dot{N}^T + \dot{N} = 0$, as the total population is fixed.

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links with the recovery rate in the SIR model, γ, which indicates that the average duration of an epidemic is γ^{-1}. In Ponzi schemes, we translate γ^{-1} as the lock-up period of the scheme. This assumption makes the investment product in a Ponzi scheme similar to a fixed-income instrument with a predetermined investment term.

A direct benefit of the two restrictions is that they significantly simplify the real cash balance process. Otherwise, investors could enter and leave the scheme in a random fashion, hence leading to overly complicated inflow and outflow dynamics, and examples of such complications can be found in Mayorga-Zambrano (2011) and Zhu, et al. (2017).[26]

5.3.2 The basic model

Our basic model involves two structural parameters: the transmission rate β and the recovery rate γ, both of which are assumed constant. Moreover, we assume the scheme only offers one product, promising a continuously compounded return of r. The risk-free rate, or any rate used as the benchmark, is r_f. We disregard any exogenous shocks, such as the possibility that the Ponzi scheme collapses due to forced liquidation after regulatory detection. The existence of a lock-up period, which is of length γ^{-1}, divides the Ponzi scheme into two stages.

Stage 1.

Before the lock-up period expires for the first batch of investors, the recovery or redemption rate γ is equal to zero. The dynamics of the three groups of investors are then characterized by a susceptible-infected (SI) model:

$$\begin{aligned}\dot{N}^S(t) &= -\beta \cdot \tfrac{N^I(t)}{N} \cdot N^S(t),\\ \dot{N}^I(t) &= \beta \cdot \tfrac{N^I(t)}{N} \cdot N^S(t),\\ \dot{N}^R(t) &= 0.\end{aligned}$$

These specifications are simply Equations (32) and (33) without the redemption term $\gamma \cdot N^I$. As before, β is the transmission rate. The actual wealth or balance of the scheme, W, has the following dynamics:

$$\dot{W}(t) = -c_0 - K \cdot \dot{N}^S(t), \tag{35}$$

where K is the average amount invested in the scheme, and the initial wealth level is $W(0) = K \cdot N^I(0)$; c_0 is a fixed rate of takeout by the promoter. To set the Ponzi game in motion, the *basic reproduction rate*, $\mathcal{R}_0 = \beta/\gamma$, must be greater than one.

Equation (35) shows that the changes in wealth are composed of two terms: $-c_0$, the promoter's withdrawal, which is fixed; and $-K \cdot \mathrm{d}N^S/\mathrm{d}t$, the money brought by new investors, whose population is the negative of the change in the number of susceptible investors, $\dot{N}^S$. This

[26]For example, Mayorga-Zambrano (2011) uses a transition matrix to account for the random entry and random withdrawal of investors. In addition, the transmission and withdrawal rates are also random, making the model difficult to solve since it necessitates the use of simulations. Zhu et al. (2017) developed the potential-investor-divestor (PID) model, where the transmission rate β depends on a representative agent's average degree of connection network, denoted as $\beta\langle k\rangle$. Over each time interval, $[m\tau,\ (m+1)\tau)$, the promised interest rate is fixed; at the end of $m\tau + \tau$ $(m = 0,\ 1,\ 2, \ldots)$, the interest payment accrued over that interval is distributed to all investors. Consequently, the cash inflow is $I(m\tau,\ \tau) = K \int_{m\tau}^{m\tau+\tau} \left(-\dot{N}^S\right)\mathrm{d}t$ and the cash outflow is $O(m\tau,\ \tau) = K \int_{m\tau}^{m\tau+\tau} \dot{N}^R \mathrm{d}t + K \cdot r \cdot N^I(m\tau)$. Here, the cash outflow, O, comprises two parts: the principal redeemed by investors and the interest given to all investors. The cash balance change over $[m\tau,\ (m+1)\tau)$, defined by $I(m\tau,\ \tau) - O(m\tau,\ \tau)$, can be obtained using approximations. However, the last term in $O(m\tau,\ \tau)$ is at variance with empirical observations: investors do not often withdraw the interest payments in full amount; instead, they tend to keep the proceeds on the fictitious account, $F(t)$. Therefore, Zhu et al. (2017) exaggerate the magnitude of the cash outflow.

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means that N^S decreases with time so that $\dot{N}^S$ is negative and the last term in Equation (35) is positive.

Stage 2.

After the lock-up period ($t \geq \gamma^{-1}$), earlier investors begin to redeem. The demographic dynamics are restored to the following SIR model:

$$\begin{aligned}
\dot{N}^S(t) &= -\beta \cdot \tfrac{N^I(t)}{N} \cdot N^S(t), \\
\dot{N}^I(t) &= \beta \cdot \tfrac{N^I(t)}{N} \cdot N^S(t) - \gamma \cdot \tfrac{N^I(t-\gamma^{-1})}{N} \cdot N^S\big(t-\gamma^{-1}\big), \\
\dot{N}^R(t) &= \gamma \cdot \frac{N^I(t-\gamma^{-1})}{N} \cdot N^S\big(t-\gamma^{-1}\big).
\end{aligned}$$

Here, the second and third equations become *delay* differential equations (DDE) that must be solved numerically.[27] Compared with the standard SIR model in Equation (33), the dynamics of N^I now depend on the current N^I and N^I with a delay. With a delay equating the lock-up period (γ^{-1}), the promoter's actual wealth evolves according to

$$\dot{W}(t) = -c_0 - K \cdot \dot{N}^S(t) - \mathrm{e}^{r/\gamma} \cdot K \cdot \dot{N}^R\big(t-\gamma^{-1}\big), \tag{36}$$

where the first two terms are the same as in Equation (35). The last term is the amount withdrawn, in which $\mathrm{e}^{r/\gamma} \cdot K$ is the future value of an investment K made γ^{-1} unit of time before t. As long as $W(t)$ remains non-negative, it evolves according to Equation (35) within the lock-up period and Equation (36) after the period.

Due to the takeout by the promoter and withdrawal by investors, the cash balance will eventually approach zero, leading to the scheme's demise. As before, the duration of the scheme is the maximum time length that the wealth $W(t)$ stays non-negative, i.e., $T = \mathrm{argmax}_t \{W(t) \geq 0\}$.

The demographic dynamics and the wealth process of our bond-like Ponzi scheme are illustrated in Panels (a) and (b) of Fig. 1. We can see that the actual wealth declines to zero in the 53rd month. This timing is earlier than that predicted by the standard SIR model, in which the wealth process is disregarded, such that the process ends when the number of susceptible investors drops to zero.

Note: In this graph, we set $N = 100,000$, $\beta = 0.2$, $\gamma = 1/12$, $r = 10\%$, $K = 5$, $c_0 = 0$. We implicitly assume that the promoter is risk-neutral. The Ponzi scheme collapsed in month 53 when the cash balance W dropped below zero.

5.3.3 Extensions: functional forms of structural parameters

There are many possible extensions to the baseline model. For example, we can also add $-c_1 W(t)$ to Equations (35) and (36), where $c_1 > 0$ is a fixed constant. This addition entails that the promoter's withdrawal has a component that increases with the real cash balance.

Additionally, it is possible to adopt other plausible forms for the structural parameters. As before, we denote the promised rate of return by r and the risk-free by r_f. We can assume that the target population depends on r and satisfies the following equation:

$$N = N_0 \cdot \exp\big(-a(r - r_f)\big), \quad \text{for } r > r_f \tag{37}$$

where N_0 is the total population in the area, and a is a positive constant. This implies that the higher the promised rate, the smaller the pool of susceptible investors.[28]

[27] See Kuang (1993) for more details on delay differential equations and Naresh et al. (2009) for a discussion of time-delayed SIR models.

[28] One explanation for this assumption is that a high interest rate is a red flag.

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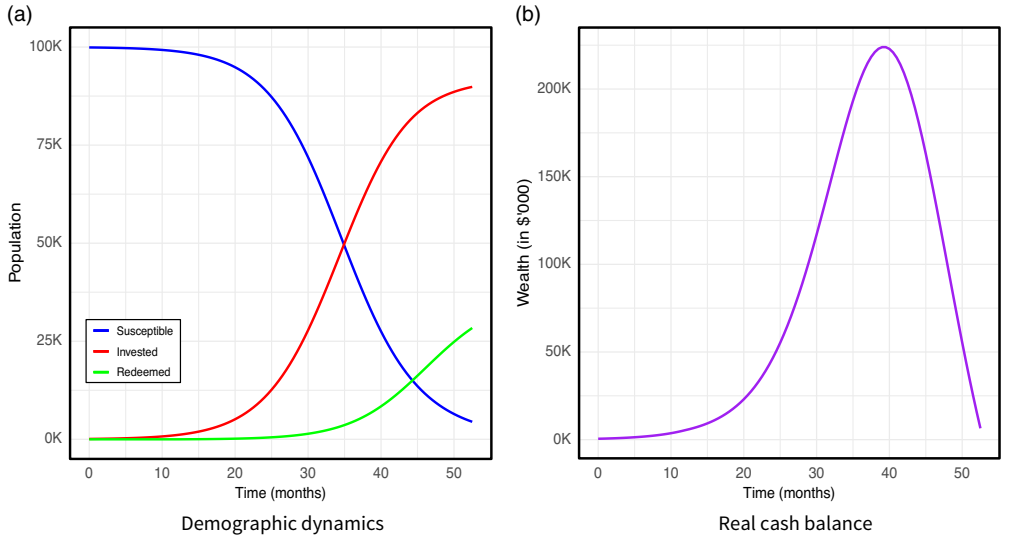


Figure 1. An illustration of the basic SIR-type Ponzi model.

Alternatively, the two key parameters – the transmission rate, β, and the recovery rate, γ – can also depend on the promised interest rate, r. For example, Amona & Oduro (2019) assume that β and γ follow the following rules

$$\beta = \lambda e^{-\lambda r}, \quad \gamma = k(1-\beta) = k(1-\lambda e^{-\lambda r}). \tag{38}$$

Using this specification, the transmission rate peaks at $r = 0$, which is inconsistent with empirical observations.[29]

By using the same structural parameters, the basic model assumes homogeneous characteristics across members of the three groups. There is evidence that this may not apply. Rantala (2019) documented that in the WinCapita scheme, a subset of influential investors, known as "highly connected hubs" in network parlance, had a much higher impact on the spread of the scam than average investors. Such influential investors correspond to the so-called *super spreader* in an epidemic, who exhibits an above-average risk of becoming infected and can infect a large number of susceptibles (Wong et al., 2015). In a recent paper, Szapudi (2020) replaces β, the transmission rate, by p_k, the probability that an individual with k connections will be infected, and explains how to modify the SIR model to include super spreaders.

6. Conclusion

According to the SEC, a Ponzi scheme is defined as "an investment fraud that involves the payment of purported returns to existing investors from funds contributed by new investors." These scams have occurred all over the world in a variety of different forms. Our paper has identified the key features of these schemes and illustrated how to model their dynamics. We demonstrated, using evidence from the literature, that Ponzi schemes harm society. It is hoped this paper can provide guidance to investors, regulators, and the financial community at large.

We examined several high-profile Ponzi schemes in Section 2. These include heterogeneous schemes that are unique, exemplified by two US schemes perpetrated by Bernard L. Madoff and Robert Allen Stanford, respectively, and the Finnish WinCapita scheme. The homogeneous schemes, which share a similar structure and the same investment story, include schemes

[29] However, Amona & Oduro (2019) do not specify a wealth process, and the duration of their scheme may be overstated.

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in Jamaica and Colombia, small US schemes, the Ezubao scheme and other P2P lending scams in China.

It is not surprising that Ponzi schemes strive to copy many of the characteristics of legitimate investment firms. In Section 3, we highlighted the main characteristics that help make these scams appear legitimate and identified several potential red flags. A prospective investor should be skeptical when a firm announces that it will deliver a relatively high promised return and explicitly claims that this return is guaranteed. The watchwords are *caveat emptor*.

In Section 4, we discussed how a Ponzi scheme originates from but deviates from a standard, intertemporal consumption-savings model. To illustrate their similarities, we start from the neoclassical economic paradigm where the agent strives to maximize his expected utility given the resource constraints. We built a stylistic Ponzi scheme to present the key differences. This simple model helps us to identify the key elements of a Ponzi scheme: the cash inflows and outflows. Unlike rational expectation models, the duration or the time to ruin is not known *a priori*, and not all cash outflow goes to the promoter.

We summarized existing models of Ponzi schemes in Section 5. These extant Ponzi models can be considered as different extensions to the standard consumption-savings problem, achieved by tweaking the resource constraints. The key to Ponzi modeling is to capture the dynamics of the real cash balance, which is affected by both the inflow and the outflow. This reformulation also helps us understand how a Ponzi scheme, with mechanical cash flow dynamics, differs from a PAYG pension system, where the pension authority can alter population policies and contribution rules thus influencing the cash flows. The inherent population dynamics in both Ponzi models and a legitimate investment fund motivate scholars to adopt a pure statistical perspective and project the key variables with an additional a demographic process. We explained how statistical models could be modified to include the SIR framework. This adaptation, nevertheless, also relies on some simplifications of the withdrawal behavior. To sum up, modeling a Ponzi scheme is a complex exercise, and while progress has been made, there is still much more to be done.

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Data availability statements. The data and codes that support the findings of this study are available from the corresponding author, Phelim Boyle, upon reasonable request.

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Appendix A. Defining Features of Major Ponzi Schemes: A Summary

The following table lists all the defining features of Ponzi schemes introduced in Section 2 and illustrates the extent to which different schemes exhibit these features. The table also includes the features of the pay-as-you-go (PAYG) system, which we discussed in Section 5.1.2.

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Table A1. Features of representative ponzi schemes

Scheme	Promoter	Plausible story	Trust-building	Investment promise	Control features	Marketing strategies	Regulation circumventions	Agency problem	How the scheme ends
Panel A: Heterogenous Ponzi schemes									
Madoff (Section 2.1.1)	Bernard L. Madoff	Mutual/hedge funds	Affinity-based, self-promotion, word-of-month, private referrals	High returns at first, then stable returns	Various	Used feeder funds	Used regulated business to cover unregulated operations	Used a dubious audit firm, no internal monitoring	Collapsed due to large redemptions during the 2008 financial crisis
Stanford (Section 2.1.2)	Robert A. Stanford	Certificates of deposits (CDs)	Government affiliates, self-promotion, charity works, sports sponsorship	High, stable returns	Fixed-term investments	Build and maintain positive personal image	Bribed (former) government officials to circumvent regulation and receive insider information	sat on regulatory committees himself	Cracked down by US government agencies and regulators
WinCapita (Section 2.1.3)	Hannu Kailajärvi	Profits earned from various investment activities	Participation by invitation from a sponsor	High returns	A lock-up period of 6-months	Sponsors serve as *songbirds*; investors grows along an affinity network	Opaque operations		Government crackdown following investigations
Panel B: Homogenous Ponzi schemes									
Schemes in Jamaica (Section 2.2.1)		Investment vehicles	Affinity-based	High returns		Successful early investors (i.e., *songbirds*)	Opaque operations		Implosion as it cannot satisfy redemptions requests

Table A1. *Continued*

Scheme	Promoter	Plausible story	Trust-building	Investment promise	Control features	Marketing strategies	Regulation circumventions	Agency problem	How the scheme ends
Schemes in Columbia (Section 2.2.2)		Prepaid cards and other instruments	Affinity-based	High returns	Fixed-term investment	Induce repeated investments	Opaque operations		Implosion or government crackdown
Small schemes in US (Section 2.2.2)		Investment vehicles	Affinity-based	High returns	A lock-up period	Word-of-mouth; induce repeated investments	Opaque operations	Dubious or no audits	Government crackdown
P2P Lending Schemes in China (Section 2.2.4)		Peer-to-peer (P2P) lending platforms appearing as investment vehicles	Affinity-based; advertisement; charity work	High, stable returns of little risk	A lock-up period; sometimes with a minimum investing threshold	Word-of-mouth, advertisement, office in prime locations, etc.	Opaque operations	Dubious or no audits	Implosion or government crackdown
Panel C: pay-as-you-go (PAYG) systems									
Pay-as-you-go (Section 5.1.2)	State-sponsored	Receive income after retirement	Sovereign guarantee	Predictable pension income	Various policy tools to influence the contribution rate, replacement ratio, dependency ratio, etc.	Compulsory participation	Operations subject to statutory disclosure; changes made after a formal/democratic process	Subject to general public scrutiny	Perpetual unless extreme demographic, political, or economic events happen

Notes: In some cases, the related features are not available or non-existent, and the corresponding cells are left as blank.

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