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Gaussian beam evolution in nonlinear inhomogeneous plasma

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The method of nonlinear complex geometrical optics (NCGO) is proposed in this paper for description of the evolution of a spatially narrow Gaussian beam (GB) in an inhomogeneous nonlinear plasma. NCGO method deals with first-order ordinary differential equations for the complex curvature of the wave front and for GB amplitude and for second-order ordinary differential equation for GB width. Thus, NCGO simplifies the description of GB diffraction and self-focusing effects as compared to the known methods of plasma physics and this way it can be assumed to be attractive and comprehensive approach in problems of plasma heating by electromagnetic waves. Moreover, we demonstrate in this paper some regularity for nonlinear inhomogeneous plasma in the framework of which central ray of a GB is not subjected to nonlinear refraction within NCGO method boundary applicability. On the contrary, the beam width, wave front curvature, and GB amplitude are modified by diffraction and self-focusing processes. General properties of the beam propagation are illustrated with results of numerical modeling for two particular cases: GB diffraction and self-focusing along curvilinear trajectory with torsion in axially symmetric plasma column and GB reflection from nonlinear inhomogeneous plasma layer. We prove in this paper that NCGO is new effective method of plasma physics, which can be applied for improvement of ray tracing techniques and plasma diagnostics.

1. Introduction

Diffraction and self-focusing of electromagnetic beams is one of the important problems both in plasma physics and in nonlinear optics. In this paper, we present new method for the problems concerning the nonlinear effects in propagation of a Gaussian beam (GB) in a layer of inhomogeneous plasma and axially symmetric plasma column. This paper is generalization of results of other authors who used quasi-optical method based on the parabolic equation (Max 1976; Lam and Lipman 1977; Anderson 1978) and the beam tracing method (Nowak and Orefice 1993) for description of linear electromagnetic waves in an inhomogeneous plasma. The nonlinear complex geometrical optics (NCGO) method can be readily applied for modeling of electron cyclotron absorption and current drive in tokamaks (Permitin

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and Smirnov 1996) similarly to the quasi-optical approach. The problem of modeling of interaction laser-plasma and comparison with experimental data is discussed in Weber et al. (2004).

The method of NCGO was developed earlier in Berczynski et al. (2010) for analysis of self-focusing in nonlinear media of the Kerr type and for GB propagation in an inhomogeneous media (Berczynski and Kravtsov 2004; Berczynski et al. 2006), including nonlinear inhomogeneous fibers (Berczynski 2011). Next this simple and effective approach was applied to describe GB evolution in nonlinear saturable media (Berczynski 2012, 2013a) and in longitudinally and transversely inhomogeneous fibers with absorption (Berczynski 2013b).

Here we make use of the analogy between the Kerr nonlinear response, which is a linear function of the electromagnetic field intensity (Akhmanov et al. 1968), and the ponderomotive nonlinearity in plasma (Tewari and Sharma 1979; Gill et al. 2010), which is a saturated exponential function of intensity. Ordinary differential equations of NCGO describing a quasi-stationary diffraction of electromagnetic beam in nonlinear plasma are much simpler than a nonlinear paraxial equation with partial derivatives often used in the physics of laser plasma interaction (Poli et al. 1999; Balakin et al. 2007a, b; Farina 2007; Bertelli et al. 2010). Propagation of intense laser beams in plasma makes a part of many applications such as laser beam ray tracing, plasma diagnostics, plasma heating, *etc*. An accurate and fast analytical or numerical description of this process could be very useful. We would like to emphasize that a combination of the quasi-isotropic approximation (QIA) of geometric optics with the complex geometrical optics (GO; Kravtsov et al. 2009) enables to describe both diffraction and polarization evolution of Gaussian electromagnetic beams in weakly anisotropic inhomogeneous media including magnetized plasma. NCGO method deals with paraxial (spatially narrow) beams whose width is small as compared to the density non-uniformity scale of a weakly inhomogeneous plasma.

The paper is organized as follows. Section 2 discusses the boundary of applicability of NCGO method for nonlinear inhomogeneous plasma. Section 3 considers equations of NCGO method for a plasma with ponderomotive nonlinearity. A new element of NCGO is pointed out for nonlinear inhomogeneous plasma, according to which the central ray of GB is not affected by the nonlinear refraction within NCGO method boundary applicability. That fact simplifies significantly the governing equations. The problem of self-focusing of GB propagating in an axially symmetric plasma is analyzed in Sec. 4. Section 5 is devoted to numerical modeling of propagation of a powerful GB in a plane inhomogeneous plasma layer.

2. Boundary of applicability of NCGO method for nonlinear inhomogeneous plasma

NCGO method is nonlinear generalization of classical GO (Kravtsov and Berczynski 2007; Kravtsov and Zhu 2009). This method deals with quasi-plane inhomogeneous waves in the form

$$u(r) = A(r)\exp[i\Psi(r)] = A(r)\exp[ik_0\psi(r)]. \tag{1}$$

In contrast to conventional GO, the eikonal (optical path) $\psi(r)$ and amplitude $A(r)$ are complex valued in the framework of NCGO. The direction of wave propagation is determined by the gradient of the real part of complex eikonal $\nabla\psi_R(r) = \nabla\mathrm{Re}\{\psi(r)\}$. The direction of exponential decay of the field's magnitude is principally given by the gradient of the imaginary part $\nabla\psi_I(r) = \nabla\mathrm{Im}\{\psi(r)\}$. The gradient of complex

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eikonal determines the ray momentum $\mathbf{p} = \nabla\psi(r)$, which satisfy the ray equations in Hamiltonian form (Kravtsov and Zhu 2009)

$$\frac{dr}{ds} = \frac{p}{\sqrt{\varepsilon}}, \quad \frac{dp}{ds} = \frac{1}{2}\nabla\sqrt{\varepsilon}(r), \tag{2}$$

where n is refractive index and ds is elementary arc length. In (2) the parameter $\varepsilon(r)$ is relative permittivity, which, for an isotropic non-magnetic plasma, is related with refractive index by the formula

$$\varepsilon(r) = n^2(r). \tag{3}$$

Besides Hamiltonian form, the equation for the ray evolution in inhomogeneous nonlinear plasma is usually presented in the following form (Holm 2011)

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}_c}{ds}\right) = \frac{1}{2}\nabla n. \tag{4}$$

Based on the assumption of quasi-plane inhomogeneous wave structure of NCGO wave field, it is natural to require (analogously like in GO) that the real and imaginary parts of complex amplitude $A(r)$ and local wave vector $\mathbf{k} = k_0\mathbf{p} = k_0\nabla\psi(r)$ do not change significantly on the scale of $\lambda' = \lambda/2\pi = 1/|\mathbf{k}|$. Thus we have that

$$\lambda'|\nabla A| \ll |A|, \ \lambda'|k_i| \ll |k_i|, \tag{5}$$

where k_i are the components of wave vector $\mathbf{k}$. A similar assumption is also made for the refractive index of inhomogeneous nonlinear plasma

$$\lambda'|\nabla n| \ll n. \tag{6}$$

Since in a medium of refractive index n the wave length λ is n times shorter than that in free space, the above inequalities in (5) and (6) are equivalent to

$$\frac{\lambda_0'}{n} \ll \frac{|A|}{|\nabla A|} \equiv L_1, \quad \frac{\lambda_0'}{n} \ll \frac{|k_i|}{|\nabla k_i|} \equiv L_2 \ \frac{\lambda_0'}{n} \ll \frac{|n|}{|\nabla n|} \equiv L_{3_i'}, \tag{7}$$

where$\lambda' = \lambda/2\pi$. Alternatively, these conditions, stating that $A(r)$, $\mathbf{k}(r)$, $n(\mathbf{r})$ vary insignificantly within region of the order of λ', may be united in the single inequality

$$\mu_{NCGO} = \frac{1}{k_0 L} = \frac{\lambda'}{L} \ll 1, \tag{8}$$

where μ_{NCGO} is the parameter of smallness in NCGO method, and L is the smallest of the characteristic lengths of the quantities $A(r)$, $\mathbf{k}(r)$, $n(\mathbf{r})$, where $L = \min(L_1, L_2, L_3)$. To derive the basic equations of NCGO method, let us take advantage of Rytov expansion (Kravtsov and Zhu 2009) of the field in a small dimensionless parameter $\mu_{NCGO} = 1/k_0 L$. Within dimensionless variables $x_1 = k_0 x$, $y_1 = k_0 y$, and $z_1 = k_0 z$, the Helmholtz equation for nonlinear inhomogeneous plasma takes the form

$$\Delta_1 u(r_1) + n^2 u(r_1) = 0, \tag{9}$$

where $\Delta_1 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial z_1^2}$ and characteristic length L for the change of parameters ε, $\mathbf{k}$, A converts into dimensionless parameter $k_0 L = 1/\mu_{NCGO}$. We can introduce this dimensionless parameter into (9) by transforming $r_2 = \mu_{NCGO} r_1 = r/L$, $n(r_2) = n(\mu_{NCGO} r_1) = n(\mathbf{r}/L)$ obtaining the equation

$$\Delta_2 u(r_2) + \frac{n^2(r_2)}{\mu_{NCGO}^2} u(r_2) = 0, \tag{10}$$

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where $\Delta_2 = \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2}$. The amplitude is assumed to vary slowly; thus, we have that $A = A(\mu_{NCGO} r_1) = A(r_2)$. It is also convenient to write the phase in the form $\Psi(r) = \Psi_1(\mu_{NCGO} r_1)/\mu_{NCGO} = \Psi_1(r_2)/\mu_{NCGO}$ so that the local wave vector as a gradient of phase $\mathbf{k} = \nabla\Psi = k_0 \nabla_I \Psi_1(\mu_{NCGO} r_1)/\mu_{NCGO} = k_0 \nabla_2 \Psi_1(r_2)$ would also assume to be a function that changes slowly with coordinates, where $\nabla_1 = \partial/\partial \mathbf{r}_1$ and $\nabla_2 = \partial/\partial \mathbf{r}_2$. As a result the wave field in coordinates r_1 and r_2 has the form

$$u = A(\mu_{NCGO} r_1) \exp[i\Psi_1(\mu_{NCGO} r_1)/\mu_{NCGO}] = A(r_2) \exp[i\Psi_1(r_2)/\mu_{NCGO}]. \quad (11)$$

Substitution of (11) into (10) leads to the equation

$$\Delta_2 u(r_2) + \frac{\varepsilon(r_2)}{\mu_{NCGO}^2} u(r_2) \equiv \left\{ \frac{1}{\mu_{NCGO}^2} [n^2 - (\nabla_2 \Psi_1)^2] A + \frac{i}{\mu_{NCGO}} [2\nabla_2 A \cdot \nabla_2 \Psi_1 + A\Delta_2 \Psi_1] + \Delta_2 A \right\} \times \exp[i\Psi_1/\mu_{NCGO}] = 0. \quad (12)$$

Comparison of the components with $1/\mu_{NCGO}^2$ in (12) leads to the eikonal equation

$$(\nabla_2 \Psi_1)^2 = n^2. \quad (13)$$

Comparing also the components of i/μ_{NCGO} we obtain the transport equation in the form

$$2(\nabla_2 A \cdot \nabla \Psi_1) + A\Delta_2 \Psi_I = 0. \quad (14)$$

Strictly speaking, (14) is the transport equation derived in zeroth-order GO approximation (Kravtsov and Zhu 2009) that enables satisfactory accuracy in wave analysis shown on the example of GB evolution in inhomogeneous and nonlinear media including optical fibers. In more sophisticated problems such as wave field reflection from weak interface (Kravtsov and Zhu 2009) that demands to describe entire wave phenomena in the framework of NCGO, one can use the expansion of the wave amplitude$A$ in powers of parameter μ_{NCGO} (Rytov expansion)

$$u(\mathbf{r}_2) = \sum_{m=0}^{\infty} A_m(\mathbf{r}_2) \left(\frac{\mu_{NCGO}}{i} \right)^m \exp(i\Psi_1(\mathbf{r}_2)/\mu_{NCGO}), \quad (15)$$

or Debye expansion of the field in inverse powers of wave number $1/k_0$ in the form

$$u(\mathbf{r}) = \sum_{m=0}^{\infty} \frac{A_m(\mathbf{r})}{(ik_0)^m} \exp(ik_0 \psi(\mathbf{r})). \quad (16)$$

Limiting ourselves to amplitude of zeroth-order in Rytov expansion in (15), rewriting (13) and (14) in dimensional variables x, y, z, and introducing eikonal $k_0\psi = \Psi_1/\mu_{NCGO}$, we obtain NCGO equations in standard form (Kravtsov and Zhu 2009)

$$(\nabla\psi)^2 = n^2, \quad (17)$$

$$2(\nabla A \cdot \nabla\psi) + A\Delta\psi = 0. \quad (18)$$

NCGO method deals with small-angle (paraxial) beams, which are localized in the vicinity of the central ray (beam trajectory) satisfying the ray equations in (2). To satisfy the condition of NCGO method applicability, we introduce the small paraxial parameter that has the form

$$\mu_{Parax} = 1/k_0 w_0 = \lambda_0'/w_0 \ll 1. \quad (19)$$

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The CGO paraxial parameter defined in (19) appear usually in problems of nonlinear wave theory, where the optical description by means of Nonlinear Helmholtz Equation reduces to Nonlinear Schrödinger Equation (NLS). It is shown in Berczynski (2011) that NCGO method supplies solutions for nonlinear inhomogeneous fiber of Kerr type, which are identical with results obtained by NLS. We demonstrated in Berczynski (2011) that CGO method supplies solutions for GB evolution in inhomogeneous and nonlinear Kerr type fibers in much simpler way comparing to standard methods of nonlinear optics such as variational method approach and method of moments. GB is self-sustained solution in the framework of NCGO method and we noticed from our numerical calculations that enough condition for GB to preserve its 'Gaussian' profile in nonlinear plasma is that GB width should be small enough with respect to characteristic nonlinear scale of the medium. To satisfy this condition let us introduce small 'nonlinear' parameter in the form

$$\mu_{NL} = w_0/L_{NL} \ll 1, \tag{20}$$

where $L_{NL} = w_0/\sqrt{\varepsilon_{NL}\,|A_0|}^2$ (w_0 is initial beam width and A_0 is initial complex amplitude) is nonlinear parameter when we limit ourselves to weakly nonlinear effects in laboratory plasma. From the other side, the nonlinear scale L_{NL} should be comparable with scales L_1, L_2, L_3 in (7). In other words, we can prove that nonlinear scale of plasma should satisfy the equation

$$\frac{1}{k_0^2 L_{NL}^2} \leqslant \mu_{CGO}^2. \tag{21}$$

Using relation among total beam power P, critical power P_{crit}, and nonlinear distance L_{NL} in the form (Akhmanov et al. 1968)

$$\frac{L_D^2}{L_{NL}^2} = \frac{P}{P_{crit}}, \tag{22}$$

where

$$P = \frac{c\sqrt{\varepsilon_0}}{8\pi}\int_0^\infty |A_0|^2 \exp\left(-\frac{\zeta^2}{2w^2}\right) 2\pi\zeta d\zeta = \frac{1}{8} c\sqrt{\varepsilon_0} w_0^2 |A_0|^2 \tag{23}$$

and

$$P_{crit} = \frac{1}{8}\frac{c}{k_0^2 \varepsilon_{NL}}\sqrt{1 - \frac{\omega_p^2(0)}{\omega^2(0)}} \tag{24}$$

is critical power of nonlinear plasma described by relative permittivity in the form

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} + \varepsilon_{NL}(|E|^2), \tag{25}$$

where ω_p is the plasma frequency and ω is the frequency of the electromagnetic (laser) field, we can notice that NCGO method is applicable only when

$$P \leqslant \mu_{CGO}^2 L_{DD}^2 P_{crit}, \tag{26}$$

where $L_{DD}^2 = k_0^4 w_0^4$ is dimensionless diffraction distance, which is related with paraxial parameter by the relation $L_{DD}^2 = (1/\mu_{Parax})^2$. Thus condition of NCGO method applicability can be presented in the convenient general form

$$P \leqslant \left(\frac{\mu_{CGO}}{\mu_{Parax}^2}\right)^2 P_{crit}. \tag{27}$$

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Taking into account the parameters of inhomogeneous axially symmetric plasma from Figs. (2–5), where $w_0 = 0.1\,\mathrm{mm}$, $\omega_p^2/\omega^2 = 0.2$, $\lambda = 0.01m$, and $L = 3\,\mathrm{mm}$, one can notice that NCGO method is applicable for total beam powers that are small or equal than $P \leqslant 4P_{crit}$, taking into account that $L_D = 6.3\,\mathrm{mm}$. To generalize description by NCGO method for nonlinear inhomogeneous plasma, we should take into account also the effect of linear refraction. Thus, introducing the refraction parameter μ_{REF} we notice that the beam preserves its 'Gaussian' form during propagation in inhomogeneous medium when this refraction parameter is small enough. Thus we obtain that

$$\mu_{REF} = w_0/L \ll 1, \tag{28}$$

where $L = n/2\,|\nabla n|$ is inhomogeneity scale of smoothly inhomogeneous plasma.

3. Nonlinear Complex Geometrical Optics (NCGO) equations for plasma

The dielectric permittivity of a plasma in the field of quasi-monochromatic electromagnetic wave with the electric field amplitude E is of the form

$$\varepsilon = \varepsilon_L + \varepsilon_{NL}(|E|^2), \tag{29}$$

where

$$\varepsilon_L = 1 - \frac{\omega_p^2}{\omega^2} = 1 - 4\pi e^2 N_e/m\omega^2 \tag{30}$$

is the linear part of plasma permittivity ε, N_e is the electron density, $-e$ and m are the electron charge and mass, ω_p is the plasma frequency, and ω is the frequency of the electromagnetic (laser) field. The nonlinear component of plasma permittivity is defined by the plasma response. In general, it is described by a non-stationary equation for the electron density or a set of hydrodynamic equations. However, we limit ourselves here to the case where the characteristic time of the laser beam evolution, t_{las}, is larger than the time of the plasma response, $w_{las}/c_s \sim 100$ ps, where $w_{las} \sim 30\ \mu$m is the characteristic width of the laser beam and $c_s \sim 300$ km s^{-1} is the ion acoustic velocity in plasma. Then the plasma is locally in equilibrium where the laser ponderomotive force is balanced by the thermal pressure gradient. The nonlinear part of the plasma permittivity is a function of the laser beam intensity $c\,|E|^2/4\pi$ and it can be presented in the form

$$\varepsilon_{NL}(|E|^2) = \frac{\omega_p^2}{\omega^2}[1 - \exp(-\gamma\,|E|^2)], \tag{31}$$

where

$$\gamma = e^2/8\omega^2 mT. \tag{32}$$

In (32), T denotes the electron temperature. The nonlinear permittivity (31) for sufficiently low intensities, $\gamma|E|^2 \ll 1$, depends quadratically on $|E|$ and it takes a form of the Kerr nonlinearity:

$$\varepsilon_{NL}(|E|^2) = \frac{\omega_p^2 \gamma\,|E|^2}{\omega^2}. \tag{33}$$

At high intensities, $\gamma|E|^2 \sim 1$, the nonlinear response saturates, which corresponds to complete expulsion of the plasma from the laser beam path. Under the condition

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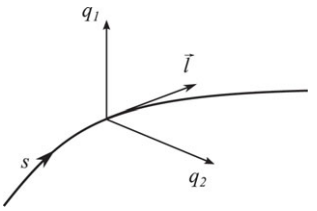


FIGURE 1. Ray-centered coordinate system.

that $t_{las} \gg w_{las}/c_s$ one may use the NCGO method that deals with a GB of the form

$$E(s, q_1, q_2) = A \exp\left(\frac{ik_0}{2} B_{ij} q_i q_j + ik_0 \psi_c\right), \tag{34}$$

where $i = 1, 2$ and summation over repeated indices is implied. In (34),

$$\psi_c = \int_o^s \sqrt{\varepsilon_c} ds' \tag{35}$$

is the eikonal along the central ray of the beam and s is the distance measured along the ray in the ray-centered coordinate system (Fig. 1). The variables q_i and q_j in (34) are distances from the central ray to the observation point in the directions of the main normal and binormal and B_{ik} is positively defined matrix of complex curvature (Kravtsov and Zhu 2009). In view of extremal properties of the central ray, the terms linear in q_i do not contribute to (34) in an isotropic plasma.

Moreover, we consider here the case of a weakly inhomogeneous plasma where the characteristic scale length of the density variation L is much larger than the width $1/|\mathrm{Im}B|$ of the laser beam defined by the imaginary part of the complex curvature matrix B. The central ray obeys the Fermat's ray equation (Holm 2011), which follows from the standard eikonal equation in the GO and has the form:

$$\frac{d}{ds}\left(n \frac{d\mathbf{r}_c}{ds}\right) = \frac{1}{2} \nabla n, \tag{36}$$

where ds is the arc-length of central ray. After simple manipulations, the equation for central ray evolution in an inhomogeneous plasma with the permittivity (36) can be presented in the form

$$\frac{1}{2} \frac{d\varepsilon}{ds} \frac{d\mathbf{r}_c}{ds} + \varepsilon \frac{d^2 \mathbf{r}_c}{ds^2} = \frac{1}{2} \left[\nabla \varepsilon_L(\mathbf{r}_c) + \nabla \varepsilon_{NL}(\mathbf{r}_c)\right]. \tag{37}$$

The nonlinear term can be neglected in (37) due to symmetry of GB with respect to the central ray and assuming a weak plasma inhomogeneity, $|\mathrm{Im}B|L \gg 1$, and a weak nonlinearity $\gamma |E|^2 \ll 1$. In that approximation the nonlinear refraction of central ray caused by the nonlinear part of plasma permittivity can be neglected. Therefore, the trajectory of central ray in a plasma with the ponderomotive nonlinearity coincides within NCGO method boundary applicability with the central ray in a plasma with a linear permittivity $\varepsilon = \varepsilon_L$. This fact is quite helpful for analysis of GB propagation in an inhomogeneous plasma. However, this result applies only to a symmetric GB. Different will be the case of asymmetric beam propagation. In that case the nonlinear refraction and aforementioned gradient need to be accounted for (Tomlinson et al. 1982). Assuming the beam to be of an axial symmetry, one can obtain from the

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eikonal equation the following equation for wave front curvature matrix B_{ij}:

$$\frac{dB_{ij}}{ds} + B_{ik}B_{kj} = \alpha_{ij} + \beta_{ik}, \tag{38}$$

where

$$\alpha_{ij}(s) = \left[\frac{1}{2}\frac{\partial^2 \varepsilon_L}{\partial q_i \partial q_j} - \frac{3}{4\varepsilon}\frac{\partial \varepsilon_L}{\partial q_i}\frac{\partial \varepsilon_L}{\partial q_j}\right]_{\mathbf{r}=\mathbf{r}_c} \tag{39}$$

characterizes the influence of plasma inhomogeneity and the coefficient

$$\beta_{ij}(s) = \left[\frac{1}{2}\frac{\partial^2 \varepsilon_{NL}}{\partial q_i \partial q_j} - \frac{3}{4\varepsilon}\frac{\partial \varepsilon_{NL}}{\partial q_i}\frac{\partial \varepsilon_{NL}}{\partial q_j}\right]_{\mathbf{r}=\mathbf{r}_c} \tag{40}$$

describes a nonlinear contribution. The general form of coefficients $\alpha_{ij}(s)$ and $\beta_{ij}(s)$ presented in (40) enables one to take into account such effects as evolution of GB cross-section along helical rays in graded-index fibers (Berczynski et al. 2006) and rotation of GB in a nonlinear Kerr type media (Marburger 1975; Goncharenko et al. 1991; Yariv and Yeh 2006). Knowing the matrix elements B_{ij}, one can calculate the amplitude of paraxial wave field in the form

$$\tilde{A}(s) = \tilde{A}^0 \exp\left(-\frac{1}{2}\int Tr B_{ij} ds\right), \tag{41}$$

where $\tilde{A} = \sqrt[4]{\varepsilon_c}A$, $\tilde{A}^0 \equiv \tilde{A}(0)$ is the initial amplitude and $TrB_{ij} = B_{11} + B_{22}$. The real part of matrix B_{ij} characterizes the wave front curvature, and the principal values R_i of the curvature

matrix $R_{ij}(\tau) = \mathrm{Re}B_{ij}$ define the principal curvatures κ_i,

$$R_i = \kappa_i. \tag{42}$$

The principle values I_i of matrix $I_{ij} = \mathrm{Im}B_{ij}$ determine the principle widths of the GB $w_i(\tau)$ in the following way (Berczynski et al. 2006):

$$I_i = \frac{1}{k_0 w_i^2}. \tag{43}$$

As a result, (42) and (43) determine the principle widths w_1 and w_2,

$$w_{1,2}^2 = \frac{2}{k_0\left[I_{11} + I_{22} \pm \sqrt{(I_{11} - I_{22})^2 + 4I_{12}^2}\right]}, \tag{44}$$

and the principle curvatures of the wave front κ_1 and κ_2 in the form

$$\kappa_{1,2} = \frac{R_{11} + R_{22} \pm \sqrt{(R_{11} - R_{22})^2 + 4R_{12}^2}}{2}. \tag{45}$$

In condition, that the coordinate q_1 lays in the plane of the central ray and the coordinate q_2 is orthogonal to this plane, the above equations take a simpler form because the matrix elements B_{12} and B_{21} disappear:

$$B_{ij} = \begin{bmatrix} \kappa_1 + \dfrac{i}{k_0 w_1^2} & 0 \\ 0 & \kappa_2 + \dfrac{i}{k_0 w_2^2} \end{bmatrix}. \tag{46}$$

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4. Self-focusing of GB in inhomogeneous axially symmetric plasma

As first example of application of the NCGO technique, let us consider propagation of a GB in an axially symmetric plasma waveguide with the density minimum on the axis. The relative permittivity profile in that case has the form

$$\varepsilon(z,q) = 1 - \frac{\omega_p^2}{\omega^2}\left(1+\frac{q^2}{L^2}\right) + \frac{\omega_p^2}{\omega^2}\left(1+\frac{q^2}{L^2}\right)[1-\exp(-\gamma\,|E|^2)], \tag{47}$$

where L is the characteristic radius of plasma density and $q = \sqrt{q_1^2+q_2^2}$ is the distance from plasma axis z (radius in cylindrical coordinates). The GB incidents on the base surface of a plasma cylinder at the distance q_c is defined by a unit tangent vector $\mathbf{1}$ with respect to the central ray that posses a component in the azimuthal direction $\mathbf{l}_\varphi$ and along the plasma symmetry axis $\mathbf{l}_z$. Because the nonlinearity does not influence the central ray within NCGO method boundary applicability, the GB propagates along a helical ray with a constant radius q_c and nonlinearity influences only the amplitude, the wave front curvature, and the beam width. Consequently, we define the plasma frequency at the central ray ω_{pc} as ${\omega_{pc}}^2 = {\omega_p}^2\,(1+{q_c}^2\,/L^2\,)$, and neglect the explicit coordinate dependence of the plasma frequency in the nonlinear term of (29). From (2) and using formulas from (Berczynski 2013a) one can find out the parameters

$$\alpha_{11} = -\frac{\omega_p^2\left(1+3l_\varphi^2\right)}{\omega^2L^2}\cos^2\theta, \quad \alpha_{22} = -\frac{\omega_p^2\left(1+3l_\varphi^2\right)}{\omega^2L^2}\sin^2\theta,$$

$$\alpha_{12} = \alpha_{21} = \frac{\omega_p^2\left(1+3l_\varphi^2\right)}{\omega^2L^2}\sin\theta\cos\theta. \tag{48}$$

The complex equation (38) can be presented as a system of six real equations for quantities $R_{ij} \equiv \mathrm{Re}\ B_{ij}$ and $I_{ij} \equiv \mathrm{In}\ B_{ij}$:

$$\frac{dR_{ij}}{ds} + R_{ik}R_{kj} - I_{ik}I_{kj} = \alpha_{ij} + \beta_{ik}, \quad \frac{dI_{ij}}{ds} + R_{ik}I_{kj} + I_{ik}R_{kj} = 0. \tag{49}$$

The parameters β_{ik} in (38) are equal to

$$\beta_{11(22)} = -k_0^2 w_{10}w_{20}I_{11(22)}\sqrt{I_{11}I_{22}-I_{12}^2}\gamma\exp\left[-\gamma k_0 w_{10}w_{20}\sqrt{I_{11}I_{22}-I_{12}^2}\,|A_0|^2\right] \times \omega_p^2 q_c^2/\omega^2L^2,$$

$$\beta_{12} = \beta_{21} = -k_0^2 w_{10}w_{20}I_{12}\sqrt{I_{11}I_{22}-I_{12}^2}\gamma\exp\left[-\gamma k_0 w_{10}w_{20}\sqrt{I_{11}I_{22}-I_{12}^2}\,|A_0|^2\right] \times \omega_p^2 q_c^2/\omega^2L^2. \tag{50}$$

This system of equations has been solved numerically for the following parameters:

$$w_0 = 0.1\,\mathrm{mm}, \quad \omega_p^2/\omega^2 = 0.2, \quad \lambda = 0.01\,\mathrm{m}, \quad L = 3\,\mathrm{mm}, \quad l_\varphi = 1/\sqrt{2}. \tag{51}$$

They correspond to a beam of CO_2 laser propagating in a channel of plasma with the density 2×10^{18} cm^{-3}. The obtained solutions of the system (49) together with (50) are presented in Figs. 2–7. The central ray is characterized by the radius of the helix $q_c = 1.4$ mm and the torsion $\chi = 0.321\ \mathrm{mm}^{-1}$.

Figures 2–4 demonstrate the case where GB with initially a circular cross-section $w_1(0) = w_2(0)$ and initial principal curvatures of the beam equal to zero $\kappa_i(0) = 0$

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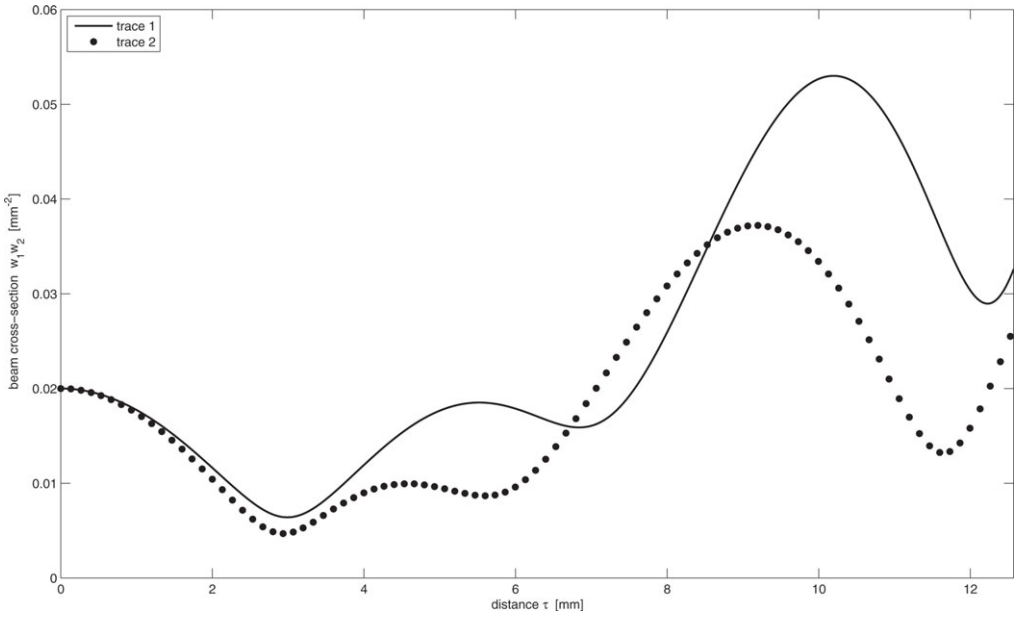


FIGURE 2. Evolution of the beam spot area $w_1 w_2$ for initially circular GB propagating in inhomogeneous axially symmetric plasma in under-critical regime (trace 1), where $\gamma |A_0|^2 = 10^{-4}$ ($P = 0.4 P_{\mathrm{crit}}$) and in over-critical regime (trace 2), where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4 P_{\mathrm{crit}}$) . Other parameters are given in (51) and initial principal curvatures of the beam are equal to zero, $\kappa_i(0) = 0$.

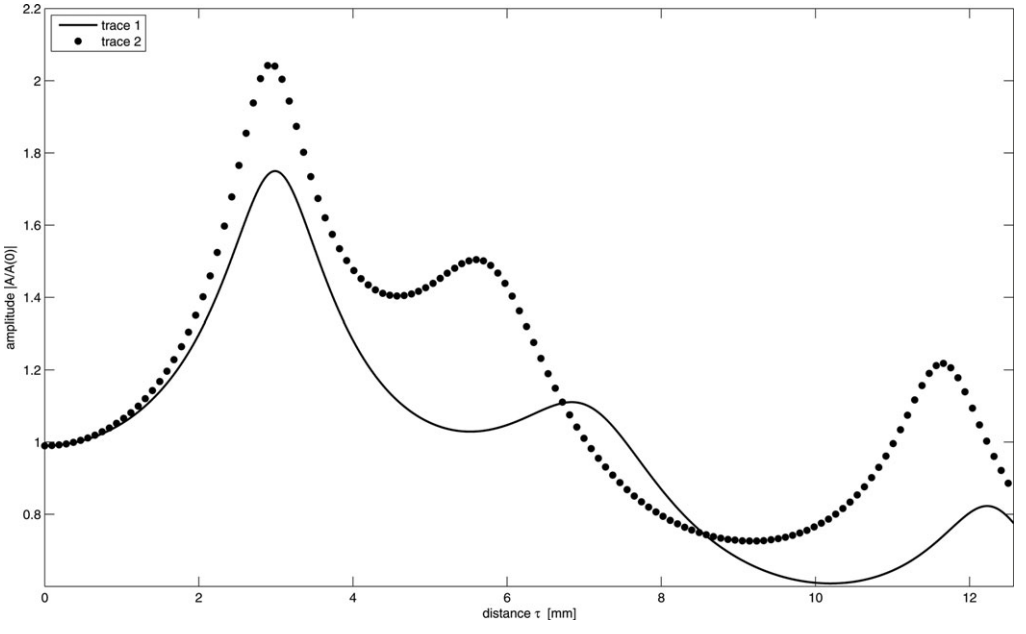


FIGURE 3. Evolution of the modulus of complex amplitude $|A| / |A_0|$ for initially circular GB propagating in inhomogeneous axially symmetric plasma in under-critical regime (trace 1), where $\gamma |A_0|^2 = 10^{-4}$ ($P = 0.4 P_{\mathrm{crit}}$), and in over-critical regime (trace 2), where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4 P_{\mathrm{crit}}$). Other parameters are given in (51) and initial principal curvatures of the beam are equal to zero, $\kappa_i(0) = 0$.

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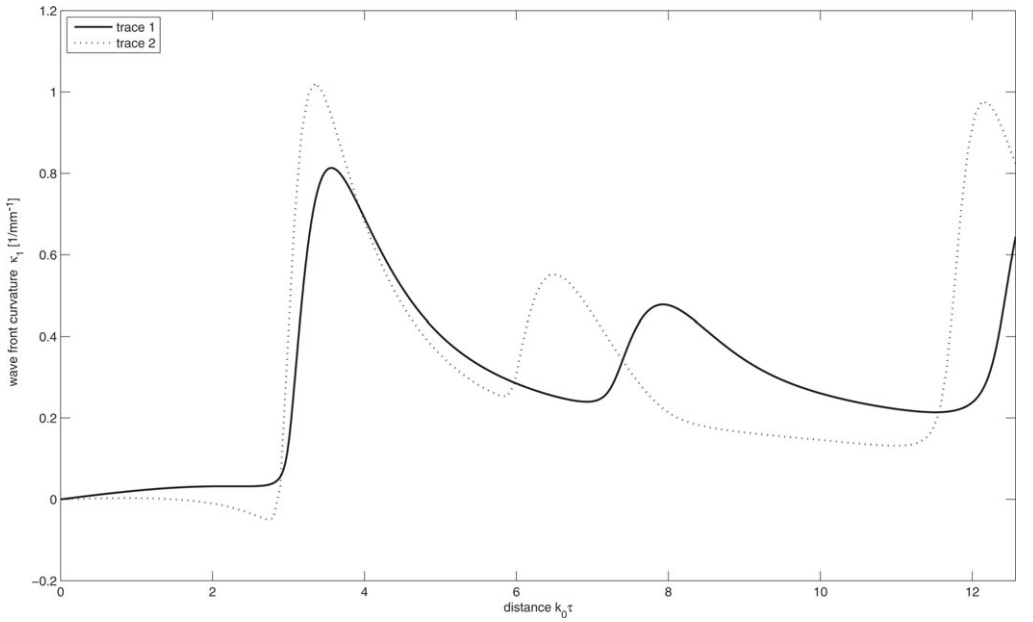


FIGURE 4. Evolution of the maximal principal curvature κ_1 for initially circular GB propagating in inhomogeneous axially symmetric plasma in under-critical regime (trace 1), where $\gamma |A_0|^2 = 10^{-4}$ ($P = 0.4P_{\text{crit}}$), and in over-critical regime (trace 2), where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4P_{\text{crit}}$). Other parameters are given in (51) and initial principal curvatures of the beam are equal to zero, $\kappa_i(0) = 0$.

propagates along a helical trajectory. The effect of self-focusing is described by the product of a nonlinear part of plasma permittivity and an initial amplitude $\gamma |A_0|^2$ (nonlinear parameter). In Fig. 2, the principal beam width increases in the azimuthal direction and decreases in the radial direction. As a result, the GB cross-section $w_1 w_2$ (trace 1 in Fig. 3) has the same value for the distance of two $s = 2L_R$ and three $s = 3L_R$ diffraction lengths, where $L_R = k_0 w_{10}^2 = 6.3 \cdot 10^{-3} m$. However, initially the effect of linear and nonlinear focusing dominates over the diffraction widening as it is shown in Fig. 2. For this case, the amplitude of GB (Fig. 3) initially increases and at the distance of one diffraction distance $s = L_R$ becomes three times greater compared to its initial value. Next, the diffraction widening dominates on linear and nonlinear focusing effects, and on the distance of two $s = 2L_R$ and three $s = 3L_R$ diffraction lengths the amplitude becomes comparable to its initial value. The maximal principal curvature κ_1 (Fig. 4) is positive on the distance of two $s = 2L_R$ and three $s = 3L_R$ diffraction lengths as it is shown. The minimal principle curvature κ_2 (Fig. 5) is initially negative and on the distance of one diffraction length is equal to zero, $\kappa_2 = 0$, and next becomes positive, $\kappa_2 > 0$, for the rest of the beam trajectory length.

Our description can be generalized for initially elliptical cross-sections of the beam. Eikonal-based NCGO approach demonstrates high ability in applications of nonlinear cylindrical plasma. To confirm it, let us perform explicitly the evolution of the beam cross-section, which is determined by the equation $k_0 I_{ij}(\tau) q_i q_j = 1$ in parallel transport coordinates (ptc)(q_1, q_2). In Fig. (6), we show the evolution of beam cross-section parallel transport coordinates (q_1, q_2) and in Fig. (7) evolution of beam cross-section in coordinates (q_n, q_b) connected to normal-binormal with unit vectors in (51), which in fact are attached to cylindrical ones.

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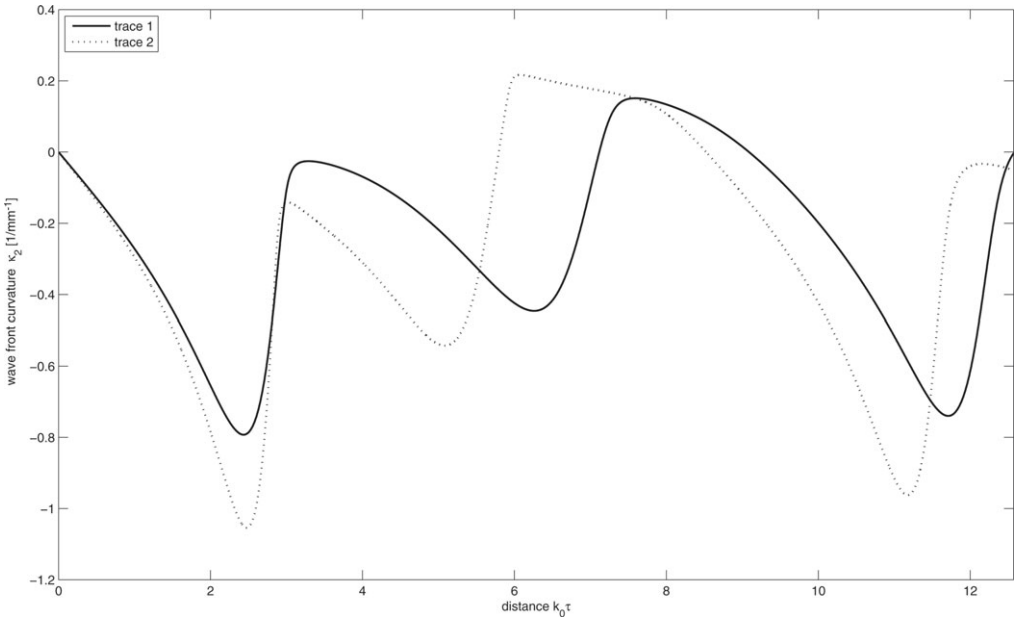


FIGURE 5. Evolution of the minimal principal curvature κ_2 for initially circular GB propagating in inhomogeneous axially symmetric plasma in under-critical regime (trace 1), where $\gamma |A_0|^2 = 10^{-4}$ ($P = 0.4P_{\text{crit}}$), and in over-critical regime (trace 2), where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4P_{\text{crit}}$). Other parameters are given in (51) and initial principal curvatures of the beam are equal to zero, $\kappa_i(0) = 0$.

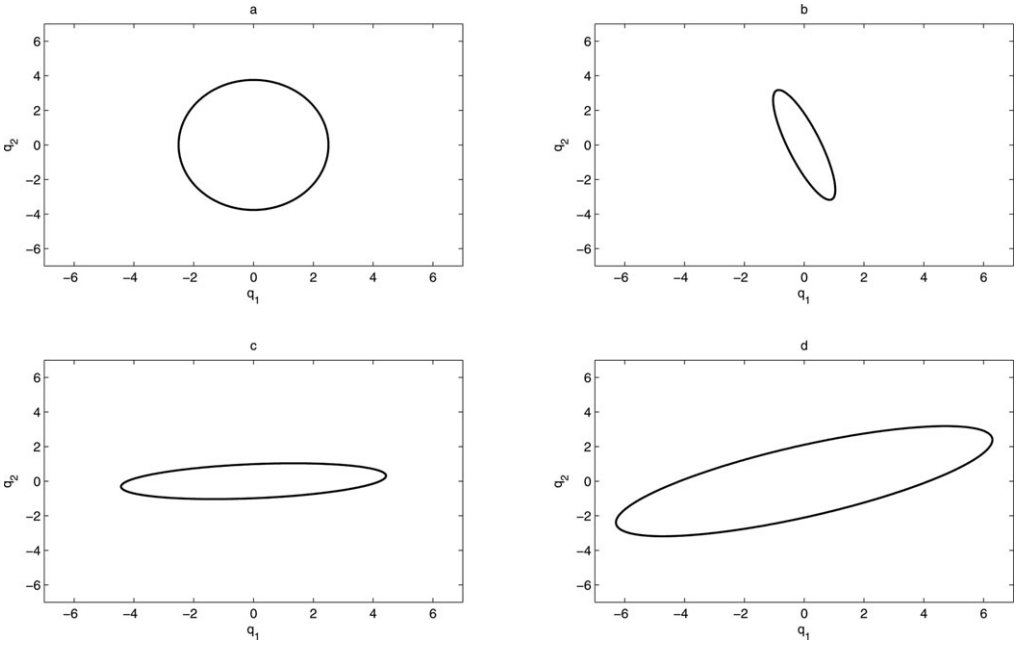


FIGURE 6. Evolution of GB (with initial ellipticity $w_{20} = 1.5w_{10}$) cross-section in parallel transport coordinates (ptc) (q_1, q_2) for GB propagating along helical ray in inhomogeneous axially symmetric plasma in over-critical regime, where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4P_{\text{crit}}$). The images of GB cross-section are shown for the values of s: (a) ($s = 0$), (b) ($s = 0.5L_D$), (c) ($s = L_D$), and (d) ($s = 1.5L_D$). Other parameters are given in (51) and initial principal curvatures of the beam are equal to zero, $\kappa_i(0) = 0$.

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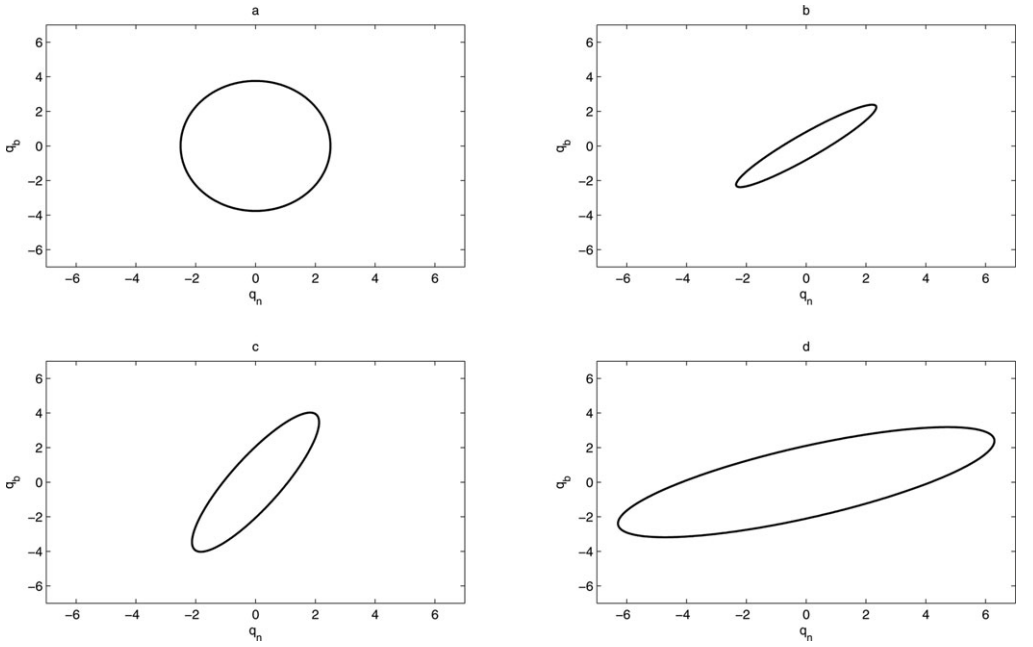


FIGURE 7. Evolution of GB (with initial ellipticity $w_{20} = 2w_{10}$) cross-section in coordinates attached to the normal and binormal (nbc) (q_n, q_b) for GB propagating along helical ray in inhomogeneous axially symmetric plasma in over-critical regime, where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4P_{\text{crit}}$). The images of GB cross-section are shown for the values of s: (a) ($s = 0$), (b) ($s = 0.5L_D$), (c) ($s = L_D$), and (d) ($s = 1.5L_D$). Other parameters are given in (51) and initial principal curvatures of the beam are equal to zero, $\kappa_i(0) = 0$.

One can notice in Fig. 6 that the large axis of beam cross-section rotates anticlockwise in parallel-transport coordinates (ptc), whereas conserve approximately direction in normal-binormal coordinates. The inclination of this cross-section in Fig. 7d expresses natural tendency to satisfy parallel transport along the ray.

5. Reflection of a high-power GB from an inhomogeneous plasma layer

In this section, we apply the NCGO method to analysis the reflection of high-power electromagnetic beam from an inhomogeneous plane plasma layer modeled by the permittivity profile

$$\varepsilon(y) = 1 - \frac{y}{L} + \frac{y}{L}[1 - \exp(-\gamma |E|^2)]. \tag{52}$$

It corresponds to a linear profile of the plasma density along the y-axis with the plasma frequency equal to the laser frequency at the distance $y = L$. The beam incidence angle θ_0 in the plane x, y is defined with respect to the y-axis and its intensity distribution is uniform in the direction of the beam polarization (z-direction). According to (4), we assume that the central ray of the GB leaves the starting point (x_0, y_0) with the initial conditions $\frac{dx(0)}{ds} = \sin\theta_0$, $\frac{dy(0)}{ds} = \cos\theta_0$. As a result the central ray satisfy the following equation, which depends on GB width and has the form

$$\frac{d^2 y_c}{ds^2} = \frac{\frac{1}{2L}\exp\left[-\frac{\gamma|A_0|^2 w_0^2}{w^2}\right]\left\{\left(\frac{dy_c}{ds}\right)^2 + \frac{2y_c\gamma|A_0|^2 w_0^2}{w^3}\frac{dw}{ds}\frac{dy_c}{ds} - 1\right\}}{1 - \frac{y_c}{L}\exp\left[-\frac{\gamma|A_0|^2 w_0^2}{w^2}\right]}. \tag{53}$$

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For such two-dimensional plasma layer the quadratic form $^1/_2 B_{ij} q_i q_j$ in (34) degenerates into a single term $^1/_2 B q^2$, where the complex curvature $B = \kappa + i/(k_0 w^2)$ satisfies the Riccati equation (Berczynski and Kravtsov 2004), which now depends on the position on the central ray y_c:

$$\frac{dB}{ds} + B^2 = \alpha,\ \alpha(\tau) = \left\{ \frac{1}{2}\left[\frac{\partial^2 \varepsilon(\mathbf{r})}{\partial q^2}\right] - \frac{3}{4\varepsilon(\mathrm{r})}\left[\frac{\partial \varepsilon(\mathbf{r})}{\partial q}\right]^2 \right\}_{\mathbf{r}=\mathbf{r}_c}, \tag{54}$$

where

$$[\varepsilon(\mathbf{r})]_{\mathbf{r}=\mathbf{r}_c} = \left(1 - \frac{y_c}{L}\right) \exp\left[-\frac{\gamma |A_0|^2 w_0^2}{w^2}\right], \tag{55}$$

$$\left[\frac{\partial \varepsilon(\mathbf{r})}{\partial q}\right]_{\mathbf{r}=\mathbf{r}_c} = \frac{3\sqrt{\varepsilon^0}\sin\theta_0}{L\left(1 - \frac{y_c}{L}\exp\left[-\frac{\gamma |A_0|^2 w_0^2}{w^2}\right]\right)^2} \exp\left[-\frac{\gamma |A_0|^2 w_0^2}{w^2}\right], \tag{56}$$

$$\begin{aligned}\left[\frac{\partial^2 \varepsilon(\mathbf{r})}{\partial q^2}\right]_{\mathbf{r}=\mathbf{r}_c} &= \frac{3\sqrt{\varepsilon^0}\sin\theta_0}{L} \exp\left[-\frac{\gamma |A_0|^2 w_0^2}{w^2}\right] \\ &\times \left[\frac{2y_c}{L^2}\left(\frac{\sqrt{\varepsilon^0}\sin\theta_0}{\sqrt{\left(\frac{dy_c}{ds}\right)^2 + \varepsilon^0 \sin^2\theta_0}}\right)\sqrt{1 - \frac{y_c}{L}\exp\left[-\frac{\gamma |A_0|^2 w_0^2}{w^2}\right]}\right] \\ &+ \left(\frac{\sqrt{\varepsilon^0}\sin\theta_0}{\sqrt{\left(\frac{dy_c}{ds}\right)^2 + \varepsilon^0 \sin^2\theta_0}}\right) \frac{\exp\left[-\frac{\gamma |A_0|^2 w_0^2}{w^2}\right]}{2L\sqrt{1 - \frac{y_c}{L}\exp\left[-\frac{\gamma |A_0|^2 w_0^2}{w^2}\right]}}.\end{aligned} \tag{57}$$

The Riccati equation in (54) can be presented in a form of two equations for the real and imaginary parts of the complex parameter $B = \kappa + i/k_0 w^2$ and as a result we obtain the equations for GB width,

$$\frac{d^2 w}{ds^2} - \alpha w = \frac{1}{k_0^2 w^3}, \tag{58}$$

and equation for wave front curvature evolution in the form

$$\kappa = \frac{1}{w}\frac{dw}{ds}, \tag{59}$$

where $\varepsilon^0 = 1 - \frac{y_c(0)}{L}\exp[-\gamma |A_0|^2]$ is the value of plasma permittivity for $s = 0$. Having calculated these two parameters from above equations, one can readily calculate the complex amplitude of GB from the equation

$$\frac{dA}{ds} + \left[\kappa + \frac{i}{k_0 w^2}\right] A = 0. \tag{60}$$

For the following parameters $L = 3\,\mathrm{mm}$, $\theta_0 = \pi/6$ (Figs. 8–11)) the numerical solutions of (53)–(60) are presented in Figs. (8–11).

In Fig. 8, one can notice that trajectory of beam central ray does not depend on plasma nonlinearity. Resulting in these trajectories are the same in under-critical regime ($P = 0.4 P_{\mathrm{crit}}$) and in over-critical regime ($P = 4 P_{\mathrm{crit}}$). In Fig. 9, one can notice that GB width initially increases to maximal value. Next dominates the plasma

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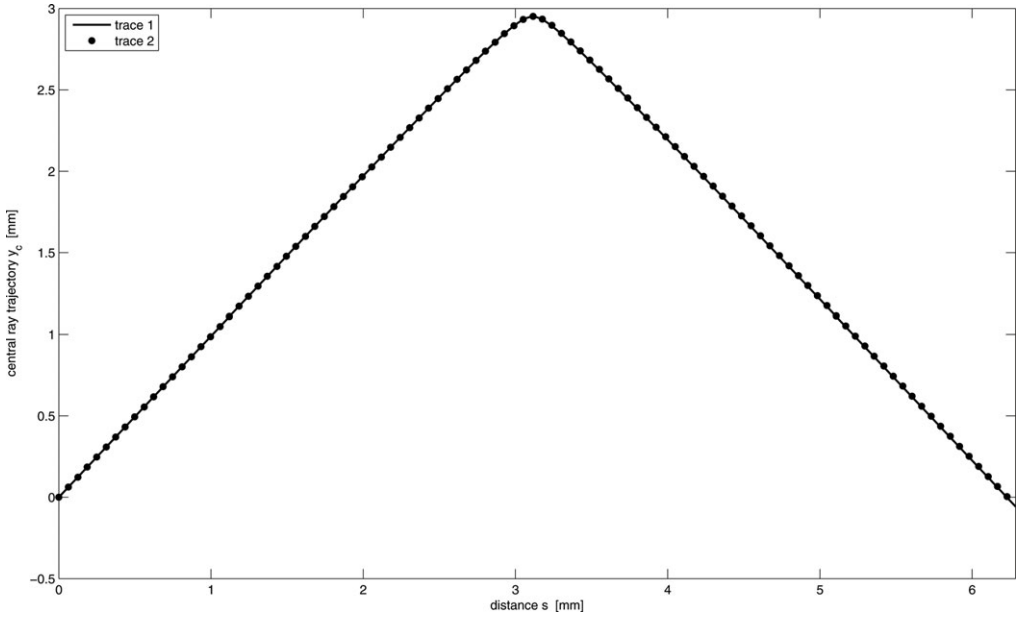


FIGURE 8. The central ray trajectory in inhomogeneous plasma layer in under-critical regime (trace 1), where $\gamma |A_0|^2 = 10^{-4}$ ($P = 0.4P_{\rm crit}$), and in over-critical regime (trace 2), where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4P_{crit}$).

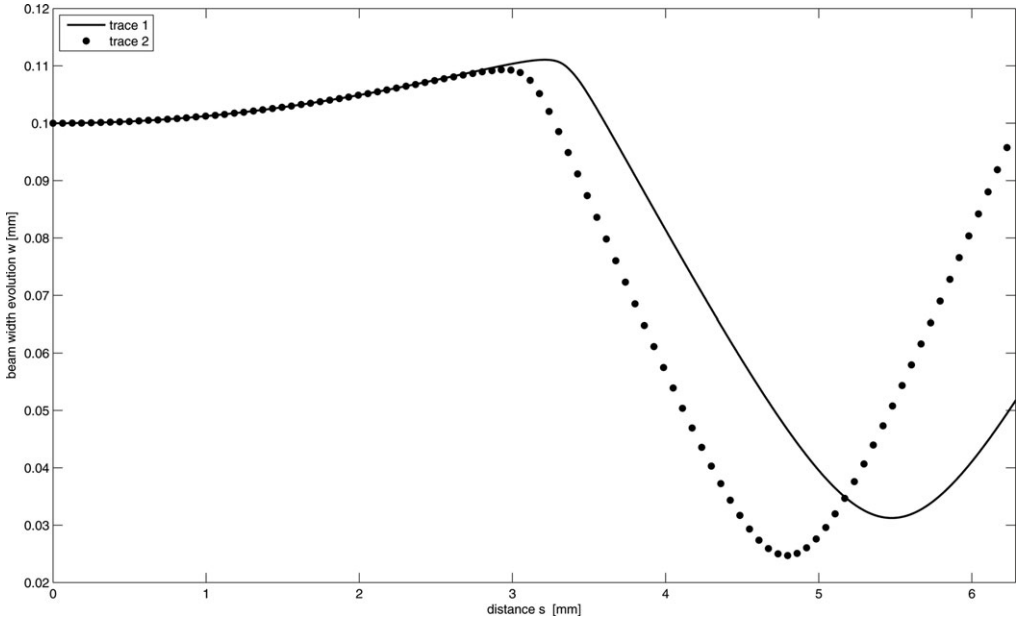


FIGURE 9. GB width evolution in inhomogeneous plasma layer in under-critical regime (trace 1), where $\gamma |A_0|^2 = 10^{-4}$ ($P = 0.4P_{\rm crit}$), and in over-critical regime (trace 2), where $\gamma |A_0|^2 = 10^{-3}$ ($P = 4P_{\rm crit}$).

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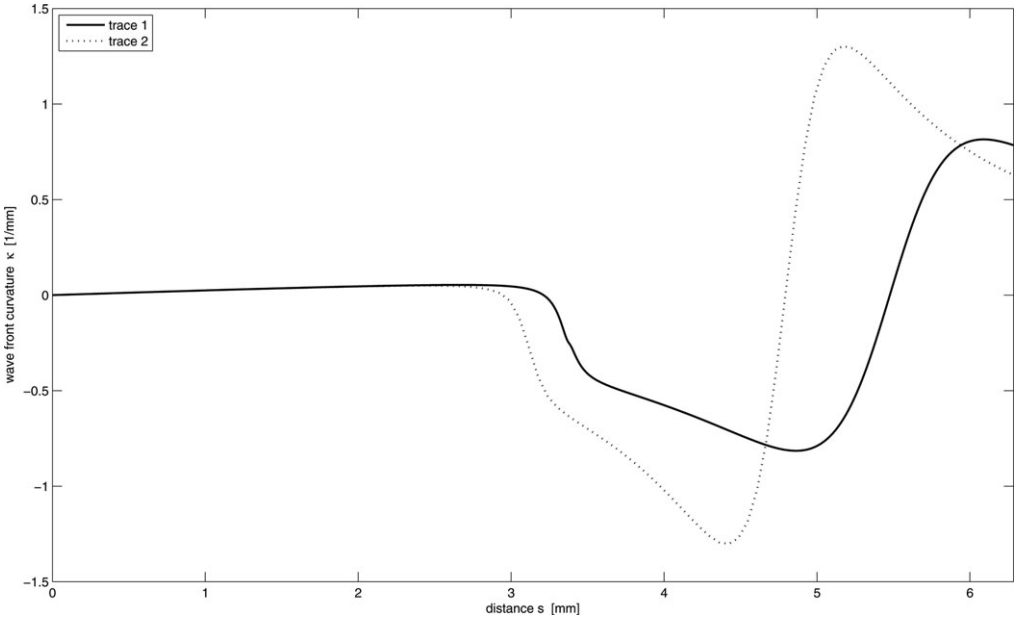


FIGURE 10. Wave front curvature evolution in inhomogeneous plasma layer in under-critical regime (trace 1), where $\gamma\,|A_0|^2 = 10^{-4}$ ($P = 0.4P_{\mathrm{crit}}$), and in over-critical regime (trace 2), where $\gamma\,|A_0|^2 = 10^{-3}$ ($P = 4P_{\mathrm{crit}}$).

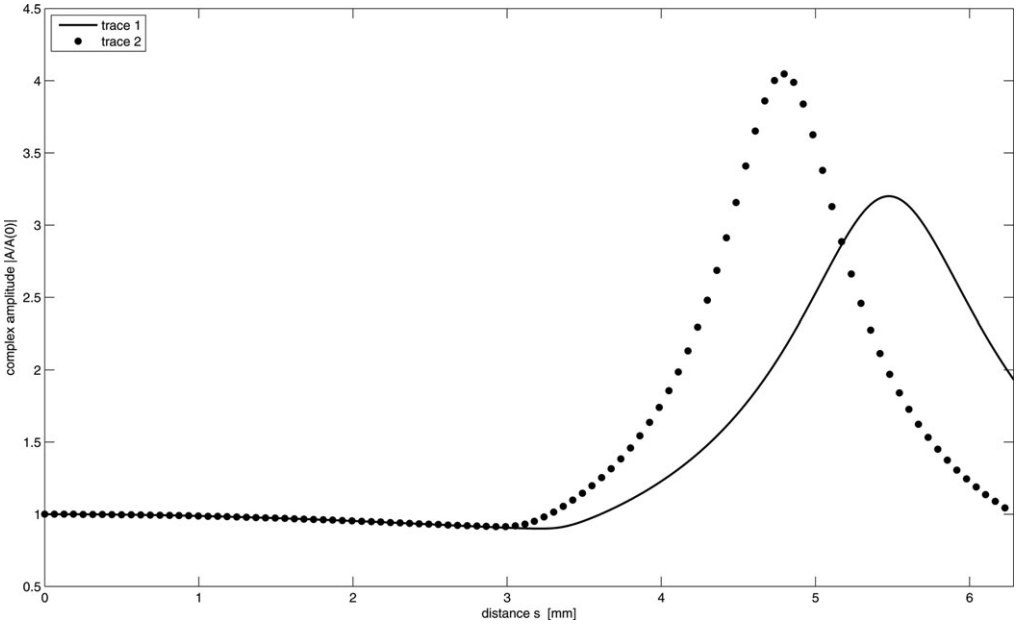


FIGURE 11. The amplitude $|A|\,/\,|A_0|$ evolution in inhomogeneous plasma layer in under-critical regime (trace 1), where $\gamma\,|A_0|^2 = 10^{-4}$ ($P = 0.4P_{\mathrm{crit}}$), and in over-critical regime (trace 2), where $\gamma\,|A_0|^2 = 10^{-3}$ ($P = 4P_{\mathrm{crit}}$).

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focusing together with self-focusing of the beam, and beam width decreases to minimal value and then increases again. The wave front curvature approaches zero three times at $s = 0$ in accordance with initial condition, at the point when the width is maximal and minimal (Fig. 10). The modulus of complex amplitude initially decreases and then becomes almost four times greater at beam waist, as is presented in Fig. 11.

6. Conclusions

Oblique incidence of a spatially narrow GB on a nonlinear plasma layer and GB diffraction along helical ray in axially symmetric nonlinear plasma column is studied in the framework of nonlinear modification of NCGO. This new technique deals with ordinary differential equations, but is able to describe the diffraction and self-focusing of the beam. It is equivalent to the paraxial equation for a symmetric GB propagating in weakly inhomogeneous plasma but it is simpler to use as it solves a set of ordinary differential equations for the beam parameters. Thus, this method enables to work out simple and fast numerical algorithms for some problems of nonlinear optics and an inhomogeneous plasma. The basic nonlinear equations of NCGO are derived for the case of ponderomotive nonlinearity in plasma. Efficiency of this method is demonstrated by numerical modeling for three-dimensional problem of GB diffraction and self-focusing along helical ray in an axially symmetric plasma and for the problem of reflection of GB from a plane plasma layer.

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