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Risk Premium Information from Treasury-Bill Yields

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Abstract

I find that short-maturity Treasury-bill yields have unique information about risk premiums that is not spanned by long-maturity Treasury-bond yields. I estimate 2 components of risk premiums: long term and short term. The long-term component steepens the slope of yield curves and has a forecastability horizon of longer than 1 year. In contrast, the short-term component affects Treasury-bill yields but is almost invisible from Treasury bonds, has a forecastability horizon of less than 1 quarter, and is related to bond liquidity premiums.

I. Introduction

I study time variation in risk premiums for holding U.S. Treasury bonds (T-bonds). It has long been known in the literature that linear combinations of yields are able to forecast excess bond returns. For example, Fama and Bliss (1987) and Campbell and Shiller (1991) show that risk premiums increase with the slope of yield curves, and Cochrane and Piazzesi (2005) show that excess bond returns of various maturities can be predicted by a single tent-shaped linear combination of forward rates.

This article extends their works by showing that short-maturity Treasury-bill (T-bill) yields have unique information about risk premiums that is not spanned by T-bond yields. I first replicate Cieslak and Povala's (2015) approach to decompose T-bond yield curves into expectations hypothesis terms and a risk premium component, use these factors to orthogonalize T-bill yields, and then show that the residuals of T-bills are able to forecast the monthly excess returns from holding T-bonds of all maturities. I find R^2 values as high as 8%, which are huge

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improvements over the R^2s of 2%–4% obtained from the well-known risk premium predictors such as Cochrane and Piazzesi's (2005) tent-shaped factor or Cieslak and Povala's (2015) cycle factor.

I call Cieslak and Povala's (2015) cycle factor and the T-bills' residuals RPL and RPS, denoting long-term and short-term risk premium factors, respectively. RPS has shorter half-life and forecast horizon than RPL. It takes 1.8 months for RPS to halve and 7.0 months for RPL. RPS can forecast monthly excess returns up to the next 2 months whereas RPL's forecastability lasts for more than a year. Both RPL's and RPS's predictive powers are robust to generalized method of moments (GMM) standard errors.

RPL and RPS also have different influences on the shapes of yield curves. RPL steepens the yield curve slope whereas RPS's effects are visible only from T-bill yields. RPS cannot affect long-maturity yields because of its short half-life in risk-neutral dynamics as well.

This article also shows why standard Gaussian affine models easily miss the predictive power of RPS. The maximum likelihood estimation (MLE) of risk-neutral dynamics trades off its predictive power in time series for cross-sectional fit. Because RPS does not affect T-bond yields, it looks like measurement errors of short-maturity yields in the eyes of MLE. Thus, MLE ignores RPS and its predictive power, and gains in the cross-sectional fit instead.

The economic basis for RPS is believed to be the status of T-bills as the safest collaterals. For example, the repo haircut rate of short-term Treasuries was fixed at 2% during the global financial crisis in 2007–2009 whereas those of long-term Treasuries and investment-grade corporate bonds soared from 5% to 6% and 5% to 20%, respectively. I find that RPS has strong correlations to the proxies of bond liquidity premiums such as Fontaine and Garcia's (2012) on-the-run and off-the-run T-bond spreads, financial commercial paper spreads, and overnight index swap spreads.

Note that this is not the first article to examine the differences between T-bonds and T-bills. For example, Duffee (1996) documents their market segmentation by using the correlations of T-bond and T-bill yields. Pearson and Sun (1994), who estimate a 2-factor Cox, Ingersoll, and Ross (1985) term structure model, also conclude that "estimates based on only bills imply unreasonably large price errors for longer maturities" (p. 1279). However, there has been no precedent that systematically compares the risk premium forecasts with and without T-bills. Many papers use both T-bonds and T-bills to estimate risk premiums but do not investigate how much improvement in forecast performance can be accounted for by T-bills independently.

The rest of this article is organized as follows: Section II explains the construction of the forecasting variables from bond yields and inflation. Section III compares the predictive powers of RPL and RPS. Section IV shows the factor loadings of bond yields on the risk premium factors. Section V explains why standard Gaussian affine models miss the predictive power of RPS. Section VI combines the factor estimation and the risk premium forecasts into a single GMM framework and finds that the results are robust. Section VII compares RPS to liquidity premiums in the bond market. Section VIII concludes.

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II. Construction of the Forecasting Variables

Let $p_t^{(n)}$ denote the natural log of the price of an n-period maturity discount bond at time t. Its continuously compounded bond yield is

$$y_t^{(n)} = -\frac{1}{n} p_t^{(n)}, \tag{1}$$

where $y_t^{(1)} = r_t$ is the 1-period risk-free interest rate.

The excess return of holding n-period maturity bonds from time t for h periods is defined as its holding return less the h-period interest rate,

$$\begin{aligned} \mathrm{EXR}_{t,t+h}^{(n)} &= \left\{p_{t+h}^{(n-h)} - p_t^{(n)}\right\} - \left\{0 - p_t^{(h)}\right\} \\ &= -(n-h)\, y_{t+h}^{(n-h)} + n\, y_t^{(n)} - h\, y_t^{(h)}. \end{aligned} \tag{2}$$

For a unit holding period, $h = 1$, the above equation can be used to derive

$$\begin{aligned} n\, y_t^{(n)} &= r_t + (n-1)\, y_{t+1}^{(n-1)} + \mathrm{EXR}_{t+1}^{(n)} \\ &= r_t + \left\{r_{t+1} + (n-2)\, y_{t+2}^{(n-2)} + \mathrm{EXR}_{t+2}^{(n-1)}\right\} + \mathrm{EXR}_{t+1}^{(n)} \\ &= \cdots \\ &= \sum_{i=0}^{n-1} r_{t+i} + \sum_{i=0}^{n-2} \mathrm{EXR}_{t+i+1}^{(n-i)}. \end{aligned} \tag{3}$$

Therefore,

$$y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} \mathrm{E}_t\left[r_{t+i}\right] + \frac{1}{n}\sum_{i=0}^{n-2} \mathrm{E}_t\left[\mathrm{EXR}_{t+i+1}^{(n-i)}\right]. \tag{4}$$

This equation implies that bond yields are essentially the sum of 2 components: the average of expected future short interest rates (r_{t+i}) and excess returns (EXR_{t+i}). This section explains how to decompose bond yields into these two components. Section II.A separates the average of expected future short rates into a persistent part of core inflation and a transitory deviation of 1-year bond yields. Section II.B splits the average of expected excess returns into 2 risk premium state variables of different frequencies. All state variables are estimated over the sample period from Jan. 1968 through Dec. 2013.

A. Decomposition of Short Interest Rates (r_{t+i})

I follow Cieslak and Povala's (2015) estimation strategy to decompose short interest rates into 2 components. One is a persistent component related to inflation, and the other is a transitory variation.

The persistent component (τ_t) is estimated as an exponentially weighted average of realized core inflation over the past 120 months:

$$\tau_t \equiv \frac{\sum_{i=0}^{120} v^i\, \mathrm{CPI}_{t-i}}{\sum_{i=0}^{120} v^i}, \tag{5}$$

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where $(1-v)$ denotes constant gain. Cieslak and Povala (2015) estimate the gain parameter at $v=0.9868$ (standard error = 0.0025) by comparing realized inflation with inflation survey forecasts.

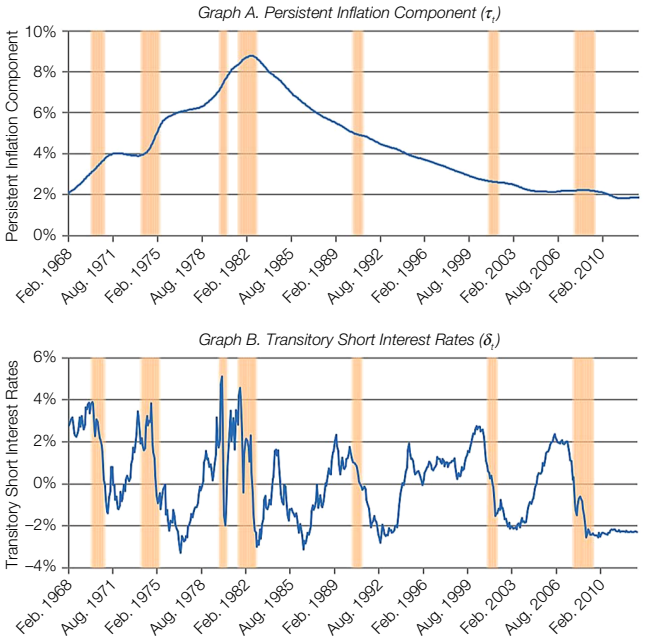
Graph A of Figure 1 shows the time series of the estimated persistent inflation component. Core inflation data are downloaded from the Federal Reserve Economic Data (FRED) database (http://research.stlouisfed.org/fred2/series/CPILFESL?cid=32424). The estimated series start in Jan. 1968 as the database provides core inflation since Jan. 1958 and 10-year histories are used to estimate the variable. According to the figure, the inflation component reached its peak in the early 1980s and has gradually declined since then. The augmented Dickey–Fuller (1979) test does not reject the null hypothesis that the persistent inflation is a unit-root process.

Provided that short interest rates are made of persistent and transitory components, the transitory component can be estimated as the residuals of short interest rates orthogonalized by the persistent inflation component. Following Cieslak and Povala (2015), Fama–Bliss (1987) 1-year bond yields are used as a proxy of the short interest rates:

$$\delta_t \equiv y_t^{(1y)} - \hat{\beta}_0 - \hat{\beta}_1 \tau_t, \tag{6}$$

FIGURE 1
Decomposition of Short Interest Rates

Figure 1 decomposes 1-year Treasury-bond yields into 2 components: persistent and transitory. The persistent component is estimated as an exponentially weighted average of realized core inflation over the past 10 years, and the transitory component as the residuals from the regression of 1-year yields on persistent inflation. Shaded areas denote National Bureau of Economic Research recessions.



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where δ_t denotes the transitory deviation of short interest rates, and $\hat{\beta}_0$ and $\hat{\beta}_1$ are ordinary least squares (OLS) coefficients of regressing $y_t^{(1y)}$ on τ_t.

Graph B of Figure 1 shows the time series of the transitory component. It drops rapidly during recession periods, which is consistent with conventional monetary policy that employs the federal funds rate to moderate business cycles. The unconditional standard deviation of δ_t is 1.834%.

B. Decomposition of Risk Premiums ($E_t[EXR_{t+i}]$)

Risk premiums can be defined as the expected excess returns of risky assets. The early literature on bond risk premiums, such as Campbell and Shiller (1991) and Dai and Singleton (2002), shows that the excess returns are predictable, and this finding has two implications. First, it rejects the expectations hypothesis that forward interest rates are equal to expected future short interest rates.[1] Second, it implies that the market price of risk is time varying and spanned by observable variables.

I replicate Cieslak and Povala's (2015) approach to estimate risk premiums in the bond market. The authors estimate the risk premium factor as the residuals from regressing T-bond yields on the two components of short interest rates. They first regress each of 2- to 5-year bond yields on the persistent inflation component (τ_t),

$$y_t^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} \tau_t + \epsilon_t^{(n)} \quad \text{for } n = 2, \ldots, 5 \text{ years}, \tag{7}$$

and then regress the average of the residuals ($\bar{\epsilon}_t \equiv 1/4 \sum_{n=2y}^{5y} \hat{\epsilon}_t^{(n)}$) on the transitory interest rate component (δ_t) without an intercept,[2]

$$\bar{\epsilon}_t = \gamma_1 \delta_t + u_t. \tag{8}$$

Cieslak and Povala (2015) show that equation (8)'s residual $\left(\hat{u}_t\right)$ outperforms both Cochrane and Piazzesi's (2005) and Ludvigson and Ng's (2009) risk premium predictors in the predictive regressions of excess returns in the bond market. I take this residual variable as the risk premium factor and call it RPL_t, denoting the long-term risk premium factor. The Center for Research in Security Prices (CRSP) Fama–Bliss (1987) discount bond yields are used for the 2- to 5-year bond yields.

So far, 3 state variables are estimated from T-bond yields and inflation data. To show that short-maturity T-bill yields have unique information that is not spanned by T-bond yields, I simply orthogonalize 3-month T-bill yields by the 3 state variables and take its residuals as RPS_t, the short-term risk premium factor.

$$RPS_t = -\{y_t^{(3m)} - \hat{\beta}_0 - \hat{\beta}_1 \tau_t - \hat{\beta}_2 \delta_t - \hat{\beta}_3 RPL_t\}, \tag{9}$$

where the $\hat{\beta}$s are OLS coefficients. RPS turns positive when 3-month T-bill yields are lower than the projected value by the 3 state variables. I specifically choose

[1] Piazzesi (2003) explains the rejection of the expectations hypothesis in detail.

[2] The latest version of Cieslak and Povala (2015) slightly changes the estimation method of the risk premium factor, but I stick to their original estimation strategy because it is more consistent with the estimation of my second risk premium factor. I also tried their new method and confirmed that the results are not changed.

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3-month T-bill yields for two reasons. First, the 3-month yields do not have mechanical correlation to monthly excess returns whereas 1-month yields do because of measurement errors. Second, they have the longest history, which starts in 1954. In comparison, the secondary market rate of 4-week T-bills is provided only since July 2001. The 3-month T-bill yields are downloaded from FRED (http://research.stlouisfed.org/fred2/series/DTB3?cid=116). RPS is winsorized at the 0.5 and 99.5 percentiles.

Figure 2 shows the time series of the two risk premium factors, RPL and RPS. They appear more mean reverting than the two earlier variables, τ and δ. According to their autocorrelations, shown in Table 1, the half-lives of δ, RPL, and RPS are approximately 17.2, 7.0, and 1.8 months, respectively. In addition, the unconditional standard deviations of RPL and RPS are 0.42% and 0.34%.

FIGURE 2
Time Series of Risk Premium Factors

Figure 2 compares the time series of long- and short-term risk premium factors. The former is estimated from Treasury-bond yields and the latter from Treasury-bill yields. Shaded areas denote National Bureau of Economic Research recessions.

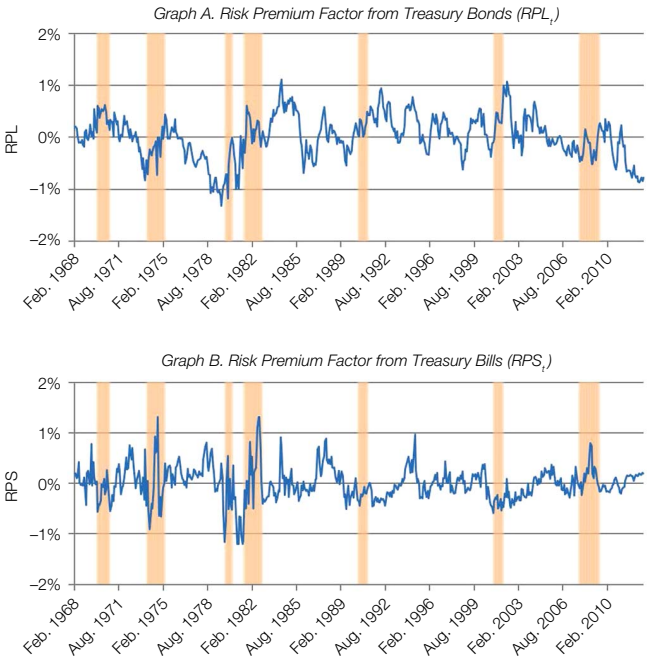


TABLE 1
Persistence of State Variables

Table 1 shows the persistence of the time series of each state variable when it is estimated as an AR(1) time-series model. Autocorrelation coefficient and half-life are measured in months.

Coefficient	τ	δ	RPL	RPS
Autocorrelation	1.00	0.96	0.91	0.68
Half-life in months	—	17.24	7.00	1.77

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In this section, 3 state variables (τ_t, δ_t, and RPL_t) are estimated based on the replication of Cieslak and Povala (2015). The current article adds 1 more state variable (short-term risk premium factor (RPS_t)) to the framework and focuses on the comparison of RPL and RPS. Although not reported here, other benchmark risk premium factors such as Fama and Bliss's (1987) forward interest rate slope and Cochrane and Piazzesi's (2005) tent-shaped factor are also used as an alternative proxy of RPL and the results are not changed. RPS is winsorized, but the unwinsorized version also yields the same results.

III. Forecast of Excess Returns in the Bond Market

Cochrane and Piazzesi (2005) and Cieslak and Povala (2015) show that their risk premium factors are able to forecast annual bond excess returns with R^2s up to 44% and 54%, respectively. However, those R^2s are likely to decline if the dependent variable is monthly excess returns instead of annual returns for two reasons. First, monthly and annual risk premiums are possibly driven by different dynamics. Second, as Boudoukh, Richardson, and Whitelaw (2007) show, R^2s are usually overestimated when forecasts are made for overlapping horizons.

Table 2 shows the forecast of monthly excess returns. The dependent variable is the excess returns from holding T-bonds for the next 1 month over 1-month risk-free interest rates, $\text{EXR}^{(n)}_{t,t+1m}$. The 1-month risk-free rates are from Ibbotson Associates.[3] Panel A uses RPL as the only predictor. As suspected, its R^2s drop to 2.7%∼3.5%, which are less than one-tenth of the R^2s found from the forecasts of annual excess returns. Although not reported here, the R^2s by other predictors such as Cochrane and Piazzesi's (2005) tent-shaped factor or Duffee's (2011) hidden factor drop even further to 1%∼2%.

TABLE 2
Forecast of Monthly Excess Returns

Table 2 forecasts Treasury-bond (T-bond) monthly excess returns using RPL and RPS. The dependent variable is excess returns of holding T-bonds over the next 1 month, $\text{EXR}^{(n)}_{t,t+1m}$, whose maturities are specified by the top row. RPL and RPS denote risk premium factors that are estimated from T-bonds and Treasury bills, respectively. Numbers in parentheses are the Huber (1967)–White (1980) t-statistics. *** indicates significance at the 1% level.

	Maturity					
Variable	2 Years	3 Years	5 Years	7 Years	10 Years	15 Years
Panel A. Forecast by RPL Only						
RPL_t	0.314***	0.475***	0.761***	1.003***	1.341***	1.863***
	(2.585)	(2.950)	(3.474)	(3.615)	(3.508)	(3.294)
No. of obs.	552	552	552	552	509	506
R^2	0.027	0.031	0.035	0.034	0.035	0.032
Panel B. Forecast by RPL and RPS						
RPL_t	0.318***	0.480***	0.769***	1.014***	1.333***	1.849***
	(2.654)	(3.030)	(3.581)	(3.751)	(3.604)	(3.414)
RPS_t	0.408***	0.549***	0.828***	1.154***	1.750***	2.860***
	(3.130)	(3.110)	(3.250)	(3.484)	(3.777)	(4.157)
No. of obs.	552	552	552	552	509	506
R^2	0.055	0.057	0.061	0.063	0.072	0.080

[3]The 1-month risk-free interest rates are obtained from Kenneth French's Web site (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

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Panel B of Table 2, however, shows that the monthly return forecast is significantly improved by adding RPS as the second predictor. R^2s almost double to 5.5%∼8.0%, and both RPL and RPS appear significant. RPL and RPS do not have a multicollinearity issue as RPS is orthogonalized by RPL and thus they have 0 correlation. According to the results, a 1-standard-deviation increase in RPL and RPS implies a 0.56% and 0.60% increase in the monthly excess returns of holding 10-year T-bonds, respectively. Although not reported here, I also project monthly excess returns on all 4 state variables and find that neither τ_t nor δ_t is significant. Therefore, Table 2 implies that RPS (or equivalently 3-month T-bills) has unique risk premium information that is not spanned by the first 3 state variables.

These results show that the predictive power of RPS is as strong as RPL. According to the Shapley decomposition of R^2s (not reported), RPL and RPS each explain half of predicted monthly excess returns. However, as shown in the previous section, RPS has a much shorter half-life than RPL, 1.8 versus 7.0 months, respectively. Because RPS has a shorter half-life, one can also expect that RPS might have a shorter forecast horizon than RPL. Table 3 tests how long RPL and RPS can present their predictive powers.

The dependent variables in each panel of Table 3 are monthly excess returns in 2, 3, and 16 months, respectively ($\text{EXR}^{(n)}_{t+1m,t+2m}$, $\text{EXR}^{(n)}_{t+2m,t+3m}$, and $\text{EXR}^{(n)}_{t+15m,t+16m}$). The table shows that RPS is equally significant for the horizon of 2 months compared to the earlier horizon but loses most of its predictive power

TABLE 3

Forecast of Monthly Excess Returns in Further Periods

Table 3 forecasts Treasury-bond (T-bond) monthly excess returns of further periods using RPL and RPS. The dependent variable is monthly excess returns of holding T-bonds in further periods, $\text{EXR}^{(n)}_{t+\phi-1,t+\phi}$ for $\phi=2$, 3, and 16 months. RPL and RPS denote risk premium factors that are estimated from T-bonds and Treasury bills, respectively. Numbers in parentheses are the Huber (1967)–White (1980) t-statistics. * and *** indicate significance at the 10% and 1% levels, respectively.

	Maturity					
Variable	2 Years	3 Years	5 Years	7 Years	10 Years	15 Years
Panel A. Monthly Excess Returns in 2 Months, $\text{EXR}^{(n)}_{t+1m,t+2m}$						
RPL_t	0.363***	0.551***	0.890***	1.180***	1.584***	2.160***
	(3.494)	(3.929)	(4.481)	(4.664)	(4.683)	(4.499)
RPS_t	0.422***	0.589***	0.876***	1.157***	1.587***	2.173***
	(3.303)	(3.422)	(3.539)	(3.564)	(3.518)	(3.174)
No. of obs.	552	552	552	552	510	507
R^2	0.066	0.071	0.076	0.076	0.080	0.071
Panel B. Monthly Excess Returns in 3 Months, $\text{EXR}^{(n)}_{t+2m,t+3m}$						
RPL_t	0.338***	0.494***	0.760***	0.989***	1.351***	1.990***
	(3.301)	(3.537)	(3.819)	(3.880)	(3.932)	(4.127)
RPS_t	0.194	0.263	0.370	0.505	0.869*	1.240*
	(1.410)	(1.438)	(1.423)	(1.500)	(1.878)	(1.790)
No. of obs.	551	551	551	551	510	507
R^2	0.037	0.039	0.039	0.039	0.045	0.046
Panel C. Monthly Excess Returns in 16 Months, $\text{EXR}^{(n)}_{t+15m,t+16m}$						
RPL_t	0.154	0.240	0.388*	0.505*	0.703*	0.926*
	(1.346)	(1.568)	(1.784)	(1.805)	(1.866)	(1.683)
RPS_t	−0.116	−0.173	−0.318	−0.475	−0.869*	−1.326*
	(−0.769)	(−0.861)	(−1.137)	(−1.349)	(−1.839)	(−1.951)
No. of obs.	538	538	538	538	510	507
R^2	0.009	0.010	0.013	0.013	0.019	0.018

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at 3 months. In contrast, RPL's forecastability remains significant for the horizon of up to 16 months. In particular, RPL appears significant at the 1% confidence level even at 12 months. Thus, Table 2 confirms that RPS's predictive power is limited to a short horizon whereas RPL's forecastability remains strong for more than 1 year.

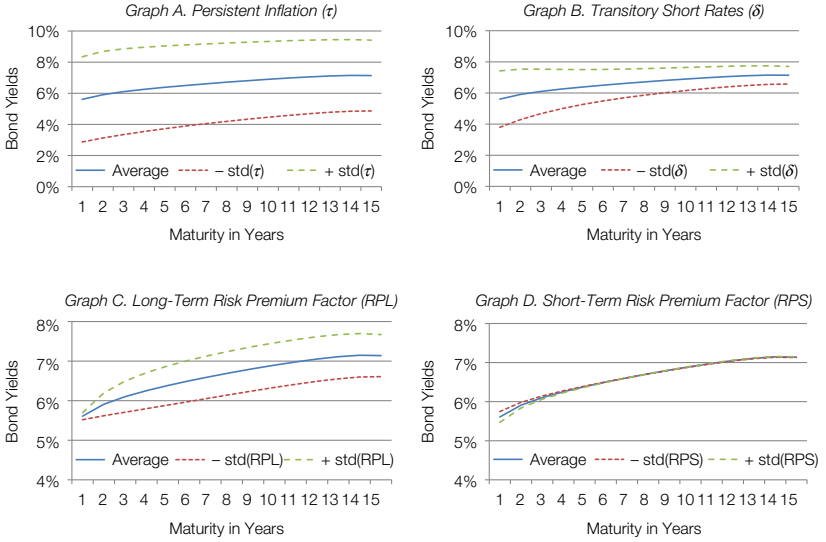
These results are related to Duffee's (2010) finding that monthly and annual expected bond excess returns are driven by 2 factors and these 2 factors operate at different frequencies. He shows that the second and fourth principal components are equally responsible for the predictable variation in monthly excess returns whereas the second component dominates the fourth in forecasts of annual excess returns. RPL and RPS demonstrate the same patterns as described in this article. However, the following section shows that RPL and RPS do not correspond exactly to the second and fourth principal components.

IV. Yield Factor Loadings and Principal Components

This section explores the relations of the 4 state variables to the shape and principal components of yield curves. Figure 3 shows how bond yields are affected when each state variable is deviated by ±1 standard deviation. Each graph is labeled according to a given state variable. The horizontal axis denotes maturities in years, and the vertical axis denotes annualized bond yields. Factor loadings of bond yields are derived from a standard Gaussian affine term structure model with τ, δ, RPL, and RPS as state variables. The model parameters are estimated by maximizing the cross-sectional fit to bond yields of 11 maturities: 1, 3, and 6 months, and 1, 2, 3, 4, 5, 7, 10, and 15 years.

FIGURE 3
Factor Loadings of Treasury-Bond Yields

Figure 3 shows how the shapes of Treasury-bond yield curves are affected by each state variable. The horizontal axis denotes maturities in years, and the vertical axis denotes annualized bond yields. Each graph corresponds to changes of ±1 standard deviation (std) in a given state variable.



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Graph A of Figure 3 shows that the persistent inflation component (τ) shifts the level of yield curves. This effect is intuitive given the near-unit-root persistence of the variable. Graphs B and C show that the slope of the yield curves is determined by the transitory short interest rate (δ) as well as the long-term risk premium factor (RPL). High δ flattens the slope by raising short-maturity bond yields whereas high RPL steepens it by raising long-maturity yields.

Graph D of Figure 3 shows that the short-term risk premium factor (RPS) has small effects only on short-maturity bond yields and is almost completely invisible from long-maturity yields. For example, the model-implied 1-year yields when RPS is ± 1 standard deviation are 5.47%–5.75% whereas the 3-year yields narrow to only 6.06%–6.14%. The yield spreads made by RPS become less than 2 basis points (bps) for maturities longer than 5 years. The effects of RPS on T-bond yields are so small that it is difficult to distinguish RPS from measurement errors.

In an earlier version of this article, yield factor loadings were estimated by simply regressing bond yields on the state variables without the no-arbitrage restriction, and the same results were found. τ moves the level, δ and RPL affect the slope, and RPS is visible only for short-maturity T-bill yields.

Many papers relate bond risk premiums to the principal components of yield curves. For example, Duffee (2002) estimates affine term structure models and finds that "an increase in the slope factor affects the price of risk of the level factor," and the twist (curvature) shock "has a strong effect on instantaneous expected returns, but it is also very short-lived. Thus, this shock is responsible for high-frequency fluctuations in expected excess returns" (p. 434). The short-term risk premium has often been related to high-order principal components but has not been pinned down to T-bill yields.

To find out the relations of the 2 risk premium factors, RPL and RPS, to the principal components, Table 4 uses the Shapley decomposition method to show the contribution of each state variable to the explained variations of principal components. Numbers in each row are summed to 100. The principal components are estimated from the bond yields of the same 11 maturities.

Table 4 shows that the variations in the slope factor (PC2) are largely driven by δ and RPL, each of which explains 43.95% and 46.29% of the variation, respectively. This result is consistent with the factor loadings of T-bond yields in Figure 3, which shows that the slope becomes steeper when δ is low or RPL is high. Moreover, it also explains why the slope factor per se is a poor proxy of

TABLE 4
Decomposition of Principal Components

Table 4 shows the contributions of the 4 state variables to the explained variance of principal components (PCs). The contribution of each state variable is computed using the Shapley decomposition. Numbers in each row are summed to 100. The PCs are estimated from the bond yields of 11 maturities: 1, 3, and 6 months, and 1, 2, 3, 4, 5, 7, 10, and 15 years.

	Factor	Persistent Inflation (τ)	Transitory Short Rates (δ)	Long-Term Risk Premium (RPL)	Short-Term Risk Premium (RPS)
PC1	(level)	78.63	20.44	0.82	0.11
PC2	(slope)	7.25	43.95	46.29	2.51
PC3	(curvature)	1.14	7.13	10.15	81.58
PC4		1.25	0.34	69.48	28.93
PC5		0.00	0.42	6.95	92.63

risk premiums. A large amount of the variations in the slope are determined by δ, which is not related to risk premiums.

Table 4 also shows that high-order principal components are dominated by RPL and RPS. For example, RPS explains 81.58% and 92.63% of the variations in PC3 and PC5, respectively, and RPL explains 69.48% of PC4. However, this result is subject to change depending on the sample from which the principal components are estimated. When more long-maturity yields are added to the sample, RPL explains more of the variations in PC3 whereas the dominance of RPS moves from PC3 to PC4 and PC5. Therefore, it is not accurate to relate RPL and RPS directly to specific principal components.

V. Comparison to an Affine Term Structure Model

In this section, I show why it is difficult to estimate RPS using an affine term structure model even when T-bill yields are used in the sample. This implication can explain why it has largely eluded the literature that the short-term risk premium factor can be easily estimated from T-bill yields.

I adopt the model specification by Joslin, Priebsch, and Singleton (2014), the term structure model with unspanned macro risks. Let $Z_t \in \mathbb{R}^N$ denote a column vector of state variables, which are made of the first 3 principal components of bond yields ($\mathcal{P}_t$) and the persistent inflation component (τ_t), $Z_t \equiv \left(\mathcal{P}_t, \tau_t\right)$. The dynamics of the state variables are governed by the following processes,

$$Z_t = K_{0Z}^{\mathbb{P}} + K_{1Z}^{\mathbb{P}} Z_{t-1} + \sqrt{\Sigma_Z}\,\eta_t^{\mathbb{P}}, \tag{10}$$

$$\mathcal{P}_t = K_{0\mathcal{P}}^{\mathbb{Q}} + K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}} \mathcal{P}_{t-1} + \sqrt{\Sigma_{\mathcal{P}\mathcal{P}}}\,\epsilon_{\mathcal{P},t}^{\mathbb{Q}}. \tag{11}$$

τ_t is an unspanned macro risk. Thus, the bond yields are affine functions of $\mathcal{P}_t$,

$$y_t^{(m)} = A_{\mathcal{P}}(m) + B_{\mathcal{P}}(m) \cdot \mathcal{P}_t, \tag{12}$$

where $y_t^{(1)} = r_t = \rho_{0\mathcal{P}} + \rho_{1\mathcal{P}} \cdot \mathcal{P}_t$ is the yield on a 1-period bond.

This specification assumes that $\mathcal{P}_t$ and τ_t are measured without errors. Its parameters are estimated with the bond yields of 11 maturities: 1, 3, and 6 months, and 1, 2, 3, 4, 5, 7, 10, and 15 years. Thus, there remains a dimension-eight linear combination of yields measured with errors.[4] The sample spans Nov. 1971 to Dec. 2013 because of the availability of 15-year yields. I use the MATLAB codes downloaded from Scott Joslin's Web site (http://www-bcf.usc.edu/~sjoslin/papers/jsz_code.zip) for the estimation, whose details are explained by Joslin, Singleton, and Zhu (2011).

After the estimation, I reconstruct model-implied yields using the model's parameters and state variables, and then replicate the estimation of RPL and RPS using those implied yields. If the affine model is able to capture all risk premium information, model-implied RPL and RPS are expected to provide equivalent predictive powers as those from empirical RPL and RPS. To test this hypothesis, I compare their predictive powers in Table 5.

[4]The design of this analysis is based on the referee's suggestion. I am very grateful for his help in improving this article.

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In Panels A and B of Table 5, the dependent variables are monthly excess returns of 2- and 5-year T-bonds for the next 1 month. The predictors in Panel A are empirical RPL and RPS and those in Panel B are model-implied RPL and RPS. As shown by the comparison of the two panels, the predictive power of implied RPL is slightly higher than that of empirical RPL, but implied RPS loses all significance. R^2s decrease from 6.3∼6.6% to 5.1∼5.4%. Implied RPL seems to absorb some but not all predictive information from RPS. Although not reported, I also regress the realized excess returns on their model-implied conditional expectations, $E_t[EXR^{(n)}_{t,t+1}]$, and their R^2s appear identical to those from Panel B. Thus, implied RPL is believed to capture all risk premium information implied by the model.

To better understand the relations between excess returns and risk premium factors, I run the Shapley decompositions in Panel C of Table 5. The dependent variables are realized excess returns in rows 1 and 2, and their model-implied conditional expectations in rows 3 and 4. The predictors in rows 1 and 3 are empirical RPL and RPS and those in rows 2 and 4 are their model-implied counterparts.

Panel C of Table 5 shows that empirical RPL and RPS explain almost equal shares of realized excess returns, 45.46% and 52.10%, respectively. However, most information from RPS is lost as it moves to the affine model. Empirical RPS explains only 9.94% of the model-implied risk premium, which is in stark contrast to the 88.28% explained by empirical RPL. Moreover, implied RPS in rows 2 and 4 does not show any meaningful relevance. Thus, these results

TABLE 5

Risk Premium Forecast by Empirical and Model-Implied Factors

Panels A and B of Table 5 report the results of regressing realized excess returns from holding 2- and 5-year T-bonds on empirical and model-implied RPL and RPS, respectively. Panel C reports the Shapley decompositions of realized excess returns and their model-implied conditional expectations by empirical and model-implied RPL and RPS. Numbers in parentheses are ordinary least squares *t*-statistics. *** indicates significance at the 1% level.

	Dependent Variable			
Variable	RPL	RPS	No. of Obs.	R^2
Panel A. Realized Excess Returns Regressed on Empirical RPL and RPS				
$EXR^{(2y)}_{t,t+1}$	0.496***	0.625***	505	0.063
	(3.982)	(4.160)		
$EXR^{(5y)}_{t,t+1}$	0.774***	1.008***	505	0.066
	(4.030)	(4.345)		
Panel B. Realized Excess Returns Regressed on Model-implied RPL and RPS				
$EXR^{(2y)}_{t,t+1}$	0.729***	0.255	505	0.051
	(5.017)	(1.300)		
$EXR^{(5y)}_{t,t+1}$	1.181***	0.311	505	0.054
	(5.265)	(1.025)		

	τ	δ	RPL	RPS
Panel C. Shapley Decomposition of R^2				
Row 1. Realized $EXR^{(5y)}_{t,t+1}$ decomposed by empirical RPL and RPS	0.65%	1.79%	45.46%	52.10%
Row 2. Realized $EXR^{(5y)}_{t,t+1}$ decomposed by model-implied RPL and RPS	0.53%	0.94%	94.94%	3.59%
Row 3. Model-implied $E_t[EXR^{(5y)}_{t,t+1}]$ decomposed by empirical RPL and RPS	0.80%	0.97%	88.28%	9.94%
Row 4. Model-implied $E_t[EXR^{(5y)}_{t,t+1}]$ decomposed by model-implied RPL and RPS	0.51%	0.72%	94.80%	3.97%

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confirm that the affine model is not able to recover the predictive information of RPS.

The failure of the affine model to recover RPS is due to two model setups. First, the model uses 3 principal components as pricing factors. Second, the model's risk-neutral dynamics are measured by the maximum likelihood (ML) of cross-sectional fits to bond yields. Because RPS can be observed only from the very short end of yield curves, the principal components and ML methods easily trade off RPS for a better fit of longer maturity yields. These methods may find it difficult to distinguish RPS from measurement errors. However, the loss of predictive power is modest (as shown by Table 5, the R^2s drop from 6.3∼6.6% to 5.1∼5.4%) and the gain in cross-sectional fit overwhelms this lost predictive power.

In addition, the loss can be mitigated by raising the number of state variables. Although not reported here, implied RPS is found to gain predictability when the affine model is estimated with 6 state variables: 5 principal components and inflation. However, using more comprehensive sample does not always help. Implied RPS starts losing predictive power when more long-maturity T-bond yields are added to the sample.

VI. Robustness Test with GMM

The analysis so far is made in two separate steps. The first step is to estimate state variables, and the second is to show that the latter two state variables are related to risk premiums in the bond market. In this section, these two steps are combined using GMM.

GMM estimation is important because it fixes OLS standard errors when explanatory variables are generated regressors. This feature is all the more important because T-bond excess returns have strong cross-sectional correlations. For example, a simple principal component analysis suggests that around 70% of the variation in excess returns from holding T-bonds of various maturities can be explained by the first principal component.

The first stage is to estimate δ_t, RPL_t, and RPS_t from the data of $y_t^{(1y)}$, $\bar{y}_t$, $y_t^{(3m)}$, and τ_t using orthogonalization.[5] The whole processes can be summarized as

$$\delta_t = y_t^{(1y)} - \begin{pmatrix} 1 & \tau_t \end{pmatrix} \boldsymbol{\beta}^{(\delta)}, \tag{13}$$

$$\text{RPL}_t = \bar{y}_t - \begin{pmatrix} 1 & \tau_t & \delta_t \end{pmatrix} \boldsymbol{\beta}^{(\text{rpl})}, \tag{14}$$

$$\text{RPS}_t = -y_t^{(3m)} + \begin{pmatrix} 1 & \tau_t & \delta_t & \text{RPL}_t \end{pmatrix} \boldsymbol{\beta}^{(\text{rps})}. \tag{15}$$

The superscripts of the $\boldsymbol{\beta}$s are determined by their corresponding dependent variables.

The second stage is to forecast monthly excess returns from holding T-bonds using RPL and RPS,

$$\text{EXR}_{t,t+1}^{(n)} = \begin{pmatrix} 1 & \text{RPL}_t & \text{RPS}_t \end{pmatrix} \boldsymbol{\beta}^{(n)} + \epsilon_{t+1}^{(n)}, \tag{16}$$

[5] $\bar{y}_t$ denotes the average of 2- to 5-year T-bond yields, $\bar{y}_t = 1/4 \left(y_t^{(2y)} + y_t^{(3y)} + y_t^{(4y)} + y_t^{(5y)} \right)$.

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for $n = 2, 3, 5, 7, 10$, and 15 years. The superscripts of the $\boldsymbol{\beta}$s denote the maturities of the excess returns. The sample begins in Nov. 1971 because 15-year T-bonds are not available earlier.

These equations are put into a vector of GMM moments,

$$g_T(\boldsymbol{\beta}) = \mathrm{E}_T\begin{bmatrix} \delta_t \otimes \begin{pmatrix}1 & \tau_t\end{pmatrix}^\top \\ \mathrm{RPL}_t \otimes \begin{pmatrix}1 & \tau_t & \delta_t\end{pmatrix}^\top \\ \mathrm{RPS}_t \otimes \begin{pmatrix}1 & \tau_t & \delta_t & \mathrm{RPL}_t\end{pmatrix}^\top \\ \boldsymbol{\epsilon}_{t+1} \otimes \begin{pmatrix}1 & \mathrm{RPL}_t & \mathrm{RPS}_t\end{pmatrix}^\top \end{bmatrix} \in \mathbb{R}^{27}, \tag{17}$$

where $\boldsymbol{\beta}$ is a vector of 27 parameters,

$$\boldsymbol{\beta} \equiv \begin{bmatrix}\beta_0^{(\delta)} & \beta_1^{(\delta)} & \beta_0^{(\mathrm{rpl})} & \beta_1^{(\mathrm{rpl})} & \beta_2^{(\mathrm{rpl})} & \beta_0^{(\mathrm{rps})} & \beta_1^{(\mathrm{rps})} & \beta_2^{(\mathrm{rps})} & \beta_3^{(\mathrm{rps})} \\ \beta_0^{(2\mathrm{y})} & \beta_1^{(2\mathrm{y})} & \beta_2^{(2\mathrm{y})} & \cdots & \beta_0^{(15\mathrm{y})} & \beta_1^{(15\mathrm{y})} & \beta_2^{(15\mathrm{y})}\end{bmatrix}^\top. \tag{18}$$

The GMM moments are exactly identified; thus, the GMM criterion is 0 in the OLS estimates. Mechanical autocorrelations between $\boldsymbol{\epsilon}_{t+1}$ and $\boldsymbol{\epsilon}_t$ due to measurement errors are controlled by the Newey–West (1987) spectral density matrix with a lag of 2 months. The standard errors of $\boldsymbol{\beta}$ are computed using the spectral density matrix and the delta method. Further details about the GMM estimation are presented in the Internet Appendix (available at www.jfqa.org).

Table 6 compares the OLS and GMM t-statistics of the second-stage parameters, the forecasts of monthly excess returns from holding T-bonds. GMM t-statistics are slightly lower than those from OLS, but they are still statistically significant across all maturities. Therefore, the forecastability by RPL and RPS is found to be robust to GMM estimation.

VII. Relation to Liquidity in the Bond Market

T-bills are special. Their values rise during a financial crisis because they are considered the safest collateral in the world. For example, the yield on 3-month T-bills turned negative on Dec. 9, 2008, 3 months after the Lehman Brothers' collapse, as investors sought a safe haven (http://blogs.wsj.com/marketbeat/2008/12/09/three-month-bill-yield-goes-negative/). One-month T-bill yields briefly went

TABLE 6

GMM Forecast of Monthly Excess Returns

Table 6 compares the ordinary least squares (OLS) and generalized method of moments (GMM) t-statistics of monthly excess return forecasts. The dependent variables are the excess returns from holding Treasury bonds over the next 1 month, $\mathrm{EXR}^{(n)}_{t,t+1m}$, whose maturities are specified by the top row. RPL and RPS denote risk premium factors. Numbers in parentheses are OLS and GMM t-statistics. *** indicates significance at the 1% level.

	Dependent Variable					
Explanatory Variable	$\mathrm{EXR}^{(2y)}_{t,t+1}$	$\mathrm{EXR}^{(3y)}_{t,t+1}$	$\mathrm{EXR}^{(5y)}_{t,t+1}$	$\mathrm{EXR}^{(7y)}_{t,t+1}$	$\mathrm{EXR}^{(10y)}_{t,t+1}$	$\mathrm{EXR}^{(15y)}_{t,t+1}$
RPL_t	0.309***	0.470***	0.764***	1.020***	1.358***	1.884***
OLS t-stat.	(3.814)	(4.126)	(4.426)	(4.474)	(4.386)	(4.231)
GMM t-stat.	(2.467)	(2.823)	(3.347)	(3.536)	(3.496)	(3.311)
RPS_t	0.431***	0.576***	0.849***	1.161***	1.716***	2.802***
OLS t-stat.	(4.419)	(4.192)	(4.083)	(4.226)	(4.599)	(5.221)
GMM t-stat.	(3.385)	(3.275)	(3.273)	(3.451)	(3.822)	(4.314)

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negative again on Aug. 4, 2011, as the European woes cast a shadow over the market (http://blogs.wsj.com/marketbeat/2011/08/04/from-one-crisis-to-another-one-month-t-bill-yields-go-negative-again/).

Even when compared to other types of credit risk-free assets such as other government securities, T-bills still look different. For example, Duffee (1996) reports that the market segmentation between T-bills and T-bonds has increased since the early 1980s. Krishnamurthy (2010) documents that the repo haircut rate of short-term Treasuries was fixed at 2% in 2007–2009 whereas those of long-term Treasuries and investment-grade corporate bonds soared from 5% to 6% and 5% to 20%, respectively.

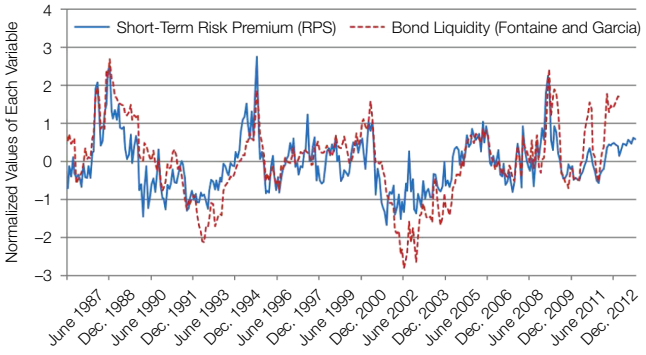
Considering the role of T-bills as the safest haven and how their price is affected by RPS, we can expect RPS to be related to liquidity premiums. Motivated by the intuition, this section compares RPS to various liquidity measures in the bond market.

Fontaine and Garcia (2012) estimate bond liquidity premiums as the difference in yields between on-the-run (most recently issued) and off-the-run (seasoned) T-bonds.[6] Because on-the-run bonds are more liquid and thus have higher value as collateral than off-the-run bonds, the yield differential between on-the-run and off-the-run bond is likely to increase when there are strong demands for liquidity. Figure 4 compares the time series of bond liquidity premiums to RPS.

Figure 4 shows that the two time series overlap each other. They both peaked in 2008 after the Lehman Brothers collapse and in 1987 after the infamous stock market crash on Black Monday. They also soared during the Tequila crisis in 1994 and remained high throughout the Asian currency crisis in 1997 and the Russian moratorium and subsequent demise of Long-Term Capital Management (LTCM) in 1998. The unconditional correlation between the two series is 0.768.

FIGURE 4
Fontaine–Garcia Bond Liquidity Premium

Figure 4 compares RPS to Fontaine and Garcia's (2012) bond liquidity premium. Fontaine and Garcia estimate their bond liquidity premium using the difference in yields between on-the-run (most recently issued) and off-the-run (seasoned) Treasury bonds. This figure compares its time series to the short-term risk premium factor (RPS).



[6]I am grateful to Jean-Sebastien Fontaine and Rene Garcia for generously sharing the data.

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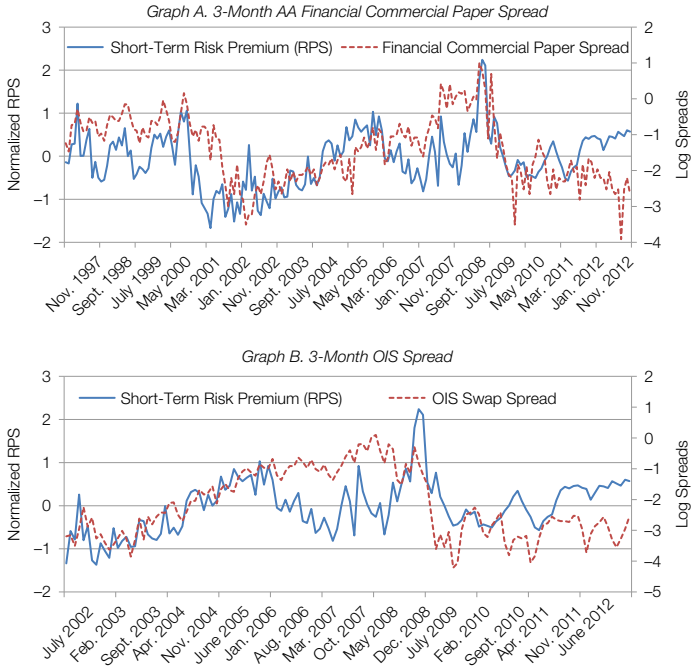
Figure 5 compares RPS to two other measures of liquidity in the bond market. Graph A uses 3-month AA financial commercial paper spreads over 3-month T-bill yields, and Graph B uses 3-month overnight index swap (OIS) spreads. Both graphs display the natural log of the values of spreads. The commercial paper spreads are downloaded from FRED (http://research.stlouisfed.org/fred2/categories/120), and the OIS spreads are from the Bloomberg. Note that the spreads are a close but not perfect liquidity measure because the spreads are determined not only by convenience yields (Grinblatt (2001)) but also by counterparty risk (Duffie and Singleton (1997)).

Again, Figure 5 shows substantial comovement of RPS with the bond market spreads. They all had gradually increased since 2002, reached to a peak during the financial crisis, and dropped rapidly and stabilized thereafter. The correlations between RPS and the two natural log of the spreads are 0.402 and 0.365, respectively.

FIGURE 5

Other Liquidity Measures in the Bond Market

Figure 5 compares the short-term risk premium factor (RPS) to two other liquidity measures in the bond market. Graph A uses 3-month AA financial commercial paper spread, and Graph B uses 3-month overnight index swap (OIS) spread. Both figures display the natural log of the values of those spreads. The scale of RPS is on the left axis and the natural log of the spreads is on the right axis.



VIII. Conclusion

This article finds that short-maturity T-bill yields have unique information about risk premiums that is not spanned by long-maturity T-bond yields. I estimate

2 components of risk premiums: long term and short term. The long-term component raises the slope of yield curve. In contrast, the short-term component affects T-bill yields but is almost invisible from T-bonds. The long-term component predicts monthly excess returns over the horizon of longer than 1 year whereas the short-term component loses its predictability in 1 quarter. The predictive power of the short-term component is likely to be missing from affine term structure models because the models are estimated using MLE, which trades off the predictive power of risk premiums with the cross-sectional fit of bond yields. The results are robust to the GMM estimation. The short-term component is found to be related to bond liquidity premiums.

References

Boudoukh, J.; M. Richardson; and R. F. Whitelaw. "The Myth of Long-Horizon Predictability." *Review of Financial Studies*, 21 (2007), 1577–1605.

Campbell, J. Y., and R. J. Shiller. "Yield Spreads and Interest Rate Movements: A Bird's Eye View." *Review of Economic Studies*, 58 (1991), 495–514.

Cieslak, A., and P. Povala. "Expected Returns in Treasury Bonds." *Review of Financial Studies*, 28 (2015), 2859–2901.

Cochrane, J. H., and M. Piazzesi. "Bond Risk Premia." *American Economic Review*, 95 (2005), 138–160.

Cox, J. C.; J. E. J. Ingersoll; and S. A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (1985), 385–407.

Dai, Q., and K. J. Singleton. "Expectation Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure." *Journal of Financial Economics*, 63 (2002), 415–441.

Dickey, D. A., and W. A. Fuller. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root." *Journal of the American Statistical Association*, 74 (1979), 427–431.

Duffee, G. R. "Idiosyncratic Variation of Treasury Bill Yields." *Journal of Finance*, 51 (1996), 527–551.

Duffee, G. R. "Term Premia and Interest Rate Forecasts in Affine Models." *Journal of Finance*, 57 (2002), 405–443.

Duffee, G. R. "Sharpe Ratios in Term Structure Models." Working Paper, Johns Hopkins University (2010).

Duffee, G. R. "Information in (and Not in) the Term Structure." *Review of Financial Studies*, 24 (2011), 2895–2934.

Duffie, D., and K. J. Singleton. "An Econometric Model of the Term Structure of Interest-Rate Swap Yields." *Journal of Finance*, 52 (1997), 1287–1321.

Fama, E. F., and R. R. Bliss. "The Information in Long-Maturity Forward Rates." *American Economic Review*, 77 (1987), 680–692.

Fontaine, J.-S., and R. Garcia. "Bond Liquidity Premia." *Review of Financial Studies*, 25 (2012), 1207–1254.

Grinblatt, M. "An Analytic Solution for Interest Rate Swap Spreads." *International Review of Finance*, 2 (2001), 113–149.

Huber, P. J. "The Behavior of Maximum Likelihood Estimates under Nonstandard Conditions." In *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability* (1967), 221–233.

Joslin, S.; M. Priebsch; and K. J. Singleton. "Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks." *Journal of Finance*, 69 (2014), 1197–1233.

Joslin, S.; K. J. Singleton; and H. Zhu. "A New Perspective on Gaussian Dynamic Term Structure Models." *Review of Financial Studies*, 24 (2011), 926–970.

Krishnamurthy, A. "How Debt Markets Have Malfunctioned in the Crisis." *Journal of Economic Perspectives*, 24 (2010), 3–28.

Ludvigson, S. C., and S. Ng. "Macro Factors in Bond Risk Premia." *Review of Financial Studies*, 22 (2009), 5027–5067.

Newey, W. K., and K. D. West. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, 55 (1987), 703–708.

https://doi.org/10.1017/S0022109017000813 Published online by Cambridge University Press

Pearson, N. D., and T.-S. Sun. "Exploiting the Conditional Density in Estimating the Term Structure: An Application to the Cox, Ingersoll, and Ross Model." *Journal of Finance*, 49 (1994), 1279–1304.
Piazzesi, M. "Affine Term Structure Models." In *Handbook of Financial Econometrics*, Y. Ait-Sahalia and L. Hansen, eds. Amsterdam, Netherlands: North-Holland (2003), 691–716.
White, H. "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity." *Econometrica*, 48 (1980), 817–838.

https://doi.org/10.1017/S0022109017000813 Published online by Cambridge University Press