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Two weak Poincaré theorems

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THEOREM 1. *Let M^3 be a compact 3-manifold and let $M^3 = E^3 \cup R$ be a standard decomposition where R is a subcomplex in a triangulation of M^3* ((2)). *If M^3 is a homology 3-sphere while R contains no arc that is wild in M^3, then $M^3 = S^3$.*

Proof. By (3) R is a 2-complex with the property that no 1-simplex in R lies on more than two simplices of dimension two. The homological triviality of R in dimension 2 follows from the construction in (2).

Since each non-zero 2-chain in R must have a non-zero boundary, there is a 1-simplex in R that lies on exactly one 2-simplex. It follows that by a finite sequence of Whitehead collapsings of 2-simplices ((4)), R can be deformed onto a graph G in its 1-skeleton. Since G is homologically trivial, G is a tree. But then G lies in an open 3-cell in M^3 and so does R. Because M^3 is a union of two open 3-cells, $M^3 = S^3$.

THEOREM 2. *If M^3 is a compact 3-manifold while $M^3 = E^3 \cup R$, let P be the 1-skeleton of R. If P lies in an open 3-cell in M^3, then $M^3 = S^3$.*

Proof. Suppose G is a polygonal graph in M^3. Then there is a pseudo-isotopy of M^3 onto M^3 taking G into a subset of P. But then G lies in an open 3-cell in M^3. It follows from (1) that M^3 is S^3.

REFERENCES

(1) BING, R. H. Necessary and sufficient conditions that a 3-manifold be S^3. *Ann. of Math.* **68** (1958), 17–37.
(2) DOYLE, P. H. and HOCKING, J. G. A decomposition theorem for n-dimensional manifolds. *Proc. Amer. Math. Soc.* **13** (1962), 469–471.
(3) PERSINGER, C. A. Thesis, Virginia Polytechnic Institute (1964).
(4) WHITEHEAD, J. H. C. Simplicial spaces, nuclei, and m-groups. *Proc. London Math. Soc.* **45** (1939), 243–327.

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