leads to the equation

$$2q^N = p^N + a_1qp^{N-1} + a_2q^2p^{N-2} + \dots,$$

for some N and integers $a_1, a_2, \dots$. This is impossible as q divides every term except p^N. From this we deduce for example that the Möbius sequence with $a = 1, b = 1, c = -2, d = 1$ cannot give an n-cycle for any n, since the ratio of eigenvalues comes to $\frac{1}{3}(-1 + 2\sqrt{2}i)$ with real part $-\frac{1}{3}$, that is $2\cos x = -\frac{2}{3}$ in the notation above. So the ratio of eigenvalues cannot be an nth root of 1 for any n. This Möbius sequence is 'chaotic'.

References

1. Matthew Temple, *Oscillating Möbius sequences*, Nuffield Bursary Project 2012, https://www.liv.ac.uk/~pjgiblin/papers/Matthew.pdf

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On 105.49: Owen Toller writes: Des MacHale starts with the apparently uncontentious statement that "If $P = Q$, then $Q = P$". But anyone who has tried to teach elementary algebra knows that actual usage of the equals sign does not always work like this. No learner would be happy with giving the last line of an equation as "$3 = x$" instead of "$x = 3$". In such a case, of course "=" means "the value of x is 3", which is not at all the same as an abbreviation for ".aeq." in Robert Recorde's usage. And this is not a trivial point – learners really struggle with it! Suppose a learner is faced with the equation $\frac{12}{x} = 18$. Many want to start by dividing both sides by 12, and obtain, wrongly, $x = 18 \div 12$. So a teacher will often advise learners to clear fractions first, going to $12 = 18x$, after which they are less likely to go wrong. (The situation is even more clear-cut with an inequality such as $12 - 5x < 2$.) But learners find it counterintuitive to put x on the right of an expression, as they are used from a very early stage to give their answers with x on the left-hand side of an equals sign. Of course there is no practical prospect of outlawing the equals sign in "assign" statements such as "$x = 3$" at this stage of mathematical history (though there might have been something to be said for it in the brief period when programming was taught more widely). Readers may care to ask themselves which equals signs in Professor MacHale's article are non-symmetric in the sense I have indicated.

On transitivity, Professor MacHale gives a good example in his Example 9; but many teachers and examiners frown upon learners who attempt to prove trigonometric identities of the form '$A \equiv B$' by showing that $A \equiv C$ and $B \equiv C$. Of course here the issue is usually claimed to be the irreversibility of the implications involved, but in most examples encountered in school the implications are all two-way anyway.

So perhaps the conclusion ought to be that some equals signs are more symmetric (or perhaps even more transitive) than others?

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