Exact strip analysis and optimum design of aerospace structures

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ABSTRACT

Minimisation of airframe mass reduces the costs of materials and manufacturing, as well as fuel consumption and atmospheric emissions. Fast, reliable analysis tools are required during preliminary design, when many alternative configurations and load cases are considered. The modelling and computational costs of finite element analysis are avoided by employing exact strip solutions of the governing differential equations, using the Wittrick-Williams algorithm to solve the resulting transcendental eigenproblems of buckling and vibration. This paper reviews recent enhancements to the exact strip method for analysis and optimum design of aerospace structures, using the specialist software VICONOPT. Lighter composite panels can be designed by obtaining reliable estimates of the reduced postbuckling stiffnesses when loaded in compression and shear. Further advances include discrete optimisation of layer thicknesses to allow for practical composite manufacturing constraints, vibration constraints, and a newly extended multi-level interface combining finite element analysis of a whole wing with exact strip postbuckling design of individual panels.

NOMENCLATURE

b, b_s	width of plate, plate strip
$\mathbf{D}$	modal displacement vector
$\mathbf{D}^*$	trial mode vector $\mathbf{D}$
$\mathbf{d}$	adjustment needed to $\mathbf{D}^*$
D_j, D^*_j, d_j	elements of $\mathbf{D}$, $\mathbf{D}^*$ and $\mathbf{d}$
F	thickness factor
f	objective function, i.e. panel mass
f_C	objective function at continuous optimum
J	number of eigenvalues exceeded by λ
J_0	number of fixed end eigenvalues exceeded by λ
J_m	contribution to J_0 from member m
$\mathbf{K}$	global stiffness matrix
$\mathbf{K}^\Delta$	upper triangular form of $\mathbf{K}$
$\mathbf{K}^*$	$\mathbf{K}$ evaluated at trial mode $\mathbf{D}^*$
K_{ij}, K^*_{ij}	elements of $\mathbf{K}$ and $\mathbf{K}^*$
$\mathbf{k}_m$	member stiffness matrix
l	length of panel
N_L, N_S, N_T	longitudinal, shear and transverse stress resultants
N_{L0}	peak value of N_L
N_{LF}	reduction in N_L due to flexure
N_{Ls}, N_{LFs},	values of N_L and N_{LF} in strip s
n	number of degrees of freedom
n_d	number of discrete design variables
n_s	number of plate strips
$\mathbf{P}$	perturbation force vector
$s\{\mathbf{K}\}$	sign count of $\mathbf{K}$
t	thickness of plate
x, y, z	principal co-ordinate directions
$\mathbf{x}$	vector of design variables
$\mathbf{x}_C$	design variables at continuous optimum
x_d	discrete elements of $\mathbf{x}$
x_d^-, x_d^+	lower and upper permitted values for x_d
α	width factoring parameter
λ	eigenparameter, i.e. load factor or frequency

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Subscripts

C	continuous optimum
d	discrete design variables
F	flexure
i, j, k	degrees of freedom
m	member
L, S, T	longitudinal, shear, transverse
s	strip

1.0 INTRODUCTION

The design of aerospace vehicles is a complex activity involving optimisation of the aerodynamics, propulsion, structures and systems. A new aircraft involves massive financial commitment to an infrastructure of design, manufacturing, marketing and support over a lifetime of decades. A major design objective is the minimisation of structural mass, which in turn reduces the costs of materials, manufacture, maintenance, and (perhaps most importantly) fuel consumption and atmospheric emissions.

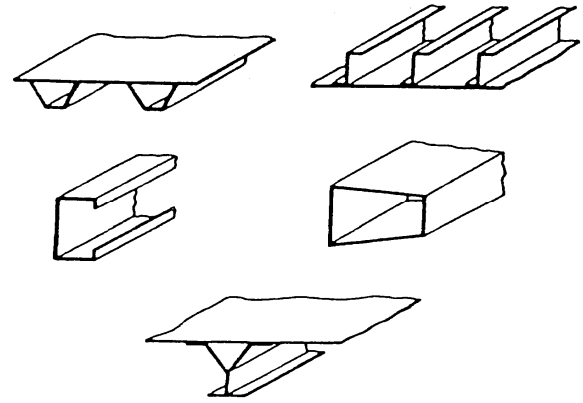
Aircraft design is conventionally defined in three consecutive stages. The conceptual design stage defines the major structural components, establishes the load paths through the structure, and determines the material and structural concepts to be used. Many alternative concepts are considered, often in a qualitative manner. A selection of promising concepts is retained for more detailed numerical analysis and optimisation in the preliminary design stage. Here a deeper understanding is sought of the structural integrity, including analysis of the stability, dynamic loads and fatigue life. Many load cases are considered, in order to model the behaviour under all anticipated payloads, manoeuvres and climatic conditions. Finally, the detailed design stage considers every aspect of the chosen configuration in minute detail, providing the data required for tooling, manufacture and certification.

The detailed design stage employs the most powerful analysis and design tools available. Large-scale finite element (FE) analysis[1] consumes a large amount of computational resources, even on today's computers. In contrast, the preliminary design stage requires less detailed analysis but needs to consider many candidate configurations and load cases. It can therefore benefit from the use of alternative, simpler yet reliable forms of analysis which are faster than the FE method, in terms of both modelling and computational time.

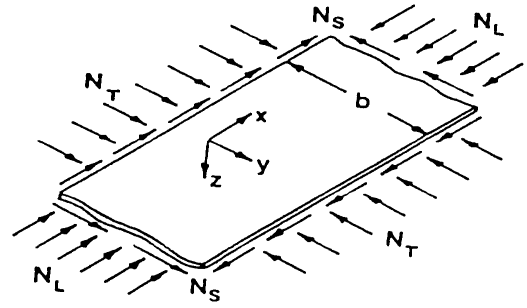
In this simplified approach, wing and fuselage structures are typically analysed at the level of individual stiffened panels or their component plates, which are assumed to be prismatic, i.e. their geometry can be represented by a cross-section. Thus the buckling behaviour can be described by analytical formulae[2-3] or semi-analytical procedures[4], which often restrict attention to simple configurations and ignore interaction with the rest of the structure. For more general configurations, the finite strip method[5-6] has proved particularly valuable, in that precision and efficiency are enhanced by combining an analytical solution in one direction with FE-like discretisation in the perpendicular direction.

Additionally, the modelling and computational costs of discretisation can be avoided altogether by using analytical solutions of the governing differential equations[7-8]. This approximate analytical approach, which is exact for certain simple cases (e.g. prismatic assemblies of orthotropic rectangular plates with simply supported ends and no shear loading), is often referred to as the 'exact strip' method. For buckling and vibration problems it results in transcendental eigenproblems, rather than the linear ones resulting from FE discretisation of the structure.

This paper reviews recent enhancements to the exact strip method for the analysis and optimum design of aerospace structures, using the specialist software VICONOPT. In line with current trends in aircraft design, the analysis assumes the use of lightweight laminated composite materials which can be tailored to specific loading requirements. Lighter structures can be designed by allowing for the postbuckling reserve of strength of slender panels. To this end, reliable estimates are obtained of the reduced postbuckling stiffnesses under compression and/or shear loading. Advances in optimisation include discrete optimum design of layer thicknesses to allow for practical composite manufacturing constraints, vibration constraints, and a newly extended multi-level interface combining FE analysis of a whole wing with exact strip postbuckling design of individual panels.



(a)



(b)

Figure 1. (a) Prismatic plate assemblies. (b) Component plate, showing in-plane loading.

2.0 REVIEW OF EXACT STRIP METHOD

2.1 Buckling and vibration analysis

The exact strip approach to structural analysis assumes a continuous distribution of stiffness and mass over the structure, so avoiding the discretisation inherent in the more widely used FE method. Analytical[9] or numerical[10] solution of the partial differential equations governing the deformations of the component members m yields exact member stiffness matrices $\mathbf{k}_m$, which are assembled into a global stiffness matrix $\mathbf{K}$ for the structure. $\mathbf{K}$ relates a finite set of displacements $\mathbf{D}$ at the nodes of the structure to their corresponding perturbation forces $\mathbf{P}$, by

$$\mathbf{KD} = \mathbf{P} \quad \ldots (1)$$

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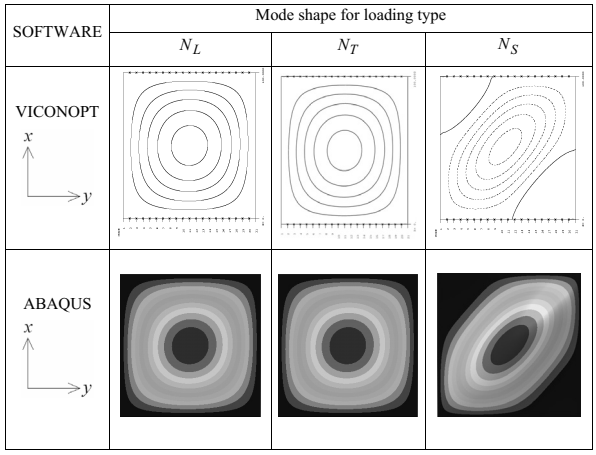


Figure 2. Critical buckling modes for a simply supported square composite plate, with longitudinal (N_L), transverse (N_T) and shear (N_S) loading.

The critical buckling loads or undamped natural frequencies of the structure correspond to the eigenvalues found by solving

$$\mathbf{KD} = \mathbf{0} \qquad \ldots (2)$$

where **K** consists of transcendental, and thus highly non-linear, functions of the eigenparameter λ, i.e. the load factor or frequency[9]. This transcendental eigenproblem is not amenable to the numerical methods available for linear eigenproblems, and is instead solved by means of an iterative search for the values of λ at which Equation (2) is satisfied. All eigenvalues, including higher natural frequencies, can be found with certainty to any required accuracy using the Wittrick-Williams algorithm[11-12]. This algorithm calculates J, the number of eigenvalues which lie between zero and any trial value of λ. Any difference in J between two trial values is equal to the number of eigenvalues lying between them.

In its simplest form the Wittrick-Williams algorithm is stated as

$$J = J_0 + s\{\mathbf{K}\} \qquad \ldots (3)$$

where $s\{\mathbf{K}\}$ is known as the sign count of **K**, and corresponds to the number of negative leading diagonal elements of the upper triangular matrix $\mathbf{K}^{\Delta}$ obtained by applying conventional Gauss elimination, without pivoting, to the matrix **K**. J_0 is the value J would have if all the freedoms corresponding to **K** were clamped, and is calculated as

$$J_0 = \sum_m J_m \qquad \ldots (4)$$

where the summation is over all members m of the structure, and J_m is calculated for each member as the number of critical load factors or natural frequencies exceeded by the trial value of λ, when the member ends are clamped.

The eigenvectors, i.e. mode shapes, are found either by subsequent solution of Equation (1) with a random force vector **P**[13], or by combining the Wittrick-Williams algorithm with a recursive Newton method based on inverse iteration[14].

2.2 Analysis and optimum design of prismatic plate assemblies

The exact strip method and Wittrick-Williams algorithm have been used for many years in the software VICONOPT[15-16] for the critical buckling and free vibration analysis and optimum design of isotropic or anisotropic prismatic plate assemblies, including composites with an arbitrary layup which is not restricted to be symmetric or balanced. Typical geometries and plate loadings are shown in Fig. 1, in which N_L, N_S and N_T are, respectively, longitudinal, shear, and transverse stress resultants, i.e. loads per unit width, which are assumed to be invariant in the longitudinal (x) direction.

In the simplest (VIPASA) analysis[9] the buckling mode is assumed to vary sinusoidally in the x direction, with a half-wavelength that divides exactly into the panel length l. Thus exact solutions are automatically obtained for simply supported isotropic and orthotropic panels without shear loading. The VIPASA analysis can only give approximate results for anisotropic and shear-loaded panels, which exhibit skewed buckling modes. More accurate results are obtained by the alternative VICON analysis[17], in which sinusoidal responses are coupled using Lagrangian multipliers to enforce the end conditions. Figure 2 shows critical buckling modes for three load cases, for a simply supported square composite plate with a 16-ply symmetric, balanced layup. There is good agreement with the modes obtained from the FE software ABAQUS[18], although the VICON analysis gives slightly (<5%) higher critical buckling loads because it assumes an infinitely long model which more closely represents conditions of continuity with adjacent parts of the structure.

In both forms of analysis, computational efficiency can be improved by using exact multi-level substructuring[9] and an infinitely wide model for transversely repetitive panels[19]. Analysis is available using classical plate theory or first order shear deformation theory[10].

VICONOPT includes substantial optimum design features for individual panels[15,20]. The principal capability is the minimisation of panel mass subject to initial buckling, strength, stiffness and geometric constraints. The design variables are plate widths, layer thicknesses and ply orientations. Continuous optimisation is performed by using a linear optimiser employing the method of feasible directions[21], in combination with a stabilisation, i.e. thickness factoring, procedure[20] based on the Wittrick-Williams algorithm which ensures a just feasible solution at each step. Sensitivities are estimated efficiently and are re-used where possible to reduce solution times. This approach does not guarantee global optimality, but it does result in low mass practical designs which can sometimes be improved by the use of alternative starting configurations.

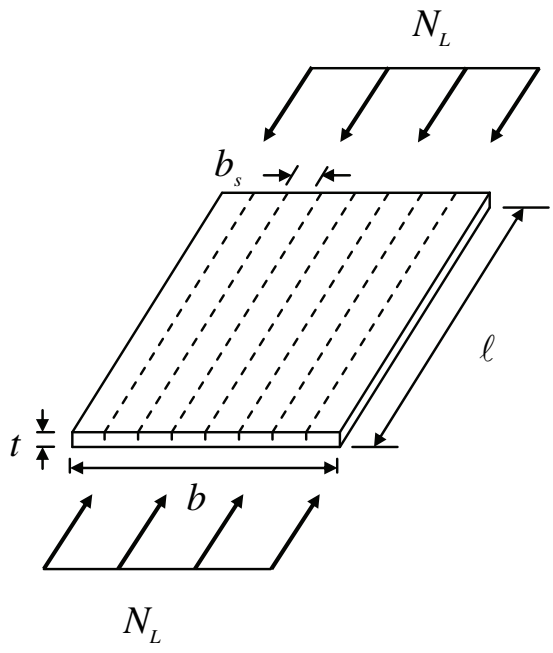
3.0 INITIAL POSTBUCKLING ANALYSIS

Slender stiffened panels can often safely carry loads in excess of their critical buckling loads, as a result of re-distribution of stress towards their supported edges or other regions where the lateral deflections are small. Taking account of this postbuckling reserve of strength enables lighter panels to be designed. Postbuckling behaviour is a complex phenomenon, e.g. being highly sensitive to initial imperfections, and is usually studied by an incremental approach, typically requiring many cycles of FE analysis.

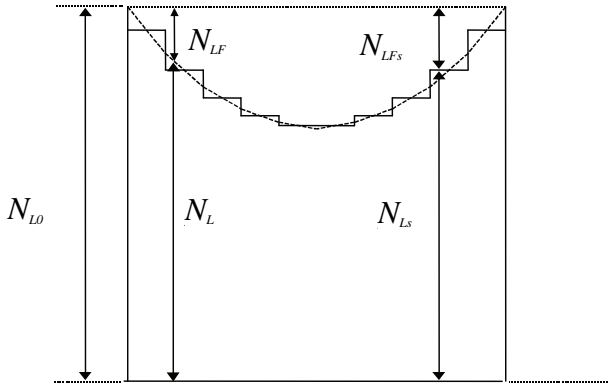
A simpler, alternative approach is provided by the extension of the VIPASA analysis option of VICONOPT to cover local postbuckling of longitudinally compressed panels. Despite various simplifying assumptions about the panel geometry, loading and end conditions, reliable estimates are obtained for the overall postbuckled stiffness of the panel and its component plates, although local variations of stress and strain cannot be captured.

Each component plate is modelled as a substructure comprising n_s longitudinal strips of equal width b_s, as shown in Fig. 3(a). The longitudinal stress resultant N_L takes a peak value N_{L0} at the supported edges where there is no out-of-plane deflection. Elsewhere, it is reduced by a component N_{LF} due to the out-of-plane flexure of its line of action; uniform values N_{LFs} and N_{Ls} are assumed across each strip s, as shown in Fig. 3(b). Any initial imperfection is conservatively assumed to have the same shape as the buckling mode for a perfect structure. If the longitudinal edges of the plate are simply supported and constrained to remain straight (i.e. they cannot

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(a)



(b)

Figure 3. (a) Plate of width b, divided into n_s strips of equal width b_s. (b) Variation of longitudinal stress resultants across the plate.

deflect in the z direction; they remain oriented in the x direction; they may or may not be able to move towards each other in the y direction), regions of transverse tension and compression develop. There is a net stiffening effect on the plate, which is approximated by applying uniform transverse tension. The postbuckling analysis comprises a number of cycles, in each of which it is required to converge on consistent values of: (*i*) the overall longitudinal strain; (*ii*) the total longitudinal load; (*iii*) the stress resultants in the strips of each component plate; (*iv*) the mode amplitude; and (*v*) the shape of the postbuckling mode. Two alternative strategies have been adopted.

In the first strategy[22], each cycle is defined by a pre-determined increment of the mode amplitude, and the remaining quantities are found by an iterative procedure. In each iteration, the Wittrick-Williams algorithm is used to find the critical buckling load and mode shape for a given stress distribution. Illustrative results have previously been presented for panels for which postbuckling is considered in analysis[23-24] and optimum design[25-26]. Although these results show good agreement with experimental and FE results, some problems with regularly spaced stiffeners exhibit slow convergence due to limited numerical accuracy of the mode shapes, interaction between similar local modes and mode jumping[27]. This difficulty can be partially avoided by using an infinitely wide panel model[19].

In the second strategy, which extends the analysis to transverse and shear loading, each cycle is defined by a pre-determined increment of longitudinal and/or shear strain. Within each cycle, it is necessary to determine the total applied loads, the variation of stress resultants across the structure, and the amplitude and shape of the postbuckling mode. Instead of using the Wittrick-Williams algorithm, Newton iterations are performed to give accurate convergence on the critical buckling load and associated mode which solve Equation (2). The mode vector $\mathbf{D} = \{D_j;\ j = 1,\ \ldots\ n\}$ includes displacements and rotations both at the longitudinal plate edges and also at the strip edges within each plate, and is ordered to permit elimination of these internal degrees of freedom by substructuring. $\mathbf{K} = \{K_{ij};\ i, j = 1,\ \ldots\ n\}$ is the corresponding exact stiffness matrix, which is a transcendental function of the stress resultants in each strip, and hence also of $\mathbf{D}$. Suppose that

$$\mathbf{D} = \mathbf{D}^* + \mathbf{d} \qquad \ldots\ (5)$$

where $\mathbf{D}^*$ is a trial mode vector and $\mathbf{d} = \{d_j;\ j = 1,\ \ldots\ n\}$ is the adjustment needed to $\mathbf{D}^*$ in order to solve Equation (2). The Newton iteration is expressed in matrix form as

$$\left(\mathbf{K}^* + \sum_{j=1}^{n} \frac{\partial \mathbf{K}^*}{\partial D_j} d_j\right)\left(\mathbf{D}^* + \mathbf{d}\right) = \mathbf{0} \qquad \ldots\ (6)$$

where $\mathbf{K}^* = \mathbf{K}(\mathbf{D}^*)$. Neglecting higher order terms, Equation (6) becomes

$$\sum_{j=1}^{n}\left(K_{ij}^* + \sum_{k=1}^{n} \frac{\partial K_{ik}^*}{\partial D_j} D_k^*\right) d_j = -\sum_{j=1}^{n} K_{ij}^* D_j^* \qquad (i = 1, \ldots n) \qquad \ldots\ (7)$$

Equation (7) is solved to obtain $\mathbf{d}$, and substitution in Equation (5) gives a new trial mode vector $\mathbf{D}$, which is used as $\mathbf{D}^*$ in the next Newton iteration. The derivatives needed in Equation (7) are found by finite difference approximations using suitably small perturbations about the trial values $D_j = D_j^*$. It should be noted that when an element D_j is perturbed, the only non-zero derivatives in Equation (7) are those of the plates (or for some loading conditions, strips) immediately adjacent to D_j, and this is allowed for in the logic of the computer coding. Convergence on the amplitude and shape of the mode vector $\mathbf{D}$ implies convergence on the adjustments to the stress resultants due to flexure, and hence the postbuckling stress distribution and total loads on the structure can be calculated.

Illustrative results are given for a simply supported square panel with four longitudinal stiffeners, whose cross-section is shown in Fig. 4(a). When loaded in longitudinal compression, initial buckling is in the sinusoidal local mode shown in Fig. 4(b), which has six half-waves along the panel length and maximum amplitude between the two central stiffeners. Figure 4(c) gives stress-strain plots, normalised with respect to values at initial buckling, for various locations in the skin and stiffeners, together with average plots for the panel. It is seen that the overall (i.e. average) stiffness of the panel is reduced by about 25% after initial buckling. There is a negligible stiffness reduction where there is little deflection, i.e. in the edge portions (location 1), the stiffener flanges (location 2) and stiffener webs (locations 5 and 6). But there is a very large reduction in the inter-stiffener skin portions (locations 3 and 4) where there is a large deflection. These observations, which are confirmed by unpublished experimental and FE results, illustrate a stress re-distribution away from the (buckled) inter-stiffener skin portions towards the (unbuckled) stiffeners and edge skin portions.

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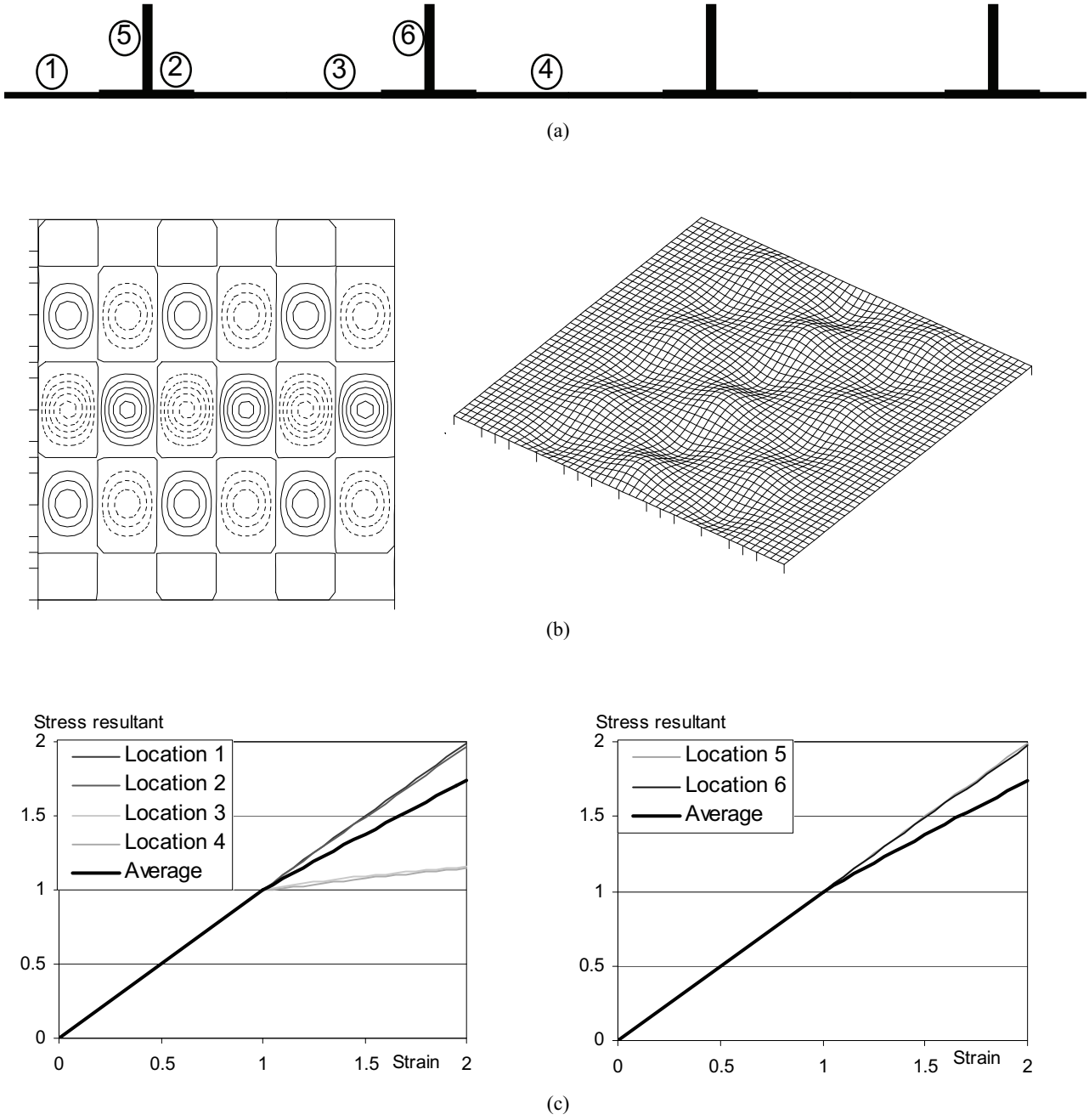


Figure 4. Postbuckling of a stiffened panel. (a) Panel cross-section. (b) Contour and isometric plots of buckling mode. (c) Normalised stress-strain plots at various locations in the skin and stiffeners.

4.0 OPTIMUM DESIGN

4.1 Discrete optimum design

Practical panel configurations are often required to satisfy discrete manufacturing constraints. Important aerospace applications include the design of metallic plate structures with a specified discrete collection of permissible plate thicknesses, and the design of laminated composite structures, where each layer must comprise an integer number of plies of a standard thickness. To handle such discrete constraints, Fleury[28] adopted a dual formulation to render the discrete domain continuous. In contrast, extensions to VICONOPT[29] follow a traditional primal approach, as follows.

Consider the mass minimisation of a panel $(\mathbf{x},f)$ with design variables $\mathbf{x}$ and mass $f(\mathbf{x})$. Some of the design variables x_d ($d = 1, ..., n_d$) are required to take values from discrete sets. The continuous optimum $(\mathbf{x}_C, f_C)$ provides a lower bound on the mass of any solution also satisfying the discrete constraints. An upper bound is found by rounding down all the discrete design variables x_d to their next lower discrete values x_d^-, and then systematically increasing them in turn to their next upper discrete values x_d^+ until a stable configuration is obtained. The combinatorial problem of size 2^{n_d} in which x_d is restricted to take the values x_d^- and x_d^+ ($d = 1, ..., n_d$) is now solved by branching and fathoming an enumeration tree[30]. All discrete

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solutions found are checked for stability using a single iteration of the Wittrick-Williams algorithm. Nodes of the tree are abandoned if it can be shown that further fathoming is unable to generate a stable solution, or to improve on the mass of the best discrete solution found so far.

Illustrative examples[29] demonstrate substantial mass savings over the simple expedient of rounding up every discrete design variable from its value at the continuous optimum to the next permitted discrete value. Areas for future work include the discrete optimisation problems associated with ply angle selection and stacking sequence design.

4.2 Vibration constraints

Fundamental natural frequency constraints and frequency free band constraints have been incorporated into the VICONOPT optimisation strategy[31-32]. As before[20], a stabilisation procedure is used in conjunction with a linear optimiser to obtain a just feasible configuration. However, in contrast to buckling design situations, simply factoring the plate thicknesses does not ensure that the fundamental natural frequency will shift monotonically and rapidly. Thus a revised method of stabilisation is used to guarantee accurate convergence on a stabilised design having a fundamental natural frequency chosen by the designer. The revised method factors both layer thicknesses and the widths of the plates containing the layers (provided the widths have also been declared as design variables). If the total thickness of a plate is thus factored by F (with $F = 1$ representing the unfactored configuration), then the plate width is factored monotonically by F^{α}, where α is a parameter chosen from the range $0{\cdot}1 \le \alpha \le 0{\cdot}4$. This range has been determined by analytical consideration of the effects of thickness and width factoring for various modes of vibration.

A further important practical requirement is the creation of frequency free bands, such constraints having being ignored by the method thus far described. A consequence is that, although the linear optimiser attempts to move natural frequencies out of a band, the stabilisation procedure could force them back into it. Thus it is desirable that the linear optimiser produces a design for which stabilisation requires minimal thickness and width factoring, and where the frequencies have been shifted a reasonable distance beyond the band limits specified by the designer. The design move made by the optimiser is guided by the sensitivities, i.e. the partial derivatives of the natural frequencies with respect to the design variables. Typically some natural frequencies will increase while others decrease. The intention is to force all natural frequencies initially in the lower half of the required band downwards and to force those in the upper half upwards, while also considering all other constraints and minimising mass. In practice more than one frequency free band constraint may be specified, and it may not always be possible for the optimiser to locate a design where all such constraints are satisfied. In such cases, a search is employed to find other frequency free bands of sufficient width that may be shifted into the position of the specified band locations. Such bands must always be shifted upwards to avoid violating fundamental frequency constraints. Mass is therefore added as though stabilisation had been further employed to obtain a design where the fundamental frequency constraint is not critical. Illustrative results have been presented elsewhere[31-32].

4.3 Multi-level optimum design

In the design of principal aircraft structures (e.g. wings and fuselage), VICONOPT provides a reliable and efficient analysis and optimisation tool for individual panels. But design changes to any panel affect the geometry of other panels and the stress distribution over the whole structure, so that other panels may prove to be far from optimal or even violate buckling or strength constraints. Following the recommendations of the Group for Aeronautical Research in Europe (GARTEUR)[33], a multi-level optimum design

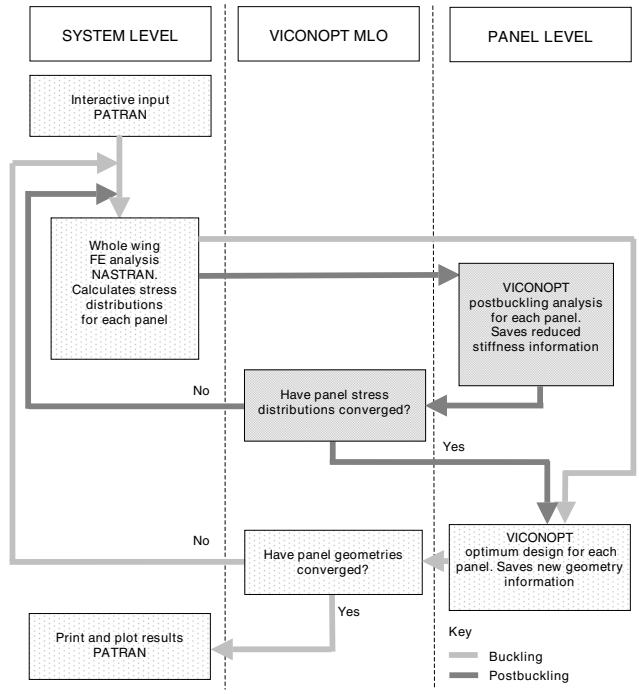


Figure 5. Multi-level postbuckling optimisation.

interface has been developed, combining 'system level' FE analysis of the whole structure with 'panel level' optimisation of individual panels. The C++ interface, known as VICONOPT MLO[34], links VICONOPT with the FE software MSC/NASTRAN[35]. MSC/PATRAN[36] or an alternative pre-/post-processor may be used to generate the geometric and overall loading data, with additional control data for the panel optimisation being supplied directly to VICONOPT MLO via a windows-based interface. VICONOPT MLO combines this data with the results of the FE analysis to generate input files for VICONOPT. Similarly, the geometric changes arising from panel optimisation are used to automatically update the data files for subsequent FE analysis.

Figure 5 is a flowchart of the multi-level optimum design strategy. The seven analysis and design components A-G shown in Fig. 6 may be regarded as building blocks, each depending on the components lying below it in the figure. Components A-D cover multi-level optimisation for initial buckling[34], while components E-G cover extensions to multi-level postbuckling optimisation[37]. System level static analysis of the whole structure is performed by the FE analysis of component A. The structure comprises a number of prismatic panels such as those shown in Fig. 1, which are each analysed at panel level by components B and E for initial buckling and postbuckling analysis, respectively. Components C and F provide corresponding panel level optimisation capabilities. Components D and G are the multi-level optimisation procedures, which entail cycling between system and panel levels to ensure consistency of the geometry and loading. The components will now be outlined in more detail.

Although the panel optimisation will include initial buckling and/or postbuckling constraints, the purpose of the system level FE analysis (component A) is simply to calculate the stresses to be carried by each of the panels. For each load case considered, a linear elastic static analysis is performed using a relatively coarse mesh which may not include all the geometric detail of individual panels. At present QUAD4 shell elements[35] are used. Once the stresses in each element have been calculated, VICONOPT MLO combines them to obtain the stresses in each panel and the plates that comprise it, as required for VICONOPT analysis and optimisation.

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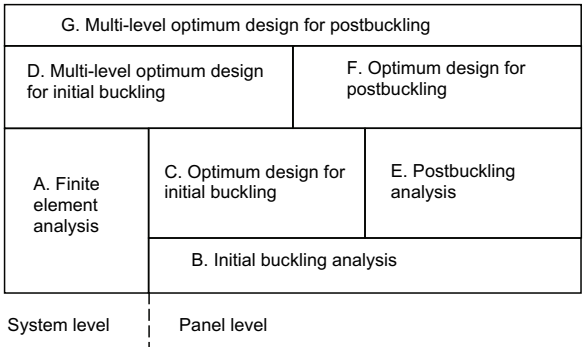


Figure 6. Components of multi-level optimum design system.

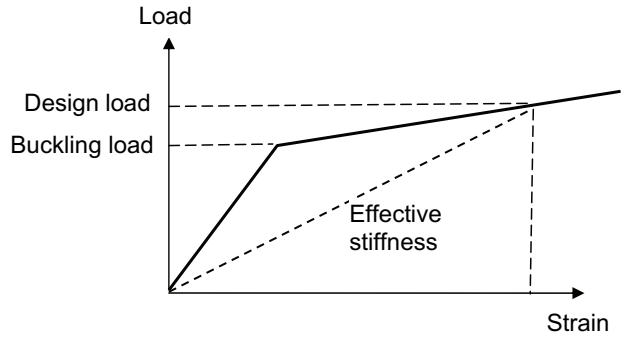


Figure 7. Calculation of effective stiffness for a postbuckled plate.

VICONOPT performs initial buckling analysis and optimum design (components B and C) of the individual panels, using the methods described earlier. Design changes to any panel must be compatible with the geometry of adjacent panels, and this can be achieved by coupling the panel optimisations using common or linked design variables. Moreover, because any panel level design change influences the stress distribution over the whole structure, it is necessary to re-run the FE analysis to calculate new stress distributions for the updated panel geometries. This process is illustrated by the lightly shaded parts of the VICONOPT MLO flowchart in Fig. 5, which represent cycling between system level FE analysis and panel level optimisation (component D).

Previously published multi-level optimum design results[34] for a wing structure with 12 skin panels have recently been verified by carrying out FE analysis using MSC/NASTRAN[35]. It was found that overall buckling of the wing occurred well above the design load. In addition, each individual panel could carry the loads predicted by VICONOPT without experiencing local buckling, apart from a discrepancy of a few percent in the tip panels. This discrepancy is accounted for by the continuous end conditions assumed by VICONOPT, and can be corrected by appropriate scaling of the skin and stiffener thicknesses in these panels.

In order to allow for postbuckling in optimum design, it is necessary to have some knowledge of the reduced postbuckling stiffness, and also of its variation across the panel cross-section. The approach adopted in VICONOPT is to carry out a postbuckling analysis (component E) on every panel before each design cycle. The in-plane stiffness of each plate is then adjusted by using an effective stiffness, taken as a weighted average of the prebuckled and postbuckled stiffnesses, as follows.

Each plate has a different effective stiffness, depending on whether or not it has buckled. It is obtained from postbuckling results such as those shown in Fig. 4(c). First, the average postbuckling stiffness for the panel is found and used to predict the overall longitudinal strain when the design load is reached. Then the individual stress/strain curves for each plate are constructed up to this level of strain, and the effective stiffness is taken as the secant stiffness as shown in Fig. 7. It should be noted that the effective stiffness is only correct at the design load, but is sufficient for evaluating stress and strain constraints and other design criteria (component F).

Finally, the multi-level design procedure of VICONOPT MLO is currently being extended to allow for postbuckling by the addition of the heavily shaded parts of the flowchart in Fig. 5 (component G). If any panel buckles below the design load, its effective stiffness is reduced and this affects the distribution of stress across the whole structure. It is therefore necessary to include additional cycling between system level FE analysis and panel level postbuckling analysis, to obtain consistent stress distributions in each panel, prior to every cycle of panel optimisation. This final building block of the multi-level design system is still under development.

5.0 CONCLUDING REMARKS

The exact strip approach provides an attractive alternative to the conventional FE method for the analysis of a range of isotropic and anisotropic prismatic plate assemblies, including typical aircraft wing and fuselage panels. By using the Wittrick-Williams algorithm, structural eigenvalues, i.e. critical buckling loads and natural frequencies of free vibration, can be found with certainty. This approach is central to the software VICONOPT, in which the eigenvalue analysis is coupled with gradient-based optimisation tools to minimise the structural mass.

Recent analytical developments to the software include the extension of the critical buckling analysis into the initial local postbuckling range. By dividing each component plate into strips, the consequent non-uniform stress distribution is approximated so that the postbuckling mode can be found using a Newton-based search strategy.

The continuous optimum design capability of the software has been extended by adjusting the optimal values of the design variables to adjacent discrete values, e.g. in order to satisfy manufacturing requirements for composites where each layer thickness must be an integer multiple of a standard ply thickness. The adjustments are made locally, by sequential rounding and binary branching, after the continuous optimisation and incur a relatively small mass penalty. Other optimum design developments include constraints on the fundamental natural frequency and the removal of all natural frequencies from specified frequency bands. Finally, VICONOPT has been linked with the FE software MSC/NASTRAN to form a powerful tool for the multi-level optimisation of a complete aircraft wing, subject to initial buckling and postbuckling constraints.

Research is continuing in most of the areas described, and also in the modelling of delamination and other forms of damage in composite structures[38].

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REFERENCES

1. ZIENKIEWICZ, O.C. *The Finite Element Method*, 1977, McGraw-Hill, London.
2. LIU, B., HAFTKA, R.T. and TROMPETTE, P. Maximization of buckling loads of composite panels using flexural lamination parameters, *Struct Multidisc Optim*, January 2004, **26**, (1-2), pp 28-36.

https://doi.org/10.1017/S0001924000003997 Published online by Cambridge University Press

3. HERENCIA, J.E., WEAVER, P.M. and FRISWELL, M.I. Initial sizing optimisation of anisotropic composite panels with T-shaped stiffeners, *Thin-Walled Struct*, April 2008, **46**, (4), pp 399-412.
4. BUSHNELL, D. PANDA2 — program for the minimum weight design of stiffened, composite, locally buckled panels, *Comput Struct*, March 1987, **25**, (4), pp 469-605.
5. CHEUNG, Y.K. *Finite Strip Method in Structural Analysis*, 1976, Pergamon, Oxford.
6. DAWE, D.J. Finite strip buckling analysis of curved plate assemblies under biaxial loading, *Int J Solids Struct*, November 1977, **13**, (11), pp 1141-1155.
7. WITTRICK, W.H. A unified approach to the initial buckling of stiffened panels in compression, *Aeronaut Q*, August 1968, **24**, (3), pp 265-282.
8. WILLIAMS, F.W. and WITTRICK, W.H. Computational procedures for a matrix analysis of the stability and vibration of thin flat-walled structures in compression, *Int J Mech Sci*, December 1969, **11**, (12), pp 979-998.
9. WITTRICK, W.H. and WILLIAMS, F.W. Buckling and vibration of anisotropic or isotropic plate assemblies under combined loadings, *Int J Mech Sci*, April 1974, **16**, (4), pp 209-223.
10. ANDERSON, M.S. and KENNEDY, D. Transverse shear deformation in exact buckling and vibration analysis of composite plate assemblies, *AIAA J*, October 1993, **31**, (10), pp 1963-1965.
11. WITTRICK, W.H. and WILLIAMS, F.W. An algorithm for computing critical buckling loads of elastic structures, *J Struct Mech*, February 1973, **1**, (4), pp 497-518.
12. WITTRICK, W.H. and WILLIAMS, F.W. A general algorithm for computing natural frequencies of elastic structures, *Quart J Mech Appl Math*, August 1971, **24**, (3), pp 263-284.
13. HOPPER, C.T. and WILLIAMS, F.W. Mode finding in nonlinear structural eigenvalue calculations, *J Struct Mech*, August 1977, **5**, (3), pp 255-278.
14. YUAN, S., YE, K., WILLIAMS, F.W. and KENNEDY, D. Recursive second order convergence method for natural frequencies and modes when using dynamic stiffness matrices, *Int J Numer Meth Engng*, March 2003, **56**, (12), pp 1795-1814.
15. WILLIAMS, F.W., KENNEDY, D., BUTLER, R. and ANDERSON, M.S. VICONOPT: program for exact vibration and buckling analysis or design of prismatic plate assemblies, *AIAA J*, November 1991, **29**, (11), pp 1927-1928.
16. KENNEDY, D., FISCHER, M. and FEATHERSTON, C.A. Recent developments in exact strip analysis and optimum design of aerospace structures, *Proc IMechE Part C: J Mech Engng Sci*, April 2007, **221**, (4), pp 399-413.
17. ANDERSON, M.S., WILLIAMS, F.W. and WRIGHT, C.J. Buckling and vibration of any prismatic assembly of shear and compression loaded anisotropic plates with an arbitrary supporting structure, *Int J Mech Sci*, August 1983, **25**, (8), pp 585-596.
18. HIBBITT, KARLSSON and SORENSEN, INC. ABAQUS Theory Manual, Version 6.8, 2008, HKS, Pawtucket, RI.
19. WILLIAMS, F.W. and ANDERSON, M.S. Buckling and vibration analysis of shear-loaded prismatic plate assemblies with supporting structures, utilizing symmetric or repetitive cross-sections, *Aspects of the Analysis of Plate Structures — A Volume in Honour of W.H. Wittrick*, DAWE, D.J., HORSINGTON, R.W., KAMTEKAR, A.G. and LITTLE, G.H., (Eds), 1985, Clarendon, Oxford, pp 51-71.
20. BUTLER, R. and WILLIAMS, F.W. Optimum design using VICONOPT, a buckling and strength constraint program for prismatic assemblies of anisotropic plates, *Comput Struct*, May 1992, **43**, (4), pp 699-708.
21. VANDERPLAATS, G.N. and MOSES, F. Structural optimization by methods of feasible directions, *Comput Struct*, July 1973, **3**, (4), pp 739-755.
22. POWELL, S.M., WILLIAMS, F.W., ASKAR, A.-S. and KENNEDY, D. Local postbuckling analysis for perfect and imperfect longitudinally compressed plates and panels, 1998, Proc 39th AIAA/ASME/ASCE/AHS/ASC Struct, Struct Dyn Mater Conf, AIAA, Reston, VA, April 1998, pp 595-603.
23. LILLICO, M., BUTLER, R., HUNT, G.W., WATSON, A., KENNEDY, D. and WILLIAMS, F.W. Analysis and testing of a post-buckled stiffened panel, *AIAA J*, May 2002, **40**, (5), pp 996-1000.
24. LILLICO, M., BUTLER, R., HUNT, G.W., WATSON, A. and KENNEDY, D. Postbuckling of stiffened panels using strut, strip, and finite element methods, *AIAA J*, June 2003, **41**, (6), pp 1172-1179.
25. ANDERSON, M.S. Design of panels having postbuckling strength, 1997, Proceedings of 38th AIAA/ASME/ASCE/AHS/ASC Struct, Struct Dyn Mater Conf, AIAA, Reston, VA, April 1997, pp 2407-2413.
26. ANDERSON, M.S. Inclusion of local post buckling response in the design of stiffened panels, 2000, Paper AIAA-2000-1661, Proceedings of 41st AIAA/ASME/ASCE/AHS/ASC Struct, Struct Dyn Mater Conf, AIAA, Reston, VA, April 2000.
27. WATSON, A. and KENNEDY, D. Mode jumping in post-buckled stiffened panels, 2004, Proceedings of 4th Int Conf Thin-Walled Struct, pp 573-580, Institute of Physics, Bristol, June 2004.
28. FLEURY, C. Reconciliation of mathematical programming and optimality methods, *Foundations of Structural Optimization: A Unified Approach*, 1982, MORRIS, A.J. (Ed), pp 363-404, Wiley, Chichester.
29. KENNEDY, D., ONG, T.J., O'LEARY, O.J. and WILLIAMS, F.W. Practical optimisation of aerospace panels, 1999, Proceedings of 1st ASMO UK/ISSMO Conf July 1999, pp 217-224, MCB University Press, Bradford.
30. GARFINKEL, R.S. and NEMHAUSER, G.L. *Integer Programming*, 1972, Wiley, New York.
31. O'LEARY, O.J., WILLIAMS, F.W. and KENNEDY, D. Optimum stiffened panel design with fundamental frequency constraint, *Thin-Walled Struct*, July 2001, **39**, (7), pp 555-569.
32. KENNEDY, D., O'LEARY, O.J. and WILLIAMS, F.W. Optimum design of prismatic plate assemblies with spectral gap constraints, 2005, Proceedings of 5th Int Symp Vib Contin Sys, Virginia Polytechnic Institute and State University, Blacksburg, VA, July 2005, pp 36-38.
33. GARTEUR. Final Report of the GARTEUR Action Group on Structural Optimization SM (AG13), 1997, Volumes 1-3, DERA, Farnborough.
34. FISCHER, M., KENNEDY, D. and FEATHERSTON, C.A. Multilevel optimization of a composite aircraft wing using VICONOPT MLO, 2002, Paper AIAA-2002-5511, Proceedings of 9th AIAA/ISSMO Symp Multidisc Anal Optim, AIAA, Reston, VA, April 2002.
35. MSC Software Corporation. MSC/NASTRAN, Version 70.7, 1999, MSC, Los Angeles.
36. MSC Software Corporation. MSC/PATRAN, Version 9.0, 1999, MSC, Los Angeles.
37. KENNEDY, D., FEATHERSTON, C.A., QU, S. and FISCHER, M. Optimum design of aerospace structures: a multi-level postbuckling approach, 2008, Proceedings of 7th ASMO UK/ISSMO Conf, University of Leeds, July 2008, pp 87-88.
38. DAMGHANI, M., FEATHERSTON, C.A. and KENNEDY, D. Critical buckling of delaminated composite plate assemblies under combined loading using an exact strip method, 2008, Paper CST2008-2007-000467, Proceedings of 9th Int Conf Comput Struct Tech, Civil-Comp Press, Stirling, September 2008.

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