ELASTIC FOUNDATION ANALYSIS OF UNIFORMLY LOADED FUNCTIONALLY GRADED VISCOELASTIC SANDWICH PLATES

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ABSTRACT

This paper deals with the static response of simply supported functionally graded material (FGM) viscoelastic sandwich plates subjected to transverse uniform loads. The FG sandwich plates are considered to be resting on Pasternak's elastic foundations. The sandwich plate is assumed to consist of a fully elastic core sandwiched by elastic-viscoelastic FGM layers. Material properties are graded according to a power-law variation from the interfaces to the faces of the plate. The equilibrium equations of the FG sandwich plate are given based on a trigonometric shear deformation plate theory. Using Illyushin's method, the governing equations of the viscoelastic sandwich plate can be solved. Parametric study on the bending analysis of FG sandwich plates is being investigated. These parameters include (i) power-law index, (ii) plate aspect ratio, (iii) side-to-thickness ratio, (iv) loading type, (v) foundation stiffnesses, and (vi) time parameter.

Keywords: FG viscoelastic sandwich plate, Uniform load, Pasternak's foundations, Trigonometric plate theory.

1. INTRODUCTION

Since the use of FGMs widely increases in many engineering applications, for example, in aerospace, automotive and biomedical applications, considerable research efforts over the past few years have be developed to characterize FGMs, and also to design and analyze structural components made of these materials. The FGM can be prepared by continuously changing the constituents of multi-phase materials in a predetermined volume fraction of the constituent material. Due to the continuous change in material properties of a FGM, the interfaces between two materials disappear but the characteristics of two or more different materials of the composite are preserved.

From this point of view, great attention has been paid to the effect of introducing a gradient in elastic properties on the indentation strength of materials. Reddy and Cheng [1] have studied three-dimensional thermomechanical deformations of simply supported, FG rectangular plates by using an asymptotic method. Reddy [2] has analyzed the static behavior of FG rectangular plates based on his third-order shear deformation plate theory. Brischetto and Carrera [3] have investigated FGM plates subjected to a transverse mechanical load using the unified formulation and the Reissner's mixed variational theorem. Zenkour [4] has derived the exact solution for FGM plates using generalized sinusoidal shear deformation theory and presented numerical results on displacement and stress response of FGM plates under uniform loading. Zenkour [5,6] has employed the refined trigonometric shear deformation plate theory to study the bending response and free vibration of simply supported FG plate. Dynamic response of initially stressed FG plates and non-linear analysis of FG plates under transverse and in-plane loads have been investigated by Yang and Shen [7,8]. Several studies have been also performed to analyze the behavior of FGM beams, plates and shells [9-12].

Sandwich structures, which are composed of three layers, find applications in the diverse fields of engineering. The middle or core layer is bounded by two outer layers or face sheets [13]. Sandwich structures with viscoelastic cores are particularly useful in suppressing vibration over a wide frequency range. Viscoelastic sandwich structures offer advantages in high

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DOI : 10.1017/jmech.2012.53
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https://doi.org/10.1017/jmech.2012.53 Published online by Cambridge University Press

specific stiffness and strength values. Furthermore viscoelastic materials have good anti-vibration and anti-noise properties and this makes them suitable for a variety of structural engineering applications. A large amount of literature has been devoted to study the viscoelastic sandwich plates (see, *e.g.*, [14-17]). However, the above traditional sandwich structures have some drawbacks. The sudden change in material properties across the interfaces among different materials can result in large interlaminar stresses. To overcome these disadvantages, FGMs composed of two or more phases with different material properties and continuously varying composition distribution have been used as a core layer or face layers. Such materials were introduced as to take advantage of the desired material properties of each constituent material without interface problems. Owing to these reasons, a number of research works about the sandwich structures with FGM face layers have been established, in particular to study their elastic [18,19] and thermo-elastic behavior [20-22]. To investigate the effect of FGM core on performance of sandwich plates, Anderson [23] and Kashtalyan and Menshykova [24] have developed the 3-D elasticity solution for sandwich composites with a FG core subjected to transverse loading.

The oldest, most famous and most frequently used elastic foundation model is the one devised by Winkler in 1867, in which the plate-supporting soil is modeled as a series of closely spaced, mutually independent, linear elastic vertical springs which, evidently, provide resistance in direct proportion to the deflection of the plate. In this model, the properties of the soil are described only by one parameter (k_1), that represents the stiffness of the vertical springs [25]. Some studies on the structures resting on Winkler's foundation have been performed as in Abdalla and Ibrahim [26]. They have employed the discrete Reissner-Mindlin element to solve problems of thin and thick plates resting on Winkler-type foundation. Ergüven and Gedikli [27] have derived a mixed formulation for Timoshenko beam element on Winkler's foundation. However, Winkler's model is unable to take into account the continuity or cohesion of the soil. Also, the assumption that there is no interaction between adjacent springs results in ignoring the influence of the soil on either side of the beam. To overcome this weakness, many two-parameter elastic foundation models have been proposed, such as Pasternak's and Vlasov's elastic foundations. Based on the two-parameter elastic foundation model, numerous reports have been performed [28-30]. Bending response and free vibration analysis of FG thick plates resting on two-parameter elastic foundations based on the 3-D theory of elasticity have been presented by Huang *et al*. [31] and Malekzadeh [32].

In this article, we focus on the formulation of a FG viscoelastic sandwich plate. The present simply supported plate is made of an isotropic material resting on Pasternak's elastic foundations and subjected to a transverse uniform load. The face sheets are considered to be elastic-viscoelastic FGMs whereas the core layer is still elastic. Here, we consider the elastic coefficients vary according to a power-law variation in terms of the volume fractions of the constituents. The material properties are assumed to be graded in the thickness direction only. The trigonometric terms are used for the displacements in addition to the initial terms of a power series through the thickness. The governing equations of a FG sandwich plate are given based on the trigonometric shear deformation plate theory (TPT) presented by Zenkour [18,19]. The results obtained as per TPT are compared with those obtained as per first-order shear deformation plate theory (FPT) and the higher-order one (HPT) presented by Reddy [2]. The viscoelastic solution can be obtained by applying Illyushin's approximation method [33]. A detailed parametric study is carried out to highlight the influences of side-to-thickness ratio, material property graded indexes, coefficients of elastic foundations and time parameter on the bending response of FG viscoelastic sandwich plates.

2. PROBLEM FORMULATION

Consider an elastic-viscoelastic sandwich plate is composed of three layers of length a, width b and thickness h as shown in Fig. 1. Rectangular Cartesian coordinates $\{x_i\}$ (i = 1, 2, 3) are used to describe infinitesimal deformations of a three-layer viscoelastic sandwich plate such that the lower and upper surfaces of the plate before deformation lie in the planes ($x_3 = \pm h/2$) while x_ξ denote the in-plane coordinates. The mid-plane of the composite sandwich plate is defined by $x_3 = 0$. A repeated subscript implies summation over the range of the subscript with Latin subscripts ranging from 1 to 3 and Greek subscripts from 1 to 2. Partial differentiation will be denoted by a comma, *i.e.* $(\,)_{,i} = \partial(\,)/\partial x_i$.

The core layer is made of a homogeneous elastic material. The face layers are made of an isotropic elastic-viscoelastic FGMs which are smoothly gradient in the thickness direction only. The vertical positions of the bottom and top, and of the two interfaces between the layers are denoted by $h_0 = -h/2$, h_1, h_2, $h_3 = h/2$. Based on the power law distribution, the material elastic coefficients of each layer is assumed to be in terms of a power-law distribution as follows:

$$\overline{c}_{ij}^{(n)}(x_3) = c_{ij}^{\mathrm{v}} + (c_{ij}^{\mathrm{e}} - c_{ij}^{\mathrm{v}})\, V^{(n)}\,, \quad (n = 1, 2, 3)\,, \tag{1}$$

where $\overline{c}_{ij}^{(n)}$ are the elastic stiffness coefficients for material of layer n and the superscripts v and e refer to the viscoelastic and elastic constituents which denote the material properties of the faces and the core, respectively. For isotropic structures, the elastic constants c_{ij}^{v} and c_{ij}^{e} are given as

$$c_{\xi\xi}^{\alpha} = \frac{E_\alpha}{1-\nu_\alpha^2}\,, \quad c_{\xi\zeta}^{\alpha} = \frac{\nu_\alpha\, E_\alpha}{1-\nu_\alpha^2}\,, \quad c_{ll}^{\alpha} = \frac{E_\alpha}{1-\nu_\alpha^2}\,, \quad (\alpha = \mathrm{e}, \mathrm{v};\ l = 4, 5, 6)\,, \tag{2}$$

https://doi.org/10.1017/jmech.2012.53 Published online by Cambridge University Press

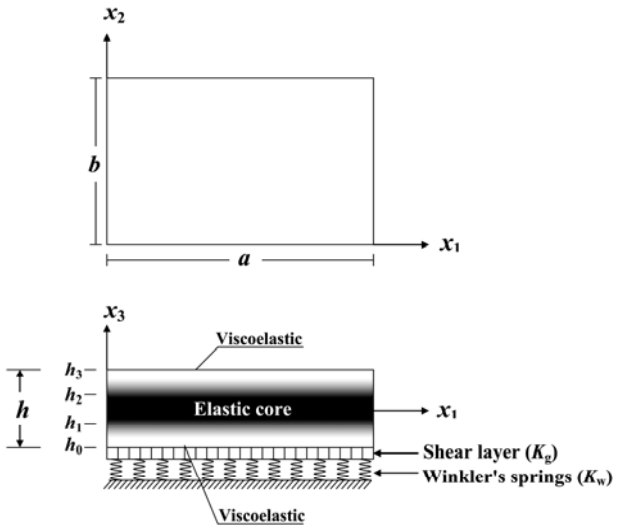


Fig. 1 Geometry of the FG viscoelastic sandwich plate resting on two parameter elastic foundations

where E_e and ν_e are Young's modulus and Poisson's ratio of the elastic material and they take constant values. The viscoelastic modulus E_v and the corresponding Poisson's ratio ν_v may be given in terms of the coefficient of volume compression K and the kernel of the relaxation function $\omega(t) = \bar{\omega}$ by

$$E_v(t) = \frac{9K\bar{\omega}}{2+\bar{\omega}}, \qquad \nu_v(t) = \frac{1-\bar{\omega}}{2+\bar{\omega}}. \tag{3}$$

The volume fraction $V^{(n)}$ is equal to unity in the core (*i.e.* $V^{(2)} = 1$, at $h_1 \le x_3 \le h_2$) while it follows a simple power-law through the thickness of the bottom and top layers that takes the form

$$V^{(1)} = \left(\frac{x_3 - h_0}{h_1 - h_0}\right)^k, \quad h_0 \le x_3 \le h_1,$$
$$V^{(3)} = \left(\frac{x_3 - h_3}{h_2 - h_3}\right)^k, \quad h_2 \le x_3 \le h_3, \tag{4}$$

where k is a parameter which denotes the power-law index and takes values greater than or equal to zero. The core is independent of the value of k which is fully elastic material. Note that the value of k equaling to zero represents a homogeneous elastic plate and the value of it equaling to infinity represents a viscoelastic-elastic-viscoelastic (v-e-v) sandwich plate. The above power-law assumption reflects a simple rule of mixtures used to obtain the effective properties of the elastic-viscoelastic sandwich plate (see Fig. 1). Also, note that the volume fraction of the elastic material is high near the interfaces, and that of the viscoelastic one is high near the bottom and top surfaces.

The FGM sandwich plate is considered to be resting on two-parameter elastic foundations and subjected to a transverse uniform load $q(x_\xi)$. The foundation is assumed to be attached to the plate and separation does not arise. The load-displacement relationship of the foundation is presumed to be $P_f = K_w u_3 - K_g \nabla^2 u_3$, where u_3 is the plate deflection, P_f is the force per unit area, K_w and K_g are the Winkler's and Pasternak's foundation stiffnesses and ∇^2 is the Laplace operator in x_ξ.

The displacements of a material point located at (x_i) in the plate may be written as follows

$$v_\xi(x_i, t) = u_\xi - x_3 u_{3,\xi} + \Psi(x_3)\,\phi_\xi,$$
$$v_3(x_i, t) = u_3, \tag{5}$$

where u_ξ and u_3 denote the in-plane and transverse displacements of points on the neutral axis of the sandwich plate and ϕ_ξ represents the rotation of a transverse normal about the plane $x_3 = 0$. All of the generalized displacements (u_i, ϕ_ξ) are independent of x_3. The displacement of the first-order shear deformation plate theory (FPT) is obtained by setting $\Psi(x_3) = x_3$ whereas the displacement of the higher-order shear deformation plate theory (HPT) presented by Reddy [2] is obtained by setting $\Psi(x_3) = zh\,(1 - 4z^2/3)$ where $z = x_3/h$. While the displacement of the trigonometric shear deformation plate theory (TPT) presented by Zenkour [18,19] is obtained by setting $\Psi(x_3) = \hbar \sin(\pi z)$ where $\hbar = h/\pi$. No transverse shear correction factors are needed for both HPT and TPT because a correct representation of the transverse shearing strain is given.

For the displacement field in Eq. (5), the linear strains $\epsilon_{ij} = (v_{i,j} + v_{j,i})/2$ are

$$\epsilon_{\xi\zeta} = \epsilon^0_{\xi\zeta} + x_3\,\epsilon^1_{\xi\zeta} + \Psi(x_3)\,\epsilon^2_{\xi\zeta},$$
$$\epsilon_{\xi 3} = \Psi_{,3}\,\epsilon^0_{\xi 3}, \tag{6}$$

where the non-vanishing strain measures $\epsilon_{ij} \equiv \epsilon_{ij}(x_\gamma)$ are expressed in terms of the displacement quantities as

$$\epsilon^0_{\xi\zeta} = \frac{1}{2}(u_{\xi,\zeta} + u_{\zeta,\xi}), \quad \epsilon^0_{\xi 3} = \frac{1}{2}\phi_\xi,$$
$$\epsilon^1_{\xi\zeta} = -u_{3,\xi\zeta}, \quad \epsilon^2_{\xi\zeta} = \frac{1}{2}(\phi_{\xi,\zeta} + \phi_{\zeta,\xi}). \tag{7}$$

The stress-strain relationships accounting for transverse shear deformation in the plate coordinates can be written as

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix}^{(n)} = \begin{bmatrix} \bar{c}_{11} & \bar{c}_{12} & 0 & 0 & 0 \\ & \bar{c}_{22} & 0 & 0 & 0 \\ & & \bar{c}_{44} & 0 & 0 \\ & & & \bar{c}_{55} & 0 \\ \text{symm.} & & & & \bar{c}_{66} \end{bmatrix}^{(n)} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{23} \\ \epsilon_{13} \\ \epsilon_{12} \end{Bmatrix}, \tag{8}$$

We introduce the following definitions of stress resultants:

$$(N_{\xi\zeta}, M_{\xi\zeta}, S_{\xi\zeta}) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} (1, x_3, \Psi)\,\sigma^{(n)}_{\xi\zeta}\,dx_3,$$
$$Q_{\xi 3} = \bar{K} \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \Psi_{,3}\,\sigma^{(n)}_{\xi 3}\,dx_3, \tag{9}$$

where $N_{\xi\zeta}$ and $M_{\xi\zeta}$ are the basic components of stress resultants and stress couples; $S_{\xi\zeta}$ are additional stress couples associated with the transverse shear effects; $Q_{\xi 3}$

https://doi.org/10.1017/jmech.2012.53 Published online by Cambridge University Press

are transverse shear stress resultants and $\bar{K}$ is the shear correction factor of FPT. Here, h_n and h_{n-1} are the top and bottom x_3-coordinates of the nth layers.

3. GOVERNING EQUATIONS

The governing equations of the equilibrium associated with the displacement field in Eq. (5) are

$$\begin{aligned} &N_{\xi\zeta,\zeta} = 0\,, \\ &M_{\xi\zeta,\xi\zeta} - P_f + q = 0\,, \\ &S_{\xi\zeta,\zeta} - Q_{\xi 3} = 0\,. \end{aligned} \tag{10}$$

Using Eqs. (8) and (9), the force and moment resultants can be related to the total strains in the following equations

$$\begin{aligned} &\begin{Bmatrix} N \\ M \\ S \end{Bmatrix} = \begin{bmatrix} A^{11} & A^{12} & A^{13} \\ & A^{22} & A^{23} \\ \text{symm.} & & A^{33} \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \varepsilon^1 \\ \varepsilon^2 \end{Bmatrix}, \\ &Q_{\xi 3} = B_{\xi 3}\, \epsilon^0_{\xi 3}\,, \end{aligned} \tag{11}$$

where

$$\begin{aligned} &N = \{N_{11} \;\; N_{22} \;\; N_{12}\}^T\,, \\ &M = \{M_{11} \;\; M_{22} \;\; M_{12}\}^T\,, \\ &S = \{S_{11} \;\; S_{22} \;\; S_{12}\}^T\,, \end{aligned} \tag{12}$$

$$\begin{aligned} &\varepsilon^0 = \{\epsilon^0_{11} \;\; \epsilon^0_{22} \;\; \epsilon^0_{12}\}^T\,, \\ &\varepsilon^1 = \{\epsilon^1_{11} \;\; \epsilon^1_{22} \;\; \epsilon^1_{12}\}^T\,, \\ &\varepsilon^2 = \{\epsilon^2_{11} \;\; \epsilon^2_{22} \;\; \epsilon^2_{12}\}^T\,, \end{aligned} \tag{13}$$

$$A^{ij} = \begin{bmatrix} A^{ij}_{11} & A^{ij}_{12} & 0 \\ & A^{ij}_{22} & 0 \\ \text{symm.} & & A^{ij}_{66} \end{bmatrix}, \quad i,\, j = 1,\, 2,\, 3\,. \tag{14}$$

The stiffness coefficients A^{ij}_{kl} and $B_{\xi 3}$ are given as

$$\begin{aligned} &\{A^{11}_{kl}\,,\;\; A^{12}_{kl}\,,\;\; A^{22}_{kl}\} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \bar{c}^{(n)}_{kl} \{1,\, x_3,\, x_3^2\}\, \mathrm{d}x_3\,, \\ &\{A^{13}_{kl}\,,\;\; A^{23}_{kl}\,,\;\; A^{33}_{kl}\} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \bar{c}^{(n)}_{kl} \Psi \{1,\, x_3,\, \Psi\}\, \mathrm{d}x_3\,, \\ &\bar{B}_{mm} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \bar{c}^{(n)}_{mm} [\Psi_{,3}]^2\, \mathrm{d}x_3\,, \\ &\qquad (k,l = 1,\, 2,\, 6\,;\;\; m = 4,\, 5\,;\;\; \bar{B}_{44} = B_{23},\;\; \bar{B}_{55} = B_{13})\,. \end{aligned} \tag{15}$$

4. TRIGONOMETRIC SOLUTION

Rectangular plates are generally classified in accordance with the type of the used support. Here we are concerned with the exact solutions of Eqs. (10) for simply supported FGM sandwich plate. The following boundary conditions are imposed at the side edges for the shear deformation theories:

$$\begin{aligned} &u_2 = u_3 = \phi_2 = N_{11} = M_{11} = S_{11} = 0 \quad \text{at} \;\; x_1 = 0,\, a\,, \\ &u_1 = u_3 = \phi_1 = N_{22} = M_{22} = S_{22} = 0 \quad \text{at} \;\; x_2 = 0,\, b\,, \end{aligned} \tag{16}$$

The following representation for the displacement quantities of the shear deformation theories, that satisfy the simply-supported boundary conditions, is appropriate in the case of our problem:

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \phi_1 \\ \phi_2 \end{Bmatrix} = \sum_{r,s=1,3,5,\ldots}^{\infty} \begin{Bmatrix} U_{rs} \cos(\lambda x_1) \sin(\mu x_2) \\ V_{rs} \sin(\lambda x_1) \cos(\mu x_2) \\ W_{rs} \sin(\lambda x_1) \sin(\mu x_2) \\ X_{rs} \cos(\lambda x_1) \sin(\mu x_2) \\ Y_{rs} \sin(\lambda x_1) \cos(\mu x_2) \end{Bmatrix}, \tag{17}$$

where $\lambda = r\pi/a$, $\mu = s\pi/b$; r and s are mod numbers and U_{rs}, V_{rs}, W_{rs}, X_{rs} and Y_{rs} are arbitrary parameters.

For further computational reasons the expressions of the stress components (8) in terms of the arbitrary parameters U_{rs}, V_{rs}, W_{rs}, X_{rs} and Y_{rs} are recorded by the help of Eqs. (6), (7) and (17). They read

$$\begin{aligned} \sigma^n_{11} = -\sum_{r,s=1,3,5,\ldots}^{\infty} &\Big[\bar{c}^n_{11}\left(\lambda U_{rs} - \lambda^2 x_3 W_{rs} + \lambda \Psi(x_3) X_{rs}\right) \\ &+ \bar{c}^n_{12}\left(\mu V_{rs} - \mu^2 x_3 W_{rs} + \mu \Psi(x_3) Y_{rs}\right)\Big] \sin(\lambda x_1) \sin(\mu x_2)\,, \end{aligned} \tag{18}$$

$$\begin{aligned} \sigma^n_{22} = -\sum_{r,s=1,3,5,\ldots}^{\infty} &\Big[\bar{c}^n_{12}\left(\lambda U_{rs} - \lambda^2 x_3 W_{rs} + \lambda \Psi(x_3) X_{rs}\right) \\ &+ \bar{c}^n_{22}\left(\mu V_{rs} - \mu^2 x_3 W_{rs} + \mu \Psi(x_3) Y_{rs}\right)\Big] \sin(\lambda x_1) \sin(\mu x_2)\,, \end{aligned} \tag{19}$$

$$\sigma^n_{23} = \sum_{r,s=1,3,5,\ldots}^{\infty} \bar{c}^n_{44} \Psi(x_3)_{,3}\; Y_{rs}\; \sin(\lambda x_1) \cos(\mu x_2)\,, \tag{20}$$

$$\sigma^n_{13} = \sum_{r,s=1,3,5,\ldots}^{\infty} \bar{c}^n_{55} \Psi(x_3)_{,3}\; X_{rs}\; \cos(\lambda x_1) \sin(\mu x_2)\,, \tag{21}$$

$$\begin{aligned} \sigma^n_{12} = \sum_{r,s=1,3,5,\ldots}^{\infty} &\bar{c}^n_{66} \big[\mu U_{rs} - \lambda V_{rs} - 2\lambda\mu x_3 W_{rs} \\ &+ \Psi(x_3)(\mu X_{rs} + \lambda Y_{rs})\big] \cos(\lambda x_1) \cos(\mu x_2)\,. \end{aligned} \tag{22}$$

The external force is presented in the following trigonometric series form

$$q(x_1,\, x_2) = \sum_{r,s=1,3,5,\ldots}^{\infty} q_{rs} \sin(\lambda x_1) \sin(\mu x_2)\,, \tag{23}$$

where the coefficients q_{rs} for the case of uniformly distributed load are defined as follows:

$$q_{rs} = \frac{16 q_0}{rs\pi^2}\,, \tag{24}$$

in which $q_0(t)$ is a transient function according to the viscoelastic response of the bending problem.

With the help of Eqs. (17) and (23), Eq. (10) becomes

$$[F]\{\Omega\} = \{L\}\,, \tag{25}$$

where $\{\Omega\}$ and $\{L\}$ denote the columns

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$$\{\Omega\}^T = \{U_{rs},\ V_{rs},\ W_{rs},\ X_{rs},\ Y_{rs}\},$$
$$\{L\}^T = \left\{0,\ 0,\ -\frac{16q_0}{rs\pi^2},\ 0,\ 0\right\}. \tag{26}$$

and the elements $F_{ij} = F_{ji}$ of matrix $[F]$ are given by:

$$\begin{aligned}
&F_{11} = -(\lambda^2 A_{11}^{11} + \mu^2 A_{66}^{11}),\\
&F_{12} = -\lambda\mu(A_{12}^{11} + A_{66}^{11}),\\
&F_{13} = \lambda\left[\lambda^2 A_{11}^{12} + \mu^2(A_{12}^{12} + 2A_{66}^{12})\right],\\
&F_{14} = -(\lambda^2 A_{11}^{13} + \mu^2 A_{66}^{13}),\\
&F_{15} = -\lambda\mu(A_{12}^{13} + A_{66}^{13}),\\
&F_{22} = -(\lambda^2 A_{66}^{11} + \mu^2 A_{22}^{11}),\\
&F_{23} = \mu\left[\lambda^2(A_{12}^{12} + 2A_{66}^{12}) + \mu^2 A_{22}^{12}\right],\ F_{24} = F_{15},\\
&F_{25} = -(\lambda^2 A_{66}^{13} + \mu^2 A_{22}^{13}),\\
&F_{33} = -\left[\lambda^4 A_{11}^{22} + 2\lambda^2\mu^2(A_{12}^{22} + 2A_{66}^{22}) + \mu^4 A_{22}^{22} + K_w + K_g(\lambda^2 + \mu^2)\right],\\
&F_{34} = \lambda\left[\lambda^2 A_{11}^{23} + \mu^2(A_{12}^{23} + 2A_{66}^{23})\right],\\
&F_{35} = \mu\left[\lambda^2(A_{12}^{23} + 2A_{66}^{23}) + \mu^2 A_{22}^{23}\right],\\
&F_{44} = -(\lambda^2 A_{11}^{33} + \mu^2 A_{66}^{33} + B_{13}),\\
&F_{45} = -\lambda\mu(A_{12}^{33} + A_{66}^{33}),\\
&F_{55} = -(\lambda^2 A_{66}^{33} + \mu^2 A_{22}^{33} + B_{23}).
\end{aligned} \tag{27}$$

5. ILLYUSHIN'S APPROXIMATION METHOD

The viscoelastic solution of the FG sandwich plates resting on elastic foundations can be obtained by using the Illyushin's approximation method [33]. For a given set of values of $\bar{\omega}$, the corresponding elastic solution is obtained numerically with $q_0(t) = \bar{q}_0$ where $\bar{q}_0$ is a constant. In the elastic composites, the stresses are functions of x_1, x_2, x_3 and $\bar{\omega}$, while in the viscoelastic composites they are operator functions of x_1, x_2, x_3 and time t. By replacing the double indices 11, 22, 23, 13 and 12 of the stress components by the single indices 1, 2, 4, 5 and 6, respectively, the stresses $\sigma_i^{(n)}$ according to Illyushin's method can be represented in the form

$$\sigma_i^{(n)}(x_1, x_2, x_3, \bar{\omega}) = \sum_{j=1}^{5} \Re_{ij}^{(n)}(x_1, x_2, x_3)\, \mathcal{X}_j(\bar{\omega}), \tag{28}$$

where $\mathcal{X}_j(\bar{\omega})$ are some known kernels given as

$$\begin{aligned}
&\mathcal{X}_1 = 1,\ \ \mathcal{X}_2 = \bar{\omega},\ \ \mathcal{X}_3 = \Pi(t) = \frac{1}{\bar{\omega}},\\
&\mathcal{X}_4 = g_{\frac{1}{2}}(t) = \frac{1}{1 + \frac{1}{2}\bar{\omega}},\ \ \mathcal{X}_5 = g_2(t) = \frac{1}{1 + 2\bar{\omega}}.
\end{aligned} \tag{29}$$

The functions $\Re_{ij}^{(n)}(x_1, x_2, x_3)$ can be determined, for example, so as to achieve a minimum squared error fit to $\sigma_i^{(n)}(x_1, x_2, x_3, \bar{\omega})$. For this reason, we have the following expression

$$J = \int_{\bar{\omega}_0}^{\bar{\omega}_1} \left[\sigma_i^{(n)}(x_1, x_2, x_3, \bar{\omega}) - \sum_{j=1}^{5} \Re_{ij}^{(n)}(x_1, x_2, x_3)\, \mathcal{X}_j(\bar{\omega})\right]^2 \mathrm{d}\bar{\omega}, \tag{30}$$

where $\bar{\omega}_0$ and $\bar{\omega}_1$ are the limits of kernel $\bar{\omega}$ and $0 \le \bar{\omega}_0 \le \bar{\omega}_1 \le 1$. Taking $\bar{\omega}_0 = 0$ and $\bar{\omega}_1 = 1$ and equating $\partial J / \partial \Re_{ij}^{(n)}$ to zero, we have the following system of algebraic equations

$$\begin{aligned}
&\int_0^1 \sum_{j=1}^{5} \Re_{ij}^{(n)}(x_1, x_2, x_3)\, \mathcal{X}_k(\bar{\omega})\, \mathcal{X}_j(\bar{\omega})\, \mathrm{d}\bar{\omega}\\
&\quad = \int_0^1 \mathcal{X}_k(\bar{\omega})\, \sigma_i^{(n)}(x_1, x_2, x_3, \bar{\omega})\, \mathrm{d}\bar{\omega}.
\end{aligned} \tag{31}$$

The viscoelastic solution may be recorded to determine explicit formulae for $\sigma_i^{(n)}$ as functions of the coordinates x_1, x_2 and x_3 and time t. Then

$$\begin{aligned}
&\sigma_i^{(n)}(x_1, x_2, x_3, t) = \Re_{i1}^{(n)} q_0(t)\\
&+\Re_{i2}^{(n)} \int_0^t \omega(t-\tau)\, \mathrm{d}q_0(\tau) + \Re_{i3}^{(n)} \int_0^t \Pi(t-\tau)\, \mathrm{d}q_0(\tau)\\
&+\Re_{i4}^{(n)} \int_0^t g_{\frac{1}{2}}(t-\tau)\, \mathrm{d}q_0(\tau) + \Re_{i5}^{(n)} \int_0^t g_2(t-\tau)\, \mathrm{d}q_0(\tau).
\end{aligned} \tag{32}$$

Taking $q_0(t) = \bar{q}_0 H(t)$ where $H(t)$ is Heaviside's unit step function,

$$H(t) = \begin{cases} 1 & \text{if } t \ge 0, \\ 0 & \text{if } t < 0. \end{cases} \tag{33}$$

Then, Eq. (32) takes the form

$$\begin{aligned}
&\sigma_i^{(n)}(x_1, x_2, x_3, t)\\
&= \bar{q}_0 \left[\Re_{i1}^{(n)} H(t) + \Re_{i2}^{(n)} \omega(t) + \Re_{i3}^{(n)} \Pi(t) + \Re_{i4}^{(n)} g_{\frac{1}{2}}(t) + \Re_{i5}^{(n)} g_2(t)\right].
\end{aligned} \tag{34}$$

Assuming an exponential relaxation function [34,35]

$$\omega(t) = c_1 + c_2\, \mathrm{e}^{-t/t_s}, \tag{35}$$

where c_1 and c_2 are constants to be experimentally determined, and t_s is the relaxation time. The Laplace-Carson transform can be used to determine the functions $\prod(t)$, $g_{1/2}(t)$ and $g_2(t)$. Since the transform of $\omega(t)$ is

$$\omega^*(s) = c_1 + c_2 \frac{s}{s + 1/t_s}, \tag{36}$$

then, we can obtain the following three equations (for more details, see Zenkour [35])

$$\Pi(t) = \frac{1}{c_1}\left[1 - \frac{c_2}{c_1 + c_2}\, \mathrm{e}^{-\frac{c_1 \tau}{(c_1 + c_2)}}\right], \tag{37}$$

$$g_{\frac{1}{2}}(t) = \frac{2}{2 + c_1}\left[1 - \frac{c_2}{2 + c_1 + c_2}\, \mathrm{e}^{-\frac{(2 + c_1)\tau}{2 + c_1 + c_2}}\right], \tag{38}$$

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$$g_2(t)=\frac{1}{1+2c_1}\left[1-\frac{2c_2}{1+2(c_1+c_2)}\mathrm{e}^{-\frac{(1+2c_1)\tau}{1+2(c_1+c_2)}}\right]. \tag{39}$$

Using Eqs. (35), (37), (38) and (39), the final form of the stresses in terms of the time parameter τ is given by

$$\begin{aligned}&\sigma_i^{(n)}(x_1,x_2,x_3,t)\\&=\bar{q}_0\left\{\Re_{i1}^{(n)}H(t)+\Re_{i2}^{(n)}[c_1+c_2\mathrm{e}^{-\tau}]+\frac{\Re_{i3}^{(n)}}{c_1}\left[1-\frac{c_2}{c_1+c_2}\mathrm{e}^{-\frac{c_1\tau}{c_1+c_2}}\right]\right.\\&\quad+\frac{2\Re_{i4}^{(n)}}{2+c_1}\left[1-\frac{c_2}{2+c_1+c_2}\mathrm{e}^{-\frac{(2+c_1)\tau}{2+c_1+c_2}}\right]\\&\quad\left.+\frac{\Re_{i5}^{(n)}}{1+2c_1}\left[1-\frac{2c_2}{1+2(c_1+c_2)}\mathrm{e}^{-\frac{(1+2c_1)\tau}{1+2(c_1+c_2)}}\right]\right\}.\end{aligned} \tag{40}$$

6. SEVERAL TYPES OF VISCOELASTIC SANDWICH PLATES

Figure 2 shows the through-the-thickness variation of the volume fraction function of the material for $k = 0.1$, 0.5, 1, 2, 5, 10. Note that the core of the symmetric and nonsymmetric plates are fully elastic while the bottom and top surfaces of the plate are viscoelastic-rich.

6.1 The (1-2-1) FGM Viscoelastic Sandwich Plate

Here the core thickness is twice the face thickness (see Fig. 2(a)). So, one obtains

$$h_1=-\frac{h}{4},\quad h_2=\frac{h}{4}. \tag{41}$$

6.2 The (1-1-1) FGM Viscoelastic Sandwich Plate

As shown in Fig. 2(b) the plate is made of three equal-thickness layers. So, one gets

$$h_1=-\frac{h}{6},\quad h_2=\frac{h}{6}. \tag{42}$$

6.3 The (2-1-2) FGM Viscoelastic Sandwich Plate

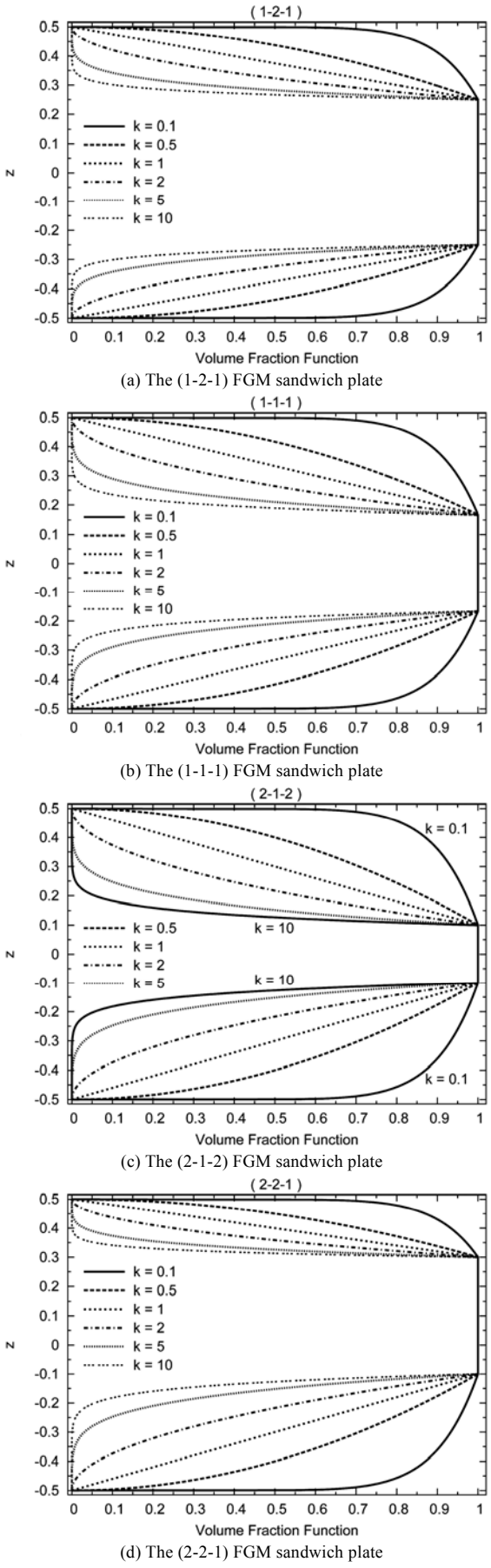
In this state the core of the plate is half the face thickness (see Fig. 2(c)). Thus,

$$h_1=-\frac{h}{10},\quad h_2=\frac{h}{10}. \tag{43}$$

6.4 The (2-2-1) FGM Viscoelastic Sandwich Plate

Here the sandwich plate is nonsymmetric about the mid-plane in which the core thickness equals the lower face thickness while it is twice the upper face thickness (see Fig. 2(d)). Thus, we have

$$h_1=-\frac{h}{10},\quad h_2=\frac{3h}{10}. \tag{44}$$



(a) The (1-2-1) FGM sandwich plate

(b) The (1-1-1) FGM sandwich plate

(c) The (2-1-2) FGM sandwich plate

(d) The (2-2-1) FGM sandwich plate

Fig. 2 Through-thickness distributions of volume fraction function for various values of the power-law index k and different types of viscoelastic sandwich plates

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7. NUMERICAL RESULTS AND DISCUSSIONS

Elastic foundation analysis of FG viscoelastic sandwich plates subjected to a uniformly distributed load is studied in this paper. The static analysis is conducted for two combinations of elastic and viscoelastic materials. Based on the analysis of the previous sections, the stresses and transverse displacements of simply supported plates are obtained by using the trigonometric shear deformation plate theory. Many examples have been solved numerically using the following fixed data (unless otherwise stated) $\upsilon_e = 0.3$, $a = 10h$, $b = 2a$, $K = 0.1E_e$, $\bar{\omega} = 0.3$, $k_1 = 10$, $k_2 = 10$, $k = 1$, $c_1 = 0.1$, $c_2 = 0.9$. Note that the shear correction factor of FPT is fixed to be 5/6. Numerical results are presented in terms of non-dimensional stresses and deflection. The various nondimensional parameters used are

$$w = \frac{10^3 D}{\bar{q}_0 a^4} u_3\left(\frac{a}{2}, \frac{b}{2}\right), \quad \sigma_{xx} = \frac{1}{\bar{q}_0}\sigma_1\left(\frac{a}{2}, \frac{b}{2}, x_3\right),$$

$$\sigma_{xz} = \frac{1}{\bar{q}_0}\sigma_5\left(0, \frac{b}{2}, x_3\right),$$

$$K_w = \frac{Dk_1}{a^4}, \quad K_g = \frac{Dk_2}{a^2}, \quad D = \frac{h^3 E_e}{12(1-\upsilon_e^2)}.$$

7.1 Comparison Studies

As a check on the numerical accuracy of the theory and formulation, results were obtained for isotropic homogeneous plate and compared with published values. For isotropic rectangular plate ($k = 0$) and from Eq. (15) we have

$$A_{ij}^{11} = hc_{ij}^e \quad A_{ij}^{12} = A_{ij}^{13} = 0, \quad A_{ij}^{22} = \frac{h^3 c_{ij}^e}{12},$$

$$A_{ij}^{23} = \frac{2h^3 c_{ij}^e}{\pi^3}, \quad A_{ij}^{33} = \frac{h^3 c_{ij}^e}{2\pi^2},$$

$$\bar{B}_{ll} = \frac{hc_{ll}^e}{2}, \quad (i, j = 1, 2, 6, \ l = 4, 5).$$

The central deflection w of an isotropic homogeneous square plate under uniformly distributed load obtained by the present theory is compared with those obtained by the finite element method [36] and the analytical solution [37] for different values of Winkler's foundation parameter k_1 as shown in Table 1. For small values of the parameter k_1, severe deformation can be clearly observed. The deflection reaches its peak value when the plate is without elastic foundation, and then a steep decays as the Winkler's parameter increases. Good agreement is achieved between the present solution and the finite element method results given by Buczkowski and Torbacki [36] and the analytical solution presented by Timoshenko and Woinowsky-Krieger [37].

We turn now our attention to the bending problem of isotropic inhomogeneous sandwich plates with or without elastic foundations and subjected to uniform load or sinusoidal load. The central deflection w, in-plane longitudinal stress σ_{xx} ($x_3 = h/2$) and transverse shear stress σ_{xz} ($x_3 = 0$) obtained by TPT are compared with those obtained by HPT and FPT as exhibited in Table 2. Results increase as the core thickness of the sandwich plate is decreasing.

Table 1 Comparison of the deflection w of a homogeneous square plate under uniformly distributed load resting on Winkler's elastic foundation ($k = 0$, $a/h = 20$)

k_1	w		
	Present	Finite element method [36]	Analytical solution [37]
0	4.1149	4.1197	4.0624
1	4.1039	4.1088	4.0517
3^4	3.3813	3.3855	3.3472
5^4	1.5094	1.5114	1.5060
10^4	0.1108	0.1096	0.1115
15^4	0.0196	0.0191	0.0195

7.2 Parametric Studies

Parametric studies have been undertaken to investigate the bending analysis of FG viscoelastic sandwich plate resting on elastic foundation. Table 3 shows the effects of the power-law index k and the side-to-thickness ratio a/h on the deflection w and the stresses σ_{xx} ($x_3 = h/2$) and σ_{xz} ($x_3 = 0$) of the FG viscoelastic sandwich square plates resting on two-parameter elastic foundations and under uniformly or sinusoidally distributed loads. As it is well known, the uniform load distribution always overpredicts the deflection and stresses magnitude. As the plate becomes more and more elastic, the deflection decreases whereas the transverse shear stress increases. However, the in-plane longitudinal stress of the FGM sandwich plate increases gradually as the power-law index k increases. Regardless the type of the plate, the deflection w is monotonically decreasing as a/h increases, while the change of the stresses σ_{xx} and σ_{xz} is reversed.

Typical results are shown in Figs. 3 ~ 5. Each figure contains four different sandwich plates: (a) The (1-2-1) FGM sandwich plate, (b) The (1-1-1) FGM sandwich plate, (c) The (2-1-2) FGM sandwich plate, (d) The (2-2-1) FGM sandwich plate. Apart from the types of the sandwich plates stated above, the deflection is minimum for the elastic plate and maximum for the v-e-v sandwich plate and the response of the FGM sandwich plate is intermediate to that of the homogeneous elastic plate and v-e-v sandwich plate as shown in Fig. 3. Also, it is to be noted that the differences between the curves may be unchanged with the increase of the side-to-thickness ratio. Figure 4 shows that the maximum compressive (tensile) in-plane longitudinal stress σ_{xx} for the homogeneous plate occurs at the top (bottom) surfaces while it occurs at the top (bottom) interfaces for the v-e-v sandwich plate. For the all types of the sandwich plate, the maximum value of the shear stress σ_{xz} takes place at the mid-plane of the plates as exhibited in Fig. 5. It is important to observe from Figs. 4 and 5 that a sudden change in the stresses takes place at the interfaces of the v-e-v sandwich plate. Thus, the FGMs are used here to overcome that problem.

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Table 2 The deflection w and stresses σ_{xx} and σ_{xz} of the FG viscoelastic sandwich square plates under uniformly or sinusoidally distributed loads and with or without elastic foundations

Scheme	k_1	k_2	Theory	UL			SL		
				w	σ_{xx}	σ_{xz}	w	σ_{xx}	σ_{xz}
1-2-1	0	0	TPT	7.89475	9.10780	7.63500	5.00158	4.42816	2.73669
			HPT	7.89428	9.08878	7.43546	5.00126	4.42578	2.65883
			FPT	7.87123	8.74923	5.77342	4.98543	4.39279	2.04241
	10	0	TPT	7.50906	8.76339	7.42059	4.76333	4.21723	2.60633
			HPT	7.50863	8.74459	7.22716	4.76304	4.21498	2.53219
			FPT	7.48796	8.40875	5.61392	4.74869	4.18419	1.94542
	10	10	TPT	3.80817	5.20488	5.02628	2.45501	2.17355	1.34330
			HPT	3.80807	5.18897	4.90001	2.45494	2.17246	1.30512
			FPT	3.80353	4.90412	3.83034	2.45112	2.15974	1.00416
1-1-1	0	0	TPT	9.40283	10.84180	8.08625	5.95519	5.28832	2.89667
			HPT	9.40346	10.82234	7.91087	5.95562	5.28590	2.82748
			FPT	9.38777	10.45770	6.37544	5.94485	5.25058	2.25538
	10	0	TPT	8.86096	10.35652	7.81849	5.62048	4.99109	2.73387
			HPT	8.86151	10.33725	7.64949	5.62086	4.98878	2.66855
			FPT	8.84772	9.97680	6.16731	5.61127	4.95595	2.12882
	10	10	TPT	4.12397	5.77104	5.07207	2.66444	2.36607	1.29601
			HPT	4.12407	5.75454	4.96712	2.66453	2.36489	1.26500
			FPT	4.12161	5.44992	4.02861	2.66237	2.35144	1.01006
2-1-2	0	0	TPT	10.61742	12.23535	8.62509	6.72349	5.97856	3.08879
			HPT	10.61901	12.21572	8.46409	6.72458	5.97611	3.02456
			FPT	10.60798	11.82807	6.95568	6.71702	5.93861	2.46065
	10	0	TPT	9.93169	11.62041	8.30506	6.29991	5.60191	2.89420
			HPT	9.93306	11.60095	8.15066	6.30088	5.59956	2.83398
			FPT	9.92352	11.21794	6.70097	6.29424	5.56482	2.30577
	10	10	TPT	4.33850	6.18044	5.22011	2.80800	2.49689	1.29000
			HPT	4.33871	6.16354	5.12775	2.80819	2.49563	1.26306
			FPT	4.33725	5.84122	4.24029	2.80687	2.48160	1.02824
2-2-1	0	0	TPT	8.65405	9.28600	7.98517	5.48188	4.51236	2.86187
			HPT	8.65426	9.26726	7.79490	5.48203	4.51000	2.78715
			FPT	8.63347	8.91443	6.12017	5.46775	4.47573	2.16507
	10	0	TPT	8.19284	8.90310	7.74054	5.19699	4.27785	2.71314
			HPT	8.19302	8.88455	7.55665	5.19712	4.27561	2.64230
			FPT	8.17456	8.53569	5.93558	5.18429	4.24370	2.05283
	10	10	TPT	3.97472	5.12334	5.12544	2.56534	2.11164	1.33926
			HPT	3.97475	5.10753	5.00846	2.56537	2.11050	1.30428
			FPT	3.97111	4.81234	3.95905	2.56224	2.09737	1.01457

Figures 6 ~ 8 depict the effect of the relaxation function $\bar{\omega}$ on the central deflection and the stresses σ_{xx} and σ_{xz}, respectively, for the plate resting on Winkler's or Pasternak's elastic foundations and under a transverse uniform load. It can be seen that the deflection increases as the relaxation function decreases and the aspect ratio b/a increases. For the plate with two-parameter elastic foundation, the difference is rather small compared with that in the case of one-parameter elastic foundation as shown in Fig. 6. The effect of $\bar{\omega}$ on the stress σ_{xx} has the same behavior whether the plate resting on one- or two-parameter elastic foundation as plotted in Fig. 7. While that effect on the stress σ_{xz} is not the same in the above two states (see Fig. 8). Note that, in the core of the sandwich plate on Pasternak's elastic foundations, the values of the transverse shear stress σ_{xz} for $\bar{\omega} = 0.3$ and $\bar{\omega} = 0.5$ are coincided.

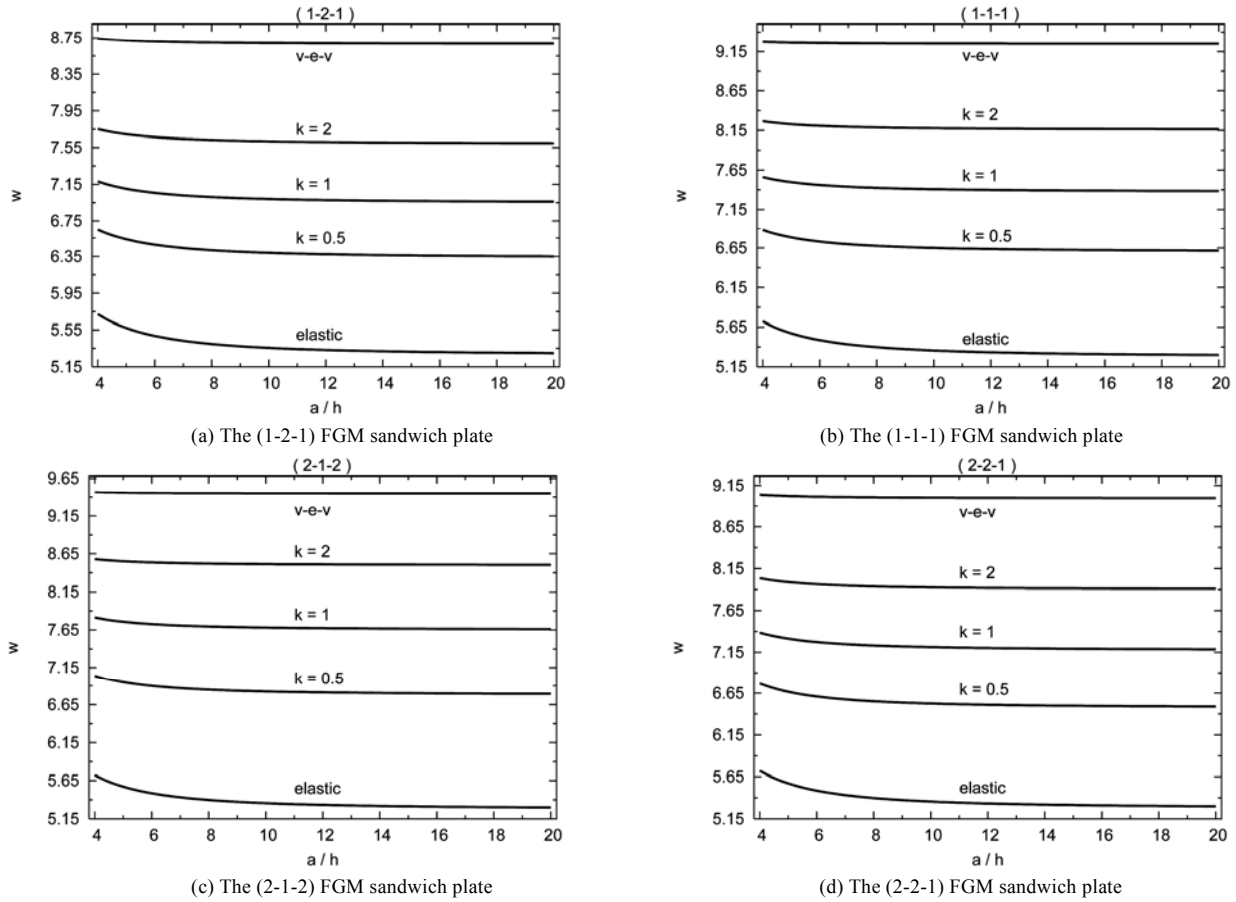
The results in Figs. 9 ~ 13 are estimated when the sandwich plate is acted by a transverse uniform load and by a sinusoidal one. Figures 9 and 10 exhibit the in-plane longitudinal stress σ_{xx} and transverse shear stress σ_{xz} through-the-thickness of the (1-2-1) FG sandwich plate $k = 0.5$ for various values of the aspect ratio b/a. It can be seen that the stresses σ_{xx} and σ_{xz} increase with the increase of b/a.

Figures 11 ~ 13 display, respectively, the variation of the deflection and the stresses σ_{xx} and σ_{xz} against the time parameter τ for different values of the foundation parameters k_1 and k_2. It is noticed that a decrement occurs for the deflection and stresses as either k_1 or k_2 increases. It is also evident that the sensitivity of the results to the variation of k_2 are greater than that to the variation of k_1. Figures 11 and 12 shows that an increment occurs for the deflection of the sandwich plate as τ increases. However, the change of longitudinal normal stress σ_{xx} is reversed. The transverse shear stress σ_{xz} of the sandwich plate under uniform load for $(k_1, k_2) = (5, 5)$ and $(15, 5)$ increases to get a constant value as the time parameter τ increases whereas it decreases slightly to get also a constant value as τ increases as investigated in Fig. 13(a). Whereas, for the plate under sinusoidal load and regardless the values of k_1 and k_2, the stress σ_{xz} decreases gradually as the increase of τ (see Fig. 13(b)).

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Table 3 Effects of the power-law index and the side-to- thickness ratio on w, σ_{xx} and σ_{xz} of the (1-2-1) FG viscoelastic sandwich square plates under uniformly or sinusoidally distributed loads

k	a/h	UL			SL		
		w	σ_{xx}	σ_{xz}	w	σ_{xx}	σ_{xz}
elastic	5	2.92671	7.69625	2.31408	1.90263	3.10955	0.74335
	10	2.70582	28.87089	5.16454	1.73529	12.77347	1.57583
	20	2.64679	112.32202	10.72197	1.69027	51.45467	3.19961
	50	2.62997	695.09263	27.14150	1.67742	322.23541	8.03326
1/2	5	3.55264	1.19817	2.29268	2.31135	0.46560	0.68219
	10	3.38083	4.47123	5.09704	2.17426	1.91395	1.44125
	20	3.33527	17.37997	10.54674	2.13754	7.71106	2.92369
	50	3.32232	107.56560	26.65262	2.12707	48.29257	7.33859
2	5	4.39362	1.64095	2.21834	2.86820	0.60101	0.58165
	10	4.28753	6.10516	4.91549	2.77350	2.46854	1.22088
	20	4.25989	23.73974	10.14130	2.74834	9.94270	2.47236
	50	4.25207	147.00260	25.59314	2.74118	62.26376	6.20268
5	5	4.85234	1.90955	2.14568	3.17749	0.67696	0.51745
	10	4.77969	7.14538	4.76253	3.10580	2.77687	1.08095
	20	4.76106	27.84774	9.82932	3.08687	11.18037	2.18623
	50	4.75581	172.57552	24.80733	3.08149	70.00673	5.48286
10	5	5.04549	2.02887	2.10999	3.30941	0.70972	0.48903
	10	4.98587	7.63110	4.69236	3.24714	2.90923	1.01922
	20	4.97071	29.79244	9.69202	3.23074	11.71099	2.06013
	50	4.96645	184.72637	24.46710	3.22609	73.32511	5.16570
v-e-v	5	5.24524	2.15501	2.08134	3.44729	0.74325	0.46100
	10	5.19804	8.16364	4.64135	3.39430	3.04459	0.95842
	20	5.18617	31.94737	9.59753	3.38039	12.25352	1.93595
	50	5.18284	198.23231	24.23787	3.37645	76.71774	4.85343



(a) The (1-2-1) FGM sandwich plate

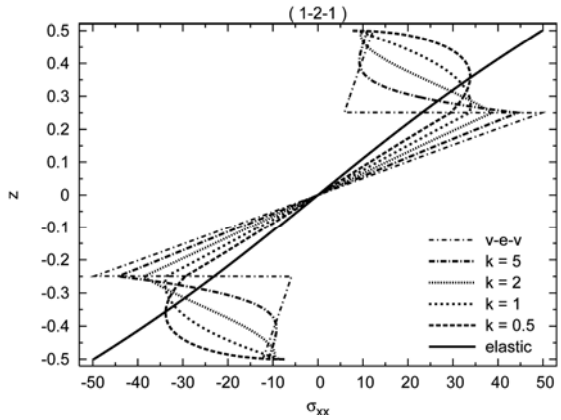
(b) The (1-1-1) FGM sandwich plate

(c) The (2-1-2) FGM sandwich plate

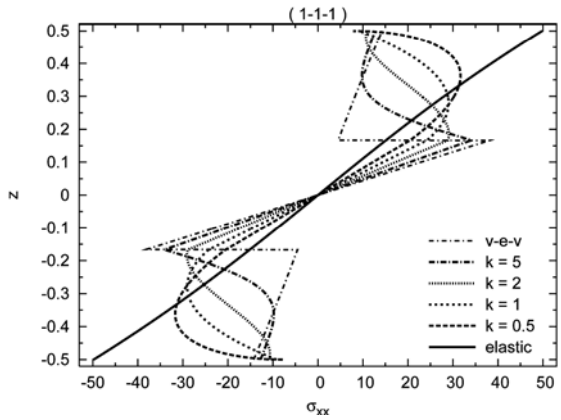
(d) The (2-2-1) FGM sandwich plate

Fig. 3 Dimensionless center deflection w versus side-to-thickness ratio a/h for various values of the power-law index k and different types of viscoelastic sandwich plates

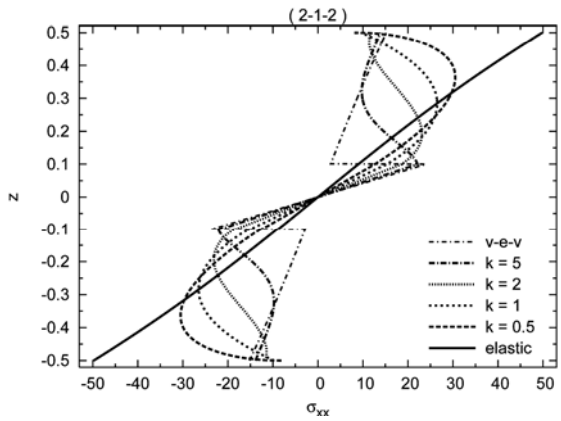
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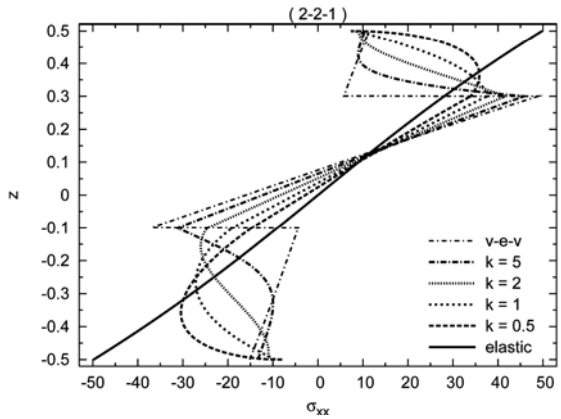
(a) The (1-2-1) FGM sandwich plate



(b) The (1-1-1) FGM sandwich plate

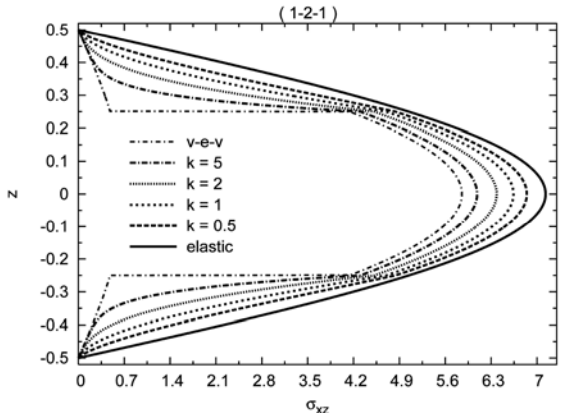


(c) The (2-1-2) FGM sandwich plate

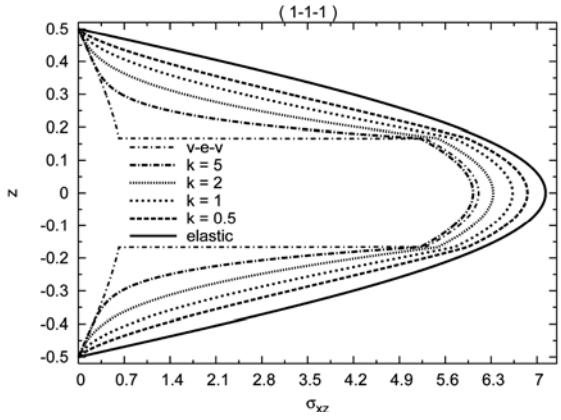


(d) The (2-2-1) FGM sandwich plate

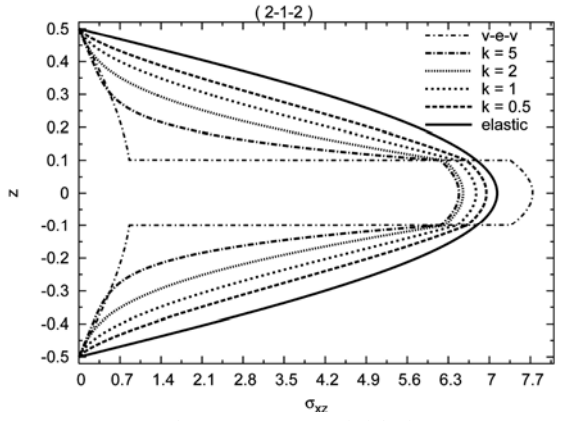
Fig. 4 Through-thickness distributions of in-plane normal stress σ_{xx} for various values of the power-law index k and different types of viscoelastic sandwich plates



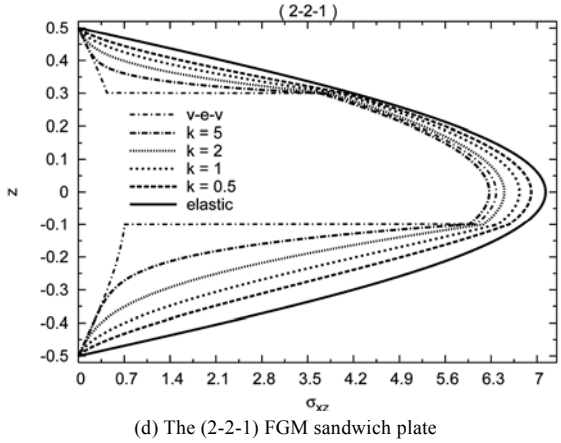
(a) The (1-2-1) FGM sandwich plate



(b) The (1-1-1) FGM sandwich plate



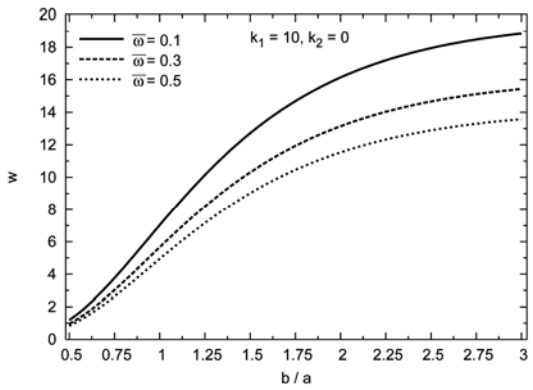
(c) The (2-1-2) FGM sandwich plate



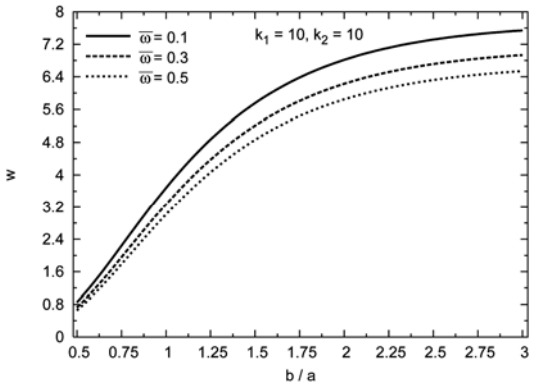
(d) The (2-2-1) FGM sandwich plate

Fig. 5 Through-thickness distributions of the transverse shear stress σ_{xz} for various values of the power-law index k and different types of viscoelastic sandwich plates

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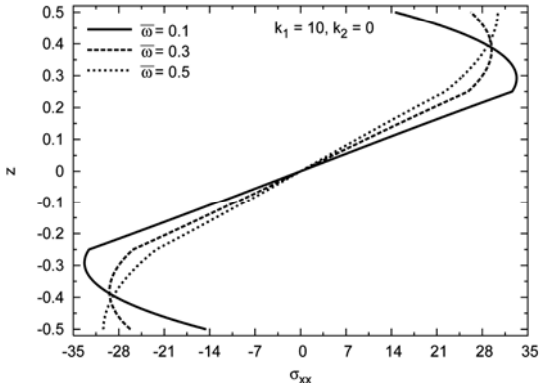


(a) On Winkler's foundation ($k_1 = 10$; $k_2 = 0$)

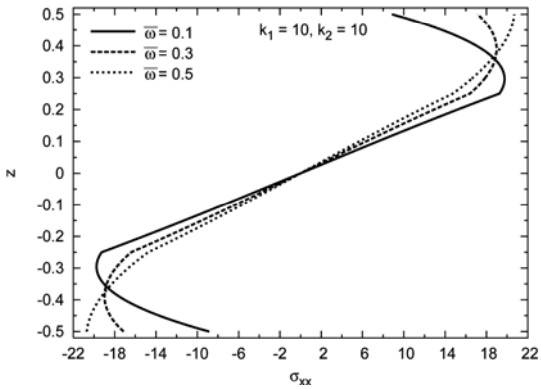


(b) On Pasternak's foundations ($k_1 = 10$; $k_2 = 10$)

Fig. 6 Dimensionless center deflection w for various values of the relaxation function $\bar{\omega}$ versus the aspect ratio b/a of (1-2-1) FGM sandwich plate ($K = 0.4E_e$)

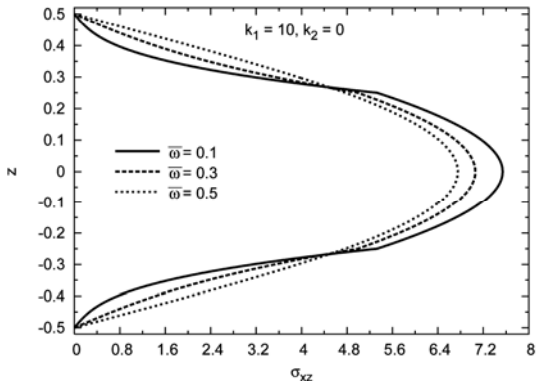


(a) On Winkler's foundation ($k_1 = 10$; $k_2 = 0$)

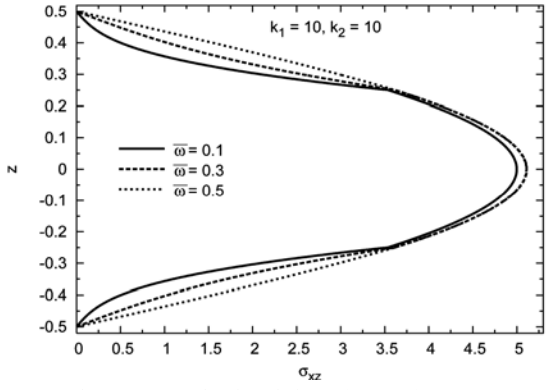


(b) On Pasternak's foundations ($k_1 = 10$; $k_2 = 10$)

Fig. 7 Through-thickness distributions of σ_{xx} for various values of the relaxation function $\bar{\omega}$ and for (1-2-1) FGM sandwich square plate ($K = 0.4E_e$)

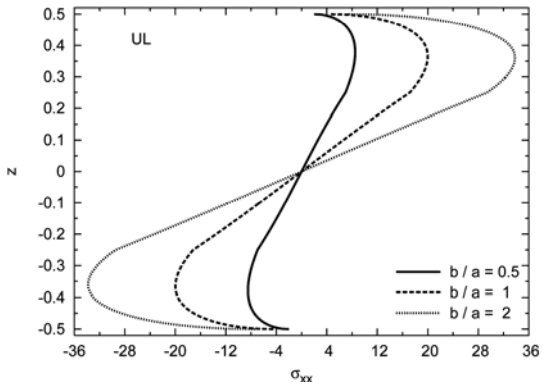


(a) On Winkler's foundation ($k_1 = 10$; $k_2 = 0$)

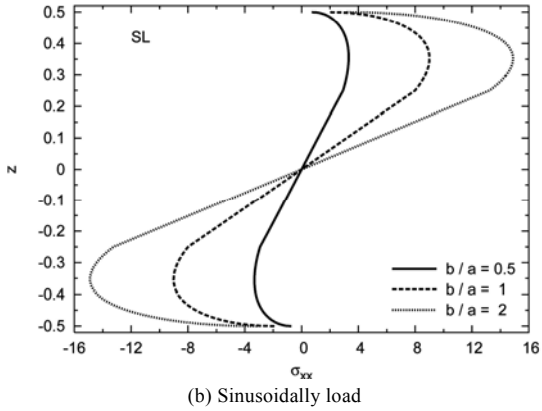


(b) On Pasternak's foundations ($k_1 = 10$; $k_2 = 10$)

Fig. 8 Through-thickness distributions of σ_{xz} for various values of the relaxation function $\bar{\omega}$ and for (1-2-1) FGM sandwich square plate ($k = 2$; $K = 0.5E_e$)



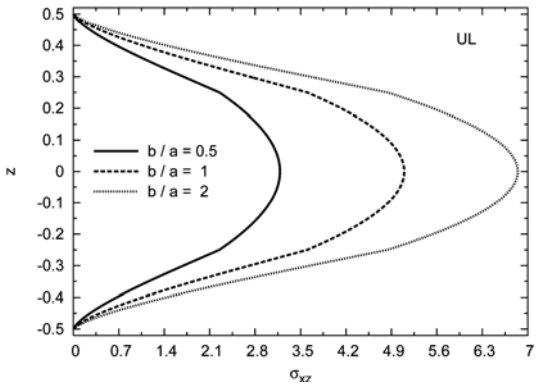
(a) Uniformly load



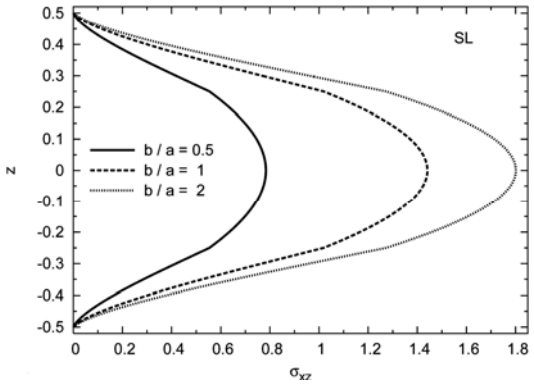
(b) Sinusoidally load

Fig. 9 Through-thickness distributions of σ_{xx} for various values of the aspect ratio b/a and for (1-2-1) FGM sandwich plate

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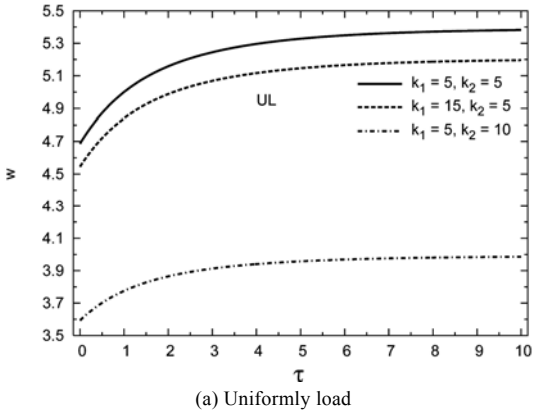


(a) Uniformly load

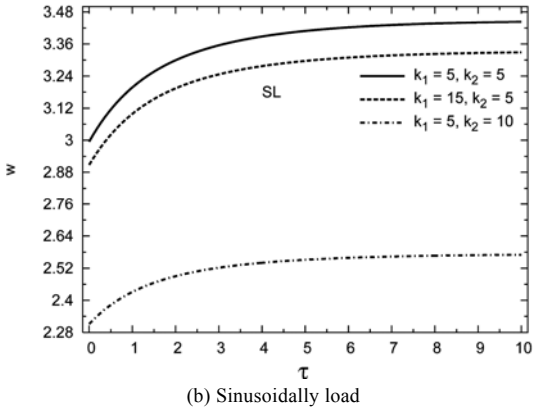


(b) Sinusoidally load

Fig. 10 Through-thickness distributions of σ_{xz} for various values of the aspect ratio b/a and for (1-2-1) FGM sandwich plate

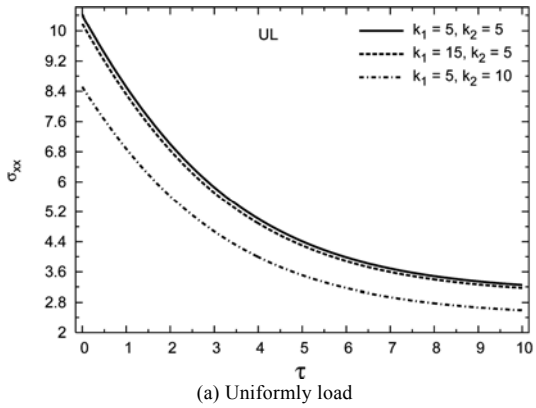


(a) Uniformly load

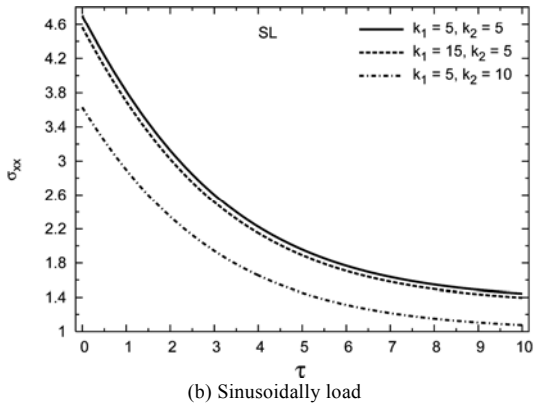


(b) Sinusoidally load

Fig. 11 Dimensionless deflection w of (1-2-1) FGM sandwich square plate as a function of the time parameter τ for different values of the elastic foundation parameters k_1 and k_2

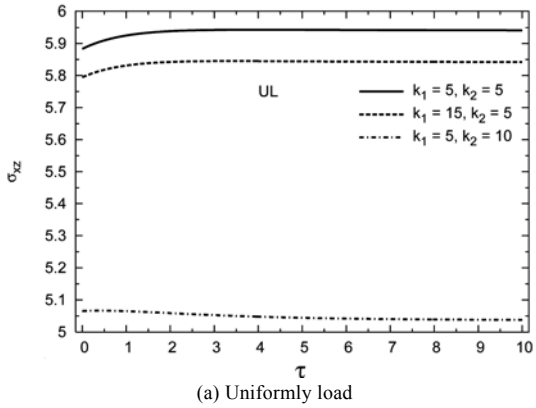


(a) Uniformly load

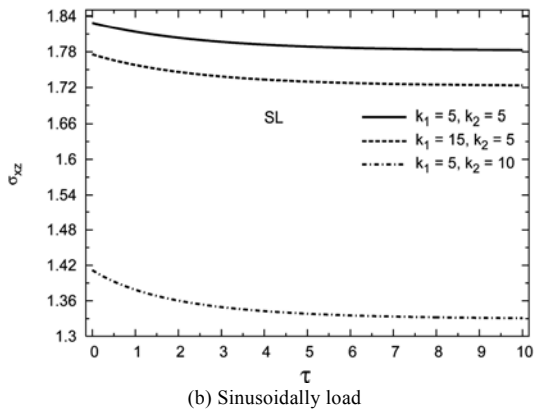


(b) Sinusoidally load

Fig. 12 Dimensionless stress σ_{xx} for (1-2-1) FGM sandwich square plate as a function of the time parameter τ for different values of the elastic foundation parameters k_1 and k_2



(a) Uniformly load



(b) Sinusoidally load

Fig. 13 Dimensionless stress σ_{xz} for (1-2-1) FGM sandwich square plate as a function of the time parameter τ for different values of the elastic foundation parameters k_1 and k_2

https://doi.org/10.1017/jmech.2012.53 Published online by Cambridge University Press

8. CONCLUSIONS

Investigations on the bending problem of uniformly loaded FG viscoelastic sandwich plates resting on two-parameter elastic foundations using the trigonometric shear deformation plate theory are presented. The present problem is also analyzed when the sandwich plate is acted by sinusoidally distributed load. The following conclusion may be drawn from the present analysis:

- In order to show the accuracy of the present results, the convergence of the numerical solutions using the trigonometric theory have been examined. It is found that the present results are very agreement with those being in literature.
- The central deflection and the stresses of the FG viscoelastic sandwich plates calculated under uniformly distributed load are usually overpredicted.
- The results show that the presence of elastic foundations leads to a significant reduction in the variation of the deflections and stresses.
- The reduction of the deflection and stresses caused by shear foundation stiffness k_2 is greater than that caused by springs stiffness k_1.
- According to the viscoelastic response, the central deflection increases while the longitudinal normal stress decreases with the increase of the time parameter.
- Effects of the power-law index, relaxation function, side-to-thickness ratio and aspect ratio on the FG viscoelastic sandwich plates with elastic cores are studied.

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(Manuscript received April 3, 2011,
accepted for publication May 17, 2011.)

https://doi.org/10.1017/jmech.2012.53 Published online by Cambridge University Press