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Nonlinear electromagnetic Eigen modes of a self created magnetized plasma channel and its stimulated Raman scattering

UPDESH VERMA[1] AND A.K. SHARMA[2]

[1]Govt. Degree College Ballarpur, Rampur, U.P., India
[2]Center for Energy Studies, Indian Institute of Technology Delhi, New Delhi, India

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Abstract

A theoretical formalism is developed to obtain the mode structure of right circularly polarized nonlinear laser Eigen mode in a self created plasma channel in the presence of an axial magnetic field. The nonlinearity in electron response arises due to relativistic mass effect and ponderomotive force induced density redistribution. The Eigen mode is seen to be unstable to stimulated Raman backscattering involving an electrostatic quasi-mode and a scattered electromagnetic wave. The growth rate increases with ambient magnetic field.

Keywords: Electromagnetic Eigen mode; Magnetized plasma channel; Ponderomotive force theoretical model; Relativistic mass effect; Stimulated Raman scattering

1. INTRODUCTION

The propagation of high power laser of finite spot size in plasmas is influenced by diffraction and self focusing effects. The former causes divergence of the beam on the scale of a Rayleigh length $R_d = kr_0^2$, where k is the wave number of the laser and r_0 is its spot size. The self focusing arises due to nonlinear permittivity that may be caused by a variety of sources, e.g., ohmic nonlinearity, ponderomotive nonlinearity and relativistic mass nonlinearity (Sodha *et al.*, 1974*a*, 1974*b*, 1976; Gill *et al.*, 2010; Saini & Gill, 2004; Singh & Tripathi, 2009; Verma & Sharma, 2011). The theories of self focusing are mostly limited to paraxial ray approximation where eikonal of the wave is expanded up to the second power of r, the radial (transverse) coordinate. Sodha *et al.* (1974*a*, 1974*b*) have surveyed the early studies on self focusing and filamentation in unmagnetized plasmas with primary focus on long time scale Ohmic and ponderomotive nonlinearities. Esaray *et al.* (1997) have studied the short-pulse laser propagation in gases undergoing ionization. They also observed optical guiding of a laser beam in plasmas by taking relativistic effects, ponderomotive effects, and preformed density channels. Kurki-Suonio *et al.* (1989) have studied the propagation of a circularly polarized laser in underdense plasma, including relativistic and ponderomotive nonlinearities, and showed that it remains intact as it propagates inside the plasma. Liu and Tripathi (2000) have developed a nonparaxial theory of laser self defocusing in a tunnel ionizing gas by expanding the eikonal up to fourth power in r.

Pathak and Tripathi (2006) have developed a formalism to study the nonlinear Eigen modes of plasma without resorting to any expansion of the eikonal. The wave equation is solved numerically. The laser intensity profile in an unmagnetized plasma under relativistic and ponderomotive nonlinearity, turns out to be close to a Lorentzian. The Eigen modes in high density plasmas are stable to stimulated Raman scattering (SRS). However, in low density plasmas, laser Eigen modes are unstable to SRS. This study is limited to unmagnetized plasma.

In this paper, we study the nonlinear Eigen modes of a laser in a self created magnetized plasma channel (Verma & Sharma, 2009*a*; Panwar & Sharma, 2009), under the combined effects of ponderomotive and relativistic mass nonlinearities, and examine their stability to stimulated Raman scattering. This study is relevant to intense short pulse laser produced plasmas where multi mega-gauss magnetic fields have been observed (Cai *et al.*, 2007; Borghesi *et al*, 1997; Verma & Sharma, 2009*b*). Horovitz *et al.* (1998) experimentally measured the generation of axial magnetic fields. They performed experiment with 1.06 μm laser of 7 ns pulse

Address correspondence and reprint requests to: Updesh Verma, Center for Energy Studies, Indian Institute of Technology Delhi, New Delhi-110016, India. E-mail: updeshv@gmail.com

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duration and 10^9 to10^{14} W/cm^2 intensity range. They observed axial fields from 500 G to 2.17 MG. Naseri *et al.* (2010) reported that inverse Faraday Effect is the main mechanism of axial magnetic field generation in inhomogeneous plasmas. Pukhov and Meyer-ter-Vehn (1998) in their three-dimensional particle-in-cell simulations of intense short pulse laser gas jet interaction have observed 100 MG magnetic fields in one-tenth critical density plasmas at $\sim 10^{19}$ W/cm^2 intensity at 0.8 μm wavelength. Frolov (2009) observed quasi-static magnetic fields generated in low-density plasma in the interaction of counter propagating laser pulses. The magnetic field introduces anisotropy and cyclotron resonance effects in the propagation of electromagnetic waves and opens up a rich variety of electrostatic modes. Parametric coupling (Liu & Tripathi, 1986; Porkolab & Chang, 1972; Short & Simon 2004) between these modes has been a fascinating subject of extensive study for quiet some time. Clark and Fisch (2005) proposed a scheme that utilizes the stimulated Raman backscattering (SRBS) in plasma to amplify a short and frequency downshifted seed pulse. Sharma and Kourakis (2009) used SRBS to investigate the pulse compression and pulse amplification mechanisms. Trines *et al.* (2004) did PIC simulation and show that suppression of SRBS increases the high energy electron yield. Sajal *et al.* (2004, 2006) have studied relativistic stimulated Raman forward scattering (SRFS) and shown that the Langmuir wave in the presence of a strong azimuthal magnetic field goes over to a localized lower hybrid wave, and the growth rate of the stimulated Raman backward scattering process is decreased. Liu and Tripathi (1996) have studied the stimulated forward Raman scattering of a laser in the channel for none relativistic case. The localization effect reduces the growth rate of Raman process. Liu and Tripathi (2001) also studied the relativistic laser guiding in a strong azimuthal magnetic field in plasma in the paraxial ray approximation. A linearly polarized laser with circular spot size is seen to acquire an elliptical spot size as it is self focused (Liu & Tripathi, 1994).

In Section 2, we deduce the nonlinear wave equation governing the propagation of a right circularly polarized mode in plasma in the presence of axial magnetic field, allowing for a transverse amplitude variation. The nonlinearity is taken to arise due to relativistic mass effect and ponderomotive force induced electron density redistribution. The ions are taken to be stationary. The wave equation is solved numerically to obtain the nonlinear Eigen modes. In Section 3, we study the stimulated Raman backscattering of a large amplitude Eigen mode. In Section 4, we discuss the results.

2. NONLINEAR ELECTROMAGNETIC PLASMA EIGEN MODE IN PRESENCE OF AXIAL MAGNETIC FIELD

Consider pre-formed plasma of electron density n_0 embedded in a static magnetic field $B_s\hat{z}$. A circularly polarized electromagnetic mode propagates through it with t, z variation of the electric field as

$$\vec{E}_0 = A_0(x)(\hat{x} + i\hat{y})e^{-i(\omega_0 t - k_0 z)}, \tag{1}$$

where A_0 has a slow dependence on x. The magnetic field of laser is

$$\omega = i\omega_0 g^{1/2}. \tag{2}$$

The laser imparts an oscillatory velocity to electrons,

$$\vec{v}_0 = \frac{e\vec{E}_0}{mi\gamma_0(\omega_0 - \omega_c/\gamma_0)}, \tag{3}$$

where $\omega_c = eB_s/m$, $-e$ and m are the electron charge and mass and $\gamma_0 = \dfrac{1}{(1 - v_0^2/c^2)^{1/2}}$ is the gamma factor. For $|v_0|^2/c^2 \ll 1$, simplifies to

$$\gamma_0 = 1 + \frac{a_0^2}{2(1 - \omega_c/\omega_0)^2} - \frac{(1 + 3\omega_c/\omega_0)}{8(1 - \omega_c/\omega_0)^5} a_0^4, \tag{4}$$

where $a_0^2 = e^2A_0^2/m^2\omega_0^2c^2$.

The laser exerts a direct current ponderomotive force on electrons,

$$\vec{F}_p = -\frac{m\gamma_0}{2}(\vec{v}_0 \cdot \nabla)\vec{v}_0^* - \frac{e}{2}(\vec{v}_0 \times \vec{B}_0^*), \tag{5}$$

$$= \frac{-e^2[\vec{E}_0 \times (\nabla \times \vec{E}_0)]}{2mi\gamma_0(\omega_0 - \omega_c/\gamma_0)i\omega_0} + \frac{e^2[(\vec{E}_0 \cdot \nabla)\vec{E}_0)]}{2m\gamma_0(\omega_0 - \omega_c/\gamma_0)^2}.$$

Using identities $\vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$ and $\nabla(\vec{A}.\vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{A}.\nabla)\vec{B} + (\vec{B}.\nabla)\vec{A}$, we get $\vec{F}_p = \frac{-e^2\omega_0[(1-\omega_c/2\omega_0\gamma_0)\nabla(\vec{E}_0.\vec{E}_0) - (\omega_c/2\omega_0\gamma_0)(\vec{E}_0 \times \nabla \times \vec{E}_0)]}{2m\gamma_0\omega_0(\omega_0 - \omega_c/\gamma_0)^2}$. But we assumed $\omega_c/\omega_0\gamma_0 < 1$, hence one can neglect the second term from above equation and hence we get

$$\vec{F}_p = \frac{e^2(1 - \omega_c/2\omega_0\gamma_0)\nabla(\vec{E}_0.\vec{E}_0)}{2m\gamma_0(\omega_0 - \omega_c/\gamma_0)^2} = e\nabla\phi_p,$$

hence

$$\phi_p = \frac{-e}{4m\gamma_0(\omega_0 - \omega_c/\gamma_0)^2}(2 - \omega_c/\omega\gamma_0)(A_0 \cdot A_0^*), \tag{6}$$

The ponderomotive force pushes the electrons away from the laser axis, creating a low electron density channel near the laser axis. For laser pulse duration shorter than the ion response time, the ions may be taken to be stationary. The ion space charge field $\vec{E}_s = -\vec{\nabla}\phi_s$, pulls the electrons back. In a time on the order of ω_p^{-1}, where $\omega_p^2 = n_0e^2/m\varepsilon_0$, a quasi-steady-state is realized, when the ponderomotive potential balances the space charge potential and further lowering of electron density at the axis stops. The time scale of

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plasma channel formation is of the order of electron plasma period ($\sim 1/\omega_p$). Hence in the quasi-steady-state

$$\phi_s = -\phi_p = \frac{e}{4m\gamma_0(\omega_0 - \omega_c/\gamma_0)}(2 - \omega_c/\omega_0\gamma_0). \tag{7}$$

Using ϕ_s in Poisson's equation, $\nabla^2\phi_s = e(n_e - n_0)/\varepsilon_0$, we obtained the modified electron density

$$n_e = n_0 - \frac{\varepsilon_0}{e}\nabla^2\phi_p. \tag{8}$$

With the relativistic modification of electron mass and ponderomotive modification of density, the non-zero component of the conductivity tensor of plasma can be written as

$$\sigma_{xx} = \sigma_{yy} = \frac{in_e e^2}{m\omega_0\gamma_0(1 - \omega_c^2/\omega_0^2\gamma_0^2)},\ \sigma_{xy} = -\sigma_{yx} = \frac{n_e e^2\omega_c}{m\omega_0^2\gamma_0^2(1 - \omega_c^2/\omega_0^2\gamma_0^2)},\ \sigma_{zz} = \frac{in_e e^2}{n_0\gamma_0\varepsilon_0 m\omega_0^2}.$$
$$\sigma_{xz} = \sigma_{zx} = \sigma_{yz} = \sigma_{zy} = 0. \tag{9}$$

Using third and fourth Maxwell's equations, we deduce the wave equation as

$$\nabla^2\vec{E}_0 - \nabla(\nabla.\vec{E}_0) + \frac{\omega_0^2}{c^2}(\underline{\varepsilon}.\vec{E}_0) = 0. \tag{10}$$

Effective dielectric constant for right circularly polarized extraordinary mode in magnetized plasma can be written as

$$\varepsilon_\pm = \varepsilon_{xx} \pm \varepsilon_{xy} = 1 - \frac{n_e\omega_p^2}{n_0\gamma_0\omega_0^2(1 \mp \omega_c/\omega_0\gamma_0)}, \tag{11}$$

where

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \frac{n_e\omega_p^2}{n_0\gamma_0\omega_0^2(1 - \omega_c^2/\omega_0^2\gamma_0^2)},$$
$$\varepsilon_{xy} = -\varepsilon_{yx} = \frac{in_e\omega_c\omega_p^2}{n_0\gamma_0^2\omega_0^3(1 - \omega_c^2/\omega_0^2\gamma_0^2)},$$
$$\varepsilon_{zz} = 1 - \frac{n_e\omega_p^2}{n_0\gamma_0\omega_0^2}$$
$$\text{and } \varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{yz} = \varepsilon_{zy} = 0. \tag{12}$$

Using $\partial E_z/\partial z = -(1/\varepsilon_{zz})[(\partial/\partial x)(\varepsilon_{xx}E_x + \varepsilon_{xy}E_y) + (\partial/\partial y)(-\varepsilon_{xy}E_x + \varepsilon_{xx}E_y)]$, and following Sodha *et al.* (1974*a*, 1974*b*, 1976) the wave equation governing the propagation of right circularly polarized wave can be written as

$$\frac{\partial^2 A_+}{\partial z^2} + \frac{1}{2}\left(1 + \frac{\varepsilon_-}{\varepsilon_{zz}}\right)\nabla_\perp^2 A_+ + \frac{\omega_0^2}{c^2}\varepsilon_- A_+ = 0, \tag{13}$$

when $\omega_c \ll \omega_0$, then $\left(1 + \frac{\varepsilon_-}{\varepsilon_{zz}}\right) \approx 2$ and hence the above equation reduces to well-known result as derived by Liu and Tripathi (1994). Here $A_+ = E_x + iE_y$ is the extraordinary mode of the laser. Let $A_+ = A_{+0}(x)e^{ik_0 z}$, where $A_{+0} = 2A_0$. In an under dense plasma $\omega_p^2 \ll \omega_0^2$ and $\left(1 + \frac{\varepsilon_-}{\varepsilon_{zz}}\right) \approx 2$. Using these values, Eq (13) becomes

$$\frac{d^2 a_0}{dx^2} = \left(k_0^2 - \frac{\omega_0^2}{c^2}\varepsilon_-\right)a_0. \tag{14}$$

Put the value of modified electron density from Eq. (8), ε_- can be written as

$$\varepsilon_- = 1 - \frac{\omega_p^2}{\omega_0^2(\gamma_0 + \omega_c/\omega_0)} - \frac{(2 - \omega_c/\omega_0\gamma_0)c^2(\partial^2 a_0^2/\partial x^2)}{4\gamma_0\omega_0^2(\gamma_0 + \omega_c/\omega_0)(1 - \omega_c/\omega_0\gamma_0)^2}. \tag{15}$$

Put the value of ε_- in the above equation, it becomes

$$\frac{d^2 a_0^2}{dX^2} = \frac{1}{2a_0^2(1 - 2\alpha a_0^2)}\left(\frac{da_0^2}{dX}\right)^2 + \frac{2a_0^2}{(1 - 2\alpha a_0^2)}\left[\left(\frac{k_0 c}{\omega_0}\right)^2 - 1\right] + \frac{2(\omega_p^2/\omega_0^2)a_0^2}{(1 - 2\alpha a_0^2)(\gamma_0 + \omega_c/\omega_0)} \tag{16}$$

where $a_0 = eA_0/m\omega_0\, c$, $X = x\omega_0/c$ and

$$\alpha = \frac{(2 - \omega_c/\omega_0\gamma_0)}{4\gamma_0(\gamma_0 + \omega_c/\omega_0)(1 - \omega_c/\omega_0\gamma_0)^2}. \tag{17}$$

We have solved Eq. (16) numerically to obtain the mode structure of the Eigen mode for parameters: $\omega_p/\omega_0 = 0.4$, $k_0 c/\omega_0 = 0.91652$ and $\omega_c/\omega_0 = 0.4$.

Figure 1 shows the mode structure of a laser in the presence of magnetic field. It resembles Gaussian profile with half width $x_0 = 0.436c/\omega_0$, $a_0 = a_{00}\psi(x)$, $\psi(x) = \left(\frac{1}{\pi^{1/4}}\right)e^{-x^2/2x_0^2}$. This is different from the Pathak and Tripathi, Lorentzian profile that have obtained in unmagnetized plasma. Figure 2 shows the variation of ω_0/ω_p with $k_0 c/\omega_p$ for different values of magnetic fields. The value of ω_0/ω_p is minimum at minimum $k_0 c/\omega_p$ and increases monotonically with increasing $k_0 c/\omega_p$. The minimum value of ω_0/ω_p increases with increasing the value of ω_c/ω_p.

3. STIMULATED RAMAN BACK SCATTERING

Now we examine the parametric decay of a large amplitude Eigen mode into a reactive quasi-mode of potential

$$\phi_\omega = A(x)\exp[-i(\omega t - kz)], \tag{18}$$

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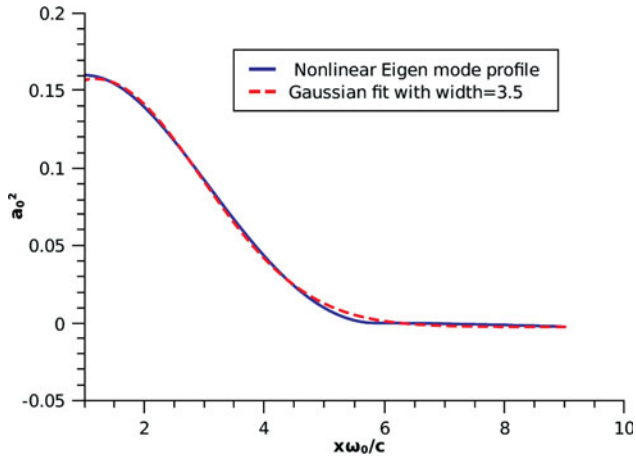


Fig. 1. (Color online) Electromagnetic mode amplitude square as a function of normalized transverse distance for $\omega_p^2/\omega_0^2 = 0.25$, $a_{00} = 0.4$, and $\omega_c/\omega_0 = 0.4$.

and a backscattered electromagnetic wave with electric and magnetic fields

$$\vec{E}_1 = \vec{A}_1(\hat{x} - i\hat{y})e^{-i(\omega_1 t - k_1 z)},\ \vec{B}_1 = \frac{\vec{k}_1 \times \vec{E}_1}{\omega_1}, \tag{19}$$

where $\omega_1 = \omega - \omega_0$, $k_1 = k - k_0$. In an underdense plasma, $\omega \ll \omega_0$ hence $|\omega_1| \approx \omega_0$, $|k_1| \approx k_0$ and $k \approx 2k_0$. The scattered wave imparts oscillatory velocity to electrons

$$\vec{v}_1 = \frac{e\vec{E}_1}{mi\gamma_0(\omega_1 + \omega_c/\gamma_0)}, \tag{20}$$

The pump and scattered waves exert a ponderomotive force on electrons at $(\omega, \vec{k})$, $\vec{F}_{pz,\omega} = e\nabla\phi_{p\omega} = ik\phi_{p\omega}\hat{Z}$, where

$$\phi_{p\omega} = \frac{e}{m\gamma_0 k}\left[\frac{k_1\omega_0(\omega_1 + \omega_c/\gamma_0) + k_0\omega_1(\omega_0 - \omega_c/\gamma_0)}{\omega_0\omega_1(\omega_0 - \omega_c/\gamma_0)(\omega_1 + \omega_c/\gamma_0)}\right]A_0A_1. \tag{21}$$

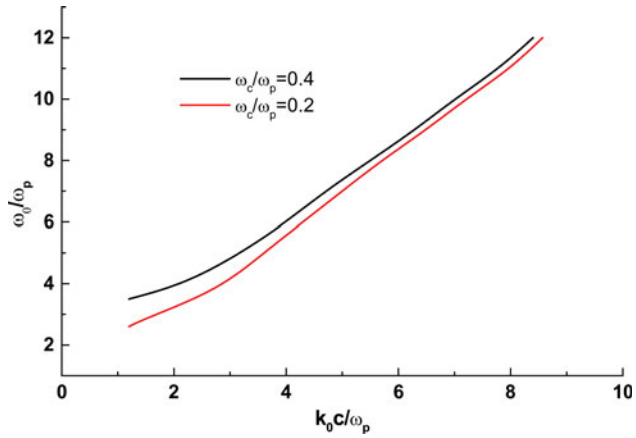


Fig. 2. (Color online) Variation of normalized frequency ω_0/ω_p with normalized wave vector k_0c/ω_p for a high power laser in plasmas. The other parameters are: $a_{00} = 0.4$, $\omega_c/\omega_p = 0.2$ and 0.4.

The electron response to ϕ_ω and $\phi_{p\omega}$ can be obtained by solving the equation of motion,

$$m\gamma_0\frac{\partial\vec{v}}{\partial t} = -e\vec{E}_\omega + \vec{F}_\omega - e(\vec{v}\times\vec{B}_s)$$

$$\vec{v} = \frac{-e\nabla(\phi_\omega + \phi_{pz})}{mi\omega\gamma_0}, \tag{22}$$

using this value in the continuity equation we get the modified density of electrons

$$n_\omega = \frac{1}{i\omega}n_o(\nabla.\vec{v}).$$

Using Poisson's equation $\nabla^2\phi_\omega = en_\omega/\varepsilon_0$, and defining $e\phi_\omega/mc^2 = \varphi_\omega$, we get

$$\frac{\partial^2\varphi_\omega}{\partial x^2}\left[1 - \frac{\omega_p^2}{\gamma_0(\omega^2 - \omega_c^2/\gamma_0^2)}\right] - k_z^2(1 - \omega_p^2/\omega^2\gamma_0)\varphi_\omega$$
$$= -\frac{\omega_p^2 k_z\omega_0^2 a_0 a_1}{\omega^2\gamma_0^2} \times\left[\frac{k_1\omega_0(\omega_1 + \omega_c/\gamma_0) + k_0\omega_1(\omega_0 - \omega_c/\gamma_0)}{\omega_0\omega_1(\omega_1 + \omega_c/\gamma_0)(\omega_0 - \omega_c/\gamma_0)}\right], \tag{23}$$

Where $a_1 = eA_1/m\omega_0 c$. For $\omega > \omega_c$, ω_p, Eq. (23) can be written as

$$\frac{\partial^2\varphi_\omega}{\partial x^2} - k_z^2\varphi_\omega = -\frac{\omega_p^2 k_z\omega_0^2 a_0 a_1}{\omega^2\gamma_0^2(1 - \omega_p^2/\omega^2\gamma_0)} \times\left[\frac{k_1\omega_0(\omega_1 + \omega_c/\gamma_0) + k_0\omega_1(\omega_0 - \omega_c/\gamma_0)}{\omega_0\omega_1(\omega_1 + \omega_c/\gamma_0)(\omega_0 - \omega_c/\gamma_0)}\right], \tag{24}$$

the current density at the side band $(\omega_1, \vec{k}_1)$ can be written as

$$\vec{J}_1 = -n_e e\vec{v}_1 - \frac{1}{2}n_\omega e\vec{v}_0^*. \tag{25}$$

The normalized wave equation for the side band, on assuming $\nabla.\vec{E}_1 \approx 0$, can be written as

$$\frac{d^2a_1}{dx^2} + \left[\frac{\omega_1^2}{c^2} - k_1^2 - \frac{\omega_p^2}{c^2\gamma_0(1 + \omega_c/\omega_1\gamma_0)} - \frac{(2 - \omega_c/\omega_0\gamma_0)(\partial^2 a_0^2/\partial x^2)}{4\gamma_0^2(1 + \omega_c/\omega_0\gamma_0)(1 - \omega_c/\omega_0\gamma_0)^2}\right]a_1 = -\frac{n_\omega e^2\omega_1 a_0^*}{2\varepsilon_0 mc^2\gamma_0(\omega_0 - \omega_c/\gamma_0)}. \tag{26}$$

The left-hand side of this equation is the same as that of Eq. (14) with ω_0 replaced by $|\omega_1|$. Hence, in the absence of the parametric coupling the mode structure of a_1 is the same as

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that of the pump Eigen mode,

$$a_1 = a_{10}\psi(x). \tag{27}$$

In low density plasma when the pump amplitude is large, the growth rate of plasma wave may become larger than the plasma frequency, $\omega^2 \gg \omega_p^2$ turning into a reactive quasi-mode. Substituting for a_1 in Eq. (26), multiplying the resulting equation by $\psi dx'$, $x' = x/x_0$, and integrating over x' from $-\infty$ to $+\infty$ we obtain

$$\begin{aligned}\omega_1^2 - k_1^2c^2 = \frac{c^2}{x_0^2}\left[1 - \int_{-\infty}^{\infty} x'^2\psi^2 dx'\right] \\ - \frac{\omega_p^2\omega_1 k^2c^2|a_{00}|^2}{2\omega^2\omega_0}\int_{-\infty}^{\infty}\frac{\psi^4 dx'}{\gamma_0^3(1-\omega_c/\omega_0\gamma_0)^2} \\ + \int_{-\infty}^{\infty}\left[\frac{\omega_p^2}{\gamma_0(1+\omega_c/\omega_1\gamma_0)}\right. \\ \left. + \frac{(2-\omega_c/\omega_0\gamma_0)c^2}{4\gamma_0^2(1+\omega_c/\omega_1\gamma_0)(1-\omega_c/\omega_1\gamma_0)^2}\frac{\partial^2}{\partial x^2}|a_0|^2\right]\psi^2 dx'.\end{aligned} \tag{28}$$

One may approximate the left-hand side as $2\omega\omega_0$. Since $\omega^3 < \omega^2$, we can neglect ω^3 terms. Hence Eq. (28) can be written as

$$\omega = i\omega g^{1/2}, \tag{29}$$

where

$$g = \frac{(\Omega_p^2\eta_1^2/4)\int_{-\infty}^{\infty}\frac{a_{00}^2\psi^4 dx'}{\gamma_0^3(1-\Omega_c/\gamma_0)^2}}{\left(\frac{0.278}{\eta_0^2}\right) + \int_{-\infty}^{\infty}\frac{\Omega_p^2\psi^2 dx'}{2\gamma_0(1-\Omega_c/\gamma_0)} + \int_{-\infty}^{\infty}\frac{(2-\Omega_c/\gamma_0)\frac{\partial^2 a_0^2}{\partial X^2}\psi^2 dx'}{8\gamma_0^2(1-\Omega_c/\gamma_0)^3}}, \tag{30}$$

other normalized quantities are as follows, $\Omega_p = \omega_p/\omega_0$, $\Omega_c = \omega_c/\omega_0$, $\Omega = \omega/\omega_0$, $\eta_0 = \frac{x_0\omega_0}{c}$, $\eta_1 = \frac{kc}{\omega_0}$, $x' = x/x_0$ and $X = \frac{x\omega_0}{c}$. Solving Eq. (29) numerically, one obtains the growth rate $\Gamma = \mathrm{Im}\omega$.

Figure 3 shows the schematic diagram of the laser plasma interaction. Figure 4 shows the variation of normalized growth rate as a function of normalized axial laser amplitude a_{00} for $\omega_c/\omega_0 = 0.4$ and $\omega_p/\omega_0 = 0.1$, 0.2 and 0.4. As ω_0 is a constant, different value of ω_p/ω_0 imply different value of background electron density. Therefore, Figure 4 shows the importance of background electron density and showed that as the plasma frequency (density of electrons) increases, Γ/ω_0 increases slowly. For larger values of laser intensity, the SRBS instability grows more rapidly. This shows that SRBS is significantly influenced by the plasma frequency. Figure 5 shows the variation of growth rate with axial laser amplitude. As a_{00} increases from 0 to 0.4, Γ/ω_0 varies from 0.001 to 0.024 for $\omega_c/\omega_0 = 0.1$. But as ω_c/ω_0 increases from 0.1 to 0.4, the value of Γ/ω_0 increases from 0.024 to

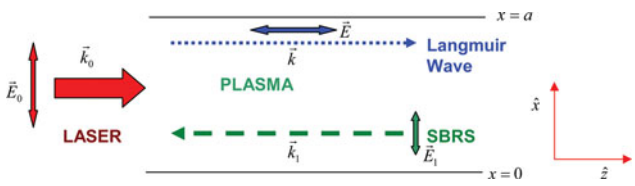


Fig. 3. (Color online) Schematic diagram of laser plasma interaction.

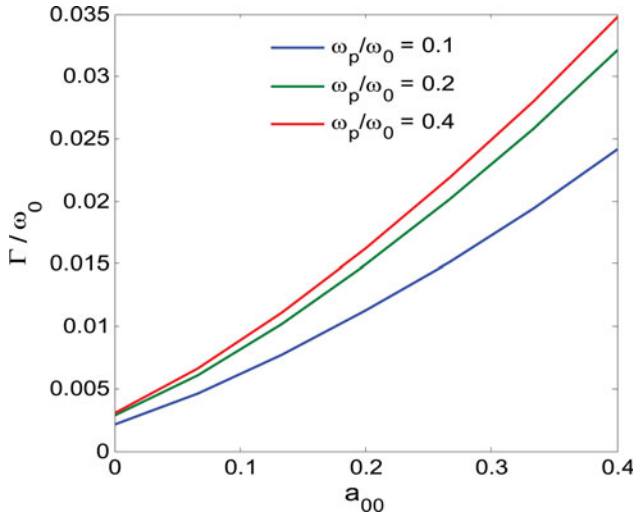


Fig. 4. (Color online) Normalized growth rate Γ/ω_0 as a function of normalized axial laser amplitude a_{00} for $\omega_c/\omega_0 = 0.4$ and $\omega_p/\omega_0 = 0.1$, 0.2 and 0.4.

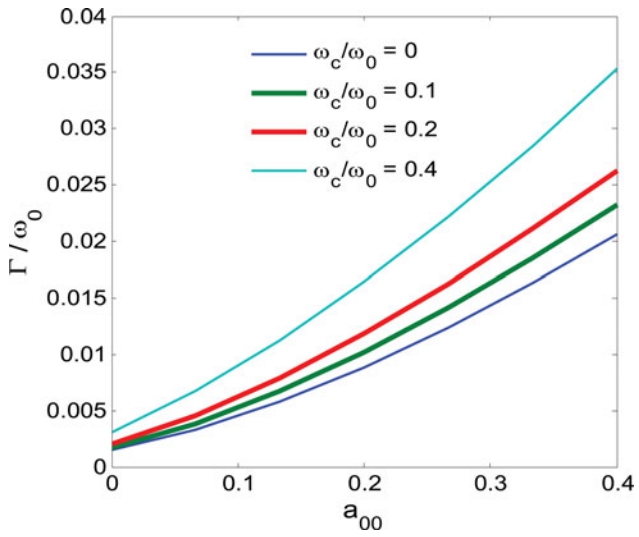


Fig. 5. (Color online) Normalized growth rate as a function of normalized axial laser amplitude for a_{00} for $\omega_p/\omega_0 = 0.4$ and $\omega_c/\omega_0 = 0.1$,0.2 and 0.4.

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0.035. This shows that magnetic field plays an important role in the growth of SRBS, and instability become stronger with increasing the magnetic field.

4. DISCUSSION

The presence of magnetic field modifies the Eigen frequency of the fundamental laser Eigen modes. The mode structure resembles a Gaussian distribution. This is different from unmagnetized plasma where Pathak and Tripathi obtain the mode structure resembling a Lorentizian profile. The growth rate of stimulated Raman backscattering in the reactive quasi-mode regime, relevant at high intensity, increases with the magnetic field. This is caused due to the cyclotron resonance effect though the theory is not applicable close to cyclotron resonance. It also increases with plasma density. As the laser intensity increases the width of nonlinear Eigen modes decreases. This is due to the fact that at higher laser intensity nonlinear self focusing effect becomes stronger, hence to compensate for it stronger diffraction effects, caused by reduced spot size, are required. Our theory is limited to $a_0 \leq 0.5$ i.e., for weakly relativistic cases.

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REFERENCES

Borghesi, M., MacKinnon, A.J., Barringer, L., Gaillard, R., Gizzi, L.A., Meyer, C., Willi, O., Pukhov, A. & Meyer-ter-Vehn, J. (1997). Relativistic channeling of a picosecond laser pulse in a near-critical preformed plasma. *Phys. Rev. Lett*. **78**, 879–882.

Cai, H-B., Yu, W., Zhu, S. & Zhou, C. (2007). Generation of strong quasistatic magnetic fields in interactions of ultraintense and short laser pulses with overdense plasma targets. *Phys. Rev. E* **76**, 036403/7.

Clark, D.S. & Fisch, N.J. (2005). Raman laser amplification in preformed and ionizing plasmas. *Laser Part. Beams* **23**, 101–106.

Esarey, E., Sprangle, P., Krall, J. & Ting, A. (1997). Self-focusing and guiding of short laser pulses in ionizing gases and plasmas. *IEEE J. Quan. Electron*. **33**, 1879–1914.

Frolov, A.A. (2009). Generation of quasistatic magnetic fields in the interaction of counter propagating laser pulses in a low-density plasma. *Plasma Phys. Rept*. **35**, 668–676.

Gill, T.S., Mahajan, R. & Kaur, R. (2010). Relativistic and ponderomotive effects on evolution of laser beam in a non-uniform plasma channel. *Laser Part. Beams* **28**, 11–20.

Horovitz, Y., Eliezer, S., Henis, Z., Paiss, Y., Moshe, E., Ludmirsky, A., Werdiger, M., Arad, B. & Zigler, A. (1998). The inverse Faraday effect in plasmas produced by circularly polarized laser light in the range of intensities 10^9–10^{14} W/cm^2. *Phys. Lett. A* **246**, 329–334.

Kurki-suonio, T., Morrison, P.J. & Tajima, T. (1989). Self-focusing of an electromagnetic beam in a plasma. *Phys. Rev. A* **40**, 3230–3239.

Liu, C.S. & Tripathi, V.K. (1994). *Interaction of Electromagnetic Waves with Electron Beams and Plasmas*. Singapore: World Scientific.

Liu, C.S. & Tripathi, V.K. (1996). Stimulated Raman scattering in a plasma channel. *Phys. Plasmas* **3**, 3410–3413.

Liu, C.S. & Tripathi, V.K. (2001). Relativistic laser guiding in an azimuthal magnetic field in a plasma. *Phys. Plasmas* **8**, 285–288.

Liu, C.S. & Tripathi, V.K. (1986). Parametric instabilities in a magnetized plasma. *Phys. Rept*. **130**, 143–216.

Liu, C.S. & Tripathi, V.K. (2000). Laser frequency upshift, self-defocusing, and ring formation in tunnel ionizing gases and plasmas. *Phys. Plasmas* **7**, 4360–4363.

Naseri, N., Bychenkov, V.Yu. & Rozmus, W. (2010). Axial magnetic field generation by intense circularly polarized laser pulses in underdense plasmas. *Phys. Plasmas* **17**, 083109/10.

Panwar, A. & Sharma, A.K. (2009). Self-phase modulation of a laser in self created plasma channel . *Laser Part. Beams* **27**, 249–253.

Pathak, V. B. & Tripathi, V.K. (2006). Nonlinear electromagnetic plasma Eigen modes and their stability to stimulated Raman scattering. *Phys. Plasmas* **13**, 082105/4.

Porkolab, M. & Chang, R.P H. (1972). Instabilities and induced scattering due to nonlinear landau damping of longitudinal plasma waves in a magnetic field. *Phys. Fluids* **15**, 283–296.

Pukhov, A. & Meyer-ter-Vehn, J. (1998). Relativistic laser-plasma interaction by multi-dimensional particle-in-cell simulations. *Phys. Plasmas* **5**, 1880–1886.

Saini, N.S. & Gill, T.S. (2004). Enhanced Raman scattering of a rippled laser beam in a magnetized collisional plasma. *Laser Part. Beams* **22**, 35–40.

Sajal, V. & Tripathi, V.K. (2004). Stimulated Raman scattering of a laser beam in a plasma with azimuthal magnetic field. *Phys. Plasmas* **11**, 4206–4212.

Sajal, V., Panwar, A. & Tripathi, V.K. (2006). Relativistic forward stimulated Raman scattering of a laser in a plasma channel. *Phys. Scr*. **74**, 484–488.

Sharma, A. & Kourakis, I. (2009). Laser pulse compression and amplification via Raman backscattering in plasma. *Laser Part. Beams* **27**, 279–285.

Short, R.W. & Simon, A. (2004). Theory of three-wave parametric instabilities in inhomogeneous plasmas revisited. *Phys. Plasmas* **11**, 5335–5340.

Singh, R. & Tripathi, V.K. (2009). Filamentation of laser in a magnetized plasma under relativistic and pondromotive nonlinearities. *Phys. Plasmas* **16**, 052108/5.

Sodha, M., Ghatak, A. & Tripathi, V.K. (1976). Self focusing of laser beams in plasmas and semiconductors. *Prog. Opt*. **13**, 169–265.

Sodha, M., Khanna, R. & Tripathi, V.K. (1974*a*). The self-focusing of electromagnetic beams in a strongly ionized magnetoplasma. *J. Phys. D: Appl. Phys*. **7**, 2188–2197.

Sodha, M., Mittal, R., Kumar, S. & Tripathi, V.K. (1974*b*). Self-focusing of electromagnetic waves in a magneto plasma. *Opto-Electron*. **6**, 167–180.

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Trines, R.M.G.M., Kamp, L.P.J., Schep, T.J., Leemans, W.P., Esarey, E.H. & Sluijter, F.W. (2004). Enhancement of high-energy electron generation through suppression of Raman backscattering. *Europhys. Lett.* **66**, 492–498.

Verma, U. & Sharma, A.K. (2009*a*). Effect of self focusing on the prolongation of laser produced plasma channel. *Laser Part. Beams* **27**, 33–39.

Verma, U. & Sharma, A.K. (2009*b*). Laser second harmonic generation in a rippled density plasma in the presence of azimuthal magnetic field. *Laser Part. Beams* **27**, 719–724.

Verma, U. & Sharma, A.K. (2011). Laser focusing and multiple ionization of Ar in a hydrogen plasma channel created by a prepulse. *Laser Part. Beams* **29**, 219–225.

https://doi.org/10.1017/S0263034611000619 Published online by Cambridge University Press