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ESSAY-REVIEW

D'al-Khwārizmī à Descartes

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Roshdi Rashed, *D'al-Khwārizmī à Descartes: Études sur l'histoire des mathématiques classiques* (Paris, Hermann, 2011), vi + 795 pp., ISBN: 978 2 7056 8191 3

The volume *D'al-Khwārizmī à Descartes* is a monumental contribution to the world history of mathematical sciences, showing clearly that Arabic mathematics was an indispensable predecessor of early modern European mathematics. Roshdi Rashed is known, first of all, as an editor of classical mathematical writings in Arabic by such authors as al-Khwārizmī, Thābit ibn Qurra, Ibrāhīm ibn Sinān, Ibn al-Haytham, al-Khayyām, Sharaf al-Dīn al-Ṭūsī, as well as of the Arabic versions of Apollonius' *Conics*, Diophantus' *Arithmetica*, and Diocles' *Burning Mirrors*. As the volume under review shows, he is also a historian of mathematics of the first class who has transformed historiography. This book is, in a sense, a manifesto of Prof. Rashed's entire *œuvre*.

Arabic mathematics is still often limited to being the agency of transmitting the ancient Greek heritage to the mathematical fruits of the European Middle Ages, Renaissance, and modern era, which led to the flowering of today's advanced mathematical sciences. This Eurocentric vision of history of science has been criticized by Prof. Rashed over the past twenty years. In the present volume, he refers to the new discipline in mathematics, written in Arabic "algebra" by al-Khwārizmī and others, and the conception of "experimentation" in Ibn al-Haytham's work of optics. Algebra opened a new horizon of the mathematical sciences, having functioned as a novel unifying bond for them. Ibn al-Haytham's experimentation in optical studies transformed the Hellenistic doctrine of vision by Euclid and Ptolemy into a new theory which was succeeded by Kepler and Descartes in early modern Europe. Thus, Ibn al-Haytham became

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well known in Latin Europe as his name Alhazen, from one of his 'given' names, al-Ḥasan.

It might be possible to subtitle Prof. Rashed's volume, "European Mathematics and the Latin Prolongation of Arabic Mathematics." In fact, he has used the title "Fibonacci et le prolongement latin des mathématiques arabes" for a chapter of the first part on arithmetic. Nevertheless, we should not misunderstand Prof. Rashed to be an Arab-centrist or Arab nationalist, since his knowledge of ancient Greek mathematics and of early modern European mathematics shown in this book is better than thorough. If Descartes is a founder of modern mathematics, he may also be thought to have been a product of the Latin prolongation of Arabic mathematics. This kind of interpretation seems natural to me, the author of *Descartes's Mathematical Thought* (Dordrecht, 2003), for Descartes's mathematics can appropriately be characterized as algebraic. Consequently, the ambitious title of this volume *D'al-Khwārizmī à Descartes: Études sur l'histoire des mathématiques classiques* is not extraordinary. The classical mathematics of early modern Europe is the prolongation of Arabic mathematics.

This book begins with methodological observations first presented as Prof. Rashed's final lecture, when he retired from the University of Tokyo in March 1997. The successive chapters on Apollonius and on founders of Arabic mathematics show how mathematicians in Islamic civilization read classical Greek texts and created their own products.

The first part consists of chapters on "Algèbre" and chapters on "Arithmétique." I believe that the first chapter on "L'algèbre et son rôle unificateur" is one of the most important chapters in this volume. This chapter is closely related to Prof. Rashed's editorial work *Al-Khwārizmī: Le commencement de l'algèbre* (Paris, 2007; English version, 2009).

Compiled about the year 820 CE, al-Khwārizmī's original monograph *Kitāb al-Jabr wa-al-muqābala* (Book of algebra and of *al-muqābala*) created a new mathematical discipline named algebra, which teaches how algorithmic calculations containing both known and unknown quantities are done. (The word "algorithm" originated from the author of the monograph al-Khwārizmī, thus a kind of Orientalism!) Al-Khwārizmī's book on algebra together with *Le calcul indien* (Algorismus) (ed. by A. Allard, in 1992), a monograph on Hindu-Arabic numerals of which only its Latin version is preserved, indicates an essential characteristic of Arabic mathematics, which was quite international, a combination of various mathematical cultures such as Greek and Indian. European mathematics in a written form after Fibonacci's *Liber abaci* of 1202 (2nd edition in 1228) is unthinkable without the contribution

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by al-Khwārizmī and his successors such as Abū Kāmil to the discipline of algebra.

The algebraic style of European mathematics was established in Descartes's *Géométrie* of 1637. Descartes unified arithmetic and geometry. Prof. Rashed (R. Rashed et B. Vahabzadeh, *Al-Khayyām mathématicien* [Paris, 1999]; English version in 2000) has shown that al-Khayyām's treatise on algebra had used the unity for overcoming the dimensionality of geometry. Accordingly, Descartes is placed in the long history of mathematics as an inheritor of the algebraic tradition of al-Khwārizmī and al-Khayyām. Prof. Rashed has suggested that Descartes could have even known the Khayyāmian treatise on the theory of equations of the 3rd and 4th degrees by two conic sections through the intervention of the mathematician and arabist Jacobus Golius in Leiden. Although an indirect influence of al-Khayyām upon Descartes is possible, I still can imagine Descartes's independent discovery of the notion of unity, in this way, emphasizing the importance of European predecessors such as Italian cossists, François Viète, and Adriaan van Roomen (Adrianus Romanus), who used the concept of *mathesis universalis*, a notion standing for a unifier of the mathematical disciplines. Nevertheless, I cannot stop admiring Prof. Rashed's historical ingenuity. From a broader perspective, Descartes must have been in the tradition of Arabic mathematics which recognized the importance of algebra as a unifying bond of the mathematical disciplines.

When we consider Descartes's *Géométrie* historically, we would have a much better image of it, if we read in parallel the algebraic treatise of al-Khayyām and *al-Mu'ādalāt* (*Les équations*) by Sharaf al-Dīn al-Ṭūsī, the most important monograph in the algebraic school of al-Khayyām. In Chapter 4 "La *Géométrie* de Descartes et la distinction entre courbes géométriques et courbes mécaniques," Prof. Rashed writes:

> Lire la *Géométrie* de Descartes, c'est aussi regarder en amont vers al-Khayyām, al-Ṭūsī; et, en aval, vers Newton, Leibniz, Cramer, Bézout et les frères Bernoulli. C'est alors que la *Géométrie* retrouve la position qui n'a jamais cessé d'être la sienne: non plus que les autres œuvres novatrices, elle n'a rien d'un commencement radical: au même titre que les autres, elle est une manière de reprendre, d'adapter, et aussi de rectifier, les traditions dont elle est l'héritière. (pp. 213–14.)

Pierre de Fermat, a contemporary of Descartes who has been regarded as a co-discoverer of algebraic (or analytic) geometry, is also placed in the algebraic tradition of al-Khayyām in Chapter 6 "Fermat et la géométrie algébrique."

Arabic mathematics restarted the Hellenistic arithmetical tradition of Euclid, Nicomachus of Gerasa, and Diophantus of Alexandria. The

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mathematician who contributed much to this new starting of arithmetical researches was, among others, the translator of the Neopythagorian Nicomachus' *Introduction to Arithmetic* into Arabic Thābit ibn Qurra. Diophantine analysis also became a subject of study with the Arabic translation of Diophantus' *Arithmetic* by Qusṭā ibn Lūqā. This translation had the new title in Arabic: *Ṣinā'at al-jabr* (The Art of Algebra), an outcome of the merger with the tradition of al-Khwārizmī's book of algebra. Thus emerged algorithmic methods of number theory. For us Fermat is thought to have been the founder of Diophantine analysis. This is true, but, only a half truth. He should be, according to Prof. Rashed, an inheritor of the Arabic tradition of algorithmic or algebraic methods of number theory.

The second part of the book is devoted to "Géométrie."

Chapter 1 is "Les Archimédiens et les problèmes infinitésimaux." According to Prof. Rashed, infinitesimal geometry flourished, first of all, in the time of Archimedes, began again in the 9th century by Arabic mathematicians, and had a new age of flowering in 17th century-Europe where its algebraic form the differential and integral calculus was created. From 1993 on Prof. Rashed published *Les mathématiques infinitésimales du IX^e^ au XI^e^ siècle*, of which volumes I–V have so far appeared. This chapter is an excellent summary of vol. I: *Fondateurs et commentateurs* (published in 1996) and vol. II: *Ibn al-Haytham* (1993). In 2012, the English edition of vol. I appeared under the title *Founding Figures and Commentators in Arabic Mathematics: A history of Arabic sciences and mathematics*, with Routledge (see my review in *Contemporary Arab Affairs*, 5.2 [April 2012]: 313–15). For me, the achievements by the Banū Mūsā (the three brothers of Mūsā), Thābit ibn Qurra, Ibrāhīm ibn Sinān, grandson of Thābit ibn Qurra, and, of course, Ibn al-Haytham were remarkable. Thābit ibn Qurra's work was a precursor of Fermat's way of the quadrature of parabolic areas, and Wallis' arithmetical approach to infinitesimal problems. Ibn al-Haytham's results were superseded only by European mathematical geniuses of the mid-seventeenth century. The last section is on Ibn al-Haytham's theory of solid angle. This theory predicted Descartes's youthful writing *Progymnasmata de solidorum elementis*.

Successive chapters describe the Arabic development of the theory of conics, a new way of the classification of curves, and Thābit ibn Qurra's discussion of the parallel postulate.

The third part of the book is "Application des mathématiques," of which there are 3 chapters on "Astronomie" and 3 chapters on "Optique."

Chapter 1 is "La cinématique céleste d'Ibn al-Haytham," which is essentially the same as the introductory part of *Les mathématiques*

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infinitésimales du IX^e au XI^e siècle, vol. V: *Ibn al-Haytham: Astronomie, géométrie sphérique et trigonométrie*, published in 2006, and its modified English version, "The celestial kinematics of Ibn al-Haytham," *Arabic Sciences and Philosophy*, 17 (2007): 7–55. As to this chapter, two points should be kept in mind. One point is the image of the astronomer Ibn al-Haytham. Our mathematician al-Ḥasan ibn al-Haytham must not be confused (*pace* A. I. Sabra) with the author of *La configuration de l'univers*, a cosmological work, "a philosopher with an interest in the sciences, but not himself a mathematician, called Muḥammad ibn al-Haytham." (Rashed, "Celestial kinematics," p. 13, cf. vol. V, pp. 5–6; see also "*The configuration of the universe*: A book by al-Ḥasan ibn al-Haytham?" *Revue d'histoire des sciences*, 60–1 [janvier-juin 2007]: 47–63.) The other point is that Ibn al-Haytham's approach to celestial kinematics was to amount to "géométriser une cinématique," or an attempt at mathematically phenomenological kinematics, as it were.

Chapter 2 "Al-Qūhī: De la météorologie à l'astronomie," and Chapter 3 "Al-Qūhī contre Aristotle: Sur le mouvement," introduce the ingenious mathematician and astronomer Abū Sahl al-Qūhī's way of addressing Aristotle's natural philosophy.

The chapters on optics show more remarkable achievements of Arabic scientists. The new development of Arabic optics began with the introduction of Hellenistic treatises on burning mirrors by Diocles and others. This kind of research lead to a study of anaclastics and dioptrics. Prof. Rashed showed in his *Geometry and Dioptrics in Classical Islam* (London, 2005): Ibn Sahl of the tenth century discovered the so-called Snellius' law of refraction by using his knowledge on conic sections. This law is known to have been also discovered almost simultaneously by Claude Mydorge and René Descartes in the mid-1620s.

Further, Ibn al-Haytham established a new theory of vision in his critical study of Ptolemy's optics under the title *Kitāb al-Manāẓir* (The Book of Optics), which was quite different from Euclid's and Ptolemy's and leading to Kepler's physiologically correct understanding.

Prof. Rashed's conclusion is "Philosophie des mathématiques." It shows that the author is a historian of mathematics animated by a deep philosophical interest.

At the beginning, Prof. Rashed introduces two philosophers. One, familiar with both philosophy and mathematics, is al-Kindī, and the other, not so familiar with mathematics, but certainly interested in it, is Maimonides. In general, philosophers of the Islamic world were equipped with the general knowledge of philosophical discussions of both Aristotelians and Neoplatonists in the Hellenistic world. For al-Kindī, the mathematical disciplines are an

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indispensable tool for philosophy. And in both al-Kindī and Maimonides, Prof. Rashed sees a precursory element of Spinoza, who made importance the order of argumentation of geometrical science.

Next come the great Islamic philosopher Ibn Sīnā (Latinized name, Avicenna) and Naṣīr al-Dīn al-Ṭūsī. From Prof. Rashed's description, we know that "le calcul indien" and "la science de l'algèbre et d'*al-muqābala*" were referred to by Ibn Sīnā, although Ibn Sīnā failed to explicate them in details. Naṣīr al-Dīn al-Ṭūsī was a mathematician who attached importance to the combinatorial art, and may be considered to have been a forerunner in the notion of *characteristica universalis* by Leibniz the German 'uomo universale' of the 17th century, although this European's idea might have emerged independently of the Arabic mathematician.

Finally, Prof. Rashed devotes several pages to the *ars analytica* and *ars inveniendi*, a pair conception of the *ars synthetica* and *ars demonstrandi*, respectively: Thābit ibn Qurra, Ibrāhīm ibn Sinān, al-Sijzī, and Ibn al-Haytham. On texts by Thābit ibn Qurra, al-Sijzī, and Ibn al-Haytham, we should consult the author's work, *Les mathématiques infinitésimales du IX^e^ au XI^e^ siècle*, vol. IV: *Ibn al-Haytham, Méthodes géométriques, transformations ponctuelles et philosophie des mathématiques*, published in 2002. On the grandson of genius of Thābit ibn Qurra, Ibrāhīm ibn Sinān, we should read *Ibrāhīm ibn Sinān: Logique et géométrie au X^e^ siècle*, co-authored by R. Rashed and Hélène Bellosta and published from Brill in Leiden in 2000. Before Prof. Rashed's works, we knew only that Hellenistic authors (Galen, Theon's commentary of Euclid, Pappus, and Proclus) wrote about the pair *ars analytica et synthetica* before the European Middle Ages and Renaissance. Now we recognize that Arabic mathematicians composed a rich corpus of writings on the methodological pair conception of analysis and synthesis as well as the *ars inveniendi*.

Indeed, without the contributions by Arabic mathematicians from the 9th through 13th centuries, classical mathematics in early modern Europe would have been impossible. The title of Descartes's monumental book published in 1637 was: *Discours de la méthode pour bien conduire sa raison & chercher la vérité dans les sciences. Plus la dioptrique, les météores, et la géométrie*. Many important parts of this book may be found in the contribution of Arabic mathematicians. Thus, the title of Prof. Rashed's volume: *D'al-Khwārizmī à Descartes*.

I conclude with thoughts about the history of mathematics in East Asia. Chinese mathematics during the Song dynasty from the tenth through the thirteenth century CE transformed itself to such an extent that the new calculating instrument *suanpan*, or Chinese abacus, came into full use, and the new art of algebra named *tian yuan*

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shu (literally translated into English, the art of celestial element) was invented. China in that period experienced a kind of "Commercial Revolution." Chinese historians of mathematics are studying whether an influence of Islamic mathematics existed, in part stimulated by Prof. Rashed's visit to Beijing, Xi'an, and Shanghai in the summer of 2010.

In the late Middle Ages, Japan imported the Chinese abacus culture along with its Commercial Revolution. Moreover, in the last decades of the 16th century, the Chinese *tian yuan* art was transmitted into Japan. This art of algebra resembled the mathematical background against which the mathematical genius Seki Takakazu and his talented disciple Takebe Katahiro invented a novel form of symbolic algebra called *tenzan jutsu* in the second half of the seventeenth century. For this reason, Seki may be called the "François Viète of early modern Japan." With the introduction of the Chinese abacus and the art of celestial element, the Japanese people arrived at a similar conception of mathematics to the European *mathesis universalis* as a unifying bond of the mathematical disciplines together with the Neo-confucianist metaphysical foundations based on the Chinese classics *I Ching*, which may have provided a kind of Pythagorean background on binary arithmetic. Equipped with this kind of symbolic algebra, around the years of the Meiji Restoration in 1868, mathematical practitioners in Japan could easily assimilate modern Western mathematics. Thus, Japan's successful introduction of modern mathematics was a kind of "translational adaptation": The European *mathesis universalis* was the art of calculation with Hindu-Arabic numerals plus the reformed art of symbolic algebra; the Japanese counterpart was the art of calculation with the abacus plus the newly invented art of *tenzan* algebra. Both were algorithmic! Traditional Chinese mathematics was an indispensable predecessor of early modern Japanese mathematics just as Arabic mathematics was for classical European mathematics.

Prof. Rashed invites us to think not simply of a historical foundational problem of European classical mathematics, but also of comparable problems in the East Asian classical mathematical heritage.

This volume should be translated into English as soon as possible.

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