Robotica (2015) volume 33, pp. 1393–1414. © Cambridge University Press 2014
doi:10.1017/S0263574714000605

Trajectory tracking of wheeled mobile robot by adopting iterative learning control with predictive, current, and past learning items

Chong Yu and Xiong Chen*

School of Information Science and Technology, Fudan University, Shanghai 200433, China

(Accepted February 13, 2014. First published online: April 1, 2014)

SUMMARY

In this paper, an iterative learning control algorithm is adopted to solve the high-precision trajectory tracking issue of a wheeled mobile robot with time-varying, nonlinear, and strong-coupling dynamics properties. The designed iterative learning control law adopts predictive, current and past learning items to drive the state variables, and input variables, and outputs variables converge to the bounded scope of their desired values. The algorithm can enhance the control performance, stability and robust characteristics. The rigorous mathematical proof of the convergence character of the proposed iterative learning control algorithm is given. The feasibility, effectiveness, and robustness of the proposed algorithm are illustrated by quantitative experiments and comparative analysis. The experimental results show that the proposed iterative learning control algorithm has an outstanding control effect on the trajectory tracking issue of wheeled mobile robots.

KEYWORDS: Wheeled mobile robot; Iterative learning control; Trajectory tracking; Predictive learning items; Current learning items; Past learning items; Proof of the convergence character.

1. Introduction

The fundamental prototype of iterative learning control was firstly put forward by an American patent[1] in the year of 1967. In 1978, by utilizing the thought of iterative learning control, Uchiyama, a Japanese scholar proposed the method to control a mechanical hand in high-speed motion.[2] Since this paper was written in Japanese, it did not cause widespread concern in academic circles. By the year 1984, Arimoto et al made improvements and introduced Uchiyama's method in English.[3] Then they explicitly defined the concept of iterative learning control.

By referring to the basic principle that the required indicators can achieve the satisfied level in the repetitive pursuit, these pioneer researches successfully controlled the industrial robots featured with multi-variable, nonlinear, and strong-coupling characteristics to complete the trajectory tracking task quickly and accurately.[4,5] The classical control strategies always take linear system as its controlled objects to begin research. However, once proposed, iterative learning control takes the nonlinear systems as its research objects. It requires fulfilling the control task that system output can fully track the target trajectory in the limited time interval. Here, the term "fully tracking" means both transient and stationary system outputs should be consistent with target trajectory. In the attempts to control the system, the iterative learning control method amends undesired input control signals based on the errors between output signals and expected targets. Then the tracking performance of the controlled system can be improved in the next control process.

Iterative learning control has some differences with classical control strategies,[6] such as adaptive control,[7,8] artificial neural networks,[9,10] and repetitive control.[11,12] An adaptive control strategy makes amendments to entire control system, while the iterative learning control method makes amendments

* Corresponding author.
E-mail: 11210720035@fudan.edu.cn, dxxzdxxz@126.com (Chong Yu)
E-mail: chenxiong@fudan.edu.cn (Chen Xiong)

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

to the input control signal. In addition, the adaptive controller does not make use of the information contained in the repetitive command signal.

Similarly, an artificial neural network learning control strategy makes amendments to relevant controller parameters instead of making amendments to the control signal. So the entire nonlinear neural network needs amendment. Amendments to large-scale neural network require numerous training data sets, and it is difficult to ensure fast convergence.[13] However, it has been proved that the iterative learning control method can acquire sufficient convergence through a certain number of iterations.

Repetitive control[14] is the most similar control strategy to iterative learning control. The most essential difference between these two control strategies is that repetitive control studies continuous operation process, while iterative learning control is focus on discontinuous operation process. Another obvious difference between these two control strategies is setting different initial conditions in experiments.[15] In a repetitive control experiment, the initial conditions are set as the termination conditions in the previous experiment. While in an iterative learning control experiment, the initial conditions of each experiment are set as the same values. The distinction between the settings of initial conditions eventually leads to two different control effects, as well as these two different control strategies.

The iterative learning control method is suitable for the controlled system featured with the nature of repetitive motions.[16–18] By utilizing the previous control experience and deviation information, the control method can amend current control actions to make the outputs of the controlled system converge to the expected values. Iterative learning control essentially adopts the feed-forward control structure. It simulates the learning and self-regulation functions of human brain, and has become a very important branch in the intelligent control field. This method does not depend on the precise mathematical model of the dynamic system. It can achieve quite a good control effect of the dynamic system with high uncertainty in a simple way. Less priori knowledge and computation consumption are required. So the iterative learning control method is easy to be implemented, and has strong adaptability. It is of great significance to the research of nonlinear, strong-coupling, and difficult-modeling systems as well as the high-precision trajectory tracking problems.

The wheeled mobile robot is featured with self-organization, self-planning, and adaptive capacities.[19] This kind of robot is able to work in complex environments. In the relevant research of the wheeled mobile robot, there is no doubt that the control method is the core technology.[20] Also, it is the key technology to realize the fully autonomous motion and intelligence of robots.[21,22] Since the wheeled mobile robot is featured with time-varying, nonlinear, and strong-coupling dynamics properties, with the influence of measurements inaccuracy, as well as external disturbances and load changes, so actually it is rarely possible to obtain the complete and accurate model of a wheeled mobile robot.[23]

The iterative learning control algorithm will determine the expected linear and angular velocities for the wheeled mobile robot, while the dynamic controller will control the actual speed and make it try to track the desired speed. Since we cannot obtain the precise kinematic model of the wheeled mobile robot, so adopting the designed dynamic controller which is based on an imprecise model cannot make the actual speed exactly be equal to the expected speed at any moment. Accordingly, we regard the error between the actual speed and expected speed as the state interference in the wheeled mobile robot kinematic model, and applying an iterative learning control algorithm with brilliant robustness capability to solve the high-precision trajectory tracking issue of the wheeled mobile robot.

The overall organization of this paper is as follows: In Section 2, we give the description of mathematical definitions and symbols' meanings involved in this paper. The system description of the wheeled mobile robot model and the trajectory tracking issue is also described. In Section 3, the design of iterative learning control law is given. Then we mainly focus on rigorous mathematical proof of the convergence character of the proposed iterative learning control algorithm. In Section 4, some quantitative experiments and comparative analyses for testing the capabilities and performance of the proposed control algorithm are shown. Finally, Section 5 summarizes the whole paper and makes closing conclusions.

2. Mathematics Backgrounds and System Description

2.1. Mathematics backgrounds

In the following sections, these mathematical definitions and symbols will be involved:

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

1. The symbol $\mathbb{R}^n$ represents the n-dimensional Euclidean space. In this space, the norm of the vector is as follows: $\|z\| = \left(z^T z\right)^{1/2}$, where the vector $z \in \mathbb{R}^n$.
2. The symbol $\boldsymbol{C} \in \mathbb{R}^{p\times m}$ represents a $(p \times m)$-dimensional matrix with elements in the real number domain. The corresponding matrix norm is as follows: $\|\boldsymbol{C}\| = \sqrt{\lambda_{\max}\left(C^T C\right)}$, where the symbol $\lambda_{\max}(\boldsymbol{A})$ represents the maximum eigenvalue of matrix $\boldsymbol{A}$.
3. The symbol $\mathbb{N}$ represents the set of positive integers, i.e. $\{1, \ldots, n\}$, then the α-norm of the positive real function $z : \mathbb{N} \to \mathbb{R}$ is as follows:

$$\|z(\cdot)\|_\alpha = \sup_{k\in\mathbb{N}} z(k)\left(\frac{1}{\alpha}\right)^k,$$

where the parameter α satisfies the condition that $\alpha \geq 1$.

2.2. System description

Among the family of all wheeled mobile robot, the simplest and most fundamental model is the differential unicycle. The mobile robot of this type adopts one axis to combine two independent driving wheels and it moves in two-dimensional space. The differential unicycle-type robot changes its advance direction by utilizing the velocity difference of two driving wheels. Due to nonholonomic constraint, the movements of this mobile robot must be vertical to the direction of connection axis. Here, we mainly based on the model of a differential unicycle-type mobile robot to discuss and solve the trajectory tracking issues by adopting the iterative learning control law.

In the following discussion, we use the symbol point $P(k)$ to represent the location of mobile robot in the k moment. In the generalized Cartesian coordinate system, the coordinate of point $P(k)$ is defined as $[x_p(k), y_p(k), \theta_p(k)]$, where the symbol $x_p(k)$ and symbol $y_p(k)$ represent the horizontal and vertical coordinates of point $P(k)$, respectively, the symbol $\theta_p(k)$ represents the azimuth angle of the robot. The azimuth angle $\theta_p(k)$ is defined as the angle between the advance direction of the robot and the positive direction of the horizontal coordinate in the world coordinate system. The linear velocity of the point $P(k)$ is represented by the symbol $v_p(k)$, while the angular velocity is represented by $\omega_p(k)$. Then in the discrete-time domain, the kinematic equation of the mobile robot based on the representative point $P(k)$ is as follows:

$$\begin{bmatrix} x_p(k+1) \\ y_p(k+1) \\ \theta_p(k+1) \end{bmatrix} = \begin{bmatrix} x_p(k) \\ y_p(k) \\ \theta_p(k) \end{bmatrix} + \Delta T \begin{bmatrix} \cos\theta_p(k) & 0 \\ \sin\theta_p(k) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_p(k) \\ \omega_p(k) \end{bmatrix}, \tag{1}$$

where the symbol ΔT represents the sampling time.

By defining the state vector of the mobile robot as $\boldsymbol{q}(k) = [x_p(k), y_p(k), \theta_p(k)]^T$, and the velocity vector as $\boldsymbol{u}_P(k) = [v_p(k), \omega_p(k)]^T$, then the kinematic Eq. (1) can be updated to the form as follows:

$$\boldsymbol{q}(k+1) = \boldsymbol{q}(k) + \boldsymbol{B}(\boldsymbol{q}(k), k)\boldsymbol{u}_P(k), \tag{2}$$

where

$$\boldsymbol{B}(\boldsymbol{q}(k), k) = \Delta T \begin{bmatrix} \cos\theta_p(k) & 0 \\ \sin\theta_p(k) & 0 \\ 0 & 1 \end{bmatrix}.$$

As illustrated in Fig. 1, the expected discrete trajectory which is represented by the symbol P_d has been set in advance. The trajectory contains n time samples of $P_d(k) = [x_d(k), y_d(k), \theta_d(k)]$, where the symbol k satisfies the condition that $1 \leq k \leq n$. The traditional control issue of kinematic trajectory tracking is to determine the input signal $\boldsymbol{u}(k) = [v(k), \omega(k)]^T$ according to $\boldsymbol{u}_P(k)$, and make $P(k)$ be able to track $P_d(k)$ in each k moment.

At present, various kinematic controllers have been developed to solve this issue. However, in order to reduce the complexity, most of the developed controllers are based on the assumption that the

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

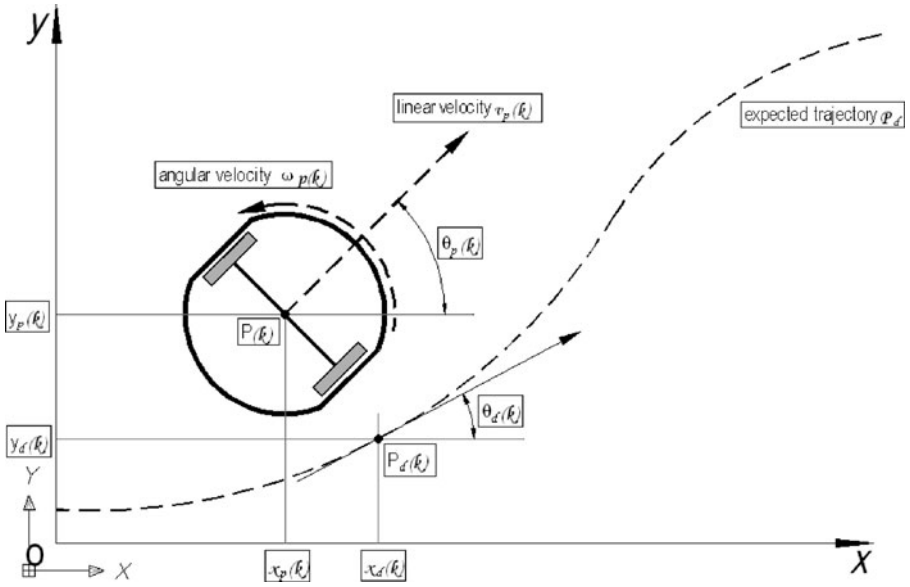


Fig. 1. Differential unicycle-type mobile robot motion model and its trajectory tracking issue.

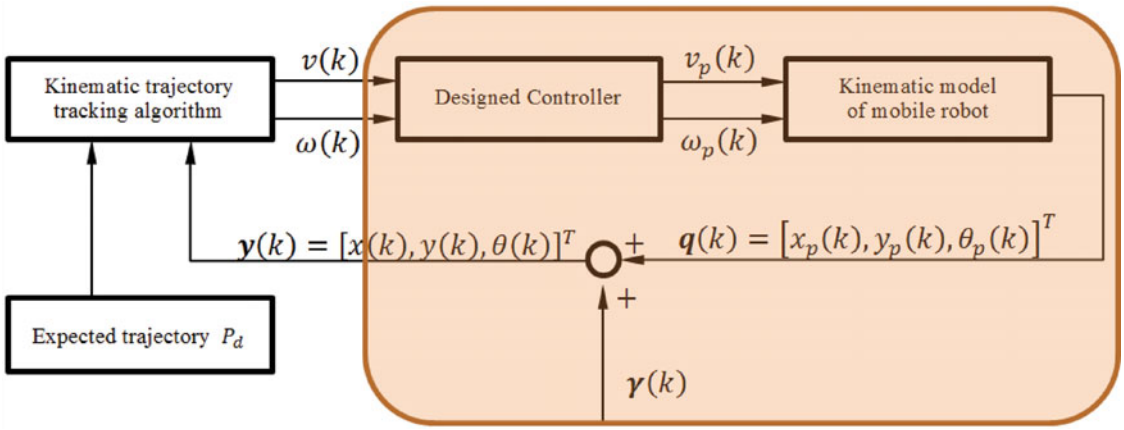


Fig. 2. Iterative learning control system structure of the mobile robot.

real velocity value $\boldsymbol{u}_p(k)$ of the mobile robot is equal to the setting value $\boldsymbol{u}(k)$ at any time, which is defined as the perfect velocity tracking assumption. Whereas, some uncertain factors and disturbances are inevitable in the real applications, therefore, the perfect velocity tracking assumption is rarely satisfied. So, it is our aim to design an appropriate controller for the dynamic system to make the velocity errors converge to bounded values when bounded uncertain factors and disturbances exist in a mobile robot.

The linear velocity error $\tilde{v}(k)$ and the angular velocity error $\tilde{\omega}(k)$ are defined as follows:

$$\tilde{v}(k) = v_p(k) - v(k), \tag{3}$$

$$\tilde{\omega}(k) = \omega_p(k) - \omega(k). \tag{4}$$

Assuming that they converge to known constants respectively, then we attempt to construct an iterative learning algorithm for $v(k)$ and $\omega(k)$. With the increase of iteration, the real location of the mobile robot $P(k)$ can track the expected discrete trajectory $P_d(k)$ in each k moment, where $1 \leq k \leq n$. This is the kinematic trajectory tracking issue based on the iterative learning control strategy.

The entire iterative learning control system is shown in Fig. 2. Our main aim is to determine the control input variables $v(k)$ and $\omega(k)$ according to the iterative learning control law. Assuming that

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

the input variables of the dynamic controller has bounded errors $\tilde{v}(k)$ and $\tilde{\omega}(k)$, which can be viewed as the state disturbances. Therefore, with the dynamic controller, which is marked by the shaded box in Fig. 2, the discrete kinematic Eq. (2) can be adapted as follows:

$$\boldsymbol{q}(k+1) = \boldsymbol{q}(k) + \boldsymbol{B}(\boldsymbol{q}(k), k)\,\boldsymbol{u}(k) + \boldsymbol{\beta}(k), \tag{5}$$

$$\boldsymbol{y}(k) = \boldsymbol{q}(k) + \boldsymbol{\gamma}(k), \tag{6}$$

where the symbol $\boldsymbol{\beta}(k)$ represents the state disturbances which is introduced by imperfect dynamic control. The symbol $\boldsymbol{\gamma}(k)$ represents output measurement noises. The vector $\boldsymbol{y}(k) = [x(k), y(k), \theta(k)]^T$ represents the system outputs for any value in the range $k \geq 1$. Considering all the disturbances and noises, we adopt the iterative learning control law to design a kinematic trajectory tracking algorithm in order to produce the control input vector $\boldsymbol{u}(k) = [v(k), \omega(k)]^T$.

Take iterative process into consideration, then the Eqs. (5) and (6) will be as follows:

$$\boldsymbol{q}_i(k+1) = \boldsymbol{q}_i(k) + \boldsymbol{B}(\boldsymbol{q}_i(k), k)\,\boldsymbol{u}_i(k) + \boldsymbol{\beta}_i(k), \tag{7}$$

$$\boldsymbol{y}_i(k) = \boldsymbol{q}_i(k) + \boldsymbol{\gamma}_i(k), \tag{8}$$

where the symbol i represents the number of iteration. The symbol k represents the discrete time, and satisfies the constraint that $k = 1, \ldots, n$. The symbols $\boldsymbol{q}_i(k)$, $\boldsymbol{u}_i(k)$, $\boldsymbol{y}_i(k)$, $\boldsymbol{\beta}_i(k)$, and $\boldsymbol{\gamma}_i(k)$ represent the state variables, input variables, output variables, state disturbances, and output noises, respectively, in the No. 1 iterative process. The Eqs. (7) and (8) satisfy the following assumptions and properties.

Property 1 Considering the ideal situation, we set the variable $\boldsymbol{\beta}_i(k)$ and $\boldsymbol{\gamma}_i(k)$ as equal to zero, where $k \in \mathbb{N}$, then the equations of the expected trajectory are as follows:

$$\boldsymbol{q}_d(k+1) = \boldsymbol{q}_d(k) + \boldsymbol{B}(\boldsymbol{q}_d(k), k)\,\boldsymbol{u}_d(k), \tag{9}$$

$$\boldsymbol{y}_d(k) = \boldsymbol{q}_d(k). \tag{10}$$

Property 2 The function $\boldsymbol{B}(\boldsymbol{q}_i(k), k)$ in matrix forms satisfies the *Lipschitz* condition:

$$\|\boldsymbol{B}(\boldsymbol{q}_1, k) - \boldsymbol{B}(\boldsymbol{q}_2, k)\| \leq c_B \|\boldsymbol{q}_1 - \boldsymbol{q}_2\|, \tag{11}$$

where $k \in \mathbb{N}$ and the symbol c_B represents a positive constant.

Property 3 The matrix $\boldsymbol{B}(\boldsymbol{q}_i(k), k)$ is bounded, i.e., $\|\boldsymbol{B}(\boldsymbol{q}_i(k), k)\| \leq b_B$, where the symbol b_B represents a positive constant. The matrix $\boldsymbol{B}(\boldsymbol{q}_i(k), k)$ is a full rank matrix in terms of $(\boldsymbol{q}_i(k), k)$.

Assumption 1

$$\max_{1 \leq k \leq n} \|\boldsymbol{u}_d(k)\| \leq b_{u_d}, \tag{12}$$

where the symbol b_{u_d} is a positive constant.

Assumption 2 The disturbances and noises are bounded, i.e.,

$$\max_{1 \leq i \leq \infty} \max_{1 \leq k \leq n} \|\boldsymbol{\beta}_i(k)\| \leq b_\beta, \ \max_{1 \leq i \leq \infty} \max_{1 \leq k \leq n} \|\boldsymbol{\gamma}_i(k)\| \leq b_\gamma, \tag{13}$$

where the symbols b_β and b_γ are positive constants.

Assumption 3 In each iterative process, the starting point of the trajectory is within the neighborhood of $\boldsymbol{q}_d(0)$, i.e.,

$$\|\boldsymbol{q}_d(0) - \boldsymbol{q}_i(0)\| \leq b_{q_0},$$

where $b_{q_0} > 0$ and $i \geq 1$.

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

3. Design of Control Law and Mathematical Proof

3.1. Design of control law

The iterative learning control law we designed and proposed is described as follows:

$$\boldsymbol{u}_{i+1}(k) = \boldsymbol{u}_i(k) + \boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1) + \boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k) + \boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}\,(k-1)\,. \tag{14}$$

In the process of No. i iteration, the tracking error is $\boldsymbol{e}_i(k) = \boldsymbol{y}_d(k) - \boldsymbol{y}_i(k)$. The symbols $\boldsymbol{L}_1(k)$, $\boldsymbol{L}_2(k)$, and $\boldsymbol{L}_3(k)$ represent the learning gain matrices which satisfy the constraints $\|\boldsymbol{L}_1(k)\| \le b_{L_1}$, $\|\boldsymbol{L}_2(k)\| \le b_{L_2}$, and $\|\boldsymbol{L}_3(k)\| \le b_{L_3}$, where $k \in \mathbb{N}$, $b_{L_1} > 0$, $b_{L_2} > 0$, $b_{L_3} > 0$. In the iterative learning control law, $\boldsymbol{u}_i(k) + \boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1)$ represents the predictive learning item, $\boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k)$ represents the current learning item, and $\boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}\,(k-1)$ represents the past learning item.

By adopting the control law shown as Eq. (14), we mean to prompt the state variable $\boldsymbol{q}_i(k)$, the control input variable $\boldsymbol{u}_i(k)$, and the system output variable $\boldsymbol{y}_i(k)$ converge to their expected values $\boldsymbol{q}_d(k)$, $\boldsymbol{u}_d(k)$, and $\boldsymbol{y}_d(k)$, respectively.

Theorem Let us consider the discrete-time system represented by Eqs. (7) and (8). If the system satisfies Assumption 1, 2, and 3 simultaneously, and the system is controlled by adopting the Eq. (14), then the following assertion is true for all $(\boldsymbol{q}_i, k) \in \mathbb{R}^n \times \mathbb{N}$

$$\|\boldsymbol{I} - \boldsymbol{L}_1(k)\boldsymbol{B}\,(\boldsymbol{q}_i, k)\| \le \rho \le 1. \tag{15}$$

If the state disturbances, output noises and initial state errors are all neglected, i.e., $b_\beta = b_\gamma = b_{q_0} = 0$, then the variables $\boldsymbol{q}_i(k)$, $\boldsymbol{u}_i(k)$, and $\boldsymbol{y}_i(k)$ converge to the values $\boldsymbol{q}_d(k)$, $\boldsymbol{u}_d(k)$, and $\boldsymbol{y}_d(k)$, respectively, where $k \in \mathbb{N}$, $i \to \infty$.

If the state disturbances, output noises and initial state errors are taken into consideration, then $\|\boldsymbol{q}_i(k) - \boldsymbol{q}_d(k)\|$, $\|\boldsymbol{u}_i(k) - \boldsymbol{u}_d(k)\|$, and $\|\boldsymbol{y}_i(k) - \boldsymbol{y}_d(k)\|$ are all bounded to the functions of parameters b_β, b_γ and b_{q_0}, respectively.

3.2. Mathematical proof

In this subsection, we will give the rigorous mathematical analysis of the convergence character of the proposed iterative learning control law, the proof of the theorem is also provided.

By combining the Eqs. (9) and (7), we can obtain the following equation:

$$\begin{aligned}
\tilde{\boldsymbol{q}}_i\,(k+1) &= \boldsymbol{q}_d\,(k+1) - \boldsymbol{q}_i\,(k+1) \\
&= [\boldsymbol{q}_d(k) + \boldsymbol{B}\,(\boldsymbol{q}_d(k), k)\,\boldsymbol{u}_d(k)] - [\boldsymbol{q}_i(k) + B\,(\boldsymbol{q}_i(k), k)\,\boldsymbol{u}_i(k) + \boldsymbol{\beta}_i(k)] \\
&= \tilde{\boldsymbol{q}}_i(k) + \boldsymbol{B}\,(\boldsymbol{q}_d(k), k)\,\boldsymbol{u}_d(k) - B\,(\boldsymbol{q}_i(k), k)\,[\boldsymbol{u}_i(k) - \boldsymbol{u}_d(k) + \boldsymbol{u}_d(k)] - \boldsymbol{\beta}_i(k) \\
&= \tilde{\boldsymbol{q}}_i(k) + [\boldsymbol{B}\,(\boldsymbol{q}_d(k), k) - \boldsymbol{B}\,(\boldsymbol{q}_i(k), k)]\,\boldsymbol{u}_d(k) + \boldsymbol{B}\,(q_i(k), k)\,\tilde{\boldsymbol{u}}_i(k) - \boldsymbol{\beta}_i(k).
\end{aligned} \tag{16}$$

According to Property 2, Property 3, Assumption 1 and Assumption 2, the inequality is obtained:

$$\|\tilde{\boldsymbol{q}}_i\,(k+1)\| \le \|\tilde{\boldsymbol{q}}_i(k)\| + c_B b_{u_d}\,\|\tilde{\boldsymbol{q}}_i(k)\| + b_B\,\|\tilde{\boldsymbol{u}}_i(k)\| + b_\beta.$$

Set the symbol $h_2 = 1 + c_B b_{u_d}$, then the inequality can be adapted as:

$$\|\tilde{\boldsymbol{q}}_i\,(k+1)\| \le h_2\,\|\tilde{\boldsymbol{q}}_i(k)\| + b_B\,\|\tilde{\boldsymbol{u}}_i(k)\| + b_\beta.$$

Make recursion to the above inequality, and consider the Assumption 3, then we can obtain:

$$\|\tilde{\boldsymbol{q}}_i(k)\| \le \sum_{j=0}^{k-1} h_2^{k-1-j}\left[b_B\,\|\tilde{\boldsymbol{u}}_i\,(j)\| + b_\beta\right] + h_2^k b_{q_0}. \tag{17}$$

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

From the control law (14), the deductive process is as follows:

$$
\begin{aligned}
\tilde{\boldsymbol{u}}_{i+1}(k) &= \boldsymbol{u}_d(k) - \boldsymbol{u}_{i+1}(k) \\
&= \boldsymbol{u}_d(k) - \boldsymbol{u}_i(k) - \boldsymbol{L}_1(k)\boldsymbol{e}_i(k+1) - \boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k) - \boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}(k-1) \\
&= \tilde{\boldsymbol{u}}_i(k) - \boldsymbol{L}_1(k)\left[\boldsymbol{y}_d(k+1) - \boldsymbol{y}_i(k+1)\right] - \boldsymbol{L}_2(k)\left[\boldsymbol{y}_d(k) - \boldsymbol{y}_{i+1}(k)\right] \\
&\quad - \boldsymbol{L}_3(k)\left[\boldsymbol{y}_d(k-1) - \boldsymbol{y}_{i+1}(k-1)\right] \\
&= \tilde{\boldsymbol{u}}_i(k) - \boldsymbol{L}_1(k)\left[\boldsymbol{q}_d(k+1) - \boldsymbol{q}_i(k+1) - \boldsymbol{\gamma}_i(k+1)\right] - \boldsymbol{L}_2(k)\left[\boldsymbol{q}_d(k) - \boldsymbol{q}_{i+1}(k) - \boldsymbol{\gamma}_{i+1}(k)\right] \\
&\quad - \boldsymbol{L}_3(k)\left[\boldsymbol{q}_d(k-1) - \boldsymbol{q}_{i+1}(k-1) - \boldsymbol{\gamma}_{i+1}(k-1)\right] \\
&= \tilde{\boldsymbol{u}}_i(k) - \boldsymbol{L}_1(k)\left[\boldsymbol{q}_d(k) + \boldsymbol{B}(\boldsymbol{q}_d(k),k)\boldsymbol{u}_d(k) - \boldsymbol{q}_i(k) - \boldsymbol{B}(\boldsymbol{q}_i(k),k)\boldsymbol{u}_i(k)\right] \\
&\quad + \boldsymbol{L}_1(k)\left[\boldsymbol{\beta}_i(k) + \boldsymbol{\gamma}_i(k+1)\right] - \boldsymbol{L}_2(k)\left[\tilde{\boldsymbol{q}}_{i+1}(k) - \boldsymbol{\gamma}_{i+1}(k)\right] \\
&\quad - \boldsymbol{L}_3(k)\left[\tilde{\boldsymbol{q}}_{i+1}(k-1) - \boldsymbol{\gamma}_{i+1}(k-1)\right] \\
&= \tilde{\boldsymbol{u}}_i(k) - \boldsymbol{L}_1(k)\tilde{\boldsymbol{q}}_i(k) - \boldsymbol{L}_1(k)\left\{\boldsymbol{B}(\boldsymbol{q}_d(k),k)\boldsymbol{u}_d(k) - \boldsymbol{B}(\boldsymbol{q}_i(k),k)\left[\boldsymbol{u}_i(k) - \boldsymbol{u}_d(k) + \boldsymbol{u}_d(k)\right]\right\} \\
&\quad + \boldsymbol{L}_1(k)\left[\boldsymbol{\beta}_i(k) + \boldsymbol{\gamma}_i(k+1)\right] - \boldsymbol{L}_2(k)\left[\tilde{\boldsymbol{q}}_{i+1}(k) - \boldsymbol{\gamma}_{i+1}(k)\right] \\
&\quad - \boldsymbol{L}_3(k)\left[\tilde{\boldsymbol{q}}_{i+1}(k-1) - \boldsymbol{\gamma}_{i+1}(k-1)\right] \\
&= \tilde{\boldsymbol{u}}_i(k) - \boldsymbol{L}_1(k)\tilde{\boldsymbol{q}}_i(k) - \boldsymbol{L}_1(k)\boldsymbol{B}(\boldsymbol{q}_i(k),k)\tilde{\boldsymbol{u}}_i(k) - \boldsymbol{L}_1(k)\left[\boldsymbol{B}(\boldsymbol{q}_d(k),k) - \boldsymbol{B}(\boldsymbol{q}_i(k),k)\right]\boldsymbol{u}_d(k) \\
&\quad - \boldsymbol{L}_2(k)\tilde{\boldsymbol{q}}_{i+1}(k) - \boldsymbol{L}_3(k)\tilde{\boldsymbol{q}}_{i+1}(k-1) + \boldsymbol{L}_1(k)\left[\boldsymbol{\beta}_i(k) + \boldsymbol{\gamma}_i(k+1)\right] \\
&\quad + \boldsymbol{L}_2(k)\boldsymbol{\gamma}_{i+1}(k) + \boldsymbol{L}_3(k)\boldsymbol{\gamma}_{i+1}(k-1) \\
&= \left[\boldsymbol{I} - \boldsymbol{L}_1(k)\boldsymbol{B}(\boldsymbol{q}_i(k),k)\right]\tilde{\boldsymbol{u}}_i(k) - \boldsymbol{L}_1(k)\tilde{\boldsymbol{q}}_i(k) - \boldsymbol{L}_1(k)\left[\boldsymbol{B}(\boldsymbol{q}_d(k),k) - \boldsymbol{B}(\boldsymbol{q}_i(k),k)\right]\boldsymbol{u}_d(k) \\
&\quad - \boldsymbol{L}_2(k)\tilde{\boldsymbol{q}}_{i+1}(k) - \boldsymbol{L}_3(k)\tilde{\boldsymbol{q}}_{i+1}(k-1) + \boldsymbol{L}_1(k)\left[\boldsymbol{\beta}_i(k) + \boldsymbol{\gamma}_i(k+1)\right] + \boldsymbol{L}_2(k)\boldsymbol{\gamma}_{i+1}(k) \\
&\quad + \boldsymbol{L}_3(k)\boldsymbol{\gamma}_{i+1}(k-1).
\end{aligned}
$$

Utilizing Property 2 and Assumption 2, the following inequality is obtained:

$$
\begin{aligned}
\|\tilde{\boldsymbol{u}}_{i+1}(k)\| \le{}& \|\boldsymbol{I} - \boldsymbol{L}_1(k)\boldsymbol{B}(\boldsymbol{q}_i(k),k)\| \cdot \|\tilde{\boldsymbol{u}}_i(k)\| + b_{L_1}\|\tilde{\boldsymbol{q}}_i(k)\| + b_{L_1}c_B b_{u_d}\|\tilde{\boldsymbol{q}}_i(k)\| \\
&+ b_{L_2}\|\tilde{\boldsymbol{q}}_{i+1}(k)\| + b_{L_3}\|\tilde{\boldsymbol{q}}_{i+1}(k-1)\| + b_{L_1}(b_\beta + b_\gamma) + b_{L_2}b_\gamma + b_{L_3}b_\gamma.
\end{aligned}
$$

Set the symbol $h_1 = b_{L_1}(1 + c_B b_{u_d}) = b_{L_1}h_2$, and $b_1 = b_{L_1}(b_\beta + b_\gamma) + b_{L_2}b_\gamma + b_{L_3}b_\gamma$, then:

$$
\begin{aligned}
\|\tilde{\boldsymbol{u}}_{i+1}(k)\| \le{}& \|\boldsymbol{I} - \boldsymbol{L}_1(k)\boldsymbol{B}(\boldsymbol{q}_i(k),k)\| \cdot \|\tilde{\boldsymbol{u}}_i(k)\| + h_1\|\tilde{\boldsymbol{q}}_i(k)\| + b_{L_2}\|\tilde{\boldsymbol{q}}_{i+1}(k)\| \\
&+ b_{L_3}\|\tilde{\boldsymbol{q}}_{i+1}(k-1)\| + b_1.
\end{aligned}
$$

Apply the Eqs. (15) and (17) to the above inequality, then:

$$
\begin{aligned}
\|\tilde{\boldsymbol{u}}_{i+1}(k)\| \le{}& \rho\|\tilde{\boldsymbol{u}}_i(k)\| + h_1\left[\sum_{j=0}^{k-1} h_2^{k-1-j}\left[b_B\|\tilde{\boldsymbol{u}}_i(j)\| + b_\beta\right] + h_2^k b_{q_0}\right] \\
&+ b_{L_2}\left[\sum_{j=0}^{k-1} h_2^{k-1-j}\left[b_B\|\tilde{\boldsymbol{u}}_{i+1}(j)\| + b_\beta\right] + h_2^k b_{q_0}\right] \\
&+ b_{L_3}\left[\sum_{j=0}^{k-2} h_2^{k-2-j}\left[b_B\|\tilde{\boldsymbol{u}}_{i+1}(j)\| + b_\beta\right] + h_2^{k-1} b_{q_0}\right] + b_1.
\end{aligned}
$$

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

Therefore

$$\|\tilde{\boldsymbol{u}}_{i+1}(k)\| \le \rho \|\tilde{\boldsymbol{u}}_i(k)\| + \left(h_1 + b_{L_2}\right) h_2^k b_{q_0} + b_{L_3} h_2^{k-1} b_{q_0} + b_1 + h_1 \sum_{j=0}^{k-1} h_2^{k-1-j} \left[b_B \|\tilde{\boldsymbol{u}}_i(j)\| + b_\beta\right]$$
$$+ b_{L_2} \sum_{j=0}^{k-1} h_2^{k-1-j} \left[b_B \|\tilde{\boldsymbol{u}}_{i+1}(j)\| + b_\beta\right] + b_{L_3} \sum_{j=0}^{k-2} h_2^{k-2-j} \left[b_B \|\tilde{\boldsymbol{u}}_{i+1}(j)\| + b_\beta\right].$$

Multiply $(\frac{1}{\alpha})^k$ to both sides of the above inequality, and compute the α-norm:

$$\|\tilde{\boldsymbol{u}}_{i+1}(k)\| \left(\frac{1}{\alpha}\right)^k \le \rho \|\tilde{\boldsymbol{u}}_i(k)\| \left(\frac{1}{\alpha}\right)^k + (h_1 + b_{L_2}) b_{q_0} \left(\frac{h_2}{\alpha}\right)^k + \frac{1}{\alpha} b_{L_3} b_{q_0} \left(\frac{h_2}{\alpha}\right)^{k-1} + b_1 \left(\frac{1}{\alpha}\right)^k$$
$$+ \left(\frac{h_1}{\alpha}\right) \sum_{j=0}^{k-1} \left(\frac{h_2}{\alpha}\right)^{k-1-j} \cdot \left[b_B \|\tilde{\boldsymbol{u}}_i(j)\| \left(\frac{1}{\alpha}\right)^j + b_\beta \left(\frac{1}{\alpha}\right)^j\right]$$
$$+ \left(\frac{b_{L_2}}{\alpha}\right) \sum_{j=0}^{k-1} \left(\frac{h_2}{\alpha}\right)^{k-1-j} \cdot \left[b_B \|\tilde{\boldsymbol{u}}_{i+1}(j)\| \left(\frac{1}{\alpha}\right)^j + b_\beta \left(\frac{1}{\alpha}\right)^j\right]$$
$$+ \left(\frac{b_{L_3}}{\alpha^2}\right) \sum_{j=0}^{k-2} \left(\frac{h_2}{\alpha}\right)^{k-2-j} \cdot \left[b_B \|\tilde{\boldsymbol{u}}_{i+1}(j)\| \left(\frac{1}{\alpha}\right)^j + b_\beta \left(\frac{1}{\alpha}\right)^j\right].$$

Set the parameter $\alpha > \max\{1, h_2\}$, then

$$\|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha \le \rho \|\tilde{\boldsymbol{u}}_i\|_\alpha + (h_1 + b_{L_2} + b_{L_3}) b_{q_0} + b_1 + (b_B \|\tilde{\boldsymbol{u}}_i\|_\alpha + b_\beta) \left(\frac{h_1}{\alpha}\right) \sum_{j=0}^{k-1} \left(\frac{h_2}{\alpha}\right)^{k-1-j}$$
$$+ \left(b_B \|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha + b_\beta\right) \left(\frac{b_{L_2}}{\alpha}\right) \sum_{j=0}^{k-1} \left(\frac{h_2}{\alpha}\right)^{k-1-j} + (b_B \|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha + b_\beta) \left(\frac{b_{L_3}}{\alpha^2}\right) \sum_{j=0}^{k-2} \left(\frac{h_2}{\alpha}\right)^{k-2-j}$$
$$\le \rho \|\tilde{\boldsymbol{u}}_i\|_\alpha + (h_1 + b_{L_2} + b_{L_3}) b_{q_0} + b_1 + (b_B \|\tilde{\boldsymbol{u}}_i\|_\alpha + b_\beta) \left(\frac{h_1 \left[1 - (h_2/\alpha)^n\right]}{\alpha - h_2}\right)$$
$$+ (b_B \|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha + b_\beta) \left(\frac{b_{L_2} \left[1 - (h_2/\alpha)^n\right]}{\alpha - h_2}\right) + (b_B \|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha + b_\beta) \left(\frac{b_{L_3} \left[1 - (h_2/\alpha)^n\right]}{\alpha - h_2}\right)$$
$$= \left(\rho + b_B h_1 \frac{1 - (h_2/\alpha)^n}{\alpha - h_2}\right) \|\tilde{\boldsymbol{u}}_i\|_\alpha + b_B (b_{L_2} + b_{L_3}) \frac{1 - (h_2/\alpha)^n}{\alpha - h_2} \|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha$$
$$+ (h_1 + b_{L_2} + b_{L_3}) b_{q_0} + b_1 + \frac{b_\beta (h_1 + b_{L_2} + b_{L_3}) \left[1 - (h_2/\alpha)^n\right]}{\alpha - h_2}.$$

That is to say:

$$\left[1 - b_B (b_{L_2} + b_{L_3}) \frac{1 - (h_2/\alpha)^n}{\alpha - h_2}\right] \|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha \le \left(\rho + b_B h_1 \frac{1 - (h_2/\alpha)^n}{\alpha - h_2}\right) \|\tilde{\boldsymbol{u}}_i\|_\alpha$$
$$+ (h_1 + b_{L_2} + b_{L_3}) b_{q_0} + b_1 + \frac{b_\beta (h_1 + b_{L_2} + b_{L_3}) \left[1 - (h_2/\alpha)^n\right]}{\alpha - h_2}. \tag{18}$$

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

According to the properties of geometric sequence, we can obtain:

$$\begin{aligned}
&\frac{1}{\alpha}\sum_{j=0}^{k-1}\left(\frac{h_2}{\alpha}\right)^{k-1-j}=\frac{1}{\alpha}\left(\frac{h_2}{\alpha}\right)^{k-1}\sum_{j=0}^{k-1}\left(\frac{h_2}{\alpha}\right)^{-j}=\frac{1}{\alpha}\left(\frac{h_2}{\alpha}\right)^{k-1}\sum_{j=0}^{k-1}\left(\frac{\alpha}{h_2}\right)^{j}=\frac{1}{\alpha}\left(\frac{h_2}{\alpha}\right)^{k-1}\frac{1-\left(\frac{\alpha}{h_2}\right)^{k}}{1-\frac{\alpha}{h_2}}\\
&=\frac{(h_2/\alpha)^{k-1}\left[(\alpha/h_2)^k-1\right]}{\alpha\,(\alpha/h_2-1)}=\frac{1-(h_2/\alpha)^k}{\alpha-h_2}\le\frac{1-(h_2/\alpha)^n}{\alpha-h_2}\\
&\frac{1}{\alpha^2}\sum_{j=0}^{k-2}\left(\frac{h_2}{\alpha}\right)^{k-2-j}\le\frac{1}{\alpha}\sum_{j=0}^{k-2}\left(\frac{h_2}{\alpha}\right)^{k-2-j}=\frac{1}{\alpha}\left(\frac{h_2}{\alpha}\right)^{k-2}\sum_{j=0}^{k-2}\left(\frac{h_2}{\alpha}\right)^{-j}=\frac{1}{\alpha}\left(\frac{h_2}{\alpha}\right)^{k-2}\sum_{j=0}^{k-2}\left(\frac{\alpha}{h_2}\right)^{j}\\
&=\frac{1}{\alpha}\left(\frac{h_2}{\alpha}\right)^{k-2}\frac{1-\left(\frac{\alpha}{h_2}\right)^{k-1}}{1-\frac{\alpha}{h_2}}=\frac{(h_2/\alpha)^{k-2}\left[(\alpha/h_2)^{k-1}-1\right]}{\alpha\,(\alpha/h_2-1)}=\frac{1-(h_2/\alpha)^{k-1}}{\alpha-h_2}\\
&\le\frac{1-(h_2/\alpha)^{n-1}}{\alpha-h_2}\le\frac{1-(h_2/\alpha)^n}{\alpha-h_2},
\end{aligned}$$

where

$$k=1,\cdots,n. \tag{19}$$

Set the parameter $\alpha>\max\{1,h_2,h_2+b_B(b_{L_2}+b_{L_3})\}$, then the Eq. (18) can be rewritten as:

$$\|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha\le\hat{\rho}\,\|\tilde{\boldsymbol{u}}_i\|_\alpha+\varepsilon, \tag{20}$$

where

$$\hat{\rho}=\frac{\rho+b_Bh_1\frac{1-(h_2/\alpha)^n}{\alpha-h_2}}{1-b_B(b_{L_2}+b_{L_3})\frac{1-(h_2/\alpha)^n}{\alpha-h_2}},\ \varepsilon=\frac{(h_1+b_{L_2}+b_{L_3})(b_{q_0}+b_\beta\frac{1-(h_2/\alpha)^n}{\alpha-h_2})+b_1}{1-b_B(b_{L_2}+b_{L_3})\frac{1-(h_2/\alpha)^n}{\alpha-h_2}}.$$

Here, in order to ensure the establishment of the condition: $1-b_B(b_{L_2}+b_{L_3})\frac{1-(h_2/\alpha)^n}{\alpha-h_2}>0$, so $\alpha^n(\alpha-h_2)>b_B(b_{L_2}+b_{L_3})(\alpha^n-h_2^n)$, i.e., $\alpha^n[\alpha-h_2-b_B(b_{L_2}+b_{L_3})]>-b_B(b_{L_2}+b_{L_3})h_2^n>0$, then the condition: $\alpha>h_2+b_B(b_{L_2}+b_{L_3})$ should be satisfied.

Make recursion to the inequality (20), we can obtain:

$$\|\tilde{\boldsymbol{u}}_{i+1}\|_\alpha\le\hat{\rho}^i\,\|\tilde{\boldsymbol{u}}_1\|_\alpha+\varepsilon\sum_{j=0}^{i-1}\hat{\rho}^j=\hat{\rho}^i\,\|\tilde{\boldsymbol{u}}_1\|_\alpha+\frac{\varepsilon\left(1-\hat{\rho}^i\right)}{1-\hat{\rho}}.$$

Set the parameter α to a value which is large enough to ensure the establishment of the condition: $\hat{\rho}\approx\rho<1$, then we finally obtain the following inequality:

$$\lim_{i\to\infty}\|\tilde{\boldsymbol{u}}_i\|_\alpha\le\frac{\varepsilon}{1-\hat{\rho}}. \tag{21}$$

Similarly, we multiply $(\frac{1}{\alpha})^k$ to both sides of the inequality (17), and obtain inequality as follows:

$$\|\tilde{\boldsymbol{q}}_i(k)\|\left(\frac{1}{\alpha}\right)^k\le\frac{1}{\alpha}\sum_{j=0}^{k-1}\left(\frac{h_2}{\alpha}\right)^{k-1-j}\left[b_B\,\|\tilde{\boldsymbol{u}}_i(j)\|\left(\frac{1}{\alpha}\right)^j+b_\beta\left(\frac{1}{\alpha}\right)^j\right]+\left(\frac{h_2}{\alpha}\right)^k b_{q_0}.$$

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

Due to the conditions that $b_\beta(\frac{1}{\alpha})^j \leq b_\beta$ and $\frac{h_2}{\alpha} < 1$, then apply inequality (19) to obtain:

$$\|\tilde{\boldsymbol{q}}_i\|_\alpha \leq (b_B\|\tilde{\boldsymbol{u}}_i\|_\alpha + b_\beta)\frac{1-(h_2/\alpha)^n}{\alpha - h_2} + b_{q_0} = b_B\frac{1-(h_2/\alpha)^n}{\alpha - h_2}\|\tilde{\boldsymbol{u}}_i\|_\alpha + b_\beta\frac{1-(h_2/\alpha)^n}{\alpha - h_2} + b_{q_0}.$$

By substituting inequality (21) into the above inequality, then:

$$\lim_{i\to\infty}\|\tilde{\boldsymbol{q}}_i\|_\alpha \leq b_B\frac{1-(h_2/\alpha)^n}{\alpha - h_2}\frac{\varepsilon}{1-\hat{\rho}} + b_\beta\frac{1-(h_2/\alpha)^n}{\alpha - h_2} + b_{q_0}. \tag{22}$$

By subtracting Eq. (6) from Eq. (10), then:

$$\tilde{\boldsymbol{y}}_i(k) = \boldsymbol{y}_d(k) - \boldsymbol{y}_i(k) = \boldsymbol{q}_d(k) - [\boldsymbol{q}_i(k) + \boldsymbol{\gamma}_i(k)] = \tilde{\boldsymbol{q}}_i(k) - \boldsymbol{\gamma}_i(k).$$

Multiply $(\frac{1}{\alpha})^k$ to both sides of the above equation, then:

$$\|\tilde{\boldsymbol{y}}_i\|_\alpha \leq \|\tilde{\boldsymbol{q}}_i\|_\alpha + b_\gamma.$$

By substituting inequality (22) into the above inequality, then we can obtain:

$$\lim_{i\to\infty}\|\tilde{\boldsymbol{y}}_i\|_\alpha \leq b_B\frac{1-(h_2/\alpha)^n}{\alpha - h_2}\frac{\varepsilon}{1-\hat{\rho}} + b_\beta\frac{1-(h_2/\alpha)^n}{\alpha - h_2} + b_{q_0} + b_\gamma. \tag{23}$$

According to inequalities (21), (22), and (23), we can make the conclusions that when the parameters $b_\beta = 0$, $b_\gamma = 0$ and $b_{q_0} = 0$, the variables $\|\tilde{\boldsymbol{u}}_i\|_\alpha$, $\|\tilde{\boldsymbol{q}}_i\|_\alpha$, and $\|\tilde{\boldsymbol{y}}_i\|_\alpha$ all converge to zero. Otherwise, the variables $\|\tilde{\boldsymbol{u}}_i\|_\alpha$, $\|\tilde{\boldsymbol{q}}_i\|_\alpha$, and $\|\tilde{\boldsymbol{y}}_i\|_\alpha$ converge to bounded values which are the functions of the parameters b_β, b_γ, and b_{q_0}.

4. Experiments and Analysis

In this section, several experiments are executed to evaluate the effectiveness and performance of the proposed iterative learning control algorithm. All experiments are performed on the MATLAB R2011b platform which runs in a laptop with Intel Core i7-2620M CPU (2.70GHz).

In the Experiment One, Experiment Two, and Experiment Three, we assume that no initial state errors exist in the simulation experiments. That is to say, the initial state values of the mobile robot are equal to the initial values of the expected trajectory in each iterative process, i.e.:

$$x_{p,i}(0) = x_d(0), \quad y_{p,i}(0) = y_d(0), \quad \theta_{p,i}(0) = \theta_d(0),$$

where $x_{p,i}(0)$, $y_{p,i}(0)$, and $\theta_{p,i}(0)$ are the initial states of the mobile robot in No. i iterative process.

*4.1. Experiment **O**ne: **C**ardioid expected trajectory*

In this experiment, we mainly discuss a cardioid expected trajectory. The coordinates of the starting point is (1, 0). The cardioid expected trajectory rotates along the counterclockwise direction and is composed of three semicircular trajectories. Firstly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the origin of coordinate system and radius is 1 unit. Then counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the point (1, 0) and radius is 2 units. Finally, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the point (2, 0) and radius is 1 unit. The cardioid expected trajectory terminates at the point (1, 0).

The parameters $x_d(k)$, $y_d(k)$, and $\theta_d(k)$ of each sample point in the expected trajectory are predefined for $k = 1, \cdots, 3000$. Due to the sampling interval is $\Delta T = 0.001$s, so completion of each iteration process takes three seconds.

In order to make a comparative illustration of the influences made by predictive learning item $\boldsymbol{L}_1(k)\boldsymbol{e}_i(k+1)$, current learning item $\boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k)$,and past learning item $\boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}(k-1)$ on the

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

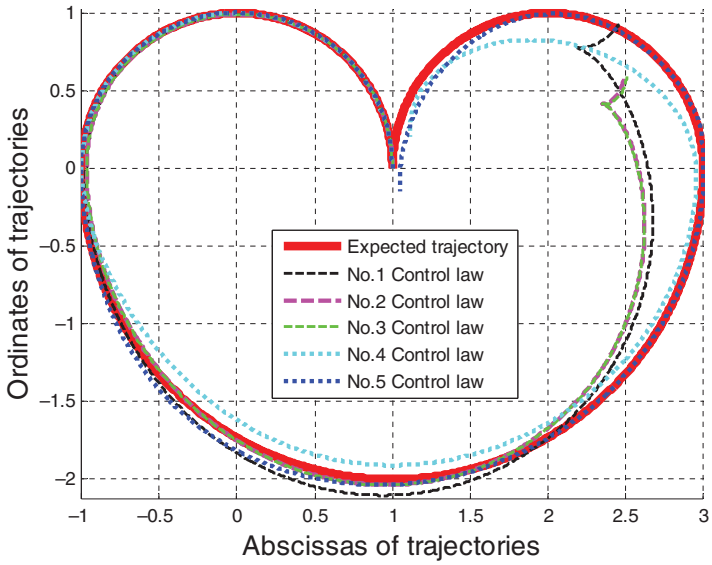


Fig. 3. The comparison of mobile robot tracking trajectories by adopting different iterative learning control laws in the last iterative process.

performance of the iterative learning control strategy, the respective learning gain matrices of three items are set to the same value, i.e.,

$$L_1(k) = L_2(k) = L_3(k) = 0.30 \begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The following comparative experiment is about the control effects of five types of iterative learning control law. In this experiment, each iterative learning control law will take 30 iterations. The five types of iterative learning control law are as follows.

No. 1: The iterative learning control law which only contains predictive learning item:

$$\boldsymbol{u}_{i+1}(k) = \boldsymbol{u}_i(k) + \boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1)\,. \tag{24}$$

No. 2: The iterative learning control law which only contains current learning item:

$$\boldsymbol{u}_{i+1}(k) = \boldsymbol{u}_i(k) + \boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k). \tag{25}$$

No. 3: The iterative learning control law which only contains past learning item:

$$\boldsymbol{u}_{i+1}(k) = \boldsymbol{u}_i(k) + \boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}\,(k-1)\,. \tag{26}$$

No. 4: The iterative learning control law which contains predictive and current learning items:

$$\boldsymbol{u}_{i+1}(k) = \boldsymbol{u}_i(k) + \boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1) + \boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k). \tag{27}$$

No. 5: The iterative learning control law which contains predictive, current, and past learning items:

$$\boldsymbol{u}_{i+1}(k) = \boldsymbol{u}_i(k) + \boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1) + \boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k) + \boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}\,(k-1)\,. \tag{28}$$

The results of comparative experiment are shown as the Figs. 3–5 below.

4.2. Experiment Two: Spiral expected trajectory

In this experiment, we mainly discuss a spiral expected trajectory. The coordinates of the starting point is (1, 0). The spiral expected trajectory rotates along the counterclockwise direction and is

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

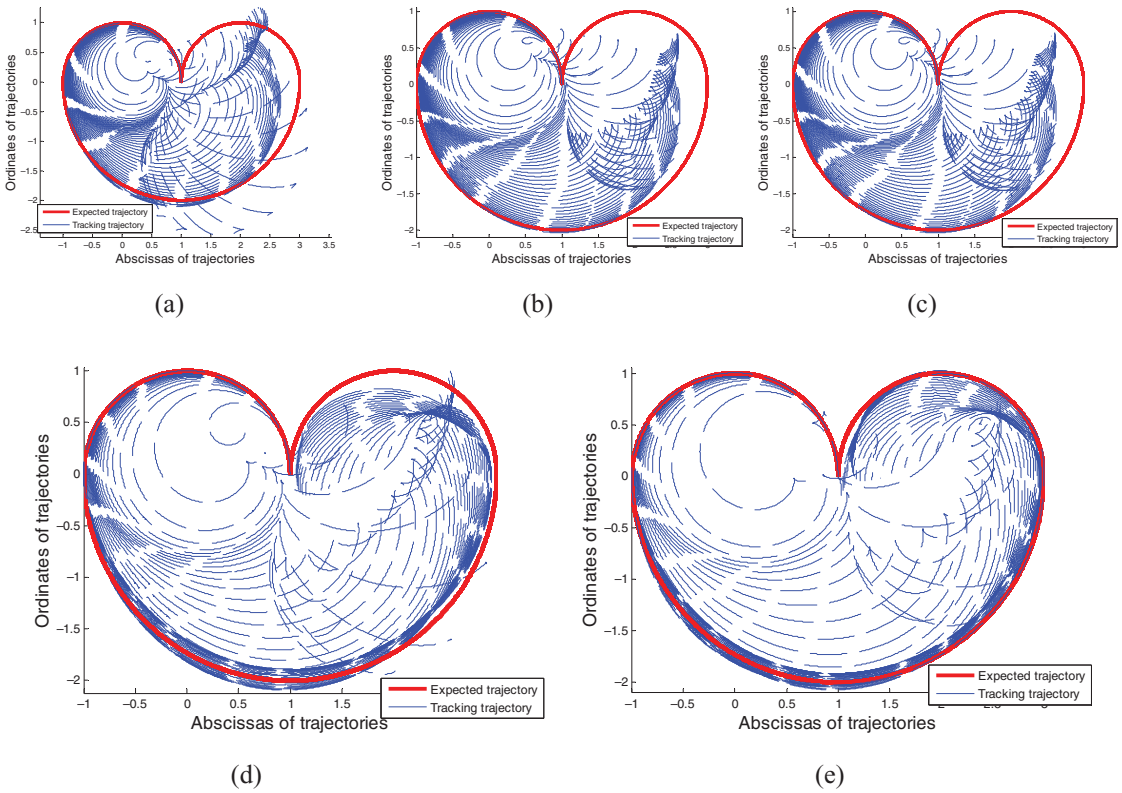


Fig. 4. The change of mobile robot trajectory in the iterative process. (a) No. 1 iterative learning control law, i.e., Eq. (24). (b) No. 2 iterative learning control law, i.e., Eq. (25). (c) No. 3 iterative learning control law, i.e., Eq. (26). (d) No. 4 iterative learning control law, i.e., Eq. (27). (e) No. 5 iterative learning control law, i.e., Eq. (28).

composed of eight semicircular trajectories. Firstly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the origin of coordinate system and radius is 1 unit. Secondly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the point (0.5, 0) and radius is 1.5 units. Thirdly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the origin of coordinate system and radius is 2 units. Fourthly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the point (0.5, 0) and radius is 2.5 units. Fifthly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the origin of coordinate system and radius is 3 units. Sixthly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the point (0.5, 0) and radius is 3.5 units. Seventhly, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the origin of coordinate system and radius is 4 units. Finally, counterclockwise rotating 180° to pass through a semicircular trajectory whose center is located at the point (0.5, 0) and radius is 4.5 units. The spiral expected trajectory terminates at the point (5, 0).

The parameters $x_d(k)$, $y_d(k)$, and $\theta_d(k)$ of each sample point in the expected trajectory are predefined for $k = 1, \cdots, 8000$. Due to the sampling interval is $\Delta T = 0.001$ s, so completion of each iteration process takes eight seconds.

In order to make a comparative illustration of the influences made by predictive learning item $\boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1)$, current learning item $\boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k)$, and past learning item $\boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}\,(k-1)$ on the performance of the iterative learning control strategy, the respective learning gain matrices of three items are set to the same value, i.e.,

$$L_1(k) = L_2(k) = L_3(k) = 0.40 \begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

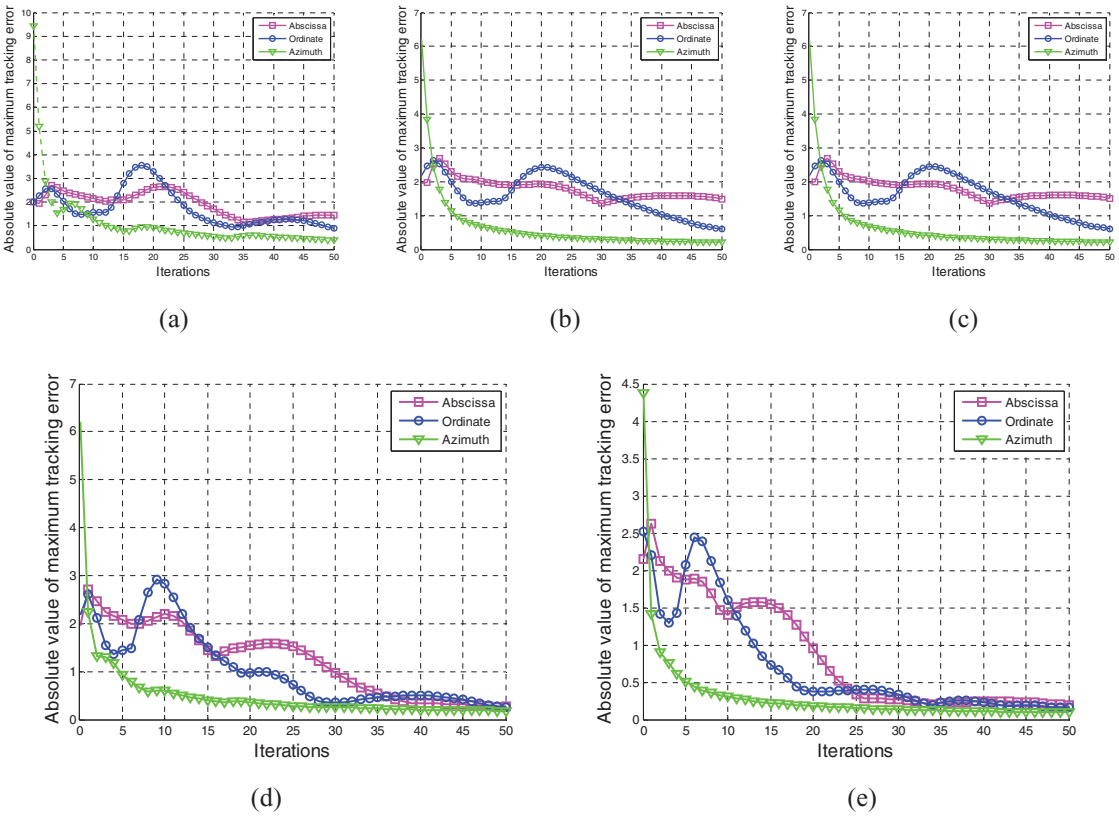


Fig. 5. With the iteration increase, the convergence process of absolute value of maximum tracking error in each iteration. (a) No. 1 iterative learning control law, i.e., Eq. (24). (b) No. 2 iterative learning control law, i.e., Eq. (25). (c) No. 3 iterative learning control law, i.e., Eq. (26). (d) No. 4 iterative learning control law, i.e., Eq. (27). (e) No. 5 iterative learning control law, i.e., Eq. (28).

The following comparative experiment is about the control effects of five types of iterative learning control law. In this experiment, each iterative learning control law will take 75 iterations. The results of comparative experiment are shown as the Figs. 6–8 below.

4.3. Experiment three: The influence of learning gain matrices values on the performance of trajectory tracking

In order to discuss the influence of learning gain matrices values of predictive learning item $\boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1)$, present learning item $\boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k)$, and past learning item $\boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}\,(k-1)$ on the performance of the iterative learning control algorithm, the comparative experiment based on No. 5 iterative learning control law, i.e., Eq. (28) is executed. In this experiment, the respective learning gain matrices of three items are set to the same value in each situation.

No. 1 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 0.10\begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

No. 2 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 0.25\begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

No. 3 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 0.50\begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

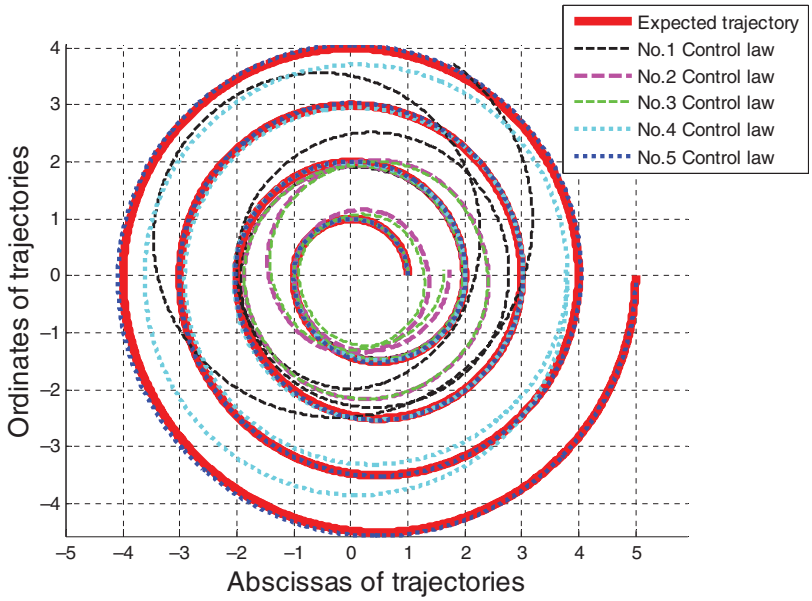


Fig. 6. The comparison of mobile robot tracking trajectories by adopting different iterative learning control laws in the last iterative process.

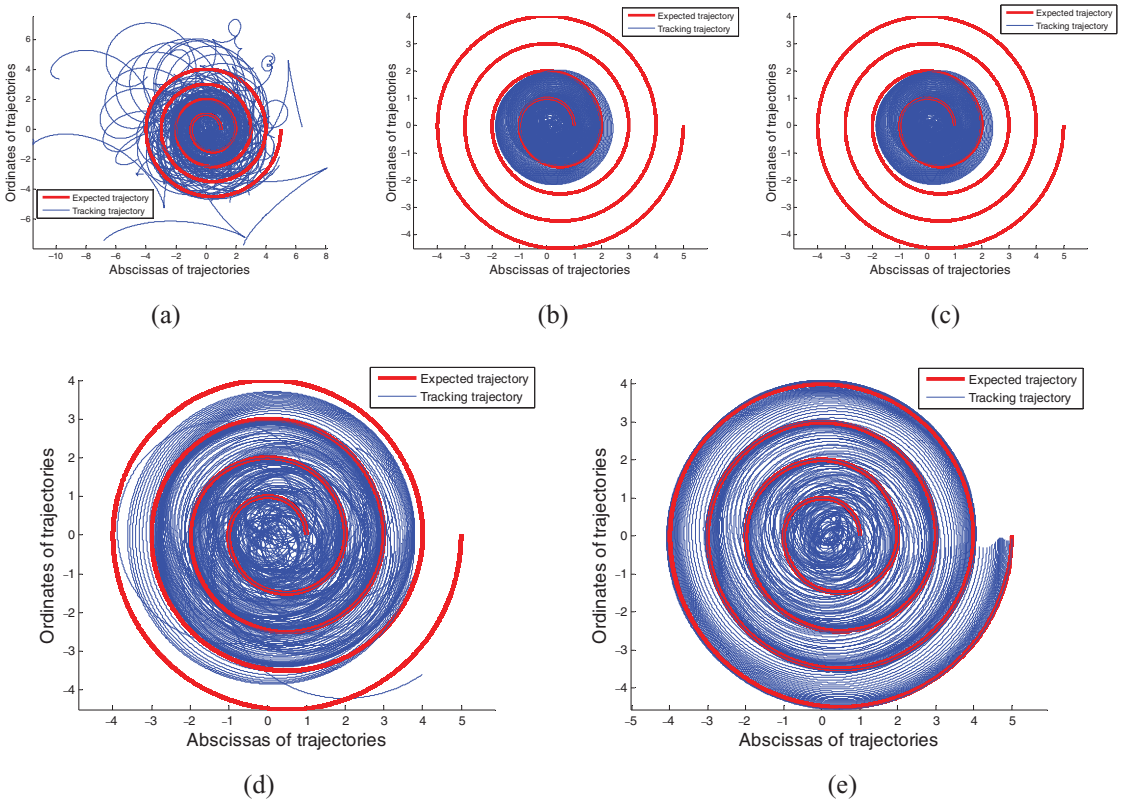


Fig. 7. The change of mobile robot trajectory in the iterative process. (a) No. 1 iterative learning control law, i.e., Eq. (24). (b) No. 2 iterative learning control law, i.e., Eq. (25). (c) No. 3 iterative learning control law, i.e., Eq. (26). (d) No. 4 iterative learning control law, i.e., Eq. (27). (e) No. 5 iterative learning control law, i.e., Eq. (28).

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

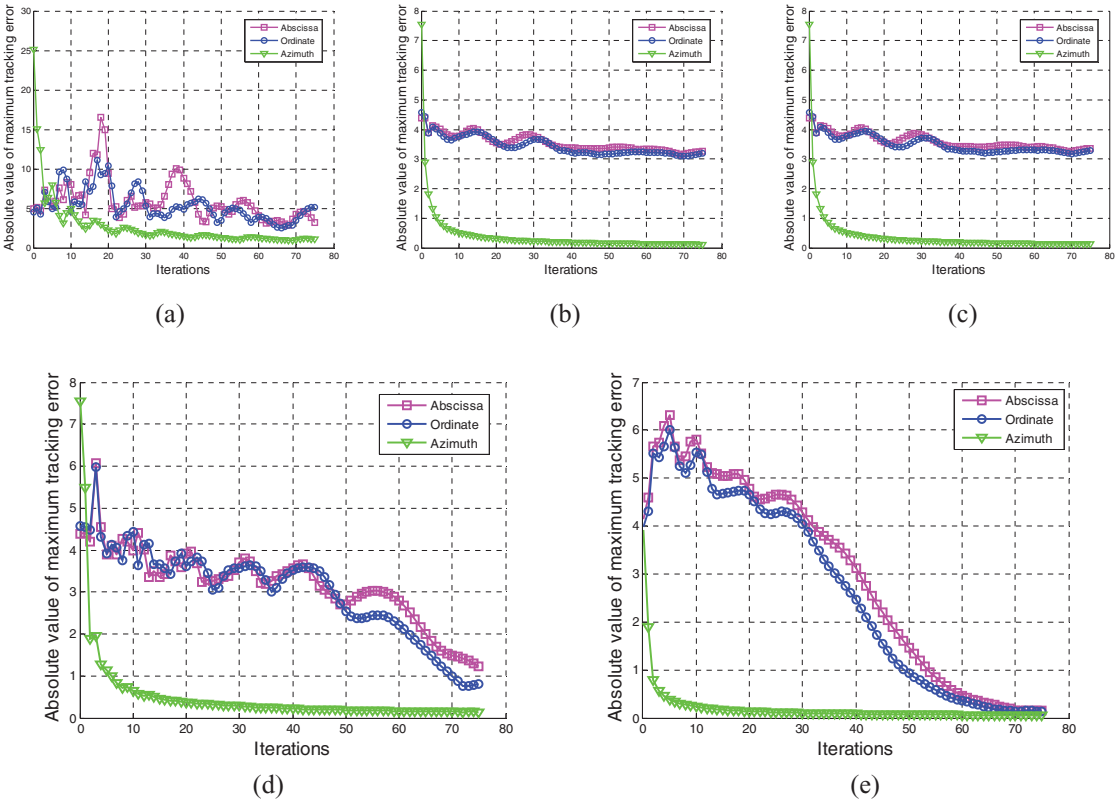


Fig. 8. With the iteration increase, the convergence process of absolute value of maximum tracking error in each iteration. (a) No. 1 iterative learning control law, i.e., Eq. (24). (b) No. 2 iterative learning control law, i.e., Eq. (25). (c) No. 3 iterative learning control law, i.e., Eq. (26). (d) No. 4 iterative learning control law, i.e., Eq. (27). (e) No. 5 iterative learning control law, i.e., Eq. (28).

No. 4 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 0.75 \begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

No. 5 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 1.00 \begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

No. 6 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 1.50 \begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

No. 7 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 2.50 \begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

No. 8 situation: The respective learning gain matrices of three items are set to the values as follows:

$$L_1(k) = L_2(k) = L_3(k) = 5.00 \begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

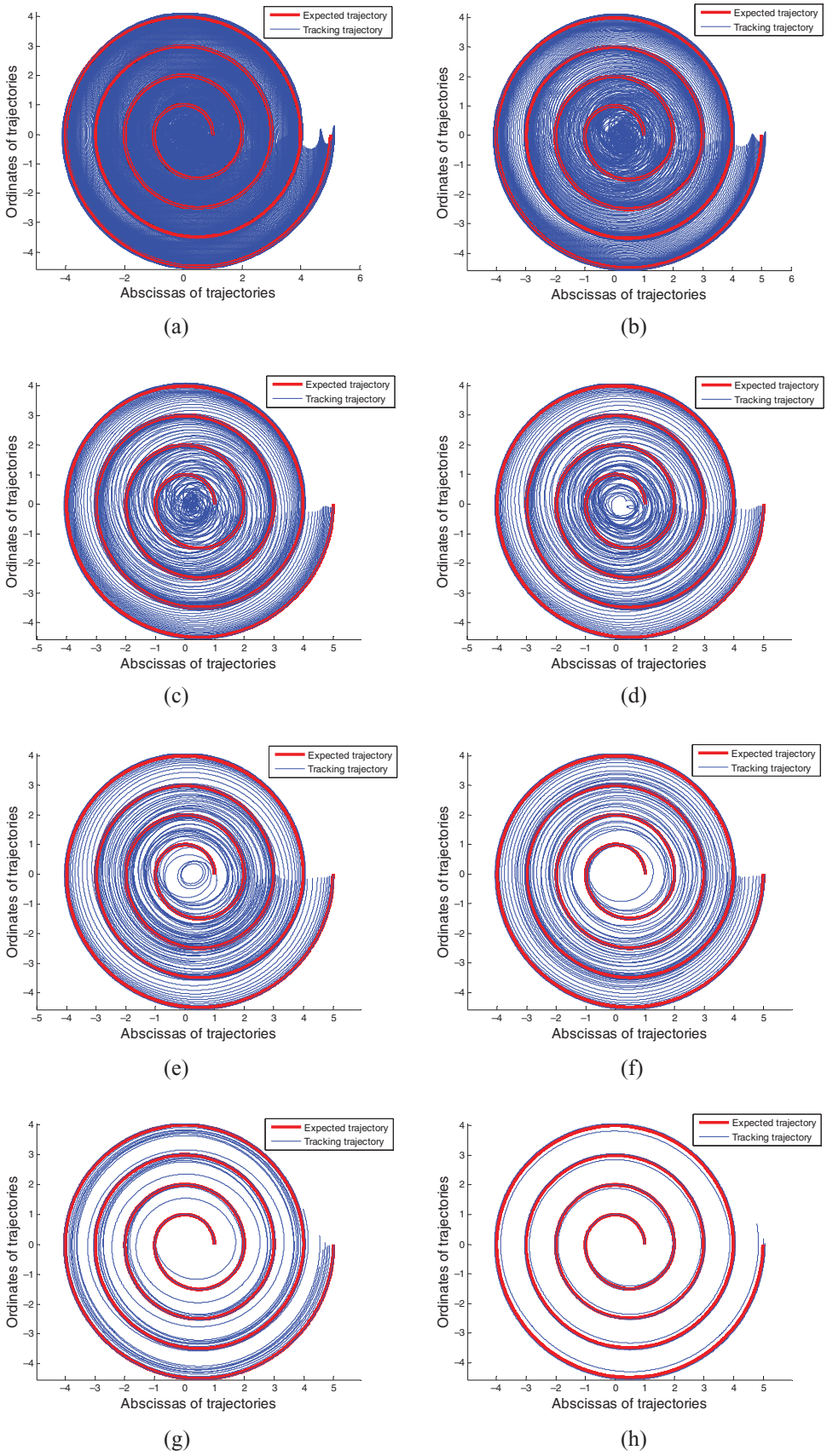


Fig. 9. The change of mobile robot trajectory in the iterative process. (a) No. 1 situation. (b) No. 2 situation. (c) No. 3 situation. (d) No. 4 situation. (e) No. 5 situation. (f) No. 6 situation. (g) No. 7 situation. (h) No. 8 situation.

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

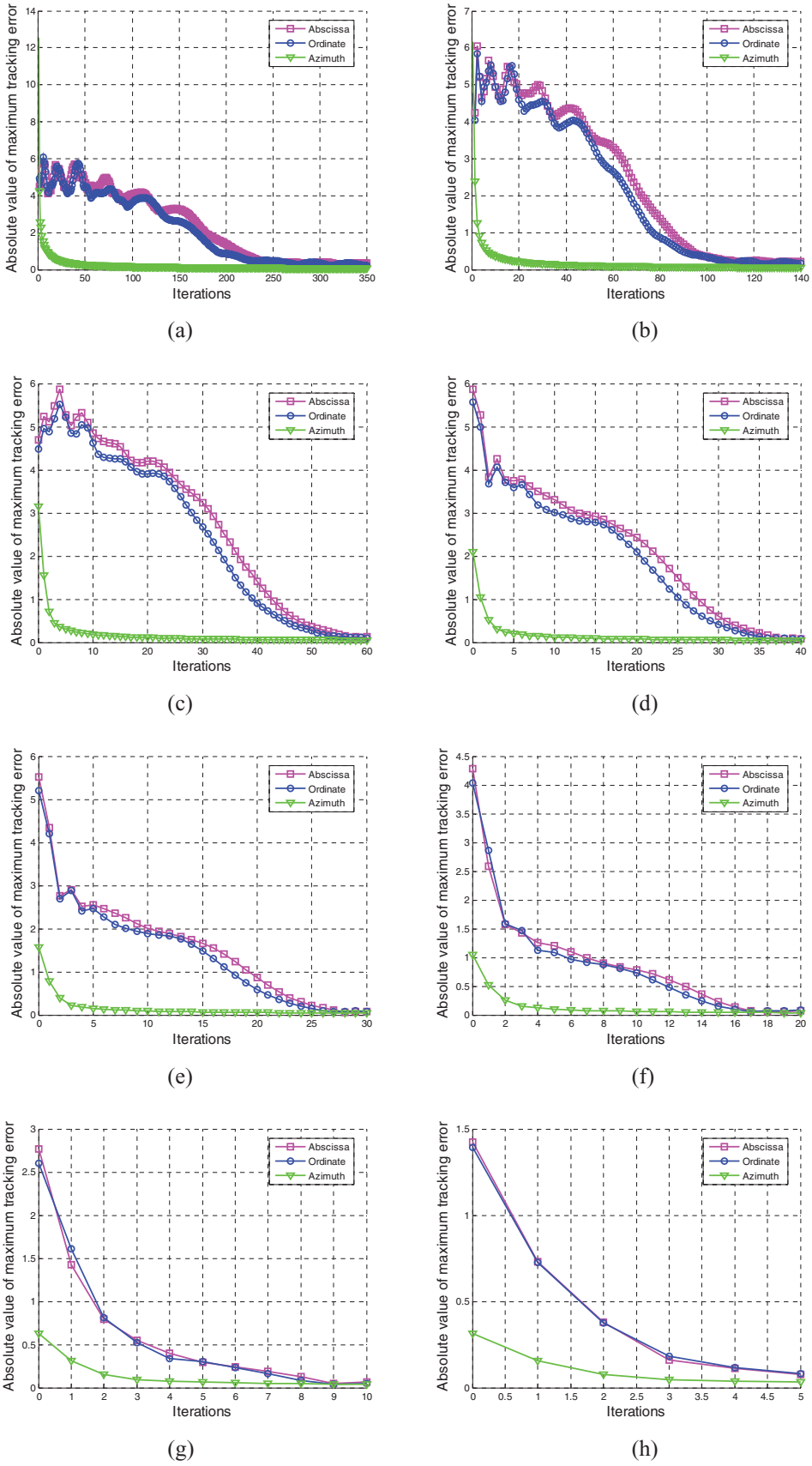


Fig. 10. With the iteration increase, the convergence process of absolute value of maximum tracking error in each iteration. (a) No. 1 situation. (b) No. 2 situation. (c) No. 3 situation. (d) No. 4 situation. (e) No. 5 situation. (f) No. 6 situation. (g) No. 7 situation. (h) No. 8 situation.

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

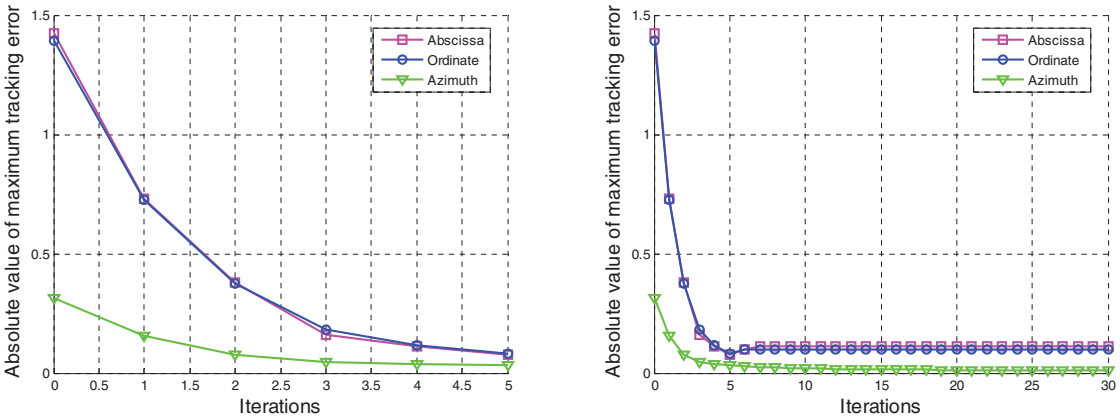


Fig. 11. With the iteration increase, the convergence process of absolute value of maximum tracking error in each iteration. (a) 5 iterations. (b) 30 iterations.

In all situations, the iterative process continues until the trajectory tracking performance and the error of the mobile robot model converge to the same value, to compare the length of convergence process in each situation, i.e., the total number of iteration needed. The specific experiment results are shown as follows in Figs. 9 and 10.

Serial number of situation	1	2	3	4	5	6	7	8
Coefficient of learning gain matrix	0.10	0.25	0.50	0.75	1.00	1.50	2.50	5.00
Iterations needed to convergence	350	140	60	40	30	20	10	5

In No. 8 situation, firstly, the experiment ends after 30 iterations. From the experimental results illustrated in Fig. 11 we find that the tracking error cannot converge to a smaller value with the increase of iterations. On the contrary, if the experiment stops after 5 iterations, the trajectory tracking error is smaller. So we can make a conclusion that in order to achieve the optimal trajectory tracking effect, the choice of the number of iterations, and the learning gain matrices values should be in a fine match.

4.4. Experiment Four: Robustness experiments against initial state errors

When the initial state errors exist, we make robustness experiment based on No. 5 iterative learning control law, i.e., Eq. (28).

In this experiment, we mainly discuss a circle expected trajectory. The coordinates of the starting point is (1, 0). Counterclockwise rotating 360° to pass through a circular trajectory whose center is located at the origin of coordinate system and radius is 1 unit. The circle expected trajectory terminates at the point (1, 0).

The parameters $x_d(k)$, $y_d(k)$, and $\theta_d(k)$ of each sample point in the expected trajectory are predefined for $k = 1, \cdots, 2000$. Due to the sampling interval is $\Delta T = 0.001$ s, so completion of each iteration process takes two seconds.

According to the convergence condition illustrated by inequality (15), the respective learning gain matrices of predictive learning item $\boldsymbol{L}_1(k)\boldsymbol{e}_i\,(k+1)$, current learning item $\boldsymbol{L}_2(k)\boldsymbol{e}_{i+1}(k)$, and past learning item $\boldsymbol{L}_3(k)\boldsymbol{e}_{i+1}\,(k-1)$ are set to the same value, i.e.,

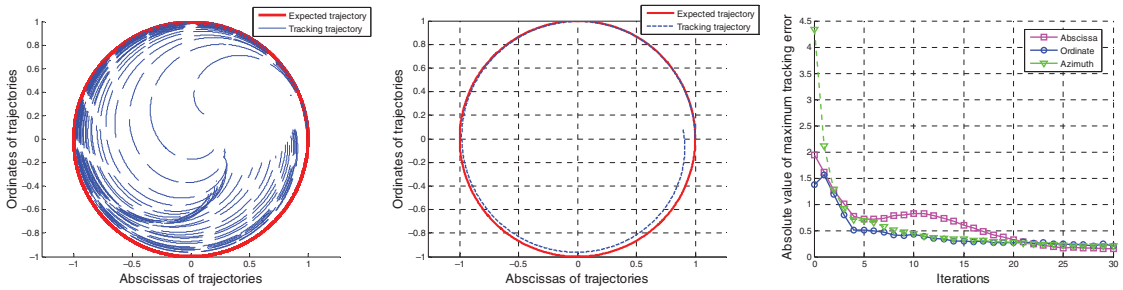
$$L_1(k) = L_2(k) = L_3(k) = 0.20\begin{bmatrix} \cos\theta(k) & \sin\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Set the initial states of the No. 1 iterative process as follows:

$$x_{p,i}(0) = x_d(0) + b_{x_0}, \quad y_{p,i}(0) = y_d(0) + b_{y_0}, \quad \theta_{p,i}(0) = \theta_d(0) + b_{\theta_0}.$$

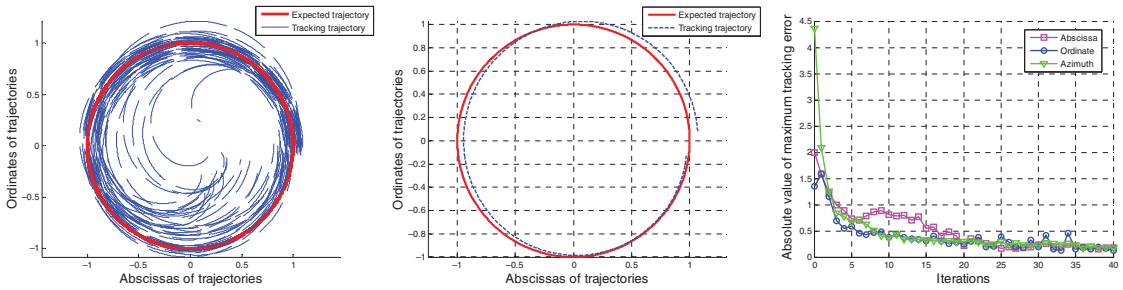
No. 1 situation: $b_{x_0} \in (-0.01, 0.01)$, $b_{y_0} \in (-0.01, 0.01)$, $b_{\theta_0} \in (-0.01, 0.01)$. The experiment results after 30 iterations are shown as follows in Fig. 12.

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press



(a) (b) (c)

Fig. 12. (a) The change of mobile robot trajectory in the iterative process. (b) The comparison of mobile robot tracking trajectory with expected trajectory in the last iterative process. (c) With the iteration increase, the convergence process of absolute value of maximum tracking error in each iteration.



(a) (b) (c)

Fig. 13. (a) The change of mobile robot trajectory in the iterative process. (b) The comparison of mobile robot tracking trajectory with expected trajectory in the last iterative process. (c) With the iteration increase, the convergence process of absolute value of maximum tracking error in each iteration.

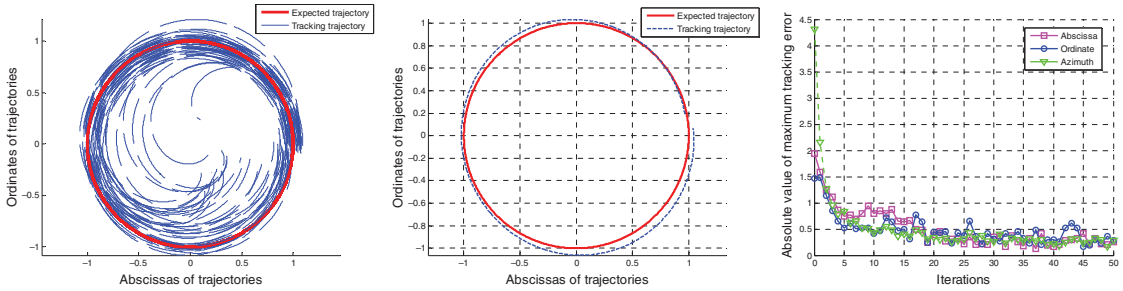


Fig. 14. (a) The change of mobile robot trajectory in the iterative process. (b) The comparison of mobile robot tracking trajectory with expected trajectory in the last iterative process. (c) With the iteration increase, the convergence process of absolute value of maximum tracking error in each iteration.

No. 2 situation: $b_{x_0} \in (-0.1, 0.1)$, $b_{y_0} \in (-0.1, 0.1)$, $b_{\theta_0} \in (-0.1, 0.1)$. The experiment results after 40 iterations are shown as follows in Fig. 13.

No. 3 situation: $b_{x_0} \in (-0.2, 0.2)$, $b_{y_0} \in (-0.2, 0.2)$, $b_{\theta_0} \in (-0.2, 0.2)$. The experiment results after 50 iterations are shown as follows in Fig. 14.

Comparing the experiment results when the initial state errors exist or not, it is obvious that when the initial state errors exist, the trajectory tracking performance of the iterative learning control algorithm is worse than that without initial state errors. However, in both cases, the trajectory tracking errors finally converge with the increase of iterations. When the initial state errors are within a bounded scope, the trajectory tracking performance of the mobile robot by adopting the iterative learning control algorithm is quite well. Moreover, the iterative learning control algorithm we proposed has a good capability in robustness.

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

4.5. Experiment results analysis and conclusions

Through the previous experiments, we can draw the following conclusions:

1. With the increase of iteration, the mobile robot model adopting the iterative learning control algorithm is able to tracking the expected trajectory. Simultaneously, the trajectory tracking error converges to a bounded value gradually.
2. After the same number of iterations, the trajectory tracking performance and error differ due to the application of different iterative learning control laws. More specifically, the tracking performance of No. 1 iterative learning control law, i.e., Eq. (24) is the worst, and the tracking error is the largest. The tracking performance and error of No. 2 and No. 3 iterative learning control laws, i.e., Eqs. (25) and (26) are relatively similar, which are both better than that of No. 1 iterative learning control law. The trajectory tracking performance of No. 4 iterative learning control law, i.e., Eq. (27) is obviously better than that of No. 1, No. 2, and No. 3 iterative learning control laws. The tracking error is also significantly reduced. The tracking performance of No. 5 iterative learning control law, i.e., Eq. (28) is the best among five iterative learning control laws. The tracking error is the smallest. Combining various learning items will improve the trajectory tracking performance. Finally, the iterative leaning control law which consists of various learning items will significantly reduce the tracking error.
3. By adopting different iterative learning control laws, the convergence speeds of the trajectory tracking errors differ with the increase of iterations. In specific, the convergence speeds of tracking errors which are arrayed from fast to slow are as follows. The iterative learning control law consists of predictive, current and past learning items, i.e., Eq. (28); the iterative learning control law consists of predictive, and current learning items, i.e., Eq. (27); the iterative learning control law only consists of current learning items, i.e., Eq. (25); the iterative learning control law only consists of past learning items, i.e., Eq. (26); the iterative learning control law only consists of predictive learning items, i.e., Eq. (24). Combining various learning items will accelerate the convergence speed of the trajectory tracking error.
4. The values of learning gain matrices have an influence on the trajectory tracking performance. When the learning gain matrices are set to small values, the convergence speed of the trajectory tracking error is slow. While the learning gain matrices are set to relatively large values, the convergence speed of the trajectory tracking error is faster. When the tracking error converges to a neighborhood which is small enough of the expected trajectory, we cannot make the tracking error converge to a smaller value with the increase of iterations. On the contrary, the trajectory tracking error may become larger finally. It illustrates that in order to achieve the optimal trajectory tracking effect, the choice of iterations and the learning gain matrices values should be in a fine match.
5. The iterative learning control law we designed adopts predictive, current and past learning items to drive the state variables, input variables, and output variables converge to their desired values, and enhance the robust characteristic of the control algorithm. Through the experiments, we can make the conclusions that if the initial state errors, state disturbances, and output measurement noises do not exist, then the errors between the state variables, input variables, output variables, and corresponding desired values will converge to zero. If the initial state errors, state disturbances and output noises are taken into consideration, then the tracking errors will finally converge with the increase of iterations. So the designed algorithm is robust and has a brilliant control performance.
6. From the comparative experiment we can find that the performance of the iterative learning control law only adopting current learning item and the performance of the control law only adopting past learning item are relatively similar. However, by introducing the past learning item, the learning gain matrices which can be adjusted are increased. So the control method is more flexible.
7. We adopt the value of current learning item in the previous sampling time as the past learning item. That is to say, there is no need to recalculate the past learning item. Therefore, the performance of the iterative learning control algorithm improves without the consumption of additional time and storage.

5. Conclusion

Since the wheeled mobile robot is featured with time-varying, nonlinear, and strong-coupling dynamics properties, with the influence of measurements and mathematical models inaccuracy, as

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

well as external disturbances and load changes, the trajectory tracking issue of the wheeled mobile robot is rather difficult to handle. In this paper, we propose an iterative learning control algorithm with brilliant robustness capability and apply it to solve the high-precision trajectory tracking issue of the wheeled mobile robot.

The designed iterative learning control algorithm adopts predictive, current, and past learning items to drive the state variables, and input variables, and output variables converge to the bounded neighborhoods of their desired values. The rigorous mathematical proof of the convergence character of the proposed iterative learning control algorithm is given. The feasibility, effectiveness and robustness of the proposed algorithm are illustrated by the quantitative experiments and the comparative analysis. The algorithm can enhance the control performance, stability, and robust characteristics than state-of-the-art control algorithms. The experiment results illustrate that the proposed iterative learning control algorithm has a marvelous control effect on the trajectory tracking issue of the wheeled mobile robots.

Acknowledgements

This work was supported by Graduate Students Innovation Foundation of Fudan University.

References

1. Y. Q. Chen and K. L. Moore, "Comments on United States Patent 3,555,252—Learning Control of Actuators in Control Systems," *Proceedings of the 2000 International Conference on Automation, Robotics, and Control* (2000).
2. M. Uchiyama, "Formulation of high-speed motion of a mechanical arm by trial," *Trans. Soc. Instrum. Control Eng.* **14**(6), 706–712 (1978).
3. S. Arimoto, S. Kawamura and F. Miyazaki, "Bettering Operation of robotics by learning," *J. Robot. Syst.* **1**(2), 123–140 (1984).
4. M.-B. Cheng, W.-C. Su,C.-C. Tsai and T. Nguyen, "Intelligent tracking control of a dual-arm wheeled mobile manipulator with dynamic uncertainties," *Int. J. Robust and Nonlinear Control* **23**(8), 839–857 (2013).
5. S. Galeani, L. Menini and A. Potini, "Robust trajectory trackingfor a class of hybrid systems: An internal model principle approach," *IEEE Trans. Autom. Control* **57**(2), 344–359 (2012).
6. D. A. Bristow, M. Tharayil and A. G. Alleyne, "A survey of iterative learning control," *IEEE Control Syst.* **26**(3), 96–114 (2006).
7. F. Hong, S. S. Ge, B. Ren and T. H. Lee," "Robust adaptive control for a class of uncertain strict-feedback nonlinear systems," *Int. J. Robust Nonlinear Control* **19**(7), 746–767 (2009).
8. D. A. Dirksz, and J. M. A. Scherpen, "Structure preserving adaptive controlof port-hamiltoniansystems," *IEEE Trans. Autom. Control* **57**(11), 2880–2885 (2012).
9. A.-M. Zou, K. D. Kumar and Z.-G. Hou, "Distributed consensus control for multi-agent systems using terminal sliding mode and Chebyshev neural networks," *Int. J. Robust Nonlinear Control* **23**(3), 334–357 (2013).
10. F. F. M. El-Sousy, "Robust wavelet-neural-networksliding-modecontrolsystem for permanent magnet synchronous motor drive," *IET Electr. Power Appl.* **5**(1), 113–132 (2011).
11. C.-L. Chen and Y.-H. Yang, "Position-dependent disturbance rejection using spatial-based adaptive feedback linearization repetitive control," *Int. J. Robust Nonlinear Control* **19**(12), 1337–1363 (2009).
12. I. Houtzager, J.-W. van Wingerden and M. Verhaegen, "Rejection of periodic wind disturbances on a smart rotor test section using lifted repetitive control," *IEEE Trans. Control Syst. Technol.* **21**(2), 347–359 (2013).
13. K. J. Hunt, D. Sbarbaro, R. Zbikowski and P. J. Gawthrop, "Neural networks for control systems—a survey," *Automatica* **28**(6), 1083–112 (1992).
14. L. Cuiyan, Z. Dongchun and Z. Xianyi, "A Survey of Repetitive Control," *Proceedings of 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2004)* **2** (2004) pp. 1160–1166.
15. R. W. Longman, "Iterative learning control and repetitive control for engineering practice," *Int. J. Control* **73**(10), 930–954 (2000).
16. K. L. Barton,A. G. Alleyne, "A cross-coupled iterative learning control design for precision motion control," *IEEE Trans. Control Syst. Technol.* **16**(6), 1218–1231 (2008).
17. C.-J. Chien, "A combined adaptive law for fuzzy iterative learning control of nonlinear systems with varying control tasks," *IEEE Trans. Fuzzy Syst.* **16**(1), 40–51 (2008).
18. A. Tayebi, S. Abdul, M. B. Zaremba and Y. Ye, "Robust iterative learning control design: application to a robot manipulator," *IEEE/ASME Trans. Mechatronics* **13**(5), 608–613 (2008).
19. B. L. Ma and S. K. Tso, "Robust discontinuous exponential regulation of dynamic nonholonomic wheeled mobile robots with parameter uncertainties," *Int. J. Robust Nonlinear Control* **18**(9), 960–974 (2008).

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press

20. J.-W. Kwon and D. Chwa, "Hierarchical Formation Control based on a vector field method for wheeled mobile robots," *IEEE Trans. Robot.* **28**(6), 1335–1345 (2012).
21. W. Ren, J.-S. Sun, R. Beard and T. McLain, "Experimental validation of anautonomouscontrol system on amobilerobotplatform," *IET Control Theory Appl.* **1**(6), 1621–1629 (2007).
22. A. V. Savkin and H. Teimoori, "Decentralized navigation of groups of wheeled mobile robots with limited communication," *IEEE Trans. Robot.* **26**(6), 1099–1104 (2010).
23. H. Fukushima, S. Satomura, T. Kawai, M. Tanaka, T. Kamegawa and F. Matsuno, "Modeling and control of a snake-like robot using the screw-drive mechanism," *IEEE Trans. Robot.* **28**(3), 541–554 (2012).

https://doi.org/10.1017/S0263574714000605 Published online by Cambridge University Press