The Law of One: recursive structures in composition*

CHRISTOS HATZIS

Faculty of Music, University of Toronto, Edward Johnson Building, 80 Queens Park Crescent, Toronto, Ontario M5S 2C5, Canada
E-mail: chatzis@chass.utoronto.ca URL: http://www.chass.utoronto.ca:8080/~chatzis

***The Law of One*, my doctoral thesis[1] composed during 1980 and 1981, was the first in an ongoing series of works which employed the numeric relations of the overtone series to determine all other compositional parameters, pitch, timbre, harmony, rhythm and several nested magnifications of form. The entire composition is a large fractal with a simple principle of construction permeating every aspect of the composition from micro to macro.**

1. THE IDEA

In 1978, while a doctoral composition student of Morton Feldman at SUNY at Buffalo, I became interested in harmonic structures which derive from the overtone series. The idea was, of course, not new at the time. The overtone series has occupied composers and theorists since the times of the Pythagoreans, and in the late 1970s Karlheinz Stockhausen's *Stimmung* and a number of works by James Tenney and other composers were circulating throughout the United States and were known to me. The re-emergence of the idea of the overtone series as a means of extended harmonic practice in the late 1960s and 1970s can be partly attributed to minimalism, but also to the desire to make a complete break with serialism, a desire deeply felt by many composers active during that time. My interest in the overtone series was holistic; I was hoping to discover a means of unifying all compositional parameters from within a single sound to overall form.

I realise in retrospect that, although antiserial in intention, my attempts were in many respects similar to serialism. The goal, the desire to arrive at a single strand of DNA code which determines all the morphological aspects of composition, was common to both approaches. The methodology, however, was different. Serialism, like the twelve-tone system before it, made an attempt to overthrow the hierarchical structures associated with western European music of the eighteenth and nineteenth centuries by introducing in their place a system of strict iteration which warrants equal rights for all elements of a set. The problems associated with this practice are well known by now, so I will only make passing reference to them. Sets extrapolated from a continuum like duration or amplitude by quantising this continuum in arbitrary segments are incompatible with musical perception. Some listeners are capable of hearing and recognising twelve-tone rows and their aggregates, but I do not know of anyone who can identify the aggregates of a set of dynamic or rhythmic values, particularly when these are superimposed on aggregates of other sets. Whatever the intellectual rigour of such an exercise is on paper, it ignores the way we hear and recognise sound. Moreover, the assignment of equal weights to all musical events has perceptually a randomising effect on the music: a piece structured by iterations of an x-element set is statistically identical to a piece the elements of which are determined by an x-sided die.

My own orientation in the late 1970s was towards perceptually legitimate structures, i.e. structures which take into account the way we listen to and/or hear music. According to this holistic way of thinking about music, the breaking up of the continuum which extends from within a single sound to the overall length of a work into various perceptual categories (pitch, rhythm, form) is simply artificial. Any repetition below the threshold of around 16 Hz is perceived as rhythm, while anything above this frequency is perceived as pitch or noise, depending on the degree of periodicity in the waveform. Furthermore, we tend to think of music as an activity in a two-dimensional continuum, the vertical axis of which is represented by pitch and the horizontal by time. Vertical versus horizontal differentiation in classical harmony and counterpoint, in the edit windows of most popular MIDI sequencers and the architectural graphs of Iannis Xenakis and other similarly minded composers conceal the fact that, while perceptually different, pitch and rhythm are manifestations of the same phenomenon: the

* I am indebted to Prof. Paul Pedersen, Prof. Gustav Ciamaga, Erik Ross and Andreas Andreopoulos for their assistance during the preparation of this paper.

[1] Copies of the score are available from UMI. Write or call UMI, 300 N. Zeeb Road, Ann Arbor, MI 48106, USA. Tel. (800) 521-0660. Ask for order no. DA8214158.

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press

high/low, fast/slow and perpendicular/horizontal continua are all time dependent.[2]

If one were to employ a structural principle which could be applied across the entire pitch–rhythm–form spectrum, one would have to start with a formulation which, when applied to microstructure, would produce recognisable and usable musical sounds. An arbitrary rhythmic or formal structure applied to microtime would most likely result in undesirable noise. I started, therefore, with a structure which is inherent to musical sound, the overtone series, and magnified the numerical relationships from within microstructure so that they became rhythmic and formal relationships in macrostructure.[3]

2. THE IMPLEMENTATION

The arbitrary partial number 40[4] of a fundamental tone of 33 Hz was chosen as the upper limit for the series. The rhythmic cycle of the work was set to 24 beats long or six measures in 4/4 time. A simple program was used to calculate the absolute positions of the harmonic nodes in this 24-beat cycle and then map these positions to the closest thirty-second note in conventional notation (figure 1). In higher harmonics with shorter durations, the imperfection of this method of approximation is more evident but the advantage of this approach is that one can notate near-perfect phasing patterns between neighbouring harmonics beyond the tuplet formulations of conventional notation, and thus bypass the human problem of performers trying to render high-numbered tuplets accurately.

The rhythmic cycle of figure 1 is never presented in its entirety. To do so would have not only cluttered the acoustic space and confused the listener but, more importantly, the work would have divulged its harmonic and rhythmic information from the outset and would have no *raison d'etre* beyond a few repetitions of the rhythmic cycle. Instead, another cycle 21 times longer than the rhythmic cycle, but otherwise identical to it, determines which notes will be played at what time. I call this cycle *infrastructure* since it is perceptually too slow to be experienced as rhythm. For the sake of illustration, let us take the 24th harmonic of the series. It is the note G which, according to the rhythmic cycle of figure 1, is one quarter note long. At the infrastructure level, the occurrences of that G would be every 21 beats, so the first one would appear at the beginning of the work, the second one would be played on the second beat of measure 6 (remember we are in 4/4 time), and so on. In order to illustrate its relative position and function within the rhythmic cycle, every time G appears it repeats for three or four times at quarter-note intervals, just enough for the listener to establish the rhythmic profile of that pitch. Then it will appear 21 beats later repeating itself at quarter-note intervals for a bit, then dropping out of the picture until 21 beats later, and so on.

This process of course happens for each one of the 40 harmonics, each one appearing at intervals 21 times larger than their time intervals in the rhythmic cycle. Every time lower harmonics occur in the larger cycle, they are accompanied by their own harmonic series. For example, every time harmonic 5 (E) occurs, so do harmonics 10(E), 15(B), 20(E), 25(G), and so on. At that point the texture will sound predominately E major-like, in fact it will sound like that five times during the course of one infrastructure cycle. The same is true for all harmonics in the cycle low enough to produce audible harmonic series of their own. The aural effect of this process is that of shifting harmonies, the linear ordering of which is determined by the hierarchical ordering of the harmonic series.

Events (notes or verticalities) may be susceptible to *Gestalt* associations made by the listener who might hear one such event as melodically related to its immediate geographical neighbours (notes or chords which happen to be physically close to the event in question but do not directly relate to it). It was imperative at the planning stage of this piece to avoid such associations. In *The Law of One* each event is related primarily to its immediate predecessor, the previous 'incarnation' of this event in the larger scheme of symmetrical repetitions, secondarily to the fundamental of the particular cycle it belongs to, and only indirectly related to the other note-members of the same cycle. Timbre and orchestration are used as a means to counter possible haphazard *Gestalt* associations made by the listener. To understand the role of timbre in *The Law of One*, imagine playing a classical piece, say a Chopin prelude, on a good piano. With a good performer one would presumably hear what Chopin intended the listener to hear: melodies, harmonic progressions, dynamic and tempo changes, all intricately interrelated. Now, suppose the piano was prepared in a way that every key had a distinctively different timbre, like, say in the *Sonatas and Interludes* by John Cage. If you played the same Chopin work on such a piano, the aural effect would be completely different. One might hear,

[2] Karlheinz Stockhausen discusses the time relations within microstructure and their magnification in the domain of rhythm in his article, 'How time passes' (*Die Reihe*, Vol. 3, pp. 10–40. Theodore Presser Company, Pennsylvania (1957)).

[3] Christos Hatzis: 'Chronochroma' (*Interface* **8**(2): 73–90 (1979)).

[4] The number 40 is in fact not as arbitrary as it seems. The integer interval 40/39 is close to the quarter-tone interval of the equal temperament system. The 40th harmonic is also the 8th member of a harmonic series based on E, the 5th harmonic of the fundamental C. For reasons of notational clarity and functionality, 40 seemed to be a good upper limit. Notes above the 40th partial are simply octave transpositions of lower partials used for emphasis only.

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press

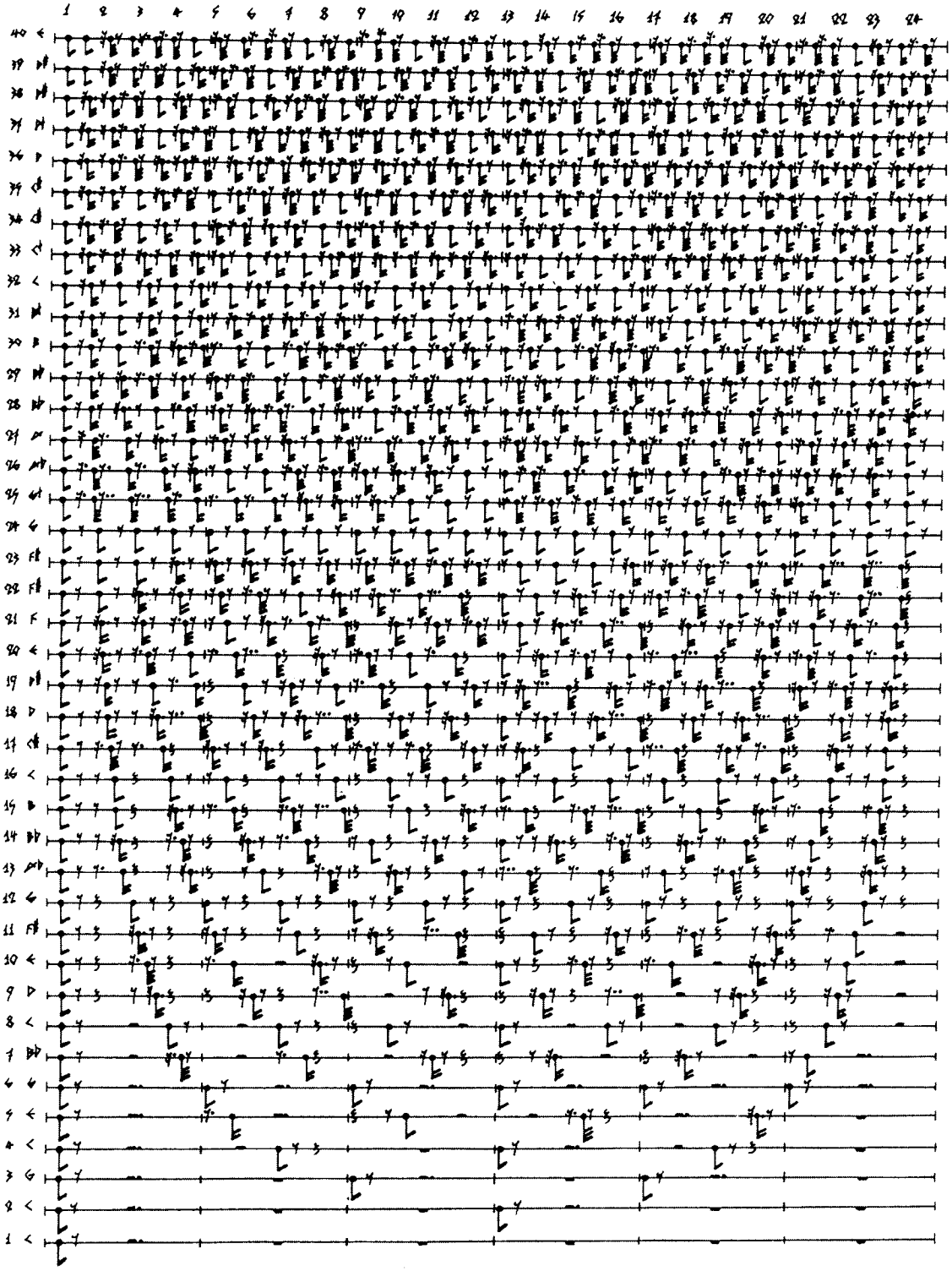


Figure 1. The rhythmic map of *The Law of One*.

for example, a discontinuous relationship between all the middle C 's in the piece and an asymmetrical rhythmic structure produced by the presumably haphazard occurrence of these C 's which is not readily audible on a normal piano. Furthermore, this new structure would be detected at the expense of melodic continuity. Timbral differentiation of this kind would counter the *Gestalt* associations we understand as melody. While in Chopin such discrepancy is not desirable, in *The Law of One* it is very much so.

The Law of One is scored for harp, two identical ensembles consisting of flute, oboe, trombone, violin, viola, double bass, percussion, piano and two vocal SATB quartets. The two ensembles are antiphonally placed to the right and left of the harp, mirroring each other: flute opposite flute, trombone opposite

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press
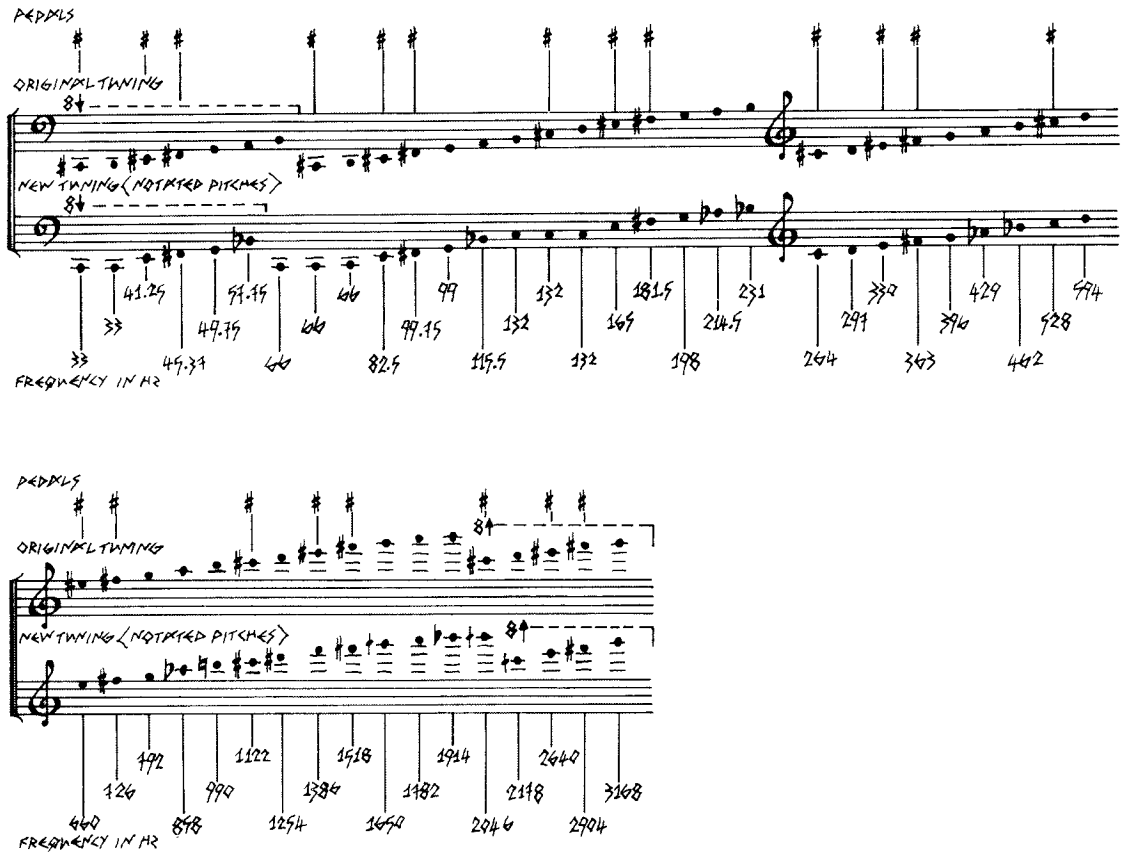


Figure 2. Tuning table for the harp.

trombone, etc. The harp and two pianos are tuned beforehand to appropriate frequencies indicated in the score (figure 2). Because of the logarithmic distribution of the overtone series across the keyboard, there are a large number of unused piano notes in the low register (muted so as not to vibrate sympathetically), whereas in the high register there are more notes required than there are keys on the piano. From middle C upwards, the two pianos have different tunings. The nonstandard pitches are notated as quarter-tones, although it should be kept in mind that neither the standard nor the quarter-tone notations adhere to the system of equal temperament. The tuning of the wind instruments is achieved by alternate fingerings for the woodwinds and nonstandard slide positions for the trombone. A limited set of notes is solicited from each wind instrument and these notes are tuned either against a frequency counter or against the pre-tuned pianos. The violins and the violas have their open strings tuned to prime number harmonics of the fundamental C = 33 Hz (figure 3). This way the upper strings and the similarly tuned double basses can obtain in natural harmonics all 40 overtones; the string writing for the entire piece is in natural harmonics.

Because of the tuning of the strings each instrument is only capable of obtaining a limited number of pitches from the total 40 available. This in fact works to advantage; it means that certain pitches will have timbres which are different than others. For reasons already explained, I conceived of the entire orchestra as a large prepared piano. The fixed-format orchestration utilised in *The Law of One* emphasises the nonconventional acoustical cause-and-effect relationships in the work, whereby pitch identity is far more important than proximity in the melodic space. As I have already stated, a given note in the piece should be heard as a 'rebirth' of the previous note of the same pitch no matter what the time interval separating the two and in no direct causal relationship with its immediate neighbours. To make this possible, I created a 'table of precompositional options' (figure 4) which is used as a guideline for the attainment of the stated goal while allowing for flexibility in dealing with problems arising during the course of composition. Within this format a number of choices are possible; in fact, almost nowhere in the piece does the orchestration of a given note utilise all the resources indicated in figure 4, and in certain instances resources different than those present are employed for purely compositional reasons.

Beyond the infrastructure ordering of the series there is a larger cycle, equal to the duration of the piece, called suprastructure. The suprastructure is less

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press

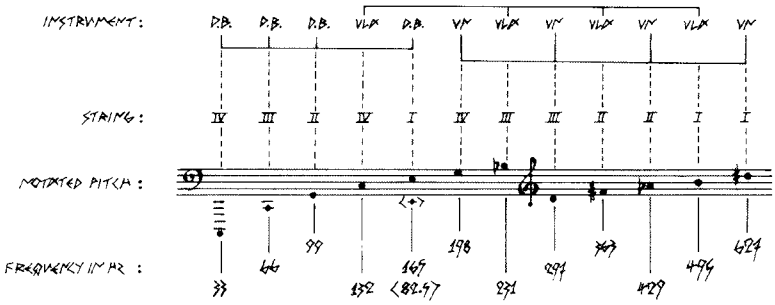


Figure 3. Tuning table for violin, viola and double bass.

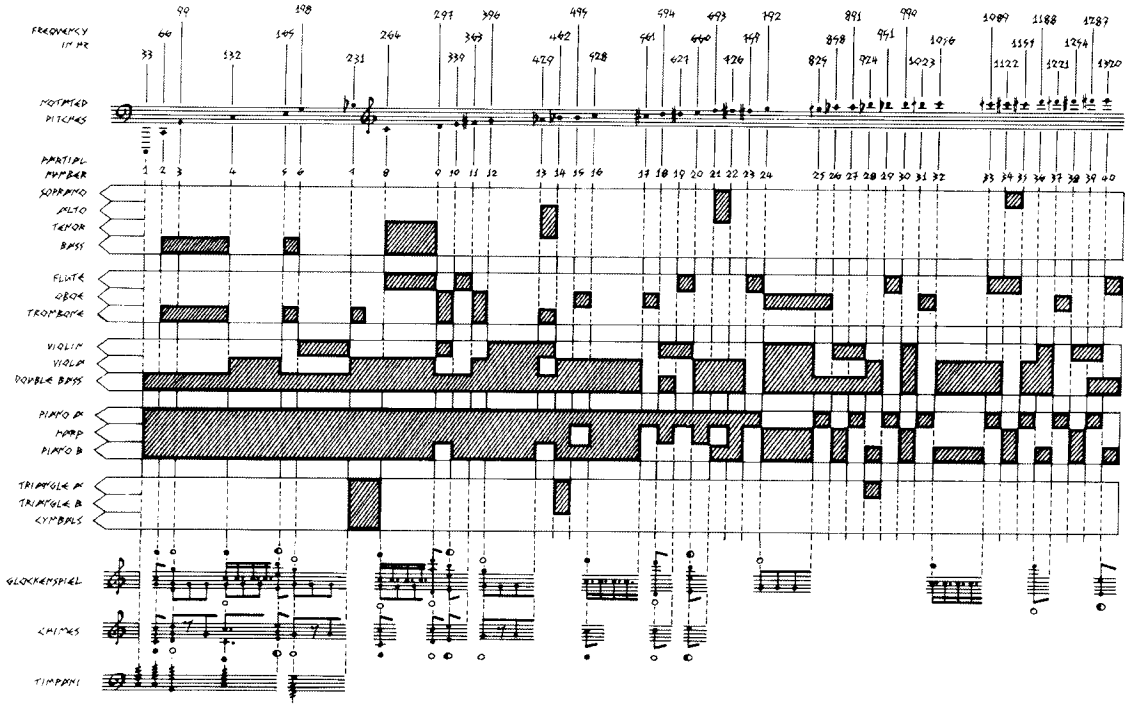


Figure 4. Table of precompositional options.

visible/audible than infrastructure, but equally important. 105 times the size of the rhythmic cycle, or 5 times the size of infrastructure, suprastructure's most discernible characteristic is that at the nodes of harmonics belonging to the Fibonacci series it overrides all rhythmic and infrastructure considerations and allows the solo voices to sing extended rhythmic presentations of the Fibonacci pitches to the words '. . . *kai zoin tou mellontos aeonos* . . .' (lit. trans. '. . . and [I believe in] life of the future aeon . . .'), the last article of the Nicene Creed. In addition to this interruptive function, the suprastructure determines the relative dynamics of the work. In general, dynamics are proportional to the duration of the rhythmic cycle: the lower harmonics with longer durations are also louder. While this principle applies to all levels of magnification, the suprastructure applies additional emphasis on the notes which belong to its own network of repetitions, i.e. every 105th occurrence of a position in the rhythmic cycle or every 5th occurrence in the infrastructure.

It is obvious from the above that, even with the best intentions on the part of the performers, some aspects of the structure would become obscure in performance, due to practical considerations alone. The intricate tuning of the harp and the strings, for example, cannot be maintained for the entire duration of the work – over an hour at quarter note = 33. *The Law of One* was conceived from the outset as a piece to be recorded in small segments on a multitrack recorder, subsequently assembled and repeatedly diffused on a six-speaker sound system in a specially designed small hexagonal space. On each of the six walls of that space there were to be panels with drawings like the one included here (figure 5) reflecting visually the architecture of the music. The idea that the final product should be a recording as opposed to a live performance is pervasive. The score

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press

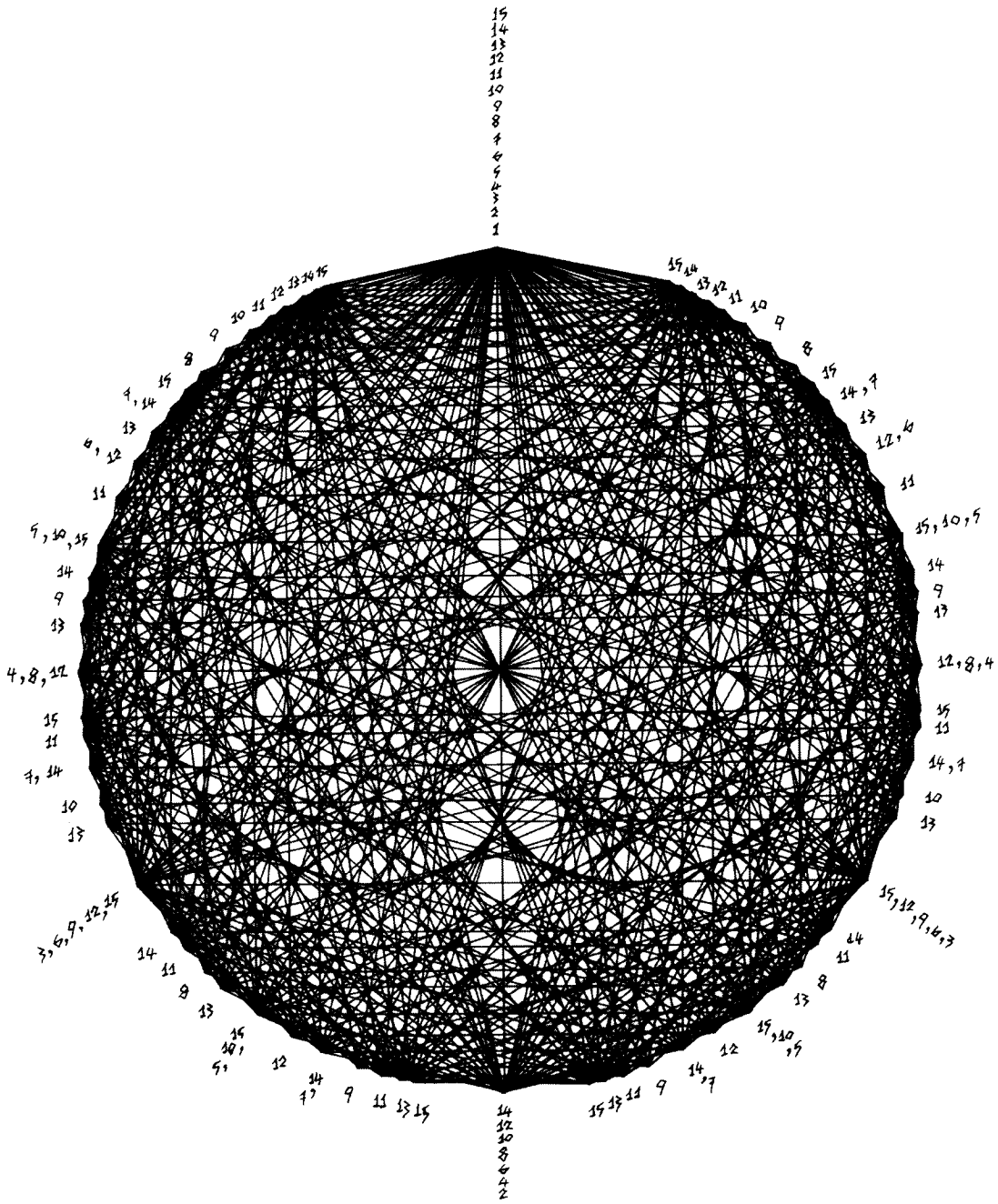


Figure 5. A graphic representation of a fundamental tone and 14 overtones. The numbers represent the nodes of each overtone.

of the work stops at the midpoint of the suprastructure cycle, the intention being that the second half of the work should be the recording of the first half played backwards. While this is quite appropriate on the conceptual level, there are problems associated with reverse envelopes of acoustic sounds, particularly percussive ones, which were not addressed at the time of the composition. Consequently, and in spite of the detailed instructions to the performers, *The Law of One* has not been realised to date. Practical considerations are partly to blame as is the fact that my interests in composition have changed since then and I have busied myself with other projects. In the meantime, however, with the advent of sampling and physical modelling technologies, an accurate realisation of this work is now possible; in fact, *The Law of One* would be quite idiomatic to these technologies. Similarly, with present-day advances in computer animation, the static graphics of figure 5 could now become animated, constantly shifting on screen(s)

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press

highlighting the aspects of the structure predominant in the music at any given time, and making the visual experience just as plastic as the aural one.

3. BEYOND THE MUSIC

The primary influence to this essentially multimedia approach was Morton Feldman's *Rothko Chapel*, a composition for viola, chorus and chamber instrumental ensemble[5] perpetually played in a space full of paintings by Mark Rothko. I had a number of conversations with Feldman about that project and the idea behind it. Feldman's attraction to abstract expressionism, traditionally attributed to his association with the prominent painters of that school in New York during the 1950s, was also related to his Jewish background[6] and its traditional aversion to pictorial representation. The abstraction of an aesthetic based entirely on the articulation of colour fields and the primacy of colour as an expressive means was very much in keeping with his heritage and his own artistic disposition. My background and disposition were quite different. The Byzantine icon, my own visual connection to the numinous, was not a representation of three-dimensional reality like in western religious painting and sculpture, but a metaphor for the metaphysical essence behind three-dimensional reality. The search for a sound appropriate to my thinking and sensibilities naturally led me to structures which transcend perceptual categories and apply to the entire micro–macro continuum, such as the one discussed here. Furthermore, I believed – and still do – in the ability of a metaphor to convey subliminally some of the power of the essence it is hinting at. A totally integrated (and integrating) structure like in *The Law of One*, where the micro and the macro combine into a whole without any part imposing on any other, should be able to perform a healing role, not just a purely aesthetic one, like in abstract expressionism. Unlike serial[7] or stochastic[8] music which start from and remain within a materialist viewpoint and are metaphors for purely physical processes, this system points to inner harmony – the makeup of one's 'timbre' – as the only means of achieving harmony at the levels of physical and social interaction.

In the late 1970s, I wrote a paper[9] which took this idea one step further. If music is to be understood as a metaphor for life, then the constant fluctuation between harmony and dissonance within and without the individual and the causal relationship between the 'within' and 'without' should also be reflected in the music, not just the aspiration for personal and social harmony, as in *The Law of One*. A system was developed whereby varying degrees of randomness could be introduced into the system and this randomness would result in asymmetries on the rhythmic and formal perceptual levels and in dissonance/noise on the level of pitch/timbre. Noise introduced in microstructure would eventually find its way into macrostructure and vice versa. The same algorithmic engine was to produce the 'sound' as well as all other aspects of the composition. The program was designed so that through a stochastic process of trial and error, the system would eventually reach a harmonic steady state devoid of randomness, the composition itself being a realtime documentation of that process. Again, even though the program was developed in the late 1970s, it is only recently, with the rapid advances in realtime sound synthesis, that it is practical. Now it can even become interactive: the end-user could affect the process on one level and witness the

[5] 'The Rothko Chapel is a spiritual environment created by the American painter Mark Rothko (1903–70) as a place for contemplation where men and women of all faiths, or of none, may meditate in silence, in solitude or celebration together. For this chapel, built in 1971 by the Menil Foundation in Houston, Texas, Rothko painted fourteen large canvasses. While I was in Houston for the opening ceremonies of the Rothko Chapel, my friends John and Dominique de Menil asked me to write a composition as a tribute to Rothko to be performed in the chapel the following year. To a large degree my choice of instruments (in terms of forces used, balance and timbre) was affected by the space of the chapel as well as the paintings. Rothko's imagery goes right to the edge of his canvas, and I wanted the same effect with the music – that it should permeate the whole octagonal-shaped room and not be heard from a certain distance. The result is very much what you have in a recording – the sound is closer, more physically with you than in a concern hall.' – Morton Feldman.

[6] Feldman was very conscious of his background and his own role in what he had often described as 'post-holocaust' music. The low dynamic levels, the absence of vibrato and the lack of goal orientation are aspects of 'mourning', a description he thought appropriate for his work.

[7] Serialism as a concept is integrated, but it is disintegrating; it causes the quantisation of continua in order to define distinct behaviours of sound (how loud, how long, how high?). Sound itself is viewed as indivisible, as a point on which the above attributes are superimposed. The system is concerned with 'social behaviour (how does each one of these points relate to the group?), but never views sound as having an inner reality. For this reason, it has been impossible for serialism to deal with the questions of orchestration and acoustics in a systematic way. Early on, Stockhausen made an attempt to reconcile this inherent contradiction in his article, 'How time passes' (see footnote 1 above). In 1957, when serialism as a movement was at its peak, he acutely detected the problems and anticipated many of the ideas discussed here. His discourse in that article fell short in only one – critical, I think – area. He took serialism and equal temperament for granted and consequently developed rhythmic structures corresponding to the tempered intervals between fundamental tones. This article points to Stockhausen's increasing disenchantment with serialism and its limitations. He ultimately abandoned the system altogether in favour of a more intuitive and acoustically legitimate approach to composition.

[8] Just like in serialism, stochastics as an organisational principle deals with sound events as indivisible units. Stochastics may be very much preoccupied with external spectra and continuous transformation, but still fails to acknowledge the inner identity of a sound event. Questions of orchestration and acoustics have been a theoretical stumbling block with stochastics as much as with serialism.

[9] Christos Hatzis: 'Towards an endogenous automated music' (*Interface* **9** (2): 83–114 (1980)).

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press

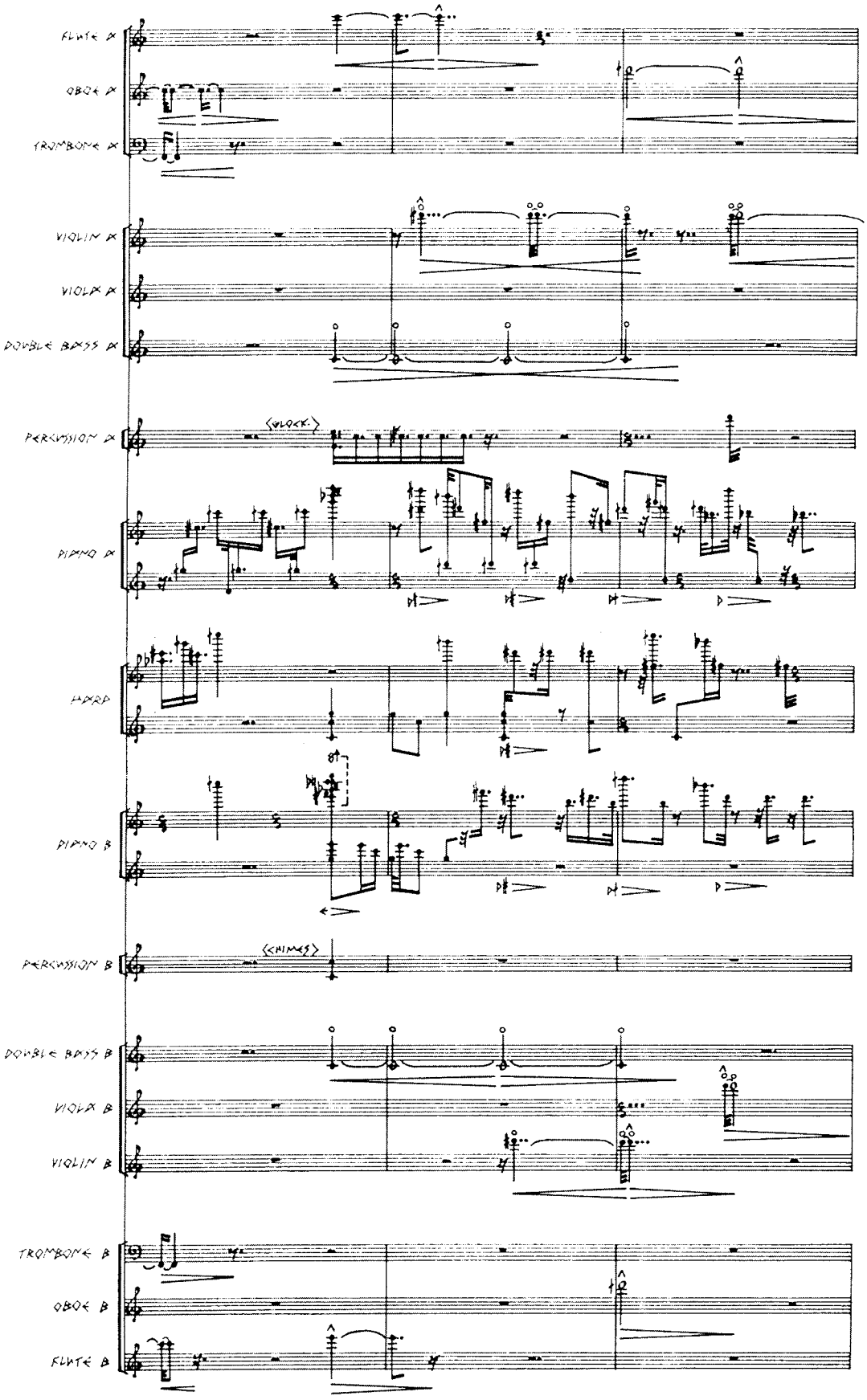


Figure 6. A sample page of the score.

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press

effect of that change on all other levels of the structure.

Since the early 1980s I have abandoned the structuralist approach to music and have opted for a more intuitive composition. I have occasionally used the ideas outlined here in later works, but none of these works display the precompositional structural rigour of *The Law of One*: they are composed 'from the bottom up' rather than 'from the top down'. This paper is therefore more like a review of my work in the late 1970s and early 1980s than a defence of my current ideas.[10] Having said that, however, I realise in retrospect that everything that I stand for today as a composer and thinker has its roots in this single work of almost twenty years ago and the ideas which generated it. The fundamental difference between then and now is that, instead of creating total musical universes like the one described here, I am now content to musically define possible individual spaces within the universe we all inhabit and share. The sacrifice in scope has been amply compensated by a better understanding of the human predicament and the role of music in improving it. *The Law of One* is an early attempt at explaining the world. My recent music is more like an aid for coping with it.

[10] For more information on my current ideas and concerns, see 'Towards a new musical paradigm', originally posted on the World Wide Web at ⟨http://www.chass.utoronto.ca:8080/~chatzis/paradigm.htm⟩ in February 1996 and published with extensive revisions by *MikroPolyphonie* in-line journal (Australia) at ⟨http://farben.latrobe.edu.au/mikropol⟩ in November 1996.

https://doi.org/10.1017/S1355771898009133 Published online by Cambridge University Press