Environment and Development Economics 3 (1998): 417–423
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Theory and Applications

Quasi-option value and hard uncertainty*

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ABSTRACT. The notion of intertemporal *h*-option value is introduced to extend to environmental decision-making problems of the concept of quasi-option value, defined by Arrow and Fisher and Henry, whenever an individual faces imprecise information, or hard uncertainty, represented by a non-additive measure over events. Under hard uncertainty and irreversibility, the decision-maker faces the intertemporal *h*-option value, that is an uncertainty premium of preservation, which represents the gain from being able to learn about the future consequences of irreversible actions. The intertemporal *h*-option value is a correction factor that has to be taken into account in evaluating the total economic value of feasible actions.

Keywords: quasi-option value, capacity, hard uncertainty, Choquet expected utility JEL classification D81; Q39

1. Introduction

The problem of how to evaluate environmental assets and resources[1] is a complex question typically because most of these assets are not marketable goods. When an environment resource is a substitute or a complement of a marketable asset, there are some approaches able to capture its use value. However, there is another source of value of environmental assets which is derived from the combination of uncertainty and learning about future use value and irreversibility in depletion. This value is an expression of flexibility, which is a 'characteristic of early decisions in a sequential chain which permits the decision-maker to adjust and take advantage of information he perceives as time elapses' (Marschak and Nelson, 1962, p. 56). Although flexibility is a relevant source of value in an intertemporal decision problem, its role in the evaluation of investment decisions had not been considered until Arrow and Fisher (1974) and Henry (1974a, 1974b) introduced the concept of

I am grateful to S. Vannucci and C. Zappia for helpful comments on earlier versions. Special thanks to A. Fisher and G. Heal for stimulating encouragement and discussion.

[1] Assets and resources are employed as synonymous.

https://doi.org/10.1017/S1355770X98000217 Published online by Cambridge University Press

quasi-option value. They proved that the coexistence of uncertainty and restriction on reversibility of actions always produces an extra value called quasi-option value, that prevents the use of the certainty equivalent method to solve decision problems, even if the payoff function is quadratic. Moreover, they argued that, in an intertemporal decision process with prospective learning that only depends on the passage of time, the quasi-option value is always positive.

The expected utility paradigm based on the Savage (1954) approach, which does not assume the existence of exogenous randomization device, prescribes that an additive probability distribution can be derived from preferences over consequences implied by any action and that feasible actions are only compared by an expected utility preference function based on this probability distribution. Nevertheless, the decision-maker can experience a lack of information either due to a misspecification of the space of the states or to complete ignorance of the set of the possible future events. The occurrence of the virus Ebola, the melting of ice caps and the hole in the ozone layer are clear examples of unanticipated events, that were omitted in the decision-maker's representation of the world. When the decision-maker has a misspecified description of the states of the world, she/he is unable to represent her/his beliefs by a unique probability distribution on the set of the events. Such a situation is referred to as hard uncertainty (see Chateauneuf, 1991). When uncertainty is hard and the use of environmental assets entails irreversible effects, an individual has to consider that environmental assets get an additional value, which has been called intertemporal *h*-option (see Basili and Vercelli, 1996). In this paper, using the same framework as Arrow and Fisher (1974), both the characteristics of the intertemporal *h*-option value and its theoretical relationship with the notion of quasi-option value are derived. The intertemporal *h*-option is independent of risk attitude, in fact it is a sort of positive uncertainty premium which derives from the asymmetry between reversible and irreversible actions and the independent learning process.

The implications for development that can be drawn from this analysis are apparent. In many less-developed countries, policies in livestock and agricultural sectors affect the cutting down of tropical evergreen and rain forests. The net revenues of converting wild land in these economic activities are compared to the valuation of forests by means of cost–benefit analysis and government policies are drawn on the basis of this. However, as far as ecosystems are involved, it is necessary to estimate the effective total economic value of forests. The intertemporal *h*-option value is a relevant component of this total economic value because it represents an additional benefit of preserving forests, that is the uncertainty premium related to preservation of the wild area, including the value of the availability of forests for future direct and indirect use value.

The paper develops as follows. Section 2 introduces capacities in order to represent the decision-maker's attitude toward hard uncertainty. Section 3 defines the notion of intertemporal *h*-option value and provides the generalization of quasi-option value. The concluding remarks make up section 4.

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2. Hard uncertainty attitude and capacities

Models explaining quasi-option value assume that environmental states of the world have an additive probability of occurring, that is the decision-maker's description of the states of the world is exhaustive. The decision-maker has (explicitly or implicitly) a unique and fully reliable probability distribution over events, moreover she/he possesses an expected utility function linear in probabilities.[2]

Consider a decision problem in which the states of the world included in the model do not exhaust the actual ones. A description of the world is considered as a misspecified model whenever omitted states are not explicitly included in the model. When the decision-maker does not know how many states are omitted, she/he can represent her/his beliefs by means of a capacity on the set of events.

Let $\Omega = \{\omega_1, \ldots, \omega_n\}$ be a non-empty set of states of the world and let $S = 2^{\Omega}$ be the set of all events. The decision-maker chooses an action in the set X. In particular, an action $x \in X$ is a function assigning a consequence to each state, $x : \Omega \to C$, where C is the set of consequences. A function $\mu : S \to R$ is a capacity or a non-additive signed measure if it has the following characteristics:

$\mu(\varnothing) = 0$, $\mu(\Omega) = 1$ (i.e., the capacity is normalized);

$\forall A, B \in S$ such that $A \subset B$, $\mu(A) \leq \mu(B)$ (i.e., the capacity is monotone).

A capacity[3] is convex (concave) if for all $A,B \in S$ such that $A \cup B$, $A \cap B \in S$, $\mu(A \cup B) + \mu(A \cap B) \geq (\leq) \mu(A) + \mu(B)$. It is superadditive (subadditive) if $\mu(A \cup B) \geq (\leq) \mu(A) + \mu(B)$ for all $A,B \in S$ such that $A \cup B \in S$, $A \cap B = \varnothing$.

Given a real-valued function $f : \Omega \to R$, f is a measurable function if for every $t \in R$, $\{\omega | f(\omega) \geq t\}$ and $\{\omega | f(\omega) > t\}$ are elements of S. Since μ is non-additive, the integration of a real-valued function f with respect to μ is impossible in the Lebesgue sense. The proper integral for the non-additive measures is the Choquet integral, originally defined by Choquet (1954) and discussed in Schmeidler (1989). The Choquet integral of f with respect to a capacity μ is defined as

$$\int f d\mu = \int_0^{\infty} \mu\left(\left\{\omega \middle| f(\omega) \geq t\right\}\right) dt + \int_{-\infty}^{0} \left[\mu\left(\left\{\omega \middle| f(\omega) \geq t\right\}\right) - \mu(\Omega)\right] dt$$

The Choquet integral coincides with the standard one if μ is additive.

[2] When the decision-maker 'possesses a unique, well-defined classical probability distribution over events, possesses a von Neumann–Morgenstern utility function over outcomes, and ranks subjectively uncertain acts according to the expected utility of their induced probability distributions over outcomes ... [she/he] is a probabilistically sophisticated expected utility maximizer' (Machina and Schmeidler, 1992, p. 747).

[3] It is worth noting that a capacity is additive or a probability measure if for all $A,B \in S$ such that $A \cup B \in S$ and $A \cap B = \varnothing$, $\mu(A \cup B) = \mu(A) + \mu(B)$.

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The standard subjective expected utility theory assumes that there exists a bounded real-valued function $u: C \rightarrow R$ and a finitely additive measure or probability p on S, such that expected utility is equal to $\int_{\Omega} u(f(\omega))dp$. In addition u is unique up to a positive linear transformation.

Given a utility function u, a capacity μ on S and a set of comonotonic[4] actions X, the Choquet integral provides the analog of the Savage expected utility and thus it yields the expected utility of an action with respect to a non-additive measure on events, which has been called Choquet expected utility or CEU. The Choquet expected utility[5] can be represented (see Wakker, 1989) as a weighted average, where the weights are not given by the non-additive measure but are derived from the cumulative distribution induced by a bijection $\pi: \{\omega_1, \ldots, \omega_n\} \rightarrow S$, such that $f(\pi_1) \geq \ldots \geq f(\pi_n)$.

The Choquet integral $\int f d\mu = \int_0^{\infty} \mu\left(\left\{\omega \middle| f(\omega) \geq t\right\}\right)dt + \int_{-\infty}^{0} \left[\mu\left(\left\{\omega \middle| f(\omega) \geq t\right\}\right) - \mu(\Omega)\right]dt$ can be written as $\sum_{k=1}^{n} f\left(\pi(k)\right)\left[\mu\left(\left\{\pi(1), \ldots, \pi(k)\right\}\right) - \mu\left(\left\{\pi(1), \ldots, \pi(k-1)\right\}\right)\right]$ and the Choquet expected utility is $\int_{\Omega} u(f(\omega))d\mu$.

The procedure just introduced allows one to represent a preference ordering over the set of the decision-maker's feasible actions by a utility function (unique up to a positive linear transformation), even in a situation in which there are omitted states.

The decision-maker expresses hard uncertainty aversion (preference) if she/he assigns larger probabilities to states when they are unfavorable (favorable), than when they are favorable (unfavorable), that is if her/his non-additive measure is convex (concave).[6] Hence, the convexity (concavity) of the capacity, that implies superadditivity (subadditivity) of the Choquet integral, captures the decision-maker's attitude toward hard uncertainty (see Chateauneuf, 1991).

3. The main result

Consider the case of a forest which can be either preserved, reversible action, or destroyed, irreversible actions, by cutting down for livestock or agricultural cultures. The hard uncertainty averse decision-maker (pessimist) assigns a convex capacity over events, whereas the hard

[4] Two actions $x, y \in X$ are comonotonic if and only if there are no $A, B \in S$, so that $x(A) > x(B)$ and $y(A) < y(B)$. Roughly speaking, two actions are comonotonic if they induce the same ordering of favorable state and the same permutation.

[5] It is worth noting that the preference function defined over acts by the Choquet integral 'is not probabilistically sophisticated, but nevertheless satisfies all the Savage axioms except the sure-thing principle' (Machina and Schmeidler, 1992, p. 758).

[6] Schmeidler (1989) points out that smoothing or averaging utility distribution and the convexity of preference ordering are equivalent to the convexity of the capacity. In fact decision-maker is uncertainty averse if for any three actions, $x, y, z \in X$ and $\alpha \in [0,1]$, such that $x \geq z$ and $y \geq z$, then $\alpha x + (1-\alpha)y \geq z$.

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uncertainty seeking decision-maker (optimist) assigns a concave capacity over events. The uncertain decision-maker faces actions all of which assign a consequence, a net monetary revenue, to each possible state of the world. At the beginning of the intertemporal decision problem, the decision-maker has to take into account the arrival of news about the future consequence of each action with the passage of time. For the sake of simplicity, it is possible to collapse all future periods in only one future period and describe the intertemporal decision problem as a two-period process, i.e., the present and the future, in which all available choices are either to cut down the forest at once or to wait for additional information. In the first period the decision-maker is uncertain about the future state of the world. Uncertainty is resolved by nature in the second period. The decision-maker can choose at the beginning of both periods, but her/his second period choice depends on the previous choice. The irreversible action is possible in both periods, that is, cutting down is not a 'now or never' opportunity, but a 'now or next period' one. In this intertemporal decision process involving hard uncertainty, learning and comonotonic actions, the notion of intertemporal *h*-option value emerges. The intertemporal *h*-option value generalizes the notion of quasi-option value. In fact, by the Choquet integral it is possible to evaluate the Choquet expected utility of an action and generalize the Arrow and Fisher result, which is obtained evaluating the expected utility of an action by the usual integral.

Proposition Because of the attitude towards hard uncertainty, the decision-maker faces a sort of uncertainty premium called intertemporal *h*-option value, independently of his risk attitude. The intertemporal *h*-option value is always positive and completes the notion of quasi-option value.

The formulation employed is as in Arrow and Fisher. Consider the development of a wild area d over two periods.

Let
$d =$ unity
$d_1 =$ the amount of land developed in the first period, such that $0 \leq d_1 \leq 1$
$d_2 =$ the amount of land developed in the second period, such that $d_2 \geq 0$ and $d_2 \leq 1 - d_1$
$b_p =$ benefits from preservation of d in the first period
$b_d =$ benefits from development of d in the first period
$\beta_p =$ expected benefits, conditional on b_p and b_d, from preservation of d in the second period
$\beta_d =$ expected benefits, conditional on b_p and b_d, from development of d in the second period
$c_1 =$ investment cost in the first period
$c_2 =$ investment cost in the second period.

Define $(b_d - b_p - c_1) = w$ and $(\beta_d - \beta_p) = z$, if $z > c_2$ then $d_2 = 1 - d_1$; if $z < c_2$ then $d_2 = 0$. There are only two states A and A^C in the world (Ω) and if A occurs then $z > c_2$, on the other hand if A^C occurs then $z < c_2$.

Differently from Arrow and Fisher, who assume the two events have an additive probability to occur and can thus derive the expected value

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associated to each possible action, in this model the events have a non-additive measure of occurring. If the measure on the set of the states is convex, the pessimistic decision-maker relatively gives great weight to the worse events. On the contrary, if the measure is concave the optimistic decision-maker relatively overvalues the better events. If A occurs, the total benefits are $b_p(1-d_1)+b_d d_1-c_1 d_1+\beta_d-c_2(1-d_1)=wd_1+c_2 d_1+b_p+\beta_d-c_2$, on the contrary if A^C occurs, the total benefits are $b_p(1-d_1)+b_d d_1-c_1 d_1+\beta_p(1-d_1)+\beta_d d_1=wd_1+zd_1+b_p+\beta_p$. The Choquet expected (*CE* in short) benefits from developing $d_1>0$ in the first period are

$$CE[(w+\min(c_2,z))d_1+b_p+\max(\beta_d-c_2,\beta_p)]$$

The Choquet expected benefits from preservation $d_1=0$ in the first period are

$$CE[b_p+\max(\beta_d-c_2,\beta_p)]$$

The difference between development and preservation (net benefits from development) is

$$CE[(w+\min(c_2,z))d_1]$$

It immediately follows that if this difference is positive, development is chosen, while if it is negative, conservation is a better choice.

When the decision-maker ignores uncertainty, he replays random variables w and z by their Choquet expected values $CE(w)$ and $CE(z)$, so that the criterion becomes $CE(w)+\min(c_2,CE(z))$. As in Arrow and Fisher, consider the case in which $c_2<CE(z)$, development is chosen, then the criterion is $CE(w)+c_2$. Since w is a fixed value, whatever happens in the second period, it is a comonotonic constant random variable[7] with respect to every random variable. Then $CE[(w+\min(c_2,z))]$ can be written as $CE(w)+\min(c_2,CE(z))$, by comonotonic additivity of the Choquet integral.[8] Since $\min(c_2,z)\leq c_2$, then

$$CE[(w+\min(c_2,z))]\leq CE(w)+c_2 \tag{1}$$

In the same way, if $c_2>CE(z)$ the criterion is $CE(w)+CE(z)$ and since $\min(c_2,z)\leq z$

$$CE[(w+\min(c_2,z))]\leq CE(w)+CE(z) \tag{2}$$

The inequalities (1) and (2) emphasize that benefits under uncertainty are at most equal to benefits under certainty and illustrate that the method based on Choquet certainty equivalents also fails to solve decision problems characterized by hard uncertainty and irreversibility. Moreover, the inequalities (1) and (2) suggest the existence of a positive intertemporal *h*-option value, whenever the decision-maker is either optimistic or pessimistic. When the decision-maker assigns an additive measure on

[7] A constant random variable is comonotone to every random variable.

[8] If x and y are comonotonic and real valued random variables then $\int\left(x+y\right)d\mu=\int x d\mu+\int y d\mu$

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events, the Choquet integral coincides with the usual one, the Choquet expected benefits are equivalent to the Savage expected benefits and intertemporal *h*-option value overlaps the quasi-option value.

4. Conclusion

The notion of intertemporal *h*-option value generalizes Arrow and Fisher's notion of quasi-option value so as to take into account hard uncertainty. When the decision-maker faces irreversible acts, hard uncertainty and learning, the intertemporal *h*-option value always exists. The intertemporal *h*-option value is positive and does not allow reducing a dynamic decision process to a timeless one. The intertemporal *h*-option value represents a further correction factor that has to be introduced into the evaluation of total economic value of environmental feasible actions. In environmental economics, the intertemporal *h*-option value is particularly important because the uncertainty faced is typically hard, and irreversibility is a very serious matter. Therefore, whenever development decisions involve depletion of natural resources and severe damage of ecosystems, more conservative policies can be induced by considering the attitude toward hard uncertainty.

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