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Adaptive Integrated Navigation for Multi-sensor Adjustment Outputs

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An adaptive integrated Kalman filtering based on the adjustment outputs of local navigation sensors and the outputs of a dynamic or kinematic state model is presented, which avoids the correlations of the local Kalman filtering outputs affected by the same disturbances of the dynamic state model. It has the advantage of rigor in theory and simple in calculation as well as adaptive in the various local navigation outputs. An integrated navigation estimator that is similar to the federated Kalman filtering is given as an initial estimate of a state based on the information sharing principle, but without any dynamic model information. An adaptive integrated fusion of the local navigation outputs and the dynamic model information is followed, in which the weights of the local navigation outputs and the dynamic model outputs are determined based on their differences from the integrated navigation results. The processing algorithms, logic, and associated computer burden are similar to those of federated filter. A simulated example is given to show the effectiveness of the new adaptive integrated navigation algorithm.

KEY WORDS

1. Integrated navigation. 2. Federated Kalman filtering. 3. Data fusion. 4. Adaptive fusion.

1. INTRODUCTION. Most navigation algorithms are based on the Kalman filter, where a dynamic or kinematic model plays an important role. In the application of multi-sensor navigation, a federated filter that combines the local filter outputs by an information sharing principle was developed (Carlson, 1988, 1996), where the local filter outputs are presumed to be independent. This method provides globally optimal or near-optimal estimation accuracy, with a high degree of fault tolerance, and is practical for real-time distributed navigation system applications (Carson, 1996). The federated filter structure employs sensor-dedicated local filters (LFs) and a master filter (MF) to combine or fuse the LF outputs. In practice however, the local filter outputs derived from different navigation sensors based on the same dynamic model prediction are usually dependent. Any dynamic model errors, especially significant disturbances, will influence all local outputs and thus influence the integrated navigation results. An integrated navigation algorithm based

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on a dynamic Kalman filter followed by several static filters integrating all outputs of navigation sensors was developed (Yang, 2003). The kinematic and static filtering does not repeatedly use the dynamic model information, thus the estimated state parameters are optimal in the minimal variance sense. It performs the dynamic filter by employing the dynamic model information and the measurement data of the main navigation sensor at each epoch, the resulting state of which is regarded as prior information of the state, and the following sensor data are fused in a static filtering manner, not repeatedly using the dynamic model information. The kinematic and static filtering procedure, however, does not make full use of the parallel processing technique nor make full use of the multiple data redundancy to control the bad influences of dynamic model errors and the outliers of some individual sensor outputs.

More recently, an adaptively robust filtering approach was developed and shown to be effective for controlling the bad influences of the measurement outliers and the disturbances of the dynamical model on the estimated state (Yang *et al.*, 2001, 2003). This paper introduces the idea of the adaptively robust filtering to the federated filter to develop a new adaptive integrated navigation approach. Emphasis is on the controlling of the outlying influence of individual sensor outputs on the kinematic state estimates. The performance of the new procedure is compared to that of an existing federated filter. A simulated example is given.

2. A DIRECT FUSION ALGORITHM FOR MULTI-SENSOR OUTPUTS.

Any navigation sensor's measurements can be adjusted to produce its local navigation outputs without using the dynamic model information if there exist enough measurements. The local navigation outputs are independent of each other, as required for a federated filter. A direct fusion algorithm can thus be constructed by combining the local navigation outputs and the dynamic model information.

Assume that there exist groups of observations from r sensors with the corresponding observation equations

$$L_i = A_i X + \Delta_i \tag{1}$$

where A_i are the design matrices corresponding to the observation vectors L_i, Δ_i are the error vectors; X is the state vector at time t. The error equation is

$$V_i = A_i \hat{X} - L_i \tag{2}$$

where V_i is the residual vector of L_i. By applying the least squares principle

$$V_i^T P_i V_i = \min \tag{3}$$

we derive the estimator vector $\hat{X}_i$ and its covariance matrix $\Sigma_{\hat{X}_i}$ as

$$\hat{X}_i = (A_i^T P_i A_i)^{-1} A_i^T P_i L_i \tag{4}$$

$$\Sigma_{\hat{X}_i} = (A_i^T P_i A_i)^{-1} \hat{\sigma}_{0i}^2 \tag{5}$$

where P_i is the weight matrix of L_i, $\hat{\sigma}_{0i}^2$ denotes the variance factor with respect to L_i. The local navigation solution $\hat{X}_i$ $(i = 1, 2, \ldots, r)$ and its covariance matrix $\Sigma_{\hat{X}_i}$ or the weight matrix $P_{\hat{X}_i} = \Sigma_{\hat{X}_i}^{-1}$ of each sensor can easily be obtained. The estimated state

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resulted from (4) is called the geometrical navigation state, since no dynamic model information is used.

Now we introduce the predicted state vector and its covariance matrix are

$$\bar{X}(t)=\phi_{t,t-1}\hat{X}(t-1) \tag{6}$$

$$\Sigma_{\bar{X}}=\phi_{t,t-1}\Sigma_{\hat{X}(t-1)}\phi_{t,t-1}^{T}+\Sigma_{w} \tag{7}$$

where $\phi_{t,t-1}$ is the transition matrix of the state from time $t-1$ to t, $\hat{X}(t-1)$ is the estimated state at time $t-1$, with covariance matrix $\Sigma_{\hat{X}(t-1)}$, Σ_w is the model error covariance matrix.

The error equation of the predicted state vector and the local navigation outputs are expressed as

$$V_{\bar{X}}=\hat{X}-\bar{X} \tag{8}$$

$$V_{\hat{X}_i}=\hat{X}-\hat{X}_i \tag{9}$$

The target function on the basis of local navigation results and the predicted state are

$$V_{\bar{X}}^{T}P_{\bar{X}}V_{\bar{X}}+\sum_{i=1}^{r}V_{\hat{X}_i}^{T}P_{\hat{X}_i}V_{\hat{X}_i}=\min \tag{10}$$

where

$$P_{\bar{X}}=\Sigma_{\bar{X}}^{-1},\quad P_{\hat{X}_i}=\Sigma_{\hat{X}_i}^{-1}=A_i^{T}P_iA_i/\hat{\sigma}_{0i}^{2}$$

Thus follows the fusion estimator

$$P_{\hat{X}^0}\hat{X}^{0}=P_{\bar{X}}\bar{X}+P_{\hat{X}_1}\hat{X}_1+P_{\hat{X}_2}\hat{X}_2+\cdots+P_{\hat{X}_r}\hat{X}_r \tag{11}$$

and the estimated state is

$$\hat{X}^{0}=P_{\hat{X}^0}^{-1}(P_{\bar{X}}\bar{X}+P_{\hat{X}_1}\hat{X}_1+P_{\hat{X}_2}\hat{X}_2+\cdots+P_{\hat{X}_r}\hat{X}_r) \tag{12}$$

where

$$P_{\hat{X}^0}=P_{\bar{X}}+P_{\hat{X}_1}+P_{\hat{X}_2}+\cdots+P_{\hat{X}_r} \tag{13}$$

If $\hat{X}_i$ in (12) is derived by local Kalman filter with the dynamic model information, then (12) is the federated filter. The reliability of the fusion state above is based on two assumptions. One is that the local navigation outputs of all sensors and the predicted state from the dynamical model are independent of each other as required by the target function (10). The other is that the weight matrices of the states derived from the local sensors and predicted from the dynamical model reflect their reliabilities. It is obvious that the first assumption is guaranteed by the estimator (4) in which no common information is included, neither is the dynamic model information. The second assumption, however, is hardly met, thus the reliability of the fusion result is problematic.

Theoretically the fusion result, based on the geometric navigation states $\hat{X}_i$ and the pure dynamic predicted state $\bar{X}$, is equivalent to that of an integrated least squares adjustment by using all measurements of the local sensors and the dynamic model predictions. The estimator (12) employs a parallel processing technique, thus the calculation can be speeded up; furthermore failure can be detected and tolerated by some strategies.

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3. AN ADAPTIVE FUSION BASED ON THE GEOMETRIC NAVIGATION SOLUTION. In order to optimally balance the weights of the local geometric navigation outputs, we can adaptively determine the weight matrices of all the navigation outputs based on the differences between local navigation estimates and the fusion navigation estimate, and obtain the last adaptive fusion estimated state. The target function is

$$w_{\bar{X}} V_{\bar{X}}^T P_{\bar{X}_i} V_{\bar{X}} + \sum_{i=1}^{r} w_i V_{\hat{X}_i}^T P_{\hat{X}_i} V_{\hat{X}_i} = \min \tag{14}$$

where $w_{\bar{X}}$ and w_i $(i=1,2,\ldots,r)$ are called adaptive factors of the weight matrices $P_{\hat{X}_i}$ and $P_{\bar{X}}$. It is easy to arrive at the information matrix of the fusion navigation solution following (14),

$$\bar{P}_{\hat{X}} = w_{\bar{X}} P_{\bar{X}} + w_1 P_{\hat{X}_1} + w_2 P_{\hat{X}_2} + \cdots + w_r P_{\hat{X}_r} \tag{15}$$

The fusion estimator is

$$\bar{P}_{\hat{X}} \hat{X} = \bar{P}_{\bar{X}} \bar{X} + \bar{P}_{\hat{X}_1} \hat{X}_1 + \bar{P}_{\hat{X}_2} \hat{X}_2 + \cdots + \bar{P}_{\hat{X}_r} \hat{X}_r \tag{16}$$

$$\hat{X} = \bar{P}_{\hat{X}}^{-1} (\bar{P}_{\bar{X}} \bar{X} + \bar{P}_{\hat{X}_1} \hat{X}_1 + \bar{P}_{\hat{X}_2} \hat{X}_2 + \cdots + \bar{P}_{\hat{X}_r} \hat{X}_r) \tag{17}$$

where

$$\bar{P}_{\bar{X}} = w_{\bar{X}} P_{\bar{X}}, \quad \bar{P}_{\hat{X}_i} = w_i P_{\hat{X}_i} \tag{18}$$

are called adaptive weight matrices. The adaptive factors w_{i} and $w_{\bar{X}}$ satisfy

$$w_{\bar{X}} + w_1 + w_2 + \cdots + w_r = 1 \tag{19}$$

The covariance matrix of the estimated state $\hat{X}$ is

$$\Sigma_{\hat{X}} = \bar{P}_{\hat{X}}^{-1} \hat{\sigma}_0^2 \tag{20}$$

Now the vital problem is how to reasonably determine the adaptive factors w_i and $w_{\bar{x}}$. An adaptive factor depending on the discrepancy between predicted state from the dynamic model and the geometric estimated state by using measurements was proposed by Yang *et al.* (2001a, b), can be introduced in the integrated navigation. An adaptive factor may be (Yang, 2001b)

$$\alpha_i = \begin{cases} 1 & |\Delta \tilde{X}_i| \leqslant c \\ \dfrac{c}{|\Delta \tilde{X}_i|} & |\Delta \tilde{X}_i| > c \end{cases} \tag{21}$$

where the value of c is chosen as 2·5 and $\Delta \tilde{X}_i$ is denoted by

$$\Delta \tilde{X}_i = ||\hat{X}_i - \hat{X}^0|| / \sqrt{tr(\Sigma_{\hat{X}_i})} \tag{22}$$

Similarly, the adaptive factor $\alpha_{\bar{X}}$ for the predicted state $\bar{X}$ is constructed only, $\hat{X}_i$ is replaced by $\bar{X}$. Considering the condition (19) we obtain w_i and $w_{\bar{X}}$ as

$$w_i = \alpha_i \Big/ \left(\alpha_{\bar{X}} + \sum_{i=1}^{r} \alpha_i \right) \tag{23}$$

$$w_{\bar{X}} = \alpha_{\bar{X}} \Big/ \left(\alpha_{\bar{X}} + \sum_{i=1}^{r} \alpha_i \right) \tag{24}$$

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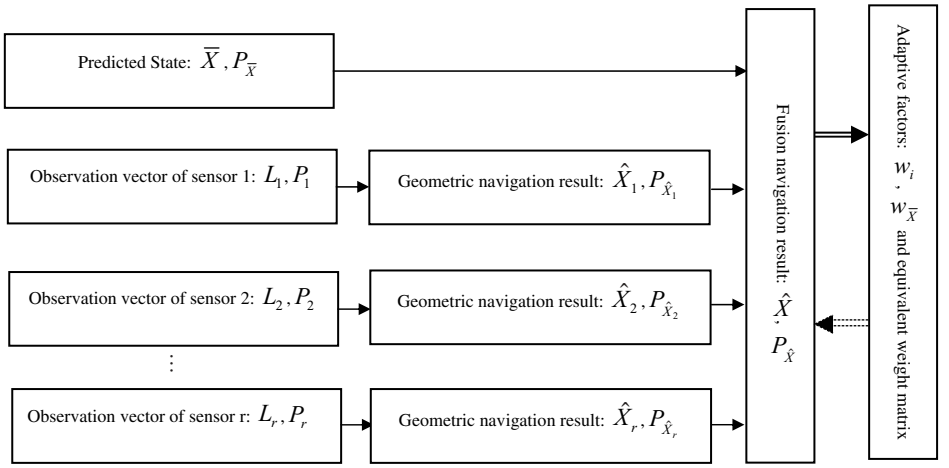


Figure 1. Adaptive integrated navigation based on multi-sensor geometric navigation.

The fusion flow of the adaptive integrated navigation based on the geometric navigation solution is shown in Figure 1.

The calculation procedure of navigation algorithm mentioned above is similar to that of a federated filter, but there exist some significant differences in concepts.

1. In the federated filter, the local filter (LF) and the main filter (MF) outputs are correlated though common use of the dynamic model information or though common use of reference outputs by some navigation sensor that is used to provide dynamic model information. The fusion state by the federated filter is not rigorous in theory since the correlations of the LF and MF are seldom considered.
2. In the new integrated navigation procedure, the local navigation outputs do not contain any common dynamic model information or reference navigation outputs, so they are independent. The fused navigation solution is thus more rigorous than that from federated filter.
3. Since the adaptive weight matrices (18) are used, the reasonable contributions of local sensor outputs are balanced.

A potential problem existing in the procedure above is that when the predicted state from dynamic model seriously deviates from the actual state, the reference fusion solution provided by (12) will be heavily influenced. The influence function (IF) is

$$IF(\hat{X}^{\mathbf{0}},\ \Delta\bar{X}) = P_{\hat{X}^{\mathbf{0}}}^{-1} P_{\bar{X}} \Delta\bar{X} \tag{25}$$

where IF denotes the influence of the deviation $\Delta\bar{X}$ of the predicted state $\bar{X}$ on the reference fusion state $\hat{X}^{\mathbf{0}}$. In turn, the heavily influenced reference state $\hat{X}^{\mathbf{0}}$ cannot act as a reference for reasonable adaptive factors α_i and $\alpha_{\bar{X}}$. A direct method to guarantee $\hat{X}^{\mathbf{0}}$ be reasonable is to avoid using the predicted state $\bar{X}$. Then a reference fusion state estimate is

$$\hat{X}^{\mathbf{0}} = P_{\hat{X}^{\mathbf{0}}}^{-1}(P_{\hat{X}_1}\hat{X}_{\mathbf{1}} + P_{\hat{X}_2}\hat{X}_{\mathbf{2}} + \cdots + P_{\hat{X}_r}\hat{X}_r) \tag{26}$$

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where, $P_{\hat{X}^0}$ is different from that expressed by formula (13),

$$P_{\hat{X}^0} = P_{\hat{X}_1} + P_{\hat{X}_2} + \cdots + P_{\hat{X}_r} \tag{27}$$

4. A SIMULATION EXAMPLE. The primary goals of the simulation tests are (1) to demonstrate the adaptation of the new adaptive integrated navigation system to the dynamic model deviation; and (2) to show the dynamic model influences on the reference state $\hat{X}^0$. The simulated dynamic models for the three components of a kinematic vehicle are

$$X = 2t + \tfrac{1}{2}a_X t^2$$
$$Y = 5t + \tfrac{1}{2}a_Y t^2$$
$$Z = 3t + \tfrac{1}{2}a_Z t^2$$

where t denotes the time, X, Y and Z are coordinate components, a_X, a_Y and a_Z are coefficients which follow $a_X \sim N(0, 0{\cdot}1)$, $a_Y \sim N(0, 0{\cdot}2)$ and $a_Z \sim N(0, 0{\cdot}15)$. Two sensors' observations are simulated.

$$\text{Sensor 1: } X_{1i} = X + V_{X1i}, \quad Y_{1i} = Y + V_{Y1i}, \quad Z_{1i} = Z + V_{Z1i}$$
$$\text{Sensor 2: } X_{2i} = X + V_{X2i}, \quad Y_{2i} = Y + V_{Y2i}, \quad Z_{2i} = Z + V_{Z2i}$$

At each epoch, six observations for each component of each sensor are simulated with zero means and the following covariance matrices

$$\Sigma_1 = \begin{bmatrix} 5 & & \\ & 10 & \\ & & 8 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 10 & & \\ & 5 & \\ & & 6 \end{bmatrix}$$

Totally 1000 groups of observations corresponding to 1000 epochs are produced. The covariance matrix of the dynamic model errors is supposed to have the following form

$$\Sigma_w = \begin{bmatrix} \frac{1}{3}Q\Delta t^3 & \frac{1}{2}Q\Delta t^2 \\ \frac{1}{2}Q\Delta t^2 & Q\Delta t \end{bmatrix}$$

where Q denotes the spectral density of the velocity which is chosen as (Schwarz *et al.*, 1989) $Q = 0{\cdot}11\ \mathrm{m^2 s^{-3}}$. The variances for initial position and velocity are supposed to be $0{\cdot}2\ \mathrm{m^2}$ and $1{\cdot}0 \times 10^{-6}\ \mathrm{m^2 s^{-2}}$.

Two data sets are simulated. In data set one, the vehicle moves stably without significant disturbing acceleration, and in data set two, the vehicle moves with disturbing acceleration during epoch 670 through epoch 710. Three calculation schemes are performed with the two simulated data sets.

Scheme 1: Direct data fusion by using formula (12) shown in Figure 2.
Scheme 2: Federated filter shown in Figure 3.
Scheme 3: Adaptive integrated navigation shown in Figure 4.

The fused results are compared with the 'true' simulated values, and the true errors of the X component only are shown in Figures 2 to 4. The RMS for X, Y and Z components of the three schemes are listed in Table 1.

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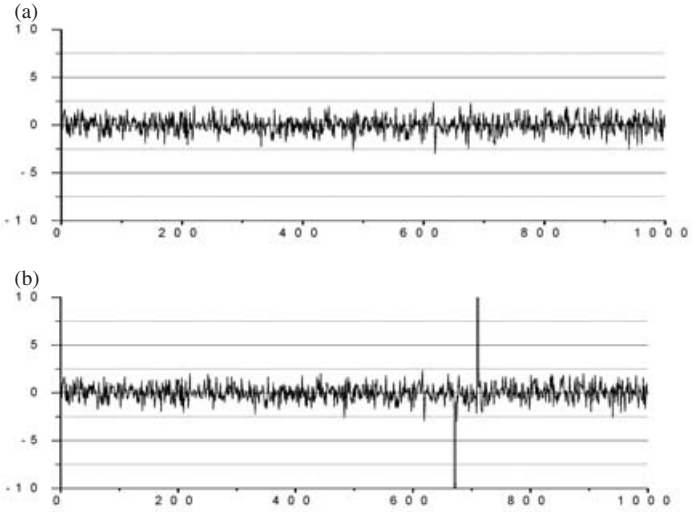


Figure 2. (a) Errors of X component derived by direct data fusion in stable situation. (b) Errors of X component derived by direct data fusion in the situation with disturbing acceleration during epoch 670 to 710.

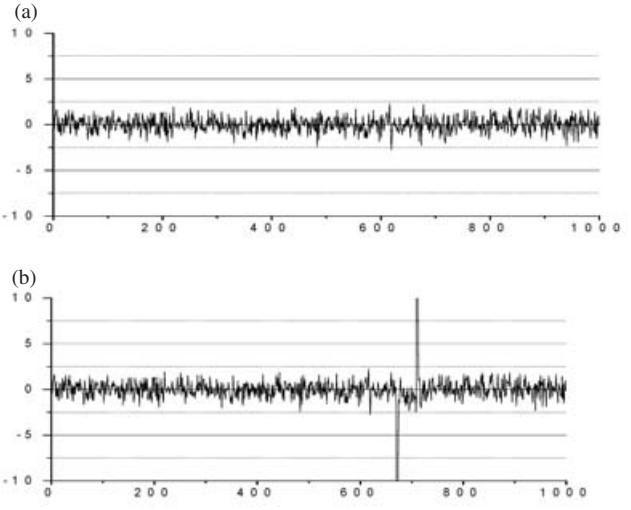


Figure 3. (a) Errors of X component derived by federated Kalman filter in stable situation. (b) Errors of X component derived by federated Kalman filter with disturbing acceleration during epoch 670 to 710.

From analysis of the results shown in Figures 2 to 4 and Table 1, the following facts can be drawn.

1. In the stable situation, the results of the direct data fusion, the federated filter and the adaptive integrated navigation have little difference. The adaptive integrated navigation is slightly better than the federated filter, and the federated filter is slightly better than the direct fusion. It shows that the influences of the

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Table 1. The RMS comparison of the three components derived by the three schemes.

	Scheme 1			Scheme 2			Scheme 3		
	X (m)	Y (m)	Z (m)	X (m)	Y (m)	Z (m)	X (m)	Y (m)	Z (m)
Data set 1	0·832	0·863	0·826	0·784	0·811	0·777	0·726	0·740	0·708
Data set 2	13·12	32·800	19·763	24·745	61·873	37·321	0·734	0·766	0·714

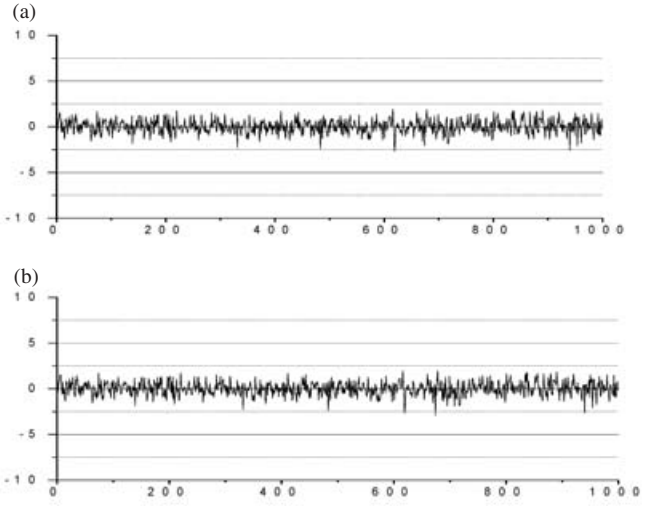


Figure 4. (a) Errors of X component derived by adaptively integrated navigation in stable situation. (b) Errors of X component derived with adaptively integrated navigation in stable situation.

correlations of the local filter outputs are not serious if the dynamic model can describe the movements of the vehicle.

2. If the vehicle moves with heavy disturbance, then the results of both the federated filter and the direct fusion are seriously distorted.
3. The adaptive integrated navigation algorithm has the potential to provide high levels of accuracy and fault tolerance. The simulated example shows that the adaptive integrated navigation gives reliable results for both stable and unstable situations of the vehicle movements. The adaptive factors play key roles to control the influences of the disturbances of the dynamic model information and the individual sensor failures.

5. CONCLUSION. The federated filter, which employs the information sharing principle, is usually effective especially when the reference dynamic model is accurate. If the dynamic model has been heavily distorted, then the fusion navigation state by the federated filter is also distorted. An essential problem of the federated filter is that the local filter outputs are usually correlated since the reference dynamic model is repeatedly used in the local filters. In order to take the advantage of the parallel calculation strategy of the federated filter and avoid the correlations

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of the LFs, it is suggested that the integrated navigation algorithm based on the local geometric adjustment outputs is used. Furthermore, in order to reasonably balance the contributions of the local geometric adjustment outputs and the dynamic model information, the adaptive integrated navigation strategy should be applied. The simulation results have shown the effectiveness of the new adaptive integrated navigation algorithm.

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